PROFESSOR MORRIS PUZZLES

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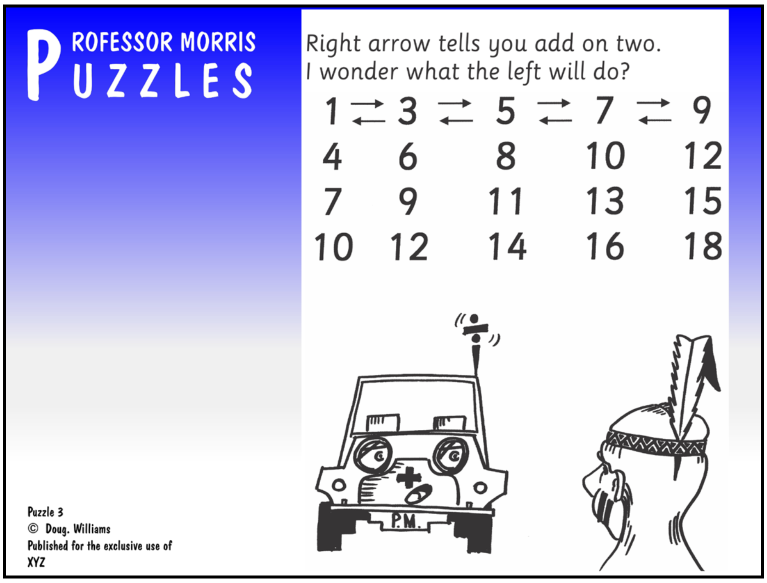
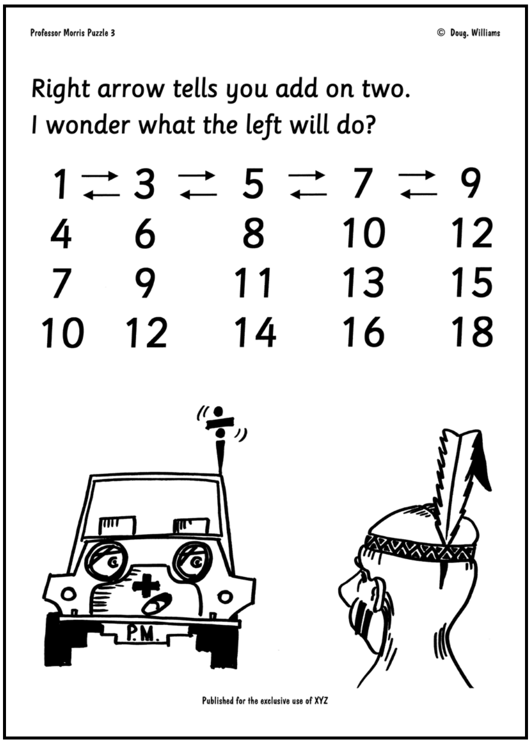
Slides and Posters



- Making mathematics talk



- 50 puzzles
- All in rhyme
- CARtoon drawings
- Years 2 to 8
- Answers and questions

- Displays and Poster Problem Clinics
- Learning to Work Like a Mathematician

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Professor Morris Puzzles is a package made up of this book and its companion puzzles provided as two PDF documents - one as poster pages and one as slides.

- *Poster pages and slides are published for the exclusive use of the purchaser.*

Design and Desktop Publishing:
Black Douglas Professional Education Services

Editions

1985
The Professor Morris Collection, Volume 1
Mathematical Association of Victoria
… Wall poster flip book

1997
Professor Morris Puzzles
Black Douglas Professional Education Services
… Folder of A4 pages with notes

2021
Professor Morris Puzzles
Black Douglas Professional Education Services
… Book, poster pages and slides designed for digital delivery as computer-based files

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Introducing Professor Morris

Professor Morris Puzzles are proving popular in Years 7 & 8.

...

My classroom cleaner commented about the rhyme and has started to think about it.

Andy Martin, Thorne Grammar, UK

"Maths is dull and boring"
is a common cry we hear,
And it often makes me wonder
if our teaching is in gear.
We put such effort into
displays of other things,
But when it comes to maths it
seems we often lose our zing.

So here we have some pages
which could be the centrepiece
Of something a bit diff'rent
to do with maths we teach.
Print A4 or larger;
show a slide upon the wall,
Then talk about the puzzles
with the children one and all.

Let them ask their daddies,
or mummies, better still,
The more they talk, the more will
grow their problem solving skill.
Then pin up all around me
maths work the children do,
And try to build an atmosphere
of fun for them and you.

... BUT ...

Don't let them spread the answer
until all have had fair turn
To ponder on the problem
and see what they can learn.
If some are very anxious
to tell you what they've thought
Ask them to come and whisper,
then all won't come to nought.

The puzzles in my pages
are graded in a way,
But which ones for which level
"ain't" a simple thing to say.
What's a puzzle to one person
to another is a breeze
So some have their extensions
to provide an extra tease.

However I will tell you
that I've used to twenty-two
Successfully with children
who were learning in Year Two.
The remainder of the puzzles
I used when in Year Five,
But some of them were toughies
which made the children strive.

In fact, they all are starters
which lead to other things.
Try looking past the answer
to see what else it brings.
Stimulate the students
then let them go their way,
You'll often get surprises
to brighten up your day.

Now last I have a thank you
for those who've been my guide.
And helped me be created
and supported me beside.
Jan McLean's suggestion
back when I first began
Was what made me a Morris
and not a Mr. Man.

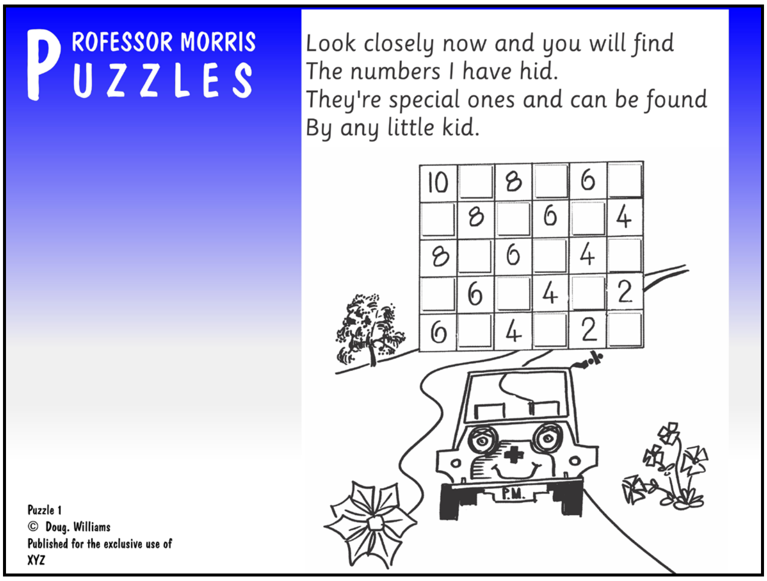
Rob Mullarvey's drawings
are fun but not a joke.
He's captured all the detail of
the quintessential Moke.
A thank you too is offered
to my friends at MAV,
Who made a publication
of an early form of me.

And lastly Douglas Williams
has had a bit to do
With the puzzles and the poems
which I present to you.
Some say he's very clever.
Some say he's just a goose.
The truth is though,
his mind's been turned
by years of Doctor Seuss!

Professor Morris 1985
(Modifications 1997, 2021)

Hints:
Print posters at A3 for display. Add colour if you wish.
Slides will ask to open full screen. Agree. Ctrl L toggles between full screen and page views.

Poster Problem Clinic



Background

A Poster Problem Clinic first wraps the Polya Problem Solving Process of:

- Read and understand the problem.
- Plan a strategy to solve the problem.
- Carry out the plan.
- Check the result.

in teaching craft. Then embeds it in the context of learning to work like a mathematician, as detailed on the next page.

A clinic begins with a puzzle poster large enough to be seen by the whole class. A slide works well for the clinic. A print copy works well as a centrepiece in a display of outcomes from the clinic. Such a display also encourages on-going consideration of the problem, which sometimes leads to additional student contributions.

Most teachers copy the Working Mathematically page for students to permanently include in their journals. Some also 'break out' the Polya steps and the toolbox of strategies to make separate displays. Example:

http://www.mathematicscentre.com/taskcentre/stratbrd.pdf

For the following model clinic, the slide above is used as the example, partly because it will help confirm that even the (apparently) simplest Professor Morris Puzzle can begin a significant mathematical conversation.

Working Mathematically

Learning to Work like a Mathematician

First give me an interesting problem.

When mathematicians become interested in a problem they:

- *Play with the problem to collect & organise data about it.*
- *Discuss & record notes and diagrams.*
- *Seek & see patterns or connections in the organised data.*
- *Make & test hypotheses based on the patterns or connections.*
- *Look in their strategy toolbox for problem solving strategies which could help.*
- *Look in their skill toolbox for mathematical skills which could help.*
- *Check their answer and think about what else they can learn from it.*
- *Publish their results.*

Questions which help mathematicians learn more are:

- *Can I check this another way?*
- *What happens if …?*
- *How many solutions are there?*
- *How will I know when I have found them all?*

When mathematicians have a problem they:

- *Read & understand the problem.*
- *Plan a strategy to start the problem.*
- *Carry out their plan.*
- *Check the result.*

A mathematician's strategy toolbox includes:

- *Do I know a similar problem?*
- *Guess, check and improve*
- *Try a simpler problem*
- *Write an equation*
- *Make a list or table*
- *Work backwards*
- *Act it out*
- *Draw a picture or graph*
- *Make a model*
- *Look for a pattern*
- *Try all possibilities*
- *Seek an exception*
- *Break a problem into smaller parts*
- *…*

If one way doesn't work, I just start again another way.

The Clinic

1. Read and understand

> *In a moment I will show you a puzzle. I just want you to read it. Don't even think about working it out. So put down your pens, close you computer lids and just read.*

Show the puzzle for just enough time to read it through, then remove it from sight. That's easy with a slide. Use a cover sheet for a poster.

> *It doesn't matter if you haven't read every single thing. We are going to work together. I'll give you a moment to think back through what you did see, then I will ask some questions.*

The aim of this first set of questions is to record key recollections on the whiteboard. Through brief discussion inviting as many students as possible to contribute, the notes are fashioned into an agreed oral hypothesis about the text and what it is asking. You will probably have to remind students that you are not asking for the answer. You are only trying to find out 'what we read'.

The process is based on a language learning technique called Read and Retell, which helps to develop reading comprehension.

> *Hamish tell me anything you noticed when you read it.*
> *...*
> *Fen can you add to what Hamish said?*
> *...*
> *Who thinks they know something else that's on the poster?*
> *...*
> *Did the picture have anything to do with it?*
> *...*
> *Great, so if we had to explain in our own words what the puzzle is asking, what would we say?*
> *...*
> *How can we check if our hypothesis is correct?*

At this point there is a usually a chorus of "Show us again." Do the reveal and review, discuss, correct and agree again as necessary.

Getting to this point should take 5 to 6 minutes.

2. Planning a strategy

Now we know what Professor Morris is asking. But don't pick up your pen yet. If you feel like picking it up, sit on your hands. If I let you start this problem I want you to know what you will do. We need to plan how we will start.

Ask partners/groups to discuss their possible starting strategy.

We don't all think the same way, so if you can think of more than one way to start that would be great.
You might want to look through the strategy toolbox on the Working Mathematically page to see if that helps.
Remember, I am not asking you to solve the problem. I only want you to plan where and how you will start.
I'll just give you a few minutes to talk.

Allow a short time for this discussion, ask for strategies and note them on the board. If you notice anyone picking up a pen remind them, with good humour, that they need to sit on their hands. Compliment and encourage as much as possible.

Sometimes you can use a statement like this to encourage wider thinking:

I used this puzzle with my Year 1 last year and one pair said they would use 'Make a model'. When I asked what they meant they said they would draw the Professor Morris chart on a piece of paper and stack blocks on the numbers. Then they could see what was missing in between.

It's almost guaranteed that someone will ask if they can try that way too.

So now you know what Professor Morris is asking and you know what you are going to do first. Would you like me to let you start?

Again a chorus of agreement is the usual response.

Okay but for this problem you must find two ways to fill the gaps.

The flurry of focussed activity at this point is usually significantly more than what follows from "Open your books to page ... and do exercise ...", or its equivalent in computer based mathematics practice.

Ten to twelves minutes have passed and all the students are involved.

3. Carry out the plan

The students are already in this phase. Let it run for few minutes. Circulate, asking students to show you what they have been doing. At all levels, regardless of the puzzle, ensure that those students who have planned materials into their planning can easily access them.

For this puzzle, which, as Professor Morris says, '...can be done by any little kid', encourage the mathematician's questions:

> *How do I know that those are the missing numbers?*
> *Can I check them another way?*

A range of strategies have already been listed to help check another way.

What you choose to do after these few minutes will depend on how you are using the clinic. You have choices. Are you intending to:
- *spend the whole lesson digging deeper into the problem?*
- *set the problem up as a class display problem for the week?*
- *encourage children to take the problem home for 'maths around the kitchen table'?*
- *use the clinic as a lead into a hands-on maths tasks lesson in which pairs choose to continue with it or change to a task?*

Whatever your plan, confirm with the students what is known to this point and make your expectations clear for how to continue.

> *I introduced a puzzle each Monday maths lesson and worked it through until the kids were engaged in the problem. Then I stopped them with the comment:*
>
> > *Now you can continue to work on this puzzle through the week whenever you have time. You may take it home and work with your family if you wish. You can work on it with someone else if you wish, BUT if you have anything to say about it you must come and WHISPER it to me.*
>
> *They knew why I wanted them to whisper. In Friday's maths session we had a plenary discussion of the problem. On many occasions I already knew the children who could help me run the discussion.*

4. Check

The Working Mathematically page states that mathematicians:

- Check their answer and think about what else they can learn from it.

and this process is supported by the questions:

- Can I check this another way?
- What happens if …?
- How many solutions are there?
- How will I know when I have found them all?

So, checking an answer is more than finding out if you are right.

Importantly too, since a mathematician's work begins with an interesting problem, they are always working with a situation in which the answer is unknown. If the answer was known the situation wouldn't be a problem. So there is no one else of whom a mathematician can ask "Am I right Miss?". The question "Can I check this another way?" encourages learners to take responsibility for their own learning and to consider that there is often more than one right way in mathematics.

In Puzzle 1 it will most likely be noticing two or more different patterns working together that convinces students their answers are right.

> I got my answers by going this way in the chart. But when I looked this way it all still fitted together.

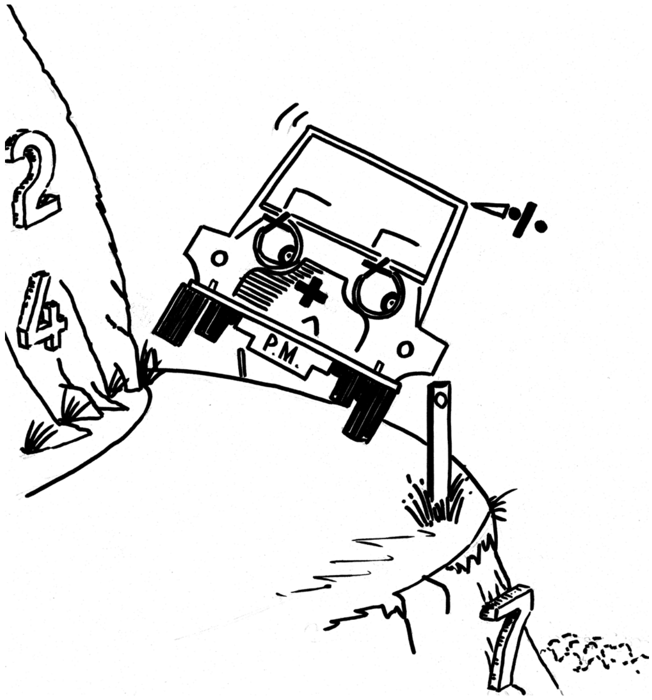
The "What happens if …?" questions for this puzzle usually pivot around pattern, but there are also calculation possibilities. Try to create some extension questions for yourself before exploring Puzzle 1 further in the Seeking a Solution section. You will also find an extensive lesson plan built around this puzzle on the Maths At Home site at:

http://www.mathematicscentre.com/mathsathome/challenges/patchart.htm

The other thing to check in the plenary session of a Problem Solving Clinic is the question:

- In what ways have we worked like a mathematician with this problem?

Learning to work like a mathematician is the core of the mathematics curriculum. This question brings that core into regular focus and the Working Mathematically page provides the framework for assessing it.



Seeking a Solution

Many Professor Morris Puzzles involve content similar to hands-on tasks from Mathematics Task Centre. The connections have been listed in these teaching notes. Find any listed task by name or number in the Task Library link below.

Also Puzzles 1, 23 & 49 are the source of Maths At Home activities Pattern Charts, Four Odd Numbers and The Farmer's Puzzles.

If you discover any new Start Talking ideas for these puzzles please send them to us. These notes can easily be extended and shared.

Puzzles 1 & 2

Topics
Counting, number patterns, addition & subtraction

Task Centre Connection
Task 4: Window Frames
Task 8: Addition Totals
Task 225: Add The Pack
Task 230: Pack Up Your Bears

Strategy
Look for pattern, work backwards

Solution
Start in one corner and count horizontally and vertically by 1s.

Start Talking

- What type of numbers are missing (or showing)?

http://www.mathematicscentre.com/mathematicscentre/task_lib.htm

http://www.mathematicscentre.com/mathsathome/choose.htm

- How can you tell if a number is odd or even?
- What patterns can you find diagonally?
- Can you extend the chart up or down, left or right?
- Professor Morris's chart counts back by ones. What happens if you make a chart which counts back by 2s or 3s or 4s... horizontally and vertically?
- What happens if you make a chart which counts back by different amounts horizontally and vertically?
- Add up all the numbers in the first row. ...the second row ... the third row. Can you predict the total for the fourth and fifth rows?
- What about in the columns?
- Add together the totals of each row. Use the answer to help predict the answer of adding the totals of each column. Check and explain.
- Design a spreadsheet that creates Professor Morris's chart.
- Design a spreadsheet that creates a chart twice as big in each direction as the Professor Morris chart.

Puzzles 3 & 4

Topics
Counting, number patterns, inverse relationships

Task Centre Connection
Task 4 Window Frames
Task 10: Find My Pattern
Task 15: Domino Trails
Task 46: Duelling Dice
(non-transitive relationships)
Task 52: Which Floor?

Strategy
Look for a pattern, work backwards

Solution
Puzzle 3: Subtract 2
Puzzle 4: Add 3

Start Talking
- Can you explain why the rows of numbers alternate odd and even?
- The arrows shown only refer to adjacent numbers. If you draw arrows to say every second number, what do these new arrows mean?
- Which rows or columns can you also find in a times tables chart?
- Can you relate arrows drawn diagonally to combinations of horizontal and vertical arrows?
- Can you extend the chart left, right, up and down?
- Can you create your own charts?
- Can you chalk your own giant chart on the playground and design a game to use the numbers?

- Each row (or column) has a number in the 1st, 2nd, 3rd ... position. What happens if you graph each row as (position, value)?

Puzzles 5, 6 & 7

Topics

Number patterns, inverse relationships, operations on number, difference, solving equations (mentally)

Task Centre Connection

Task 28: Plate Triangles
Task 34: Dice Differences
Task 56: Challenge
Task 144: Pascal's Triangle In Asia
Task 188: Arithmagons 1
Task 194: Arithmagons 2
Task 229: Animal Farms

Strategy

Look for a pattern, work backwards, look for extra information

Solution

Puzzle 5: 3, 5
Puzzle 6: 4
Puzzle 7: 6, 1

Start Talking

Puzzle 5

- Can you connect the number of balloons in the base line to the number of rows in the triangle?
- Start your own triangle with balloons 1 to 5 in the base line. Predict the number of rows there will be in your triangle. Finish the triangle to check. What is your apex number?
- What happens if you start with balloons 1 to 6?
- Cover the 1 to 6 balloons in a way that shows 1 to 5, then 1 to 4, then 1 to 3, then... Make ordered pairs (Base Balloons, Apex Number) and graph the pairs. Perhaps these two numbers are connected?
- Start with one balloon as the apex. Choose any number to go in the apex balloon. What will you put in the next row ...the next row ...the next row ...? How far down can you go?

Puzzles 6 & 7

- Both these difference triangles have an apex of 1.
- Do all difference triangles have an apex of 1?
- What happens if you extend the base line?
- What happens if you put 'really big' numbers in the base line?
- Draw an apex balloon and choose a number to go in it. How many rows can you create below it?

Puzzles 8 & 9

Topics
Multiplication, commutative law

Task Centre Connection
Task 9: Row Points
Task 17: Truth Tiles 2
Task 19: Cookie Count
Task 30: Truth Tiles
Task 123: Bob's Buttons
Task 235: Tables for 25

Strategy
Make a model, write an equation

Solution
Clearly the answer to each puzzle is 20. Also the pair of puzzles shows that 4 x 5 = 5 x 4. More importantly the puzzles demonstrate the importance of context, for in these cases 4 x 5 does not mean the same as 5 x 4.

Start Talking

- Can you create another story which could be represented by 4 x 5 or 5 x 4?
- Can you create another story in which 4 x 5 and 5 x 4 have the same answer, but do not mean the same thing?
- When you can swap the numbers in an operation and still get the same answer, the operation is called Commutative. Which other operations are commutative?
- Make up a new equation. Now try to invent a story which could be represented by this equation.
- Make up a 'some' story like this:
  There were 'some' dogs in the yard. Four dogs went for a walk. Three dogs were left behind.
  How many dogs were in the yard to start with?
  Can you write an equation that represents this story?
- Repeat for a 'some' story like this:
  There were 'some' dogs in the yard. Some dogs went for a walk. Double that number stayed behind. The total number of dogs was less than 20.
  How many dogs might there have been altogether?
  How many might have gone walking?
- Make a class book of stories.

Puzzles 10, 11, 12

Topics
Addition and subtraction, average

Task Centre Connection
Task 37: Fraction Magic Square
Task 92: Magic Square
Task 174: Magic Cube
Task 190: Magic Hexagon
Task 239: Money Charts
Strategy

Write an equation, work backwards, try all possibilities, look for extra data

Solution

Puzzle 10:

6	7	2
1	5	9
8	3	4

Puzzle 11:

10	3	8
5	7	9
6	11	4

Puzzle 12:

8	15	14	5
9	11	18	4
12	6	7	17
13	10	3	16

To solve this Puzzle 12, first fill in the two obvious lines, then consider the right hand column. Only one pair of numbers will make the 33 required (16, 17). Which goes in which space is governed by working out the diagonal. Filling these two in leads to more lines being completed until there are only four numbers left. Considering possibilities for the second diagonal leads to completion.

Start Talking

- *Write out the list of numbers used in Puzzle 10. Beside it write the list of numbers used in Puzzle 11. What do you notice?*
- *Starting from Puzzle 10, can you create another magic square which is different from Puzzle 11?*
- *Starting with Puzzle 10, can you create a magic square which uses fractions?*
- *Can you create a new magic square from Puzzle 12?*
- *Write the digits 3 to 18 in a line. Sum the two ends. Now sum the two numbers second from each end; then third from each end and so on. Can you see why the Magic Total for these numbers has to be 42?*

Puzzles 13 & 14

Topics

2D spatial perception, pattern, square numbers, odd numbers, the concept of infinity

Task Centre Connection

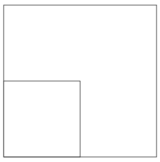
Task 29: The Farmer's Puzzles
Task 68: Six Square Puzzle
Task 115: Dividing Shapes

Strategy

Try all possibilities

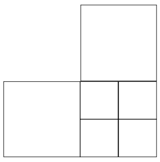
Solution

Puzzle 13



Take away matches 10 & 11.

Puzzle 14



Move matches 2 & 3.

Start Talking

- In Puzzle 13 compare the two squares. What is the length and area of each square?
- Suppose you made a third square which fitted 'around' the second square just like the second square fits 'around' the first. The length of its side would be three times more than the side length of the first square. Can you predict the area relationship?
- What do you predict for the fourth square in the pattern?
- Now you have four squares 'wrapped around' each other. They make a pattern of (upside down) L shapes. How many squares the size of the first square are in the first L? ...the second L? ...the third L?
- Can you predict a pattern?
- In Puzzle 14 there are two sizes of square. If the larger square is worth 1, what is the value of the smaller square?
- Break some matches to divide the smaller square in the bottom right hand corner into four squares. If the largest square is worth 1, what is the value of this new smallest square?
- Repeat the previous question to make four more even smaller squares inside the one which is now in the bottom right hand corner. If the largest square is worth 1, what is the value of this new smallest square?
- If you kept doing this could you make a square with a value of zero? Draw a graph of what you know and make a prediction?
- How does a calculator write one quarter? ... one quarter of one quarter? ... one quarter of one quarter of one quarter? ...

Puzzles 15 & 16

Topics

2D spatial perception, networks

Task Centre Connection

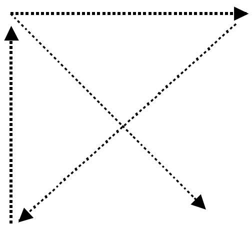
Task 96: Networks

Strategy

Look for extra information, try all possibilities

Solution

Puzzle 15



Puzzle 16: b, c, d

Start Talking

- Have you heard of the Königsberg Bridge problem? Look it up in the library or on your computer. It was solved by the mathematician Leonhard Euler. He realised that whether or not a network could be traversed depended on the number of odd and even junctions it contained. Why?

Puzzles 17 & 18

Topics
Logic

Task Centre Connection
Task 21: Tactical
Task 74: Button Sort
Task 94: Crossing The Desert
Task 110: Who Lives Where
Task 116: Who Owns The Monkey
Task 119: Police Line Up
Task 124: Human Moves Monster
Task 129: Farmyard Friends
Task 211: Soft Drink Crates
Task 228: Koala Karts

Strategy
Look for extra information

Solution
Puzzle 17: None. The dirt has been removed from the hole.
Puzzle 18: Empty the 2nd glass into the 5th. Replace it in the line.

Start Talking

- If we fill the hole with dirt, it isn't a hole any more. What could the hole be filled with?
- How many litres of water would fill the hole? How many millilitres would that be?
- How many MAB 10 tinies would it take to fill the hole?
- Puzzles like this rely on careful interpretation of the words. Another one is the story about the plane that crashes exactly on the border of two countries, which is followed by the question: "Where do they bury the survivors?". What other puzzles/riddles can you find like this?
- There are two types of glasses in Puzzle 18. Full (F) and Empty (E).
  The glasses begin as:
  F F F E E E
  and finish as:
  F E F E F E
  Find five other ways to arrange these six glasses in a line.
- Use just one of each glass. How many ways can they be arranged in a line?

- *Use two of each glass. How many ways can they be arranged in a line?*
- *Think again about three of each glass. Predict the total number of ways to arrange them in a line. How many ways can you find?*

Puzzles 19 & 20

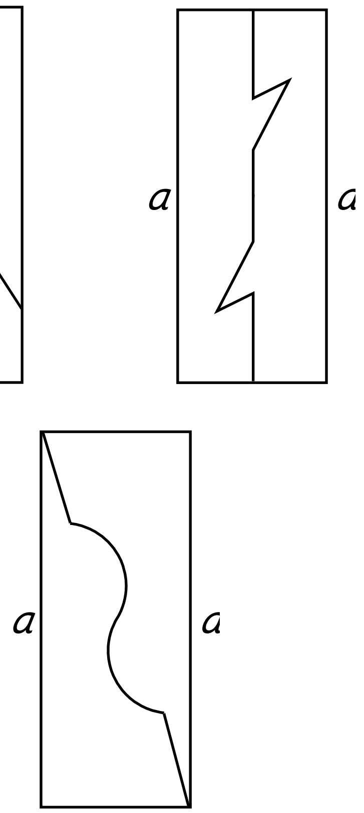
Topics
Fractions, pattern, area, tessellations

Task Centre Connection
Task 5: Make A Snake
Task 38: Mushroom Hunt
Task 146: Haberdasher's Problem
Task 166: Sphinx
Task 177: Making Fractions 2
Task 238: Growing Trisquares

Strategy
Look for a pattern, draw a diagram, work backwards, look for extra information

Solution
Puzzle 19: After 19 years.
The problem relies on halving and doubling being inverses.

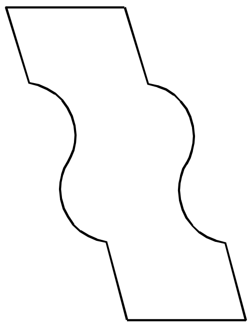
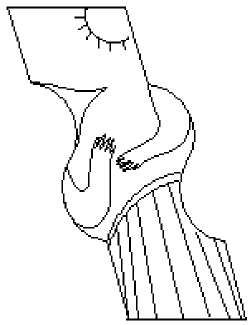
Puzzle 20: There are an infinite number of ways of cutting the rectangle in half. The line symmetry cuts are obvious (and limited) but there are also cuts which result in rotational symmetry, e.g.:



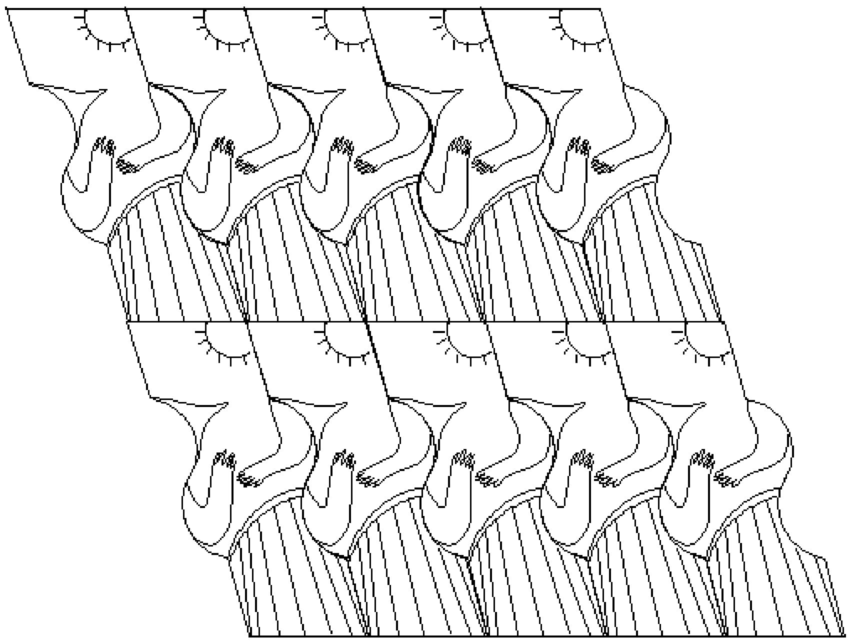
Start Talking
Puzzle 19
- *If we say that after the twentieth year we have the one whole tree, what fraction of this was one year before that?... and a year before that? ... and a year before that? ...*
- *Suppose we make the fully grown tree as high as the doorway. Can we make all the sizes of trees back to the start?*
- *How does a calculator write one half? ... one half of one half? ... one half of one half of one half? ...*
- *How does this connect to a similar question in Puzzle 14?*

Puzzle 20

If you draw the rectangle on thin card, cut and slide so that the sides marked 'a' touch you will make a new, non-regular shape which tessellates, e.g. the third one above becomes:

 then 

- Then, because all the decoration is inside the shape, we can make a stage full of opera singers to rival the three tenors. See below.
- What is the connection between the area of the original rectangle and the area of the opera singer?
- Could you design a tessellation from a transformed rectangle in the same way as the opera singer has been designed?
- What other shapes could transform in this way?
- Have you explored Escher's tessellations?
- Have you explored the Golden Rectangle?
- Have you thought of using tangrams in your class?



Puzzles 21, 22, 23 & 24

Topics
Operations on number, equations, odd and even numbers

Task Centre Connection
Task 8: Addition Totals
Task 33: Dominoes
Task 43: Number Tiles
Task 54: Fay's Nines
Task 89: Doctor Dart
Task 163: Eureka
Task 165: Change
Task 194: Arithmagons 2
Task 216: Square Pairs
Task 225: Add The Pack

Strategy
Look for a pattern, try all possibilities

Solution
Puzzle 21
(2 x 2) - 2 - 2/2, or
(2 x 2) - 2 - (2 ÷ 2)
Are these the only answers?

Puzzle 22
One of several ways to do this is:
(4 + 4) ÷ 4 - (4 ÷ 4)

Puzzle 23
1, 3, 7, 9

Puzzle 24
[20,13], [19,14], [18,15], [17,16]
Put a number in one of the circles, then you know its opposite.

Start Talking
Puzzles 21 & 22
- For each puzzle use the five to make all numbers 0 to 20.
- Using the digits of this year, in order, what other numbers can you make?
- Using the last 4 digits of your phone number, in order, what other numbers can you make?

Puzzle 23
- Make a list of odd numbers up to 20. What is the smallest total four of them could make? What is the largest total four of them could make?
- Which four odd numbers from this list would sum to 40?
- If you choose any four of these odd numbers and add them, what do the totals have in common?
- Can you explain why adding any four numbers from this list gives an answer from the four times table?
- Why do four terms taken in sequence from the list give an answer from the 8 times table?

Puzzle 24
- The solution is pairs of numbers between 10 and 20 which add to 42 when used with 9. There are 4 pairs. Suppose you still used the numbers 10 to 20 and the total 42, but they were used with 8 in the centre? How many pairs? ...7 or 10 in the centre?

- *The number 42 was also used in Puzzle 10. Is there something special about this number?*

Puzzles 25 & 26

Topics
Place value, order

Task Centre Connection
Task 39: Criss-Cross Numbers
Task 43: Number Tiles
Task 98: Martian Maths
Task 113: Calendar
Task 127: Highest Number 1
Task 128: Highest Number 2
Task 144: Pascal's Triangle in Asia
Task 215: Take Away Tiles

Strategy
Make a table, make a list

Solution
Puzzle 25
9,876,543,210 and 1,023,456,789

Puzzle 26
103 pages

Start Talking
- *Can your students read and say the largest and smallest number?*
- *What is the difference between the largest and smallest number?*
- *If your telephone number was read as a place value number, who would have the largest telephone number in the class? Who would have the smallest? What would be the difference between the largest and smallest?*
- *Until 1974 the United Kingdom and the United States did not mean the same thing when they used the words 'one billion'. Find out what each country meant by 'one billion' and the mathematical difference between them.*
- *If a teacher spent none of the money they were paid, how many years would it take for them to become a billionaire?*
- *How big is 1 million? Have you considered a class experiment counting to 1 million by 1s using the constant function on a calculator?*
- *What is a googolplex?*
- *If you are also allowed to use operations on the digits 0 to 9, what would be the largest number you could make by (a) adding, (b) subtracting (c) multiplying (d) dividing (e) using other number operations?*
- *How many 9s are used in the numbering of all the book pages in Puzzle 26?*
- *A book uses exactly ... 9s in numbering its pages. How many pages in the book? How many answers to this question?*

Puzzles 27, 28 & 29

Topics
Sorting and classifying, organised counting, pattern, large numbers

Task Centre Connection
Task 2: Cars In A Garage
Task 118: Ice Cream Flavours
Task 141: Flags From A Ship
Task 183: Pizza Toppings
Task 221: Triangles & Colours
Task 224: Matching Faces
Task 231: Flowers In The Field

Strategy
Draw a diagram, try all possibilities, make a list

Solution
Puzzle 27: 6
If there were only 2 Mokes, there would be 2 ways to arrange the parking, viz:
[L, W] or [W, L]
For each of these R could go on either side or in the middle:
[R, L, W], [L, W, R], [L, R, W]
[R, W, L], [W, L, R], [W, R, L]

Puzzle 28: 24
The blue Moke can be parked at either end of any arrangement of 3 or in either of the two 'betweens' in any arrangement of 3. So, 4 variations for each of the 6 possibilities in Puzzle 27.

Puzzle 29: 26,000
For letter A there are 1000 plates - A000 to A999. Similarly for each of the other 25 letters in the English alphabet.

Start Talking
- Five Mokes in a line? Six Mokes In a line? ... Ten Mokes in a line?
- If I asked you any number of Mokes in a line could you tell me how to work out the number of arrangements?
- Three Mokes parked in a circle (look at it from above)? Four Mokes in a circle? ... Ten Mokes in a circle?
- If I asked you any number of Mokes in a circle could you tell me how to work out the number of arrangements?
- What is the minimum number of Mokes it would take to park so that they spelled the word Moke?
- How many number plates could be made with three letters and three digits?
- How many of these number plates would have a 9 as the middle digit?
- Melbourne landline telephone numbers used to have 7 digits. Then everyone was told to put an extra 9 in front of their number. How many numbers could have been assigned under the old system? How many could be assigned under the new system?

Puzzles 30, 31 & 32

Topics
Sorting and classifying, organised counting, pattern, square numbers, triangle numbers

Task Centre Connection
Task 28: Plate Triangles
Task 48: How Many Triangles?
Task 101: Pyramid Puzzle
Task 111: Square Numbers
Task 108: How Many Squares?
Task 139: Squound
Task 216: Square Pairs
Task 236: Star Numbers

Strategy
Draw a diagram, act it out, make a list, try all possibilities, look for a pattern, look for extra information

Solution
Puzzle 30:
1 + 4 + 9 = 14

Puzzle 31:
A with B, C, D, E, F and B with C, D, E, F (but not with A because that has already been counted) and so on leads to
5 + 4 + 3 + 2 + 1 = 15

Puzzle 32:
9 + 8 + 7 + 6 + 5 + 4 + 3 + 2+ 1 = 45

Start Talking
- How many squares of Size 1? Size 2? Size 3?
- Squares in a 4 x 4 square?
  A 5 x 5 square?
  ... A 10 x 10 square?
- If I asked you any size square, could you tell me how to work out the number of squares of any size within it?
- In the squares puzzle do you notice anything special about the numbers which add to the answer each time?
- Both Puzzle 31 and Puzzle 32 require adding up a sequence of natural numbers starting from 1. If I tell you the highest number in the sequence, could you tell me how to work out the sum?
- Can you find a formula for:
  $1 + 2 + 3 + 4 + \ldots + (n-1) + n$?
- Can you find a formula for:
  $1^2 + 2^2 + 3^2 + 4^2 + \ldots + (n-1)^2 + n^2$?
  Very tough - try thinking spatially. That is, thinking about shapes the numbers represent rather than the numbers.
  Task 101, Pyramid Puzzle has some clues and so does Task 138, Rectangle Of Squares. If you are a member, Maths300 Lesson 138, Pyramid Puzzle, has the full solution.

Puzzles 33 & 34

Topics
Number sense, equations

Task Centre Connection
Task 39: Criss-Cross Numbers
Task 52: Which Floor
Task 62: 4 & 20 Blackbirds
Task 65: Shape Algebra
Task 147: Garden Beds
Task 154: 4 Arm Shapes
Task 178: Match Triangles
Task 180: Making Monuments
Task 233: Money, Money, Money

Strategy
Simplify the problem, make a model, write an equation

Solution
Puzzle 33
19 boys
Algebra (Let G = girls, then boys = G + 8) works, but I prefer the solution of one of my students who reasoned:

If all the extra boys were away one day, there would be equal numbers of girls and boys making 22 children. So on a normal day there must be 19 (11 + 8) boys.

Puzzle 34
Cork 5¢ ... Bottle $1.05
You could use algebra. Let the cork = C, then the bottle is C + 1, so C + (C + 1) = 1·10

However, why not try a trial and improve strategy. Perhaps start with: 'Suppose the cork cost 2¢...'

Start Talking

- Tell me how you worked it out. Has anyone done it a different way? How many ways can we find?
- Can you rewrite Puzzle 33 so that the numbers in the rhyme change, but the answer is still the same? Which numbers can't be used? How many possible ways are there of rewriting the rhyme?
- Can you rewrite Puzzle 34 so that the numbers in the rhyme change, but the answer is still the same for the cork? How many possible ways are there of rewriting the rhyme?

Puzzles 35, 36, 37 & 38

Topics
Measurement, doubling and halving, money, fractions, ratio

Task Centre Connection
Task 22: Time Together
Task 25: In Between Time
Task 41: Scale Drawing
Task 58: See-Saw
Task 88: Rice, Rice, Rice
Task 99: How Many Things?
Task 126: Planets
Task 136: Four Bead Mix
Task 166: Sphinx

Strategy
Draw a diagram

Solution
Puzzle 35: $400

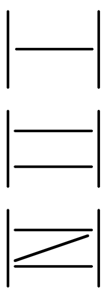
Puzzle 36: $1.40
(7 cuts at 20¢ each)

Puzzle 37: 30 metres

Puzzle 38: 21 kilos

Start Talking

- Have you tried 'fence post' problems with pop sticks? How many pop sticks are needed to build 10 sections of each of these fence designs?



- Have you tried 'class classifying' problems like:
Half the class are boys. Half of that half wear school jumpers. One third of the school jumper group also wear black shoes. Two boys wear black shoes. How many children in the class?
- Have you tried the children making up their own pop stick 'fences' or 'class classifying' problems?

Puzzle 39

Topics
Doubling and halving, equations

Task Centre Connection
Task 3: Doug's Tablecloth
Task 19: Cookie Count
Task 212: Monkeys & Bananas

Strategy
Work backwards, write an equation

Solution
36 apples in the case

Start Talking

- Professor Morris got one apple. How many apples did each of the other customers get?
- The problem has several steps, but mathematically they are all the same. What are the mathematical operations used at each step?
- How do you think the inventor of this problem thought it up?
- Can you think up a problem like this which uses halves?
- Can you think up a problem like this which uses fourths?
- Can you think up a problem like this which uses two sorts of fractions?
- Could we make a class book of problems like this?

Puzzles 40, 41 & 42

Topics

Pattern, addition and multiplication, division, average

Task Centre Connection

Task 4: Window Frames
Task 7: Consecutive Sums
Task 8: Addition Totals
Task 10: Find My Pattern
Task 28: Plate Triangles
Task 91: Pick A Box
Task 101: Pyramid Puzzle
Task 111: Square Numbers
Task 135: Famous Math'maticians
Task 172: Cover Up
Task 210: Division Boxes

Strategy

Look for extra information, work backwards, write an equation

Solution

Puzzle 40

The 3 numbers will be almost the same, so each will be about $^{1}/_{3}$ of 114. Therefore 37, 38, 39.

Puzzle 41

5 x 6 = 30

Five because there are 5 numbers equal steps apart. Six because the middle one of them is 6.

Puzzle 42

100 x 50·5 = 5050 *for the same reason, or look at the 'rainbow' in the Start Talking section.*

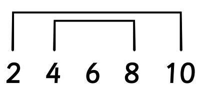
Start Talking

Puzzle 40

- *Choose any 3 digit number. Can you find 3 consecutive whole numbers which add to that total?*
- *Keep investigating 3 digit numbers until you find some that do work.*
- *What's special about the ones that do work?*
- *Can you explain why the totals have to be that special sort of number?*
- *Professor Morris only has three consecutive numbers in mind. What happens if it was five consecutive numbers?*
- *What happens if he has an even number of consecutive numbers in mind?*

Puzzle 41

- *Why does this 'rainbow' for Puzzle 41 also show that the answer must be 5 groups of 6?*



- *Can you find five other numbers that are equal jumps apart and also sum to 30?*
- *Choose five other numbers that are two apart. Can you find their total by 'times-ing'?*
- *What happens if the list has an even number of numbers that are two apart? Can you find their total by 'times-ing'?*

- Can you find the total by 'time-sing' if the numbers are equal jumps apart and the jump is greater than 2? For example: 9, 12, 15, 18, 21
- What about if the numbers are the same fraction apart?
- Have you tried Predict-a-Count on the calculator?
  For example:
  35 $\xrightarrow{2}$ means start at 35 and count forward by 2 [+, 2] BUT don't press = until you first guess what the screen answer will be. The start number and the guesses should be recorded. Stop after ten terms. Find the total of the ten numbers in two ways.

Puzzle 42

- Can you find the total of all the numbers in a 1-10 addition chart?
- Can you find the sum of all the numbers in a 1-10 multiplication chart?

Puzzle 43

Topics
Doubling and halving, powers of 2, binary numbers

Task Centre Connection
Task 38: Mushroom Hunt
Task 142: Tower of Hanoi
Task 144: Pascal's Triangle...Asia

Strategy
Look for extra information, simplify the problem

Solution
1023
Look at the first four fingers. In each case, the 'next' finger is double the one before it AND one more than the total of all the ones before it. These two observations continue to be true as you increase the number of fingers. So, the total must be (2 x 512) -1.

Start Talking

- The rhyme says that a finger up is counted, but a finger down is not. If you put some fingers down and you ARE allowed to total the up fingers, what number do you make?
- How can you use the fingers to show the number 3? or 5? ...6? ...7? ...8?
- Choose any number. Make it with the finger numbers. How many ways can you make that number with the finger numbers?
- Can you write all the 'normal' numbers from 1 - 20 as the addition of finger numbers? For example: 5 = 4 + 1
- If a finger is up call it 1. If down, it's 0. (Sometimes you will find [1, 0] on switches. It means [ON, OFF].) Write each of the 'normal' numbers from 1 - 20 as a string of 1s and 0s.

For example, looking at the fingers from the way Professor Morris would see them in his puzzle picture:

6 <---> [1 1 0]
4 and 2 are up and 1 is down
The fingers on the other side of 4 are down too.

- What is the code number for 1024?
- What is special about each of the finger numbers?
- Write each finger number as a product of 2s. Can you devise a shorthand code to save all the writing? Can you write 8192 in your code?

Puzzles 44 & 45

Topics
Operations on number, equations

Task Centre Connection
Task 14: Heads And Legs
Task 63: Fried Rice
Task 91: Pick A Box

Strategy
Trial and improve, make a table, make a model

Solution
Puzzle 44: 32 chickens
Puzzle 45: Their age difference is always the same.

Mum is now 45, I am 9 (36 years apart) and it is 6 years since she was 39 and I was 3.

Start Talking
Puzzle 44
- The number of legs is still 100, but the number of creatures is different. How many could there be?
- If there are 28 creatures and 92 legs, how many were pigs and how many were chickens?
- Can you make up some of your own? Are there some numbers which don't work?
- If you were told any number of creatures and any number of legs, how would you work out the number of each creature?

Puzzle 45
- Has mum ever been five times my age before? Will she ever be five times my age again?
- How old will you be when you are half of your mother's age?

Puzzles 46 & 47

Topics
Operations on number, fractions, ratio

Task Centre Connection
Task 63: Fried Rice
Task 99: How Many Beans?
Task 136: Four Bead Mix

Strategy
Look for extra information, simplify the problem

Solution
Puzzle 46:
12 minutes.
Sue can dig the garden twice in an hour. Jan can dig it three times - as long as she doesn't stop for that shower. So, together they could dig it five times in one hour, so it would be dug once in 12 minutes. There is another way to work it out using fractions.

Puzzle 47
Start both timers. When the 4 minute one is finished, start cooking. The seven minute one will run for another 3 minutes. When it finishes turn it over again to get another 7 minutes, making a total of 10 minutes. There is another way.

Start Talking
- *There are other fractions of an hour which could be used as work rates for Sue and Jan. Make up a similar puzzle using two other work rates.*
- *What other timers could be used to time 10 minutes?*
- *Could one of them be a timer which ran for more than 10 minutes? Could both of them be timers which ran for more than 10 minutes?*

Puzzle 48

Topics
Pattern, operations on number, equations

Task Centre Connection
Task 5: Make A Snake
Task 51: Staircase
Task 61: Double Staircase
Task 137: Training For Maths
Task 178: Match Triangles
Task 179: Unseen Triangles

Strategy
Make a table, look for a pattern

Solution
It depends how you interpret the puzzle.
If sharing means to count all the staircases already in the picture then the largest stair which can be built with 100 matches which includes these is the 5 stair. In fact, building the 1, 2, 3, 4 & 5 stairs uses exactly 100 matches.
However, if the puzzle is interpreted as building one staircase only and building it as big as possible with 100 matches, then the stair will have 8 steps, use 88 matches and leave 12 matches unused.
The same number pattern is involved in either case and the formula to find the number of matches (M) for a given number of steps (S) is:

M = 2 x {(1 + 2 + 3 + ... [S-2] + [S-1] + S) + S}

which is not simplified so you can ponder how the matches were visualised to arrive at this.

Start Talking

- Count the matches which run vertically. Count the matches which run horizontally. How can you use this information to count all the matches in a 10 stair? A 100 stair?
- Can you visualise the match counting a different way? How would you work out the matches in a 10 stair with this second method? A 100 stair?
- Can you use matches to build other sequences of shapes? What number patterns develop? How can you use them to work out the number of matches in 10th or 100th shape of your sequence?

Puzzle 49

Topics

Operations on number, equations

Task Centre Connection

Task 45: Eric The Sheep
Task 72: Farmyard
Task 75: What's It Worth?
Task 80: A Dollar To Spend

Strategy

Trial and improve, look for extra information, make a table

Solution

One solution is 88 chickens, 8 pigs and 4 sheep. When you realise that the pricing means that chickens must be bought in a certain size group, you reduce the number of explorations necessary to obtain the answer.

Start Talking

- Is there another solution?
- Is 100 such a special number? Could a similar problem be constructed with say 80 animals for $80? How about for other numbers?

Puzzle 50

Topics

Measurement, ratio, operations on numbers, estimation, scale drawing, large numbers

Task Centre Connection

Task 126: Planets
Task 204: Decimals With A Tape
Task 205: Fraction Estimation

Strategy

Make a model, look for extra information, write an equation

Solution

It will be necessary as part of the introductory discussion to refine the size of the problem which will be tackled locally. Do you really want to begin by working on all the newspapers in the world? It might be best to begin with one major local paper; even the local community paper might bring surprises. The questions raised will soon spread to include 'all newspapers in the world'.

As an example one class calculated that one issue of the Melbourne Saturday Age (when it was a broadsheet paper) would stretch around one half to two thirds of the Australian coastline. Another class calculated that one issue of the London Sun (tabloid size) would make a pathway from London to Melbourne.

However, since those times digital news has reduced the hard copy print run of newspapers. Still, the calculations from these classes off a bench mark for a comparison that gives some indication of the benefit to the environment of reducing the number of hard copy newspapers.

Start Talking

- How many trees are used to make the paper for a one issue pathway? (Search: 'How many trees to make a newspaper?')
- What percentage of the paper used in a one issue pathway is made from recycled paper?
- What percentage of a one issue pathway becomes recycled paper?
- How many issues of your paper would it take to make a pathway that goes once around the world?
- How many issues would it take to completely wrap the world in paper envelope?
- Compare the results of your one issue pathway with one of the examples given here. Since those examples were investigated digital news delivery has become more common. Use your comparison to get an idea of the level of reduction in the use of newsprint paper.
- If we decided printing newspapers was no longer necessary at all, what effect would ceasing production have on the economy?
- What effect would it have on people's lives?

At the link below see sample student work from Year 7, Bacchus Marsh Grammar School students exploring Puzzle 48.

http://www.mathematicscentre.com/taskcentre/bmg_pm.htm