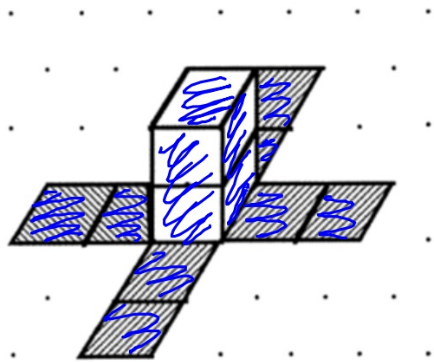
Making Monuments: Investigation Sheet No.1

Monuments are scattered all through the parks of Stonia. They are made so that the length of the paths leading up to them is the same as the height of the monument. This is a Size 2 monument:

8 + 8 + 1

17



Once the monuments and paths have been built they are tiled. The monuments *and* paths are both tiled with the same size square tiles.

1. Make a Size 1 monument and its paths. How many tiles would be needed?

2. Make Sizes 2, 3, 4 & 5 and for each one work out the number of tiles needed to tile the monument and its paths.

3. Organise the results you have so far. Can you predict the number of tiles needed for the Size 10 monument? Check by drawing.

4. Imagine the Size 100 monument. How many tiles does it need? Explain your answer.

S	1	2	3	4	5	6	10	100
T	9	17	25	33	41	49	81	801

For the tiles we use 8 times the number of blocks for the walls and paths, and and 1 for the roof.

$$8x + 1$$

The number of tiles is a function of the height and paths of the monument.

The number of tiles is a function of the height only.

$$f: \mathbb{Z}^+ \to \mathbb{Z}^+, \ f(x) = 8x + 1$$

$$f : \mathbb{Z}^+ \to \mathbb{Z}^+, \ \underline{f(x) = 8x + 1}$$

the <u>name</u> of the function

domain

what are the possible inputs?

mapped into

codomain

what type of values can we expect for the outputs?

the <u>rule</u> for the function.

How many tiles for a size 30 monument?

$$f(30) = 8(30) + 1$$
$$= 241$$

30 is mapped into 241

(30, 241)

Working backwards

5. What if you had, say, 150 tiles. Can you tell the largest size monument you would be able to completely tile using these?

$f(x) = 8x + 1$

$f(19) = 8(19) + 1$
$= 153$

too high!

$f(18) = 8(18) + 1$
$= 145$

You can build a size 18 monument

$\frac{150 - 1}{8}$

$= 18.625$

$\underline{\underline{18}} < 18.625 < 19$

18 is the largest

solve x when

$f(x) = 150$

$f(x) = 8x + 1 = 150$
$8x = 149$
$x = \frac{149}{8}$
$= 18.625$.

solve $f(x) \leq 150$, $x \in \mathbb{Z}^+$

$8x + 1 \leq 150$
$8x \leq 149$
$x \leq \frac{149}{8}$
$x \leq 18.625$

$\therefore x = 18$ for the largest monument.

Continuous v Discrete problems

7. What if a monument was built to a height of 5½ blocks, or maybe 6.2 blocks? How could you find the number of tiles needed?

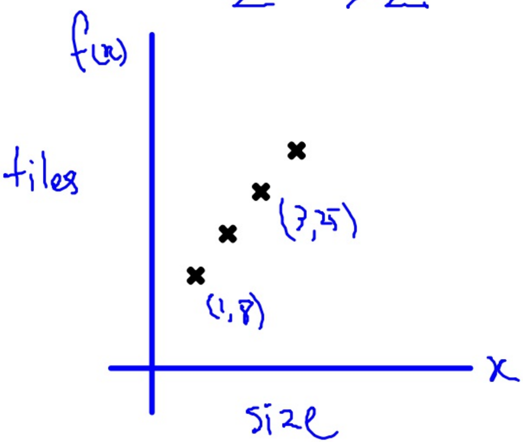
$$x = 5\tfrac{1}{2}$$

$$T = 8\left(5\tfrac{1}{2}\right) + 1$$

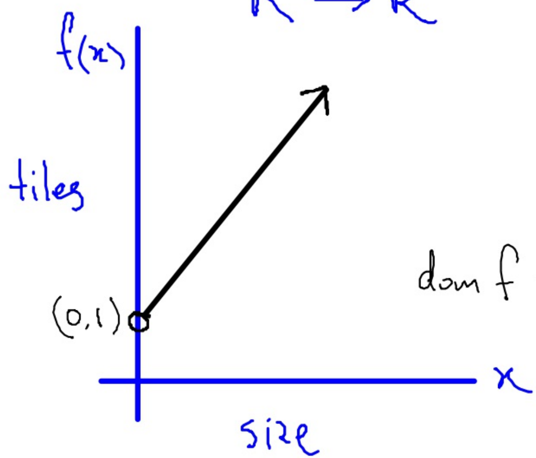
$$= 45$$

$$f : \mathbb{R}^+ \to \mathbb{R}^+,\ f(x) = 8x + 1$$

$\mathbb{Z}^+ \to \mathbb{Z}^+$

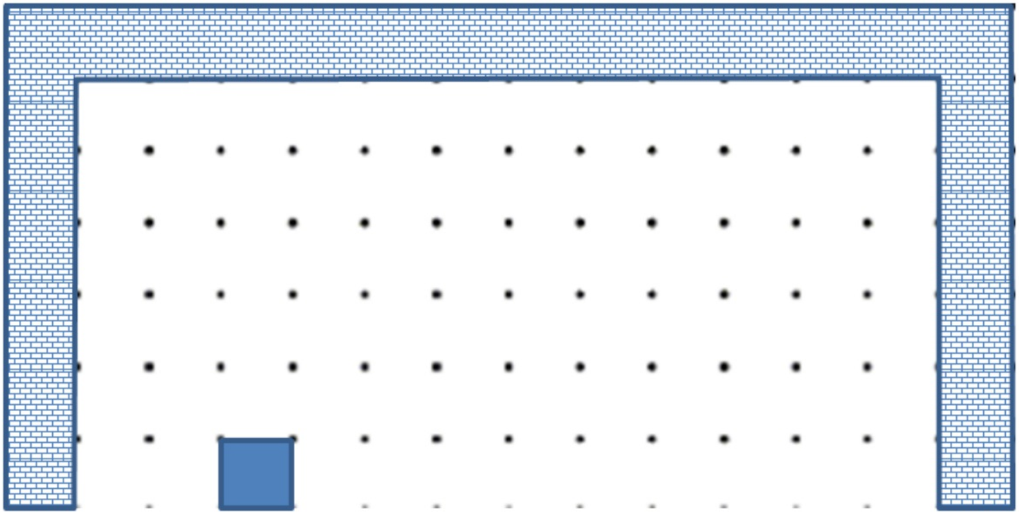


$\mathbb{R}^+ \to \mathbb{R}^+$

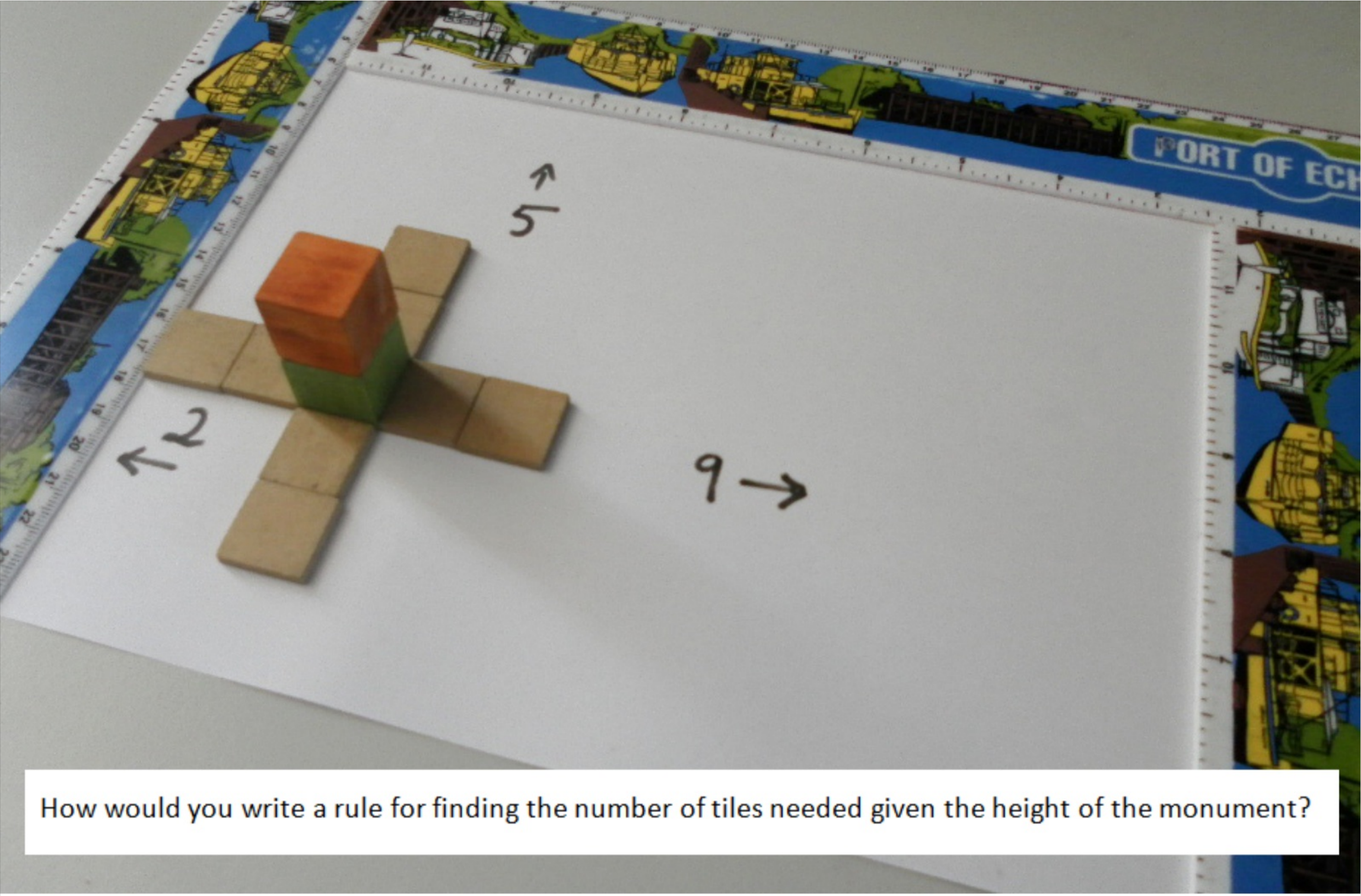


$\operatorname{dom} f = (0, \infty)$

What if a monument is placed within an unenclosed courtyard with walls on three of the sides? Here is a possible top view:



How would you write a rule for finding the number of tiles needed given the height of the monument?



How would you write a rule for finding the number of tiles needed given the height of the monument?

Make a table:

	1	2	3	4	5	6	7	8	9	10			
	9	17	24	32	40	46	54	62	70	77			
				31	38								

Break the problem into parts.

① Until we hit the '2' wall. $f(x) = 8x + 1$

$x \in \{1, 2\}$

② After the '2' wall, until we hit the '5' wall.

$f(x) = 7x + 3 \quad , x \in \{3, 4, 5\}$

Can we check the $7x+3$ rule another way?

paths walls roof 'gap'

$$38 = 3\times\underline{\underline{5}} + 4\times\underline{5} + 1 + 2$$

$$= 7\times\underline{\underline{5}} + 3$$

$$7x + 3$$

3 After the '5' wall, until the '9' wall.

$$f(x) = 6x + 8 \, , \quad x \in (6, 7, 8, 9)$$

4 After the '9' wall:

$$f(x) = 5x + 17 \, , \quad x \in [10, \infty)$$

$$x \in \mathbb{Z}^+ \, , \; x \geq 10$$

Tom says: Isn't there a shorter way of writing this?

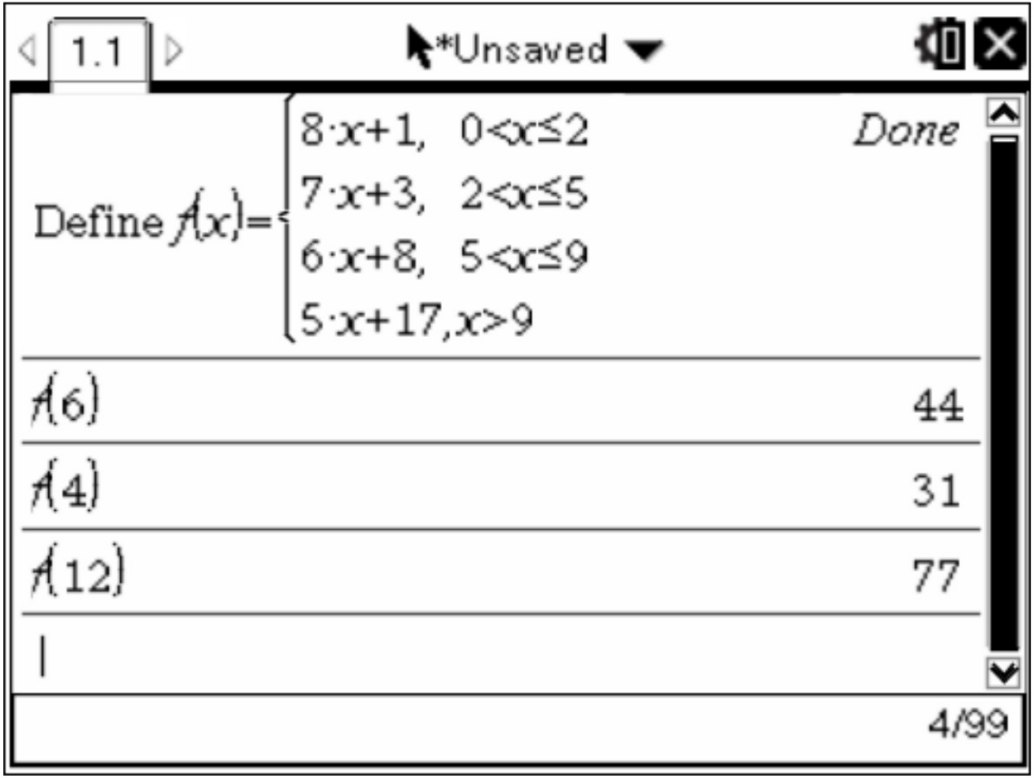
The function for tiling our monument in the courtyard would be

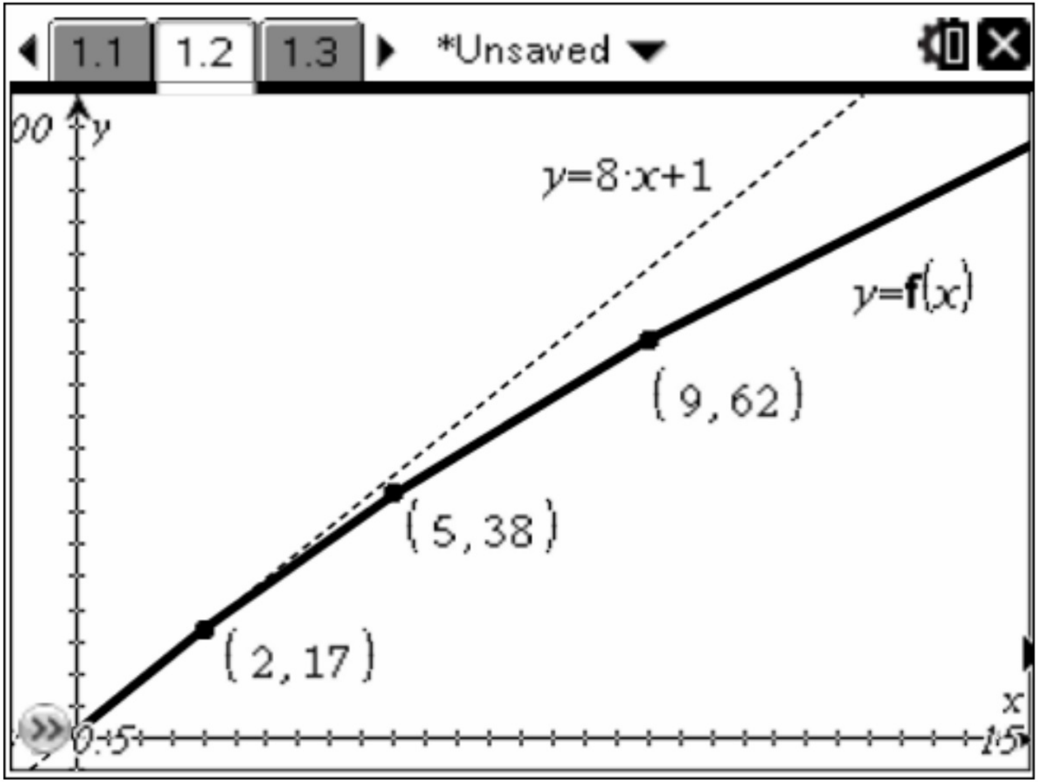
$$f : R^+ \to R^+, \quad f(x) = \begin{cases} 8x+1, & 0 < x \leq 2 \\ 7x+3, & 2 < x \leq 5 \\ 6x+8, & 5 < x \leq 9 \\ 5x+17, & x \geq 10 \end{cases}$$

When we express functions this way they are called HYBRID functions (or piecewise functions).

$$f(x) = \begin{cases} 8x+1, & 0 < x \le 2 \\ 7x+3, & 2 < x \le 5 \\ 6x+8, & 5 < x \le 9 \\ 5x+17, & x > 9 \end{cases}$$

x	S	1	2	3	4	5	6	7	8	9	10	11	12	13
$f(x)$	T	9	17	24	31	38	44	50	56	62	67	72	77	82


1.1
*Unsaved
Define f(x)= { 8·x+1, 0<x≤2 ; 7·x+3, 2<x≤5 ; 6·x+8, 5<x≤9 ; 5·x+17, x>9
Done
f(6)
44
f(4)
31
f(12)
77
4/99


1.1
1.2
1.3
*Unsaved
y
y=8·x+1
y=f(x)
(9,62)
(5,38)
(2,17)
x
0.5
15