Jack and Jill's Buckets

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What was the problem about?

Jack and Jill had two buckets. One had a capacity of 4 litres, the other a capacity of 9 litres.

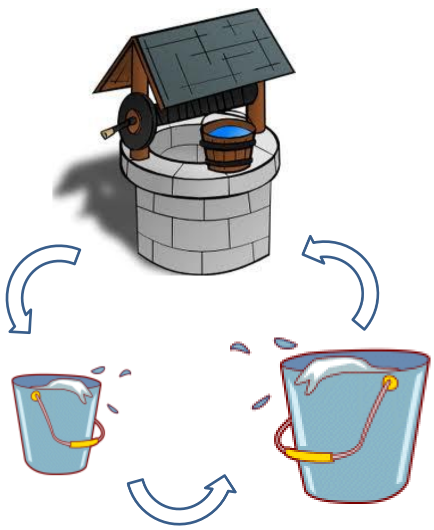
Using these buckets they managed to collect *exactly* 6 litres of water from the well.

Find out how they could have done this.

(Hint: They did not need to estimate or guess. For example, they couldn't half-fill a bucket directly from the well because they had no way of knowing exactly where half was.)

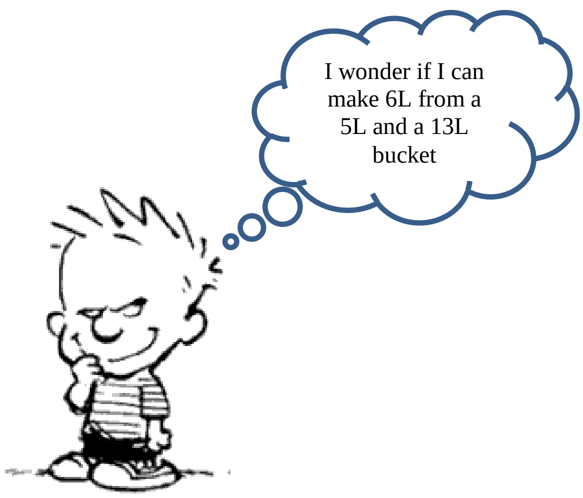
What did we do?

To get started on the problem, we acted out a solution with some models of the buckets. Eventually we collected exactly 6 litres in the larger bucket and recorded our solution in a chart.



After that, we found that there was another solution. The second solution came about by reversing the technique used the first time.

Next, we set new challenges and solved those. We either changed the sizes of the small and large buckets, or changed the target amount.



After a while, we decided to find out what would happen if we didn't stop at the target amount and kept collecting water and throwing it back. This turned out to be a good question and revealed a great deal about how these types of problems are solved.

Finally, we took all the amounts you could possibly collect in the larger bucket and mapped them out in a diagram. The result was very interesting and showed that there was a pattern at work.

What did we find out?

The original problem to collect 6 litres using a 4 and a 9-litre bucket can be solved two ways. Each solution has an equation that goes with it to show how 6 litres can be collected from the well.

1st Solution

4	**9**
0	0
4	0
0	4
4	4
0	8
4	8
3	9
3	0
0	3
4	3
0	7
4	7
2	9
2	0
0	2
4	2
0	6

$$6 \times 4l - 2 \times 9l = 6l$$

2nd Solution

4	**9**
0	0
0	9
4	5
0	5
4	1
0	1
1	0
1	9
4	6
0	6

$$2 \times 9l - 3 \times 4l = 6l$$

There is a technique used to move the water with the buckets so that target amounts can always be found. The two solutions come about because you can start with either bucket. For each "move", follow the following steps in order:

- **If you can pour water from the first bucket into the second bucket then do so.**
- **When the second bucket is full, empty it into the well**
- **When the first bucket is empty, fill it from the well.**

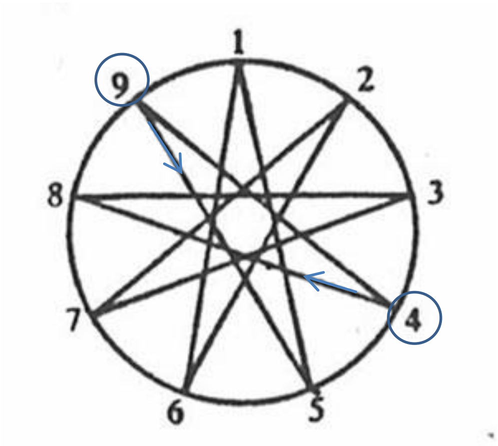
It is usually possible to collect *all* whole number amounts in the larger bucket, as seen in the chart below. This chart shows that the buckets are eventually empty again, as they were at the start. It also shows that the two solutions can be seen by reading the chart from both ends.

* Start here when beginning with the smaller bucket

4	**9**
0	0
*4	0
0	**4**
4	4
0	**8**
4	8
3	9
3	0
0	**3**
4	3
0	**7**
4	7
2	9
2	0
0	**2**
4	2
0	**6**
4	6
1	9
1	0
0	**1**
4	1
0	**5**
4	5
0	**9***
0	0

Start here when beginning with the larger bucket *

A star-diagram will reveal how all of the amounts are collected in the larger bucket and whether it is quicker to start by filling the small bucket or the large bucket. By starting at the bucket that is collecting water from the well, moving away from the other bucket and following the lines, we can see the order in which all of the amounts will be found in the large bucket.



Conclusion

This was an interesting problem. It seemed impossible at the start, so it was surprising to find out that so much can be achieved with just two buckets. We now know that if we have, say, a 6-litre and an 11-litre bucket, then all the amounts from 7 to 10 litres (and from 1 to 5 litres) can be collected exactly with no estimation or guessing at all.

Working through the problem showed that keeping tables or making charts can help find patterns and see what is going on. The star-shaped diagrams can be used to solve any problem of this type and will also show which of the two possible solutions is the quickest.