Mathematics Task Centre

Heads & Legs

Common Assessment Across Year Levels with a focus on Working Mathematically Moderation

The following material was prepared by
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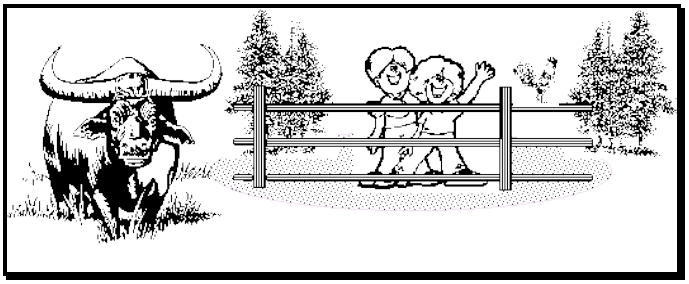
Mathematics Task Centre
Managed by Black Douglas Professional Education Services

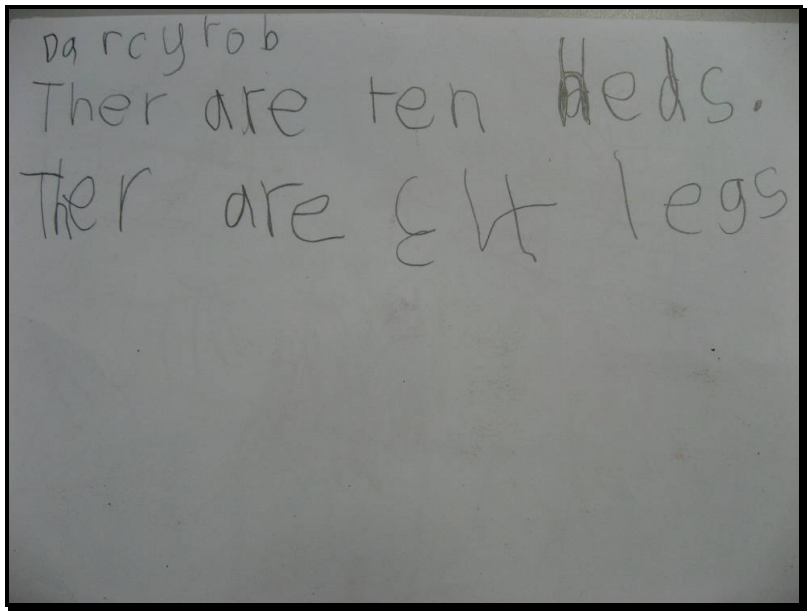
In order to paint a school-wide "picture" of our students' ability to undertake open-ended tasks and work like a mathematician, we modified the task "Heads & Legs" to meet the needs and abilities of students P – 6.



The story shell

Two children, Sarah and Sam, go to a farm. There are some roosters in the cow paddock. Sarah counts the heads they see. Sam counts the legs they see. When they run back to the farmhouse to tell Mum what they found, she is able to use the two pieces of information to tell them how many animals there were altogether.
This is a problem which can be solved with a variety of strategies and has a strong algebra link.

Our modifications



Level one and two students are studying "The farm". The Prep teachers introduced the concepts of animals with pairs of legs or groups of four through a drawing activity (draw some paddocks, draw some animals, write about how many heads and legs you can count).

When the story was introduced to the Prep classes, the number of legs and heads was reduced:

Sarah and Sam went to their uncle's farm at the weekend. While they were there watching the cows they noticed there were roosters in the cow paddock too. Sarah counted the legs they could see: there were 12. Sam counted the heads they could see: there were 4.

Students were provided with counters to represent heads and matchsticks to represent legs. They modelled the problem, drew the picture to show the problem and wrote a story about what they found out.

The "What if" questions were also modified:

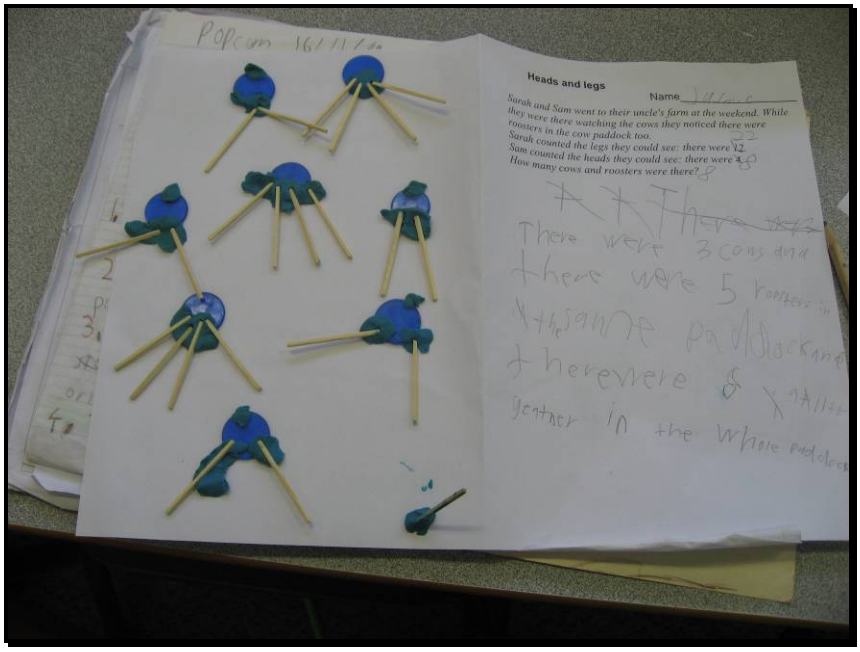
What if Sarah and Sam had counted 3 heads and 10 legs?
What if Sarah and Sam had counted 3 heads and 12 legs?
What if Sarah and Sam had counted 6 heads and 14 legs?

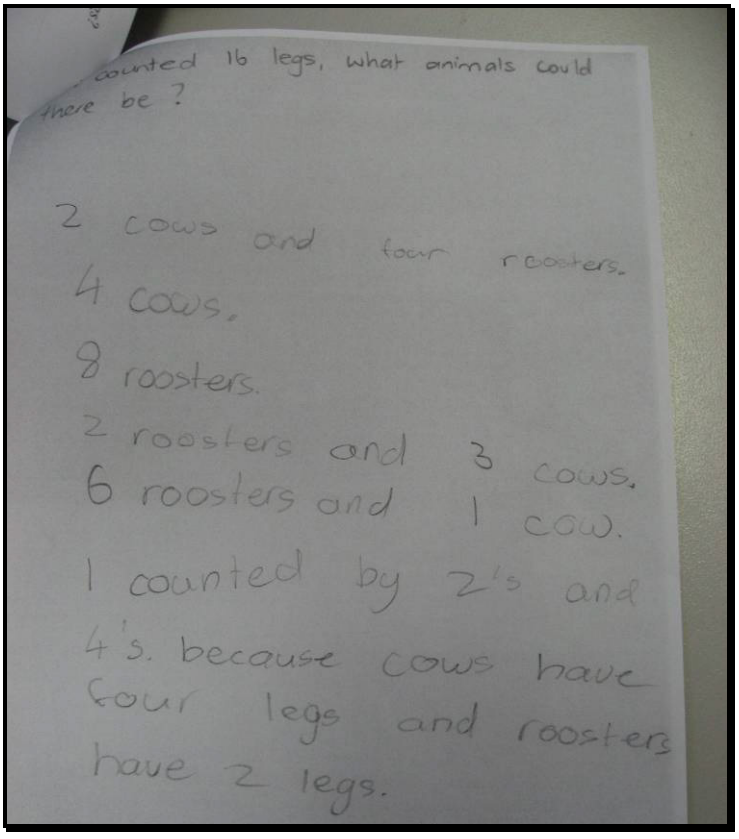
These questions were also used with some older integration students in the school, in order to help them achieve success.

Level two students were also given counters and matchsticks to represent the heads and legs, but were given the original story shell and "What if" questions.

The children who were working above Level Two were also given a couple of the questions from the investigation sheet to answer:

> *Make up all the heads and legs problems you can which use 16 legs.*
> *Make up all the heads and legs problems you can which use 9 heads.*



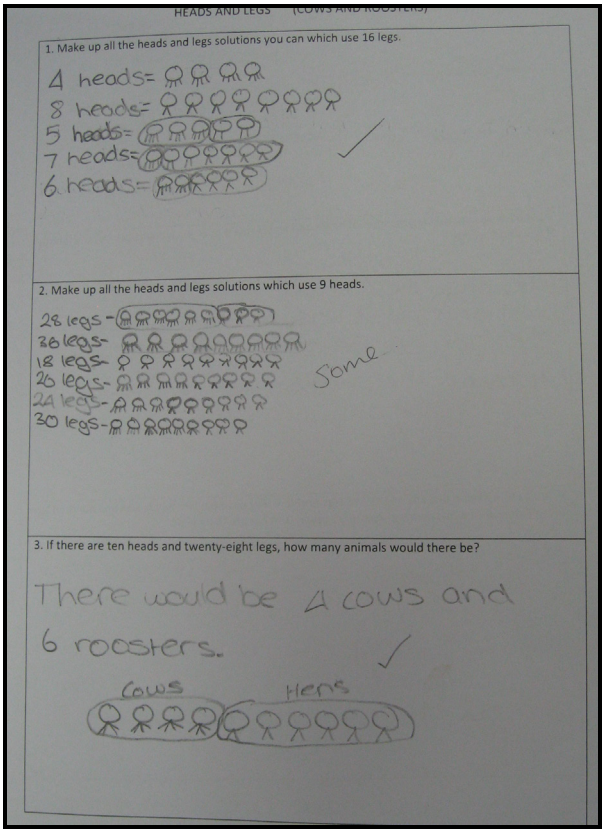


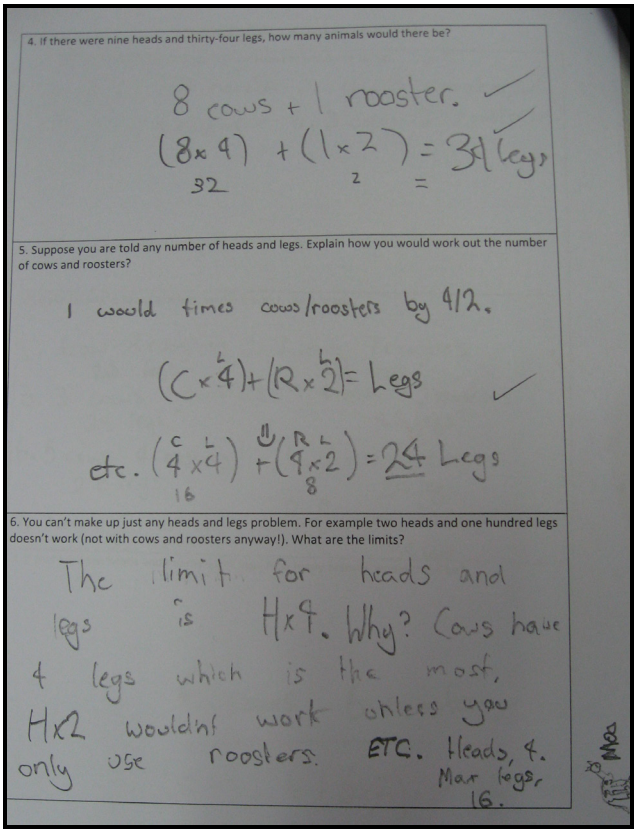
Level three and four students were given the cards from the task and the Investigation Sheet which accompanies it, although this was also modified to give students room to show their working out and write down their learning.

A rubric was developed, based on the Maths Developmental Continuum and the Working Mathematically Progression Points, to assess the students from VELS 0.5 to beyond VELS level 4.

(Editor's Note: These documents are specific to Victoria, Australia at a moment in time. However the rubrics developed from them support teachers everywhere to prepare in a similar manner.)

Whole school moderation revealed there was a wide range of abilities in Working Mathematically across the school, and gave teachers the knowledge and confidence to mark their students at appropriate levels, even when this was well below or well above that expected. It also highlighted the need for more differentiated mathematics planning and the importance of open ended problem solving activities, particularly in the upper levels of the school.





The Prep problem

Heads and Legs

Sarah and Sam went to their uncle's farm at the weekend. While they were there watching the cows they noticed there were roosters in the cow paddock too. Sarah counted the legs they could see: there were 12. Sam counted the heads they could see: there were 4. Then they ran back to the farmhouse to tell Mum what they counted.

Clever Mum. She listened to the number of heads and the number of legs and worked out the number of animals of each type in the paddock.

Now you try to work out the number of each animal. The counters and toothpicks are to help you.

Criteria	**Below level 1**	**Level 1.0**	**Above level 1**
Problem solving – models	Tries to make the problem using counters and pairs of toothpicks but has some difficulty	Model problems using materials – counters and pairs of toothpicks helps the student find solution	Is able to look at an alternative "what if" question and model a solution
Problem solving - drawing	An attempt to draw the problem	Drawing reflects the problem and successfully shows the solution	Is able to draw the "what if" problem
Multiplicative thinking	Does not recognise that each counter needs either two or four toothpicks – may not match each counter to the toothpick	Can show pairs and groups of four (legs)	Articulates the basic rule that each head needs a pair or group of four
Explanation	Explanation does not match the model or the picture	Can explain the need for pairs and groups of four	Can write an explanation using numbers and/or symbols

What if questions:

- *What if Sarah and Sam had counted 3 heads and 10 legs?*
- *What if Sarah and Sam had counted 3 heads and 12 legs?*
- *What if Sarah and Sam had counted 6 heads and 14 legs?*

The grade 1 and 2 problem

Heads and legs

> *Sarah and Sam went to their uncle's farm at the weekend. While they were there watching the cows they noticed there were roosters in the cow paddock too. Sarah counted the legs they could see: there were 22. Sam counted the heads they could see: there were 8. Then they ran back to the farmhouse to tell Mum what they counted.*
> *Clever Mum. She listened to the number of heads and the number of legs and worked out the number of animals of each type in the paddock.*
> *Now you try to work out the number of each animal. These counters and toothpicks might help.*

Criteria	**Just above level 1**	**Below level 2**	**Level 2.0**	**Above level 2**
Problem solving – models	Models problem Using materials – counters and toothpicks helps the student find the solution	Is able to look at an alternative "what if" question and model a solution	Use diagrams and models to show the problems and the solutions	Uses cards to help find the solution to the problem
Problem solving - drawing	Drawing reflects the problem and successfully shows the solution	Is able to draw the "what if" problem	A drawing and matching number sentence for each problem	A number sentence and detailed drawing showing the groupings and pairs for each problem
Multiplicative thinking	Can show pairs and groups of four (legs)	Articulates the basic rule that each head needs a pair or group of four	Can count by twos	Multiply by twos or fours to find the solutions
Explanation	Can explain the need for pairs and groups of four	Can write an explanation using numbers and/or symbols	Can record a number sentence to reflect the problems	An attempt to show multiplication in the number sentences

What if questions:

- *What if Sarah and Sam had counted 6 heads and 18 legs?*
- *What if Sarah and Sam had counted 6 heads and 24 legs?*
- *What if Sarah and Sam had counted 6 heads and 14 legs?*

The grade 3 and 4 problem

Heads and legs

> *Sarah and Sam went to their uncle's farm at the weekend. While they were there watching the cows they noticed there were roosters in the cow paddock too. Sarah counted the legs they could see: there were 22. Sam counted the heads they could see: there were 8. Then they ran back to the farmhouse to tell Mum what they counted.*
> *Clever Mum. She listened to the number of heads and the number of legs and worked out the number of animals of each type in the paddock.*
> *Now you try to work out the number of each animal. These picture cards might help. Then you can try the investigation sheet.*

Criteria	Below level 3	Level 3.0	Above level 3
Problem solving	Uses cards to help find the solution to the problem	Trial and error to find the pattern that will lead to a solution	Make lists in an effort to find a pattern An understanding that the number of heads is equal to the number of animals and the trialling of a range of addends to find the correct number of legs to match this
Multiplicative thinking	Multiply by twos or fours to find the solution	Multiply by twos and fours to find the solution	Multiply and/or divide to find the solution
Investigation sheet	Does not attempt any questions from the investigation sheet	Makes reasonable and mostly successful attempts at questions 1 - 4	Successfully completes questions 1 – 4 and attempts questions 5 and/or 6
Explanation	A number sentence and detailed drawing showing the groupings and pairs	Can reason from the legs or heads (see below – focus on strategies)	A mathematically correct answer with all steps clearly explained

Focus on strategy

How many ways can students work out the answers? This is an application of the mathematician's question: *Can I check it another way?*. They could:

- Use the cards to make a model. Draw a diagram.
- Reason from legs: *Everything has to have at least 2 legs so that uses up 16 legs for the 8 heads. There are 6 legs left, so put 2 extra legs on three heads to make buffalo. That gives 3 buffalo and 5 hens.*
- Reason from heads: *Pretend all the 8 heads are buffalo. That means there are 32 legs. That is 10 too much, so take 2 legs off 5 heads to make hens. That gives 5 hens and 3 buffalo.*
- Try all possibilities in a table.

The grade 5 and 6 problem

Heads and legs

> *Sarah and Sam went to their uncle's farm at the weekend. While they were there watching the cows they noticed there were roosters in the cow paddock too. Sarah counted the legs they could see: there were 22. Sam counted the heads they could see: there were 8. Then they ran back to the farmhouse to tell Mum what they counted.*
> *Clever Mum. She listened to the number of heads and the number of legs and worked out the number of animals of each type in the paddock.*
> *Now you try to work out the number of each animal. Then you can attempt the investigation sheet.*

Criteria	Below level 4	Level 4.0	Above level 4
Investigation sheet	Successfully completes questions 1 – 4 and attempts questions 5 and/or 6	Successfully completes questions 1 – 4 and finds the rules (limits) for question 6	Successfully completes all questions from the investigation sheet
Problem solving	Make lists in an effort to find a pattern	Makes tables in an effort to see the pattern in the solutions	Represents the known information using letters (eg: C for cow, R for roosters / L for legs, H for heads)
Multiplicative thinking	An understanding that the number of heads is equal to the number of animals And an understanding of the associative equation that the number of legs is four times the number of cows plus two times the number of roosters	Uses division and/or multiplication, with reference to the total and the legs.	An algebraic representation of the following: An understanding that the number of heads is equal to the number of animals, or H = C + R Plus an understanding of the associative equation that the number of legs is four times the number of cows plus two times the number of roosters, or L = (4 x C) + (2 x R)
Explanation	A mathematically correct answer with all steps clearly explained	Articulates strategies that help find the solution Eg: *Multiply the number of heads by two. Take this away from the number of legs. This answer is twice the number of cows*	The explanation includes at least two different ways of finding the solution