Menu Maths & Other Models for Making Mathematicians

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Working Like a Mathematician

School mathematics is about learning to work like a mathematician. Mathematicians tell us their work begins with an interesting problem ... which they tackle, often in community, through a process (HREF1, 2, 3) that includes selecting from a strategy toolbox (HREF4) and a skill toolbox.

Teaching school mathematics deserves much more than the traditional three step process ... the last three steps before the classroom door, as my colleague John Hibbs describes them:

Step 3 ... Open: the textbook I am carrying.
Step 2 ... Think: *Where did we get to yesterday?*
Step 1 ... Decide: *Ah, that's it. We'll start from there today.*

And you know what happens in the classroom after that ... much of it being about polishing items from the skill toolbox, not investigating problems.

Mathematicians say their work begins with an *interesting* problem, therefore, one implication for a curriculum based around learning to work like a mathematician is that the teacher has a *responsibility* to select teaching craft likely to interest the student(s) in the problem. See HREF5 for a list of possible teaching craft features.

One of these features is to encourage students to own the problem - ownership - and one way to do that is to invite students to choose their own problem. Mathematics Centre currently has 241 problems in its Task Library (see HREF6). About two thirds are supported by linked teachers' notes (Task Cameos), and that fraction increases every month.

When you go to a restaurant you are offered choice - a menu - and, in fact, if you weren't, it's very likely that you wouldn't go there to eat because you can get 'no choice' at home very easily. Also, within the context of their restaurant, the chef has chosen the best possible items to include on the menu. Why would they not? To do otherwise than select the best possible would be a deliberate abdication to poorer quality ... and a recipe for going out of business. Making the best selection for a menu takes preparation. Once created, enticing people to visit and partake of the menu requires marketing.

This metaphor is what has led teachers to create Menu Maths units of work within their Working Mathematically curriculum.

Menus from Tasks

Schools purchase any number of tasks, but most often it is as a kit of 100 different tasks suitable for a wide range of abilities and which fit into a range of broad content areas such as number, pattern, computation, algebra, chance, space & logic. Subsets of these can be chosen at different points in the year to build units with more specific content aims. To assist in selecting these 'best ingredients' Mathematics Centre provides a Task Cameo Content Finder (HREF7). Again, since the Task Cameo Library of teachers' notes is extended each month, not every task is represented on the list yet, but 180+ still provides a rich support base.

On the other hand, some schools build their units using a random selection of tasks, rather than a content based selection, on the grounds that it matters more that students choose a problem that interests them and as a result become involved in the Working Mathematically process. A view perhaps driven by providing opportunity for students to display their higher order reasoning abilities without being (potentially) 'scared off' by content.

Once the menu is chosen, it is published and students are invited to select from it. Staff working together to create the menu is a professional development exercise in itself. Here is an example of a menu of tasks selected for a Replacement Unit in Pattern & Algebra, so called because the three week unit replaces all the content that would have been taught more traditionally over the same three weeks. It is not an extra; it *is* the curriculum.

Pattern & Algebra Task Menu

- [ ] 4 Arm Shapes
- [ ] Addition Totals
- [ ] Algebra Through Geometry
- [ ] Crossing The River 1
- [ ] Eric The Sheep
- [ ] Garden Beds
- [ ] Lining Up
- [ ] Making Monuments
- [ ] Match Triangles
- [ ] Mirror Patterns 2
- [ ] Painted Rods
- [ ] Pointy Fences
- [ ] Shape Algebra
- [ ] Smooth Edge Tiles
- [ ] Snail Trail
- [ ] Sphinx
- [ ] Square Numbers
- [ ] The Mushroom Hunt
- [ ] Time For Tiling
- [ ] Unseen Triangles

This menu is from Maths With Attitude Pattern & Algebra Years 7 & 8 kit (HREF8) in which the Replacement Unit is used in each year as a 3 week block within 8 weeks of sequenced algebra learning.

Tasks are not games. They are investigations - the legitimate work of mathematicians - and they are in the collection because they satisfy these conditions:
- The task is intrinsically motivating.
- The task provides hands-on materials to support its solution.
- The intellectual challenge is likely to result in a first level of success in 15 - 20 minutes.
- The problem on the card is the tip of an iceberg; the beginning of a deeper investigation.
- The task has three lives:
  - as an invitation for two students to work like a mathematician
  - as a whole class investigation to model how a mathematician works
  - as a deeper investigation led by an Investigation Guide.
- The task can be used to illustrate the process of Working Mathematically.

Week 1

To begin the unit the 20 tasks are spread out and the students are invited to choose one per pair. One pair might choose *Garden Beds* for example:
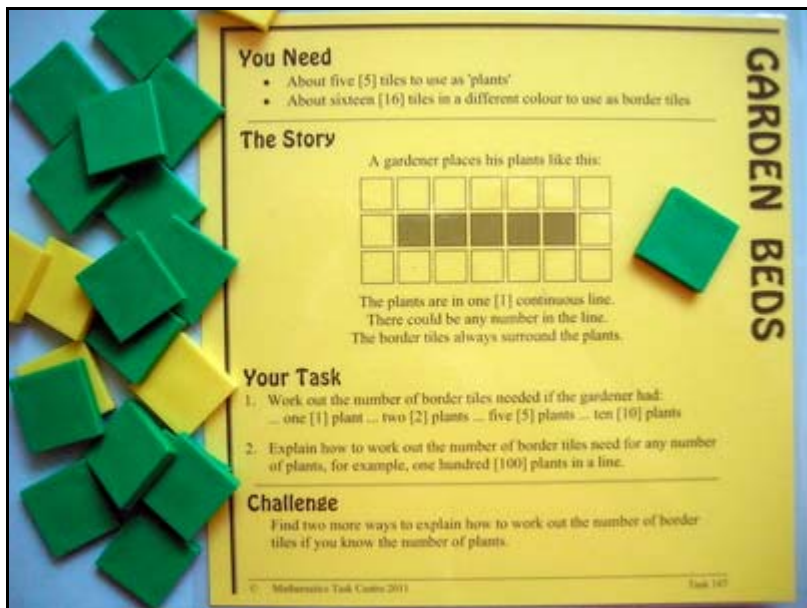
Clearly the tasks are being used as an invitation to work like a mathematician. Students explore the task and keep a record in their journal. Nothing formal, other than the name of the task and the date, but they are encouraged to make sure they will be able to understand their own entries 'three weeks from now'.
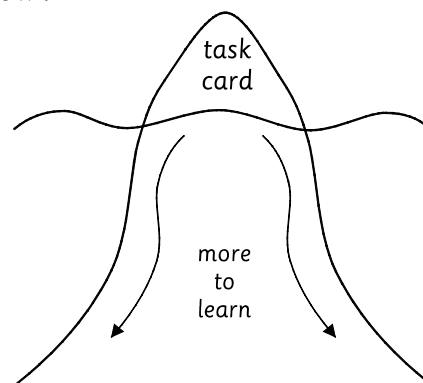
When they reach the end of card, they have completed the tip of the iceberg and are required to chat with the teacher. The teacher's role in this debrief is to encourage them to retell what they have learnt, check that key elements are recorded and help them find and record the next question. There is always a next question. No task is ever finished.

This done the students tick this task on their menu, choose another and repeat the process. This continues through the first week of the unit. Over this period, students might explore as many as five tasks.

Week 2

The second week of the unit builds on the life of a task as a whole class investigation.

Just as any reading book in the library can be chosen and explored by a student *and* the same book could be chosen by the language teacher and explored as a whole class literature adventure, so any task can be explored with the whole class by modifying the equipment.

These photos illustrate *Garden Beds* being used in this life.

1. We made a model using 4 plants in a line, then We put tiles around.
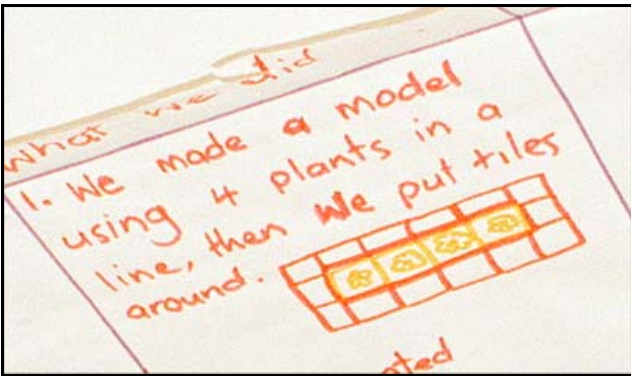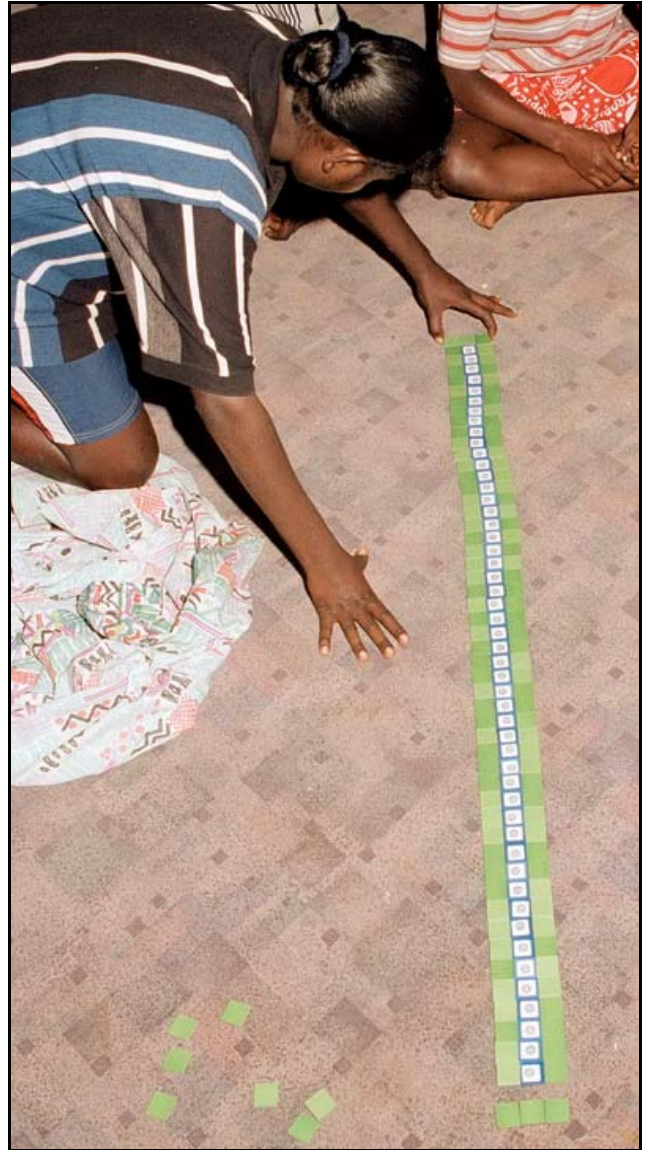
.. because he has two 50 at the two side and two 3 on the end. (Vicky)
because it has 50 plants in a line and I quickly know that 50 tiles on the side and 50 tiles on the other side and altogether is 100 and add the 3 from the two edges and is 6 so the answer altogether is 106 tiles.
(Eusebia).

I need 50 tiles down one side plus 3 tiles on one end. This makes 53. Then I need another row of 50 tiles for the other side and 3 more tiles for the other end which makes another 53. 53 plus 53 makes 106 tiles. (Juli)

Each plant needs a tile on each side, so ... this is 50 times two which

18 10 9 6 1 8 7 50 4
42 26 22 18 8 22 20 106 14

invitation
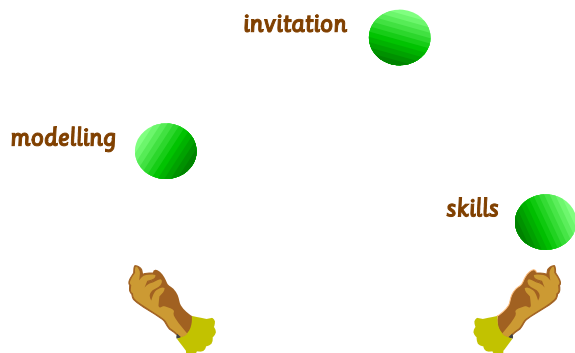
modelling

skills

Clearly the teacher is doing a lot of modelling and that's the point. To learn to do anything well, it needs to be modelled for us, we need the invitation to try it for ourselves and the time to practice key skills in context. Teachers design Replacement Units as one way to juggle the time spent on all three elements - and to help avoid the all too easy drift towards practising skills out of context.

Clearly, many skills are being called on in the above photos and this whole class investigation (which may take more than one time slot) can therefore provide the jumping off point for a Toolbox Lesson. Teachers find that by approaching skills in context like this it becomes quite acceptable in the next lesson to ask students to 'turn to page 53, exercise 5A', where the expectation of the text in this exercise can be clearly linked to the investigation. In fact, teachers report that students often comment 'this is easy' and finish the required work in double quick time. So, Week 2 becomes a combination of whole class modelling and toolbox lessons.

Week 3
The Investigation Guide life comes into its own in Week 3. Staff have prepared the Guides in advance - a great professional development exercise in itself. Guides are simply questions which assume students have completed the task and guide them deeper into its iceberg. An Investigation Guide for *Garden Beds*, again taken from the Maths With Attitude kit above, is appended to this article.

Again the tasks are laid out to start the week, but this time pairs select only one and its Guide and accept it as a project to complete. A report is expected and is assessed. This is publishing, just like a mathematician must do, but the form of the report does not have to be a written text. It could be a poster, comic strip, oral presentation, slide show, video, web site or ...

Members of Maths300 (HREF9) can see four such slide show reports about *Garden Beds* from Year 7 students at Settlebeck High School, Cumbria. And if you are not a member of that site, you can view several other reports about other tasks, including one from Settlebeck, in the Recording & Publishing link of Mathematics Centre (HREF10).

Assessment for the unit doesn't depend exclusively on the project report. In Week 1 the teacher keeps incidental records from the debriefing conversations. In Week 2 there is plenty of opportunity to 'collect and mark' or to set homework. And the assessment isn't just about skills. We are learning to work like a mathematician so we must constantly be assessed (and reported) against that one page Working Mathematically Process.

The Replacement Unit model can be represented by this diagram. Open and choice driven in the first week and third weeks and more teacher directed in the second. (Remember there is an essential Week 0 of staff preparation.) Learning through this model is encouraged by many more features than just ownership. Pause for a minute to think back through the example above and list all the planned pedagogical features likely to encourage learning. Then perhaps take a further moment to check the Lesson Features list (HREF5) for some you might not have realised.
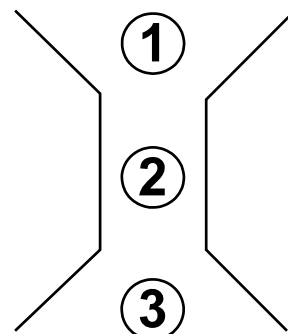
More Models and More Menus

The Replacement Unit is just one of several models teachers have developed to work like a mathematician in best practice classrooms. The Mixed Media Model is another which uses tasks, this time in a smaller menu at one work station. At another station the students are working on well chosen software related to the same topic and at a third on text-type exercises, again on the same topic. The week begins with a whole class investigation to introduce the content. Then the class is divided into three groups and one lesson in the next three periods is spent at each station. You can learn more about this model and several others at the Mathematics Centre Integrating Tasks link (HREF11).

More on Menus

Menu Maths takes many forms and Aaron Peeters has written about his and his primary children's experience with the concept in a crisp, insightful article titled Menu Maths: May I take your order? (HREF12). It begins:

> *Okay kids, what did you enjoy about our class this term?* I asked as a reflective activity on the last day of term 2.
> *I like this class because we do the Maths Menu.*
> *Me too, I like the Maths Menu.*
> It went around the circle and I was blown away by the positive responses from the students. Of course not everyone said 'Maths Menu'. There was the occasional 'I like the games,' or 'The students are friendly' but the mode was definitely 'Maths Menu.'

Please read Aaron's article to enrich your thinking about Menu Maths.

Special Menus

Some restaurants are renown for their special ingredients or culinary style. A clever variation on the menu maths concept corresponds to these. You choose investigations related to a content strand, but the extra attraction (especially in terms of teacher preparation) is that they can all be explored using the same concrete material.

You only need about five or six investigations for your menu and then you write out the essence of each on a 'proper' menu, perhaps scrawled on a board as a Pub Menu or perhaps on laminated card as in a classier venue. Each pair has a menu card (or reads the menu from the board) and the same set of equipment. They can choose to savour one menu item in depth or to treat the menu as a smörgåsbord (in either case, with their journal right beside them).

Having the same equipment automatically means that a class set of the material is in the room, so at any point the teacher can convert any of the menu items to a whole class investigation. Again, attention is being paid to invitation, modelling and skill practice in context.

It's easy to have a go at a Pub Menu. Go to the Task Library and identify the essence of *Truth Tiles 1*, *Crosses*, *Number Tiles*, *Fay's Nines*, *Magic Squares* and *Step*s. Ask the students to tear up pieces of paper to make number tiles while you scrawl your version of these on the board.

Of course a classier place, must look and feel the part and that's where Menu Maths Packs come in. They represent what is possible in Café Conundrum, a place of curious questions, tantalising tasks, remarkable riddles, perplexing puzzles and enticing enigmas.
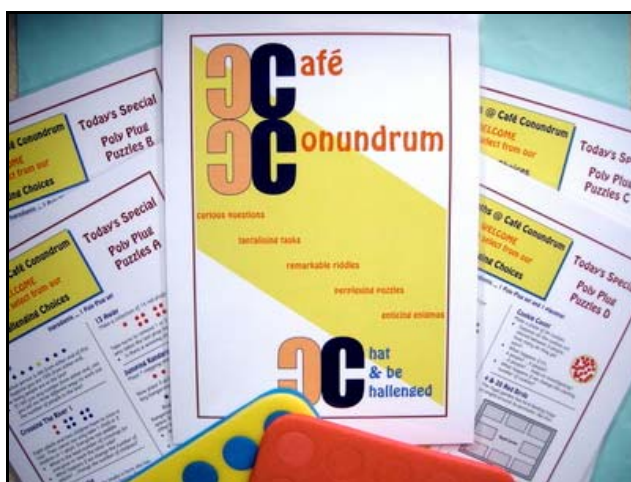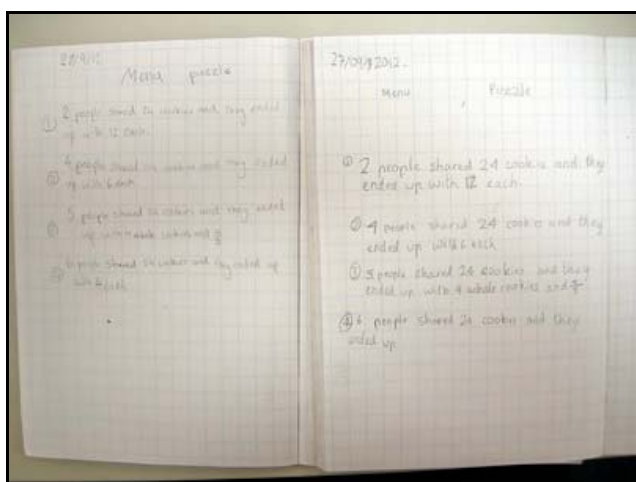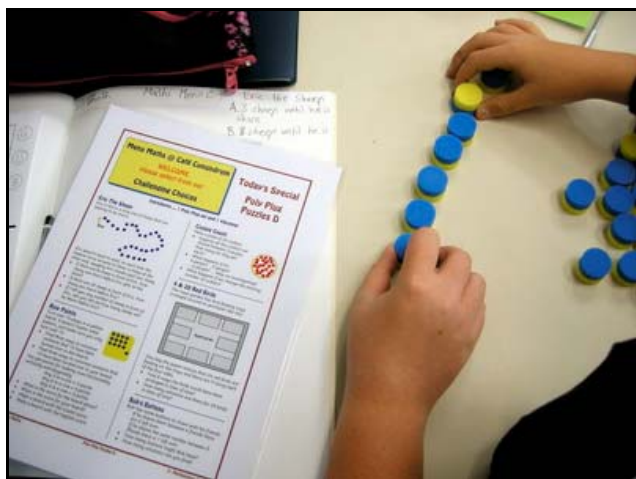
The special ingredient in this version is Poly Plug. Every investigation on the menu can be represented with this flexible, friendly material. The cards are printed in colour on both sides and securely laminated. They are just as you might expect at an à la carte venue.

At the ATM Easter Conference 2013, a room full of teachers had their first experience with Menu Maths Packs. They seemed to like the idea. It was a significant problem trying to get then to stop working and get off to morning tea. You can find out more about the Menu Maths concept and Menu Maths Packs at the Mathematics Centre Menu Maths link (HREF13). You will find plenty of support material. Bon appétit!

Acknowledgement

References

HREF1: http://www.mathematicscentre.com/taskcentre/work.htm, Working Mathematically page with examples from schools.
HREF2: http://www.mathematicscentre.com/taskcentre/workmath.pdf, Working Mathematically Process (text version).
HREF3: http://www.mathematicscentre.com/taskcentre/bemath_n.pdf, Working Mathematically Process (visual version as a flow chart - Louise Addison, St. Mary's College, New Zealand).
HREF4: http://www.mathematicscentre.com/taskcentre/stratool.pdf, Strategy Toolbox (visual version - Louise Addison, St. Mary's College, New Zealand).
HREF5: http://www.mathematicscentre.com/taskcentre/features.pdf, Lesson Features likely to encourage learning.
HREF6 - http://www.mathematicscentre.com/mathematicscentre/task_lib.htm, Task Library
HREF7 - http://www.mathematicscentre.com/mathematicscentre/content.htm, Task Cameo Content Finder
HREF8 - http://www.mathematicscentre.com/taskcentre/mwa.htm, Maths With Attitude
HREF9 - http://www.maths300.esa.edu.au, Maths300
HREF10 - http://www.mathematicscentre.com/taskcentre/record.htm, Recording & Publishing (examples of student work)
HREF11 - http://www.mathematicscentre.com/taskcentre/plans.htm, Integrating Tasks (Models & Structures)
HREF12 - http://www.mathematicscentre.com/taskcentre/menuap.htm, Menu Maths : May I take your order? (Aaron Peeters, Kingsbury Primary School, Australia)
HREF13 - http://www.mathematicscentre.com/mathematicscentre/menumaths.htm, Menu Maths

Garden Beds - Investigation Guide

Do the task first.   Use this Guide to find out more.   Prepare a report.

1. Generalising
a.   The garden bed has 50 plants in one row. Explain how to find the number of tiles.
b.   Explain in a different way if you can.
c.   If someone told you **any number** of plants in one row, explain how you would find the number of tiles.
d.   Write an equation that shows how **T** (number of Tiles) is found from **P** (number of plants).

2. Substituting

Plants	Tiles
19	
20	
35	
128	
319	
1000	

Copy & complete these tables

3. Solving - Working Backwards

Plants	Tiles
	28
	37
	65
	512
	75
	1000

Explain as much as you can about how to find the number of plants if someone tells you any number for the tiles.

4. Making Pairs
a.   Choose any five numbers up to 20 for the number of plants.
         For each number find the number of tiles and make five number pairs like this: (P, T)
b.   Choose any five numbers up to 50 that work for the number of tiles. For each tile number find the number of plants and make five more number pairs like this: (P, T)
c.   If you do the same calculation in each pair the answer is always 6.
         Explain the calculation.

5. Graphing Pairs
a.   Show your ten pairs from Question 4 on a graph and explain what you see.
b.   Make one more dot that you think belongs on your graph. Call it **My Dot**.
         Draw the Garden Bed picture that goes with My Dot.

6. Graphing in Excel
a.   Use Excel to record your ten pairs in a table.
b.   Select the table and use it to insert a chart.
         Use the XY (Scatter) graph with the first sub-type.
c.   Select the chart and choose Chart/Add Trendline. On the Options Tab select Display equation on chart. Explain how this equation links to the calculation in Question 4.

7. What happens if...?
Investigate the number of tiles to surround gardens shaped your way. You could have:
      • plants in many equal rows
      • plants in squares only
      • plants in L shapes
      or any other pattern that you want to investigate.