

Working

Mathematically

with

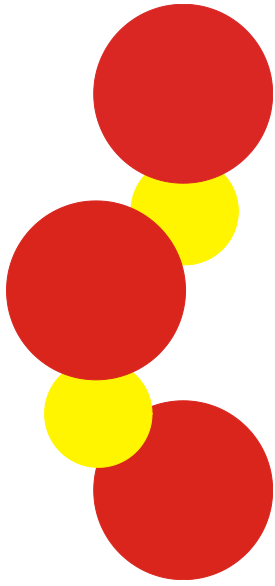
Infants



Enhancing  
Number Sense  
by  
Engineering  
'aha' Moments

Doug Williams



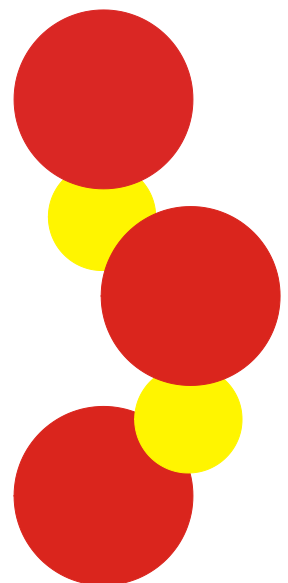


# Working Mathematically with Infants

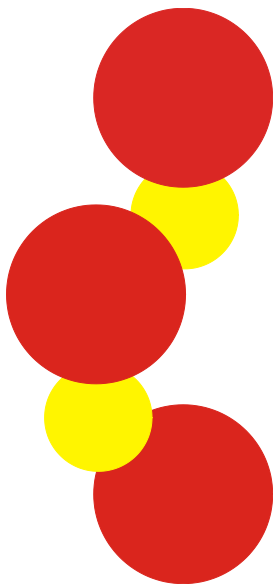
Years K, 1 & 2

Poly Plug  
Calculators  
Threaded activities  
Investigations  
Detailed curriculum planning

Doug Williams







Derived from Calculating Changes, enriched by the Mathematics Task Centre and Maths300 and integrated with Maths With Attitude. **Working Mathematically with Infants** supports teachers to construct curriculum around learning to work like a mathematician.

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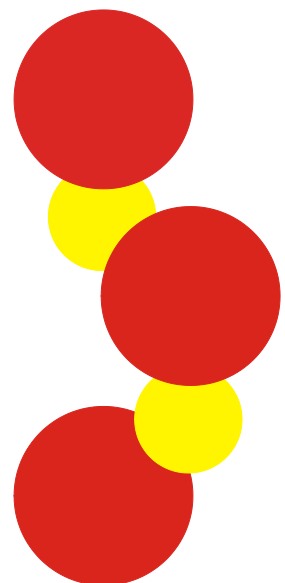
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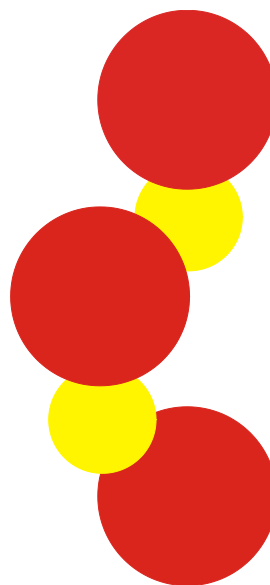
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# *Part 1: Preparing To Teach*

# Our Objectives

- ♦ To engineer the classroom so that 'aha' moments in number happen more often for more children.
- ♦ To develop children's number sense beyond what is normally expected for their age.



# Our Attitude...

- ♦ ...to learning:  
learning is a personal journey stimulated by achievable challenge
- ♦ ...to learners:  
stimulated children are creative and love to learn
- ♦ ...to pedagogy:  
the art of choosing teaching strategies to fascinate, captivate and absorb all children
- ♦ ...to mathematics:  
mathematics is concrete, visual and makes sense
- ♦ ...to learning mathematics:  
all children can learn to work like a mathematician
- ♦ ...to teachers:  
the teacher is the most important resource in education
- ♦ ...to professional development:  
teachers improve their teaching by re-enacting stories of success from the classrooms of their colleagues

***Working Mathematically with Infants*** is constructed from stories of success collected and re-enacted over years through the Calculating Changes network.

*It offers ten weeks per semester, in each semester from Year K to Year 2, of Threaded Activities and Investigations. To support the implementation of these learning experiences the kit includes a class set of Poly Plug and assumes children have free access to simple four function calculators.*

*Membership of Calculating Changes is a prerequisite for WMI as the full, and latest, information about any Threaded Activity is found on the web.*

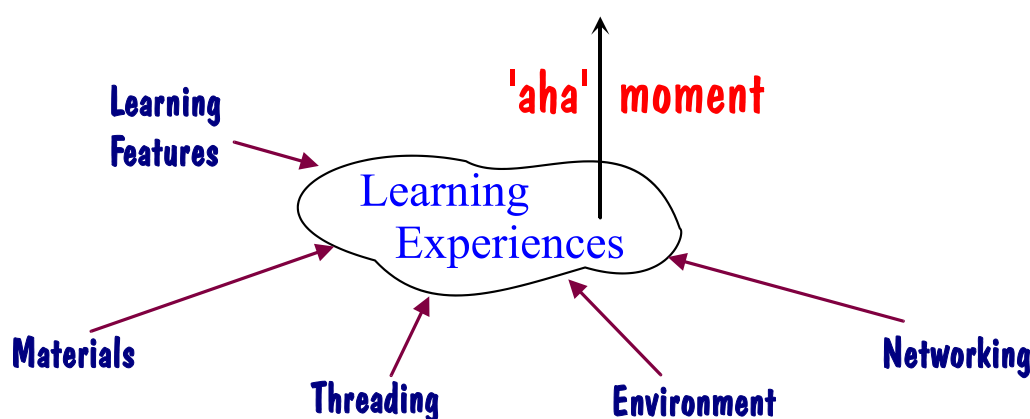
# Engineering 'aha' Moments in Number

The Calculating Changes network was established in 1996 to draw together earlier work on the use of calculators, concrete materials and constructivist theory in a web-based community of teachers.

Two major influences on the direction of the network were the:

- ◆ Calculator Aware Number project in Britain in the 1980s and early 90s
- ◆ growing recognition of the educational value of Poly Plug, a manipulative invented in 1993/4

Over time network teachers grew to see themselves as attempting to engineer best practice classroom environments to encourage children to create their own learning. Through sharing ideas and experiences across the network, teachers and children alike began experiencing more and more 'aha' moments.



Through classroom trial and reflection teachers begin to alter their vision of what children can do. Deeper and richer activities, and more open-ended teaching practices are introduced. The outcome is children's number sense is enhanced.

Drawing from these years of development *Working Mathematically with Infants* chooses to build an environment which...

- ◆ encourages open access to materials
- ◆ encourages open access to calculators
- ◆ creates time for children to construct their own learning
- ◆ builds on learner's efforts
- ◆ values learning in community
- ◆ encourages mathematical conversation
- ◆ celebrates 'aha' moments
- ◆ applauds mental strategies
- ◆ asks *Can I check this another way?*
- ◆ challenges current conceptions
- ◆ supports risk-taking

and use activities which...

- ◆ are colourful & tactile
- ◆ encourage learner ownership
- ◆ involve personal recording in a maths journal
- ◆ offer a partnership between concrete, symbolic and personal recording

- ◆ encourage mathematical conversation
- ◆ are non-threatening
- ◆ uplift the learner (they feel better about themselves)
- ◆ place number sense in problem solving situations
- ◆ offer opportunity to revisit and be challenged anew and therefore can be threaded

Problem solving situations are included in *Working Mathematically with Infants* as whole class Investigations which model learning to work like a mathematician.

## Threading

- ◆ Rich tasks
- ◆ Familiar structure
- ◆ Fresh challenge
- ◆ Short, frequent visits

**Threading** is a teaching technique requiring rich activities used 2 or 3 times a week for a few minutes each time over several weeks. The structure of the activity remains constant (and therefore familiar), but the challenge within it is fresh each time.

The activity appears as a thread in the fabric of the curriculum, as in the Planners (pp. 16-21).

**Threaded Activities** give children time to construct their own learning, eases preparation because teachers do not need to constantly look for 'something new in maths' and offers opportunity to gather assessment information as you share time with groups.

*It was like a journey that we all approached together because we were all students; it became extremely enjoyable and a very valuable learning experience. The children were keen to do these activities and even the repetition made them feel more secure and none of them complained.*

Kate Thureau, Poly Plug, Proportion & Percent, member activity for Year 3<sup>+</sup>

## Teacher Comments

The following testimonies were offered by infant teachers in their evaluation of Calculating Changes professional development courses. These teachers attended between two and five linked courses with classroom trialing sandwiched between. Comments are taken from the Evaluations section of the Calculating Changes web site.

### Teacher A ... Prep

- ◆ *Hands on fun, children confident in working through activities, chance to use mathematical language. Easily linked with other subjects - takes fear out of maths.*
- ◆ *Poly Plug is a great resource for use in most areas of maths. Can be used in both language and maths. Used almost daily.*
- ◆ *Maths is now fun - I've been able to extend and adapt activities and ideas to suit children's levels and stages of maths.*

**Teacher B ... Prep**

- ◆ *Makes maths more exciting and enjoyable.*
- ◆ *Makes children more questioning of the whys and hows of what they're doing.*
- ◆ *Children being able to explain the processes involved in solving problems*
- ◆ *A more hands on approach to maths.*
- ◆ *Poly Plugs - a fun aid for calculation.*
- ◆ *Leading children to more 'aha' moments.*

**Teacher C ... Grade 1/2**

- ◆ *Wonderful, practical ideas. Very hands on. Helps children to fully understand maths not just calculating without understanding.*
- ◆ *Poly plugs - activities are endless, really supports children.*
- ◆ *Children are often heard to say I'll just get a calculator for that.*
- ◆ *Builds my pedagogical skills, thus the children are more likely to learn!*

**Teacher D ... Grade 2/3**

- ◆ *Ideas and activities to connect kids to be interested in maths and think/articulate for themselves. Involve parents wherever possible. Promote correct understandings.*
- ◆ *Poly Plug is a quiet, attractive concrete aid to enhance understanding. Great for games.*
- ◆ *Wonderful to see how children enjoy their learning.*

## Threading Works

This report is extracted from *Reflections on Engineering 'aha' Moments in Number* by Nicholas Dale which is stored on the Maths Task Centre site. Nick was working as a Year 3/4 teacher at the time and attended this course with a representative from each level of the school. His report describes changes in his classroom and across the school.

Part of my current leadership role is to deliver professional learning to my colleagues. I have presented a number of sessions from this initiative and all teachers have taken it on. The junior primary teacher has found this to be one of the best teaching tools and uses aspects of it in her programming everyday. I have modelled lessons to staff, organised staff to visit other teachers at other sites to extend learning within their year levels, and had them share their experiences at hub meetings within the district.

I have developed a school wide mathematics proforma that incorporates Threading activities from Calculating Changes. This ensures that teachers continue to plan these rich mathematical programs each day.



Students have been more focused, engaged and interested in mathematics. They now enjoy doing maths everyday and look forward to the tasks I present to them. (Our curriculum...) has moved away from the textbook oriented lessons and into creativity in maths. Lessons in which I used to struggle to engage students for 30 minutes, now engage them for sometimes up to two hours. Therefore, this idea we have been presented with WORKS!

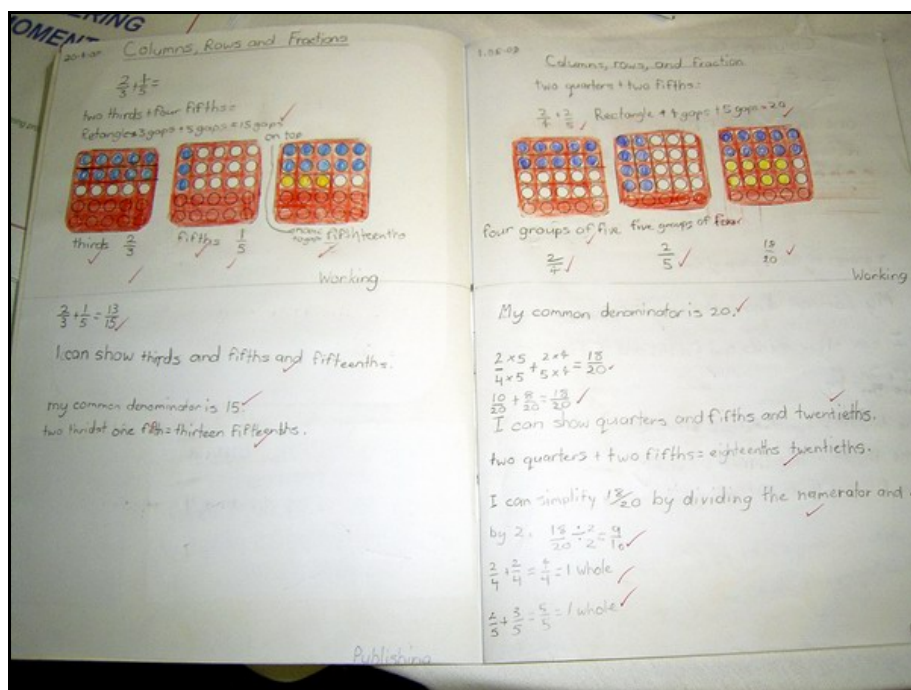
The Threading scrapbooks have been an excellent way of recording the students' work. It is a way for me to check levels of understanding, and for students to refer back when we are threading an activity for a number of weeks. Rather than asking what to do next they have a valuable tool for checking themselves. It has created students who have become more independent and take on more responsibility for their own

learning. Also I have been able to use their books as a way of presenting ideas to other teachers.

I believe recording is an important component not only for assessing and education department requirements, but for the students as well.

During this year I have attended a number of mathematical professional development sessions. *Engineering 'aha' Moments in Number* (see Professional Development Purpose p. 12) has been the most informative and practical and has shown an improvement in learning outcomes by far. I was at a point where I was not enjoying teaching maths and the students were not enjoying learning about it as well. I was looking at developing different approaches to delivering a more interesting maths program. During this program I have grown in myself and introduced a new methodology that has been successful.

When reporting to parents I talk about the student's learning in using this program and how these skills transfer. I use the Threading Book as evidence of what skills I am covering within my units and this is a good evidence based document.





# Resources

Your kit includes Calculating Changes site membership for the whole school, 25 Poly Plug and this manual. Threaded Activities for infants are outlined in the manual, and described in full on site. Companion investigations are explained in more detail in the manual and in most cases are reference to further information at Mathematics Centre or Maths300.

Planners in the manual (see Pages 16-21) detail 10 weeks of integrated number work for *each semester in each year* from K - 2. Threaded Activities and Investigations work together throughout the program to enhance number sense within the umbrella context of all children learning to work like a mathematician.

To achieve this objective we also expect you will have simple four function hand-held calculators in your school. In addition some Investigations need other materials generally found in schools (p. 15).

It is *not necessary* for your campus to belong to Maths300 to make full use of ***Working Mathematically with Infants***, but those who do will find additional support, especially in the form of software to extend some Investigations.

Further, if your school prepares a set of eTasks from Mathematics Centre, or has a set of ready-made tasks, you can continue building a Working Mathematically curriculum for Year 3<sup>+</sup> with Maths With Attitude manuals (see p. 90). ***Working Mathematically with Infants*** is designed to integrate conceptual development, content knowledge, teaching craft and documentation with Maths With Attitude. Schools that choose this suite of resources are able to build curriculum around a unified approach throughout primary school. This approach, summarised by our attitudes on Page 2, is consistent and developmental in language, teaching craft and mathematical content. All content is reflected in the requirements of any official curriculum documents of any system.

## Poly Plug

Threaded Activities assume open access to Poly Plug because nothing else offers so much in the one resource. Besides, children simply love using them. Further, where appropriate, Poly Plug is used in the Investigations, so you gain maximum benefit from the resource.

Poly Plug...

- ◆ is colourful, tactile, motivating
- ◆ is classroom savvy - noiseless, easy to pack up
- ◆ is useful as both a structured and an unstructured aid
- ◆ has applications across a broad range of mathematical content - pattern, counting, fractions, graphing, symmetry, problem solving
- ◆ has even more applications when used in conjunction with other simple, readily obtainable material, such as dice

## Calculators

Calculating Changes assumes children have free access to calculators from the day they enter school. This assumption is based on research from the Calculator Aware Number (CAN) project in England, and its derivatives elsewhere.

Given this assumption we want children to use calculators that give correct answers, and the majority of four function calculators in schools don't!

To test your calculator enter this equation in the order shown:  $2 + 3 \times 5 =$

The correct answer is 17. If your calculator gives the answer 25 it has not been programmed to interpret the order of arithmetic operations. That is, it does not have an Algebraic Operating System (AOS). If you are only using matched pairs of operations  $+/-$  or  $\times/\div$  there is no problem. However, if you are allowing free access to the machines, as assumed in Calculating Changes, children will press combinations of operations at random. Would you prefer that the calculator then gives the correct answer?

If the activity involves the possibility of mixing the operations of  $+$  or  $-$  with  $\times$  or  $\div$ , most simple four function machines will give the wrong answer to most questions. So, use your school calculators if you wish but beware of this limitation.

For further information visit the Materials link at Calculating Changes which lists the brands we currently know that are simple and operate algebraically. We are not suggesting you rush out and purchase these. We just want you to be aware of the limitations of what you may have and be aware of possibilities.

[http://mathematicscentre.com/calchange/cchpp\\_mm.htm#calculators](http://mathematicscentre.com/calchange/cchpp_mm.htm#calculators)

## Calculator Aware Number Project

The report of the Calculator Aware Number (C.A.N) project was published in 1991 under the title *Calculators, Children and Mathematics* by Simon & Schuster on behalf of the National Curriculum Council, England. Calculating Changes has been granted permission to use any part of the CAN Report (which is now out of print) to achieve its aims. The following is taken from Page 12 of the report.

From the beginning of the project, children were allowed free access to calculators alongside other apparatus. A report from one LEA (Local Education Authority), after three years work, describes the significance of this:

A significant finding from the project is that calculators should be viewed as an item of multi-purpose mathematical apparatus and that teachers do not need to design specific tasks to bring them into play. Pupils select the calculator as they feel it is appropriate to the task in hand just as they would select other material - Multilink, counters, modelling apparatus etc. to solve the problem they are dealing with. [Durham County Council 1989]

However, the frequent use of calculators has made the mathematical experience of project children different from the experience of other children. Many children have developed mathematical concepts and methods which have not in the past been expected at their age. For young children, the calculator is a toy, but it is the first toy for young children that incorporates the number system. As children play with their calculators, they find out a great deal about how numbers behave.

Ready access to calculators has also given the children great confidence. They are never faced with calculations that they cannot do.

Their problem now is to decide the appropriate calculation to do in order to solve a problem, and to interpret the results of that calculation in the problem situation. Exploration and investigation have taken over from the repetitive practice of calculation as the usual style of number work.

...

Most children in the project have also decided for themselves that they do not need, or want, to be dependent on their calculators for all calculation. It is often faster and easier to do a calculation mentally, and children sometimes vie with one another to extend their skill in mental calculation. The project has seen a great flowering of mental calculation, often led by the children.

Funded by the British national government, C.A.N began in 1985 with about 20 classes of children aged six in 15 schools. Over the years the number of children involved grew from the hundreds into the thousands as more schools joined the project. Significantly, all the originating schools and Local Education Authorities stayed with the project throughout its lifetime. The project was led by Hilary Shuard, Homerton College, Cambridge University.

The C.A.N. report is liberally sprinkled with quotes from teachers, head teachers and others who evaluated the project. For example, on Page 9 one Head Teacher comments:

*The spreading of the CAN philosophy in school has been rapidly accelerated by the movement of an experienced teacher who was involved in the first year of CAN with a class of six and seven-year-olds, to a reception class of four and five-year-olds. This teacher has used the methods, and many of the activities and ideas that she developed with her six and seven-year-olds, with great success. This approach has spread sideways, as the two other reception teachers have seen how the children have developed mathematically.*

Use your Calculating Changes membership to find more information from CAN in both the Free Tour and Members sections.

# Working Like A Mathematician

Our attitude is:

all children can learn to work like a mathematician

What does a mathematician do and why do they do it?

The Mathematics Task Centre Project has probed this question with several professional mathematicians in an attempt to identify the process for classroom use K through 12. The result is the description on the next page. In particular we are indebted to the clarity provided by Dr. Derek Holton on this matter.

Perhaps the most important aspect of Working Mathematically is the recognition that *knowledge is created by a community and becomes part of the fabric of that community.*

Recognising, and engaging in, the process by which that knowledge is generated can help children to see themselves as able to work like a mathematician.

Hence learning to work like a mathematician is the scaffolding of **Working Mathematically with Infants**. The Working Mathematically Process is described on the next page in a language which can be used with children from the day they enter school.

It is probably worth noting that one Year 1 child asked what mathematicians do replied, with her face full of wonder:

*They solve the world's hardest problems.*

By comparison, many children in later years would reply:

*Sums*

## Skills, Strategies & Working Mathematically

The Working Mathematically description places learning mathematical skills and problem solving strategies in their true context.

Lessons on skills or lessons on strategies are not an end in themselves. They provide the toolboxes that mathematicians carry in their struggle to solve problems.

- ◆ **Our skill toolbox** can be added to in the same way as the mechanic or carpenter adds tools to their toolbox. But equally, the addition of the tools is not for the sake of collecting them, but rather for the purpose of getting on with a job. A mathematician's job is to solve problems, not to collect tools that might one day help solve a problem.
- ◆ **Our strategy toolbox** has been provided by the joint wisdom of the mathematicians of the past. All mathematical problems (and indeed life problems) that have ever been solved have been solved by the application of this limited set of strategies.

# Working Mathematically

First give me an interesting problem.

**When mathematicians become interested in a problem they:**

- ◆ Play with the problem to collect & organise data about it.
- ◆ Discuss & record notes and diagrams.
- ◆ Seek & see patterns or connections in the organised data.
- ◆ Make & test hypotheses based on the patterns or connections.
- ◆ Look in their strategy toolbox for problem solving strategies which could help.
- ◆ Look in their skill toolbox for mathematical skills which could help.
- ◆ Check their answer and think about what else they can learn from it.
- ◆ Publish their results.

**Questions which help mathematicians learn more are:**

- ◆ Can I check this another way?
- ◆ What happens if ...?
- ◆ How many solutions are there?
- ◆ How will I know when I have found them all?

**When mathematicians have a problem they:**

- ◆ Read & understand the problem.
- ◆ Plan a strategy to start the problem.
- ◆ Carry out their plan.
- ◆ Check the result.

**A mathematician's strategy toolbox includes:**

- ◆ Do I know a similar problem?
- ◆ Guess, check and improve
- ◆ Try a simpler problem
- ◆ Write an equation
- ◆ Make a list or table
- ◆ Work backwards
- ◆ Act it out
- ◆ Draw a picture or graph
- ◆ Make a model
- ◆ Look for a pattern
- ◆ Try all possibilities
- ◆ Seek an exception
- ◆ Break a problem into smaller parts
- ◆ ...

If one way doesn't work, I just start again another way.



# Professional Development Purpose

Our attitude is:

the teacher is the most important resource in education

I now perceive the project as:

*... very worthwhile. Takes patience with Preps, but allows the children to leap ahead at their own pace.*

The response of my children has been:

*... enthusiastic. They use them (Poly Plug) whenever they want to help them with all sorts of tasks.*

The continuing steps for myself are:

*... trying new things; refining them; writing them up; sharing them.*

The continuing steps for the school are:

*... recording what we do; sharing what we do; keeping new staff informed; keeping the momentum going.*

This teacher's evaluation is one of many from years of Calculating Changes workshops. Their greatest success comes when they are held as linked experiences over time with classroom trials sandwiched between. The six day professional development program *Engineering 'aha' Moments in Number K-8* has proven particularly successful for primary schools. Nick Dale's report on Page 5 is an example of the shifts in personal teaching practice, children's learning and school wide curriculum development that result from this course. One day courses and after school sessions are also available.

Research suggests that the professional development most likely to succeed:

- ◆ is requested by the teachers
- ◆ takes place as close to the teacher's own working environment as possible
- ◆ takes place over an extended period of time
- ◆ provides opportunities for reflection and feedback
- ◆ enables participants to feel a substantial degree of ownership
- ◆ involves conscious commitment by the teacher
- ◆ involves groups of teachers rather than individuals from a school
- ◆ increases the participant's mathematical knowledge in some way
- ◆ uses the services of a consultant and/or critical friend

***Working Mathematically with Infants*** has been designed with these principles in mind. All the materials have been tried, tested and modified by teachers from a wide range of classrooms. If we can't be present to lead your professional development sessions, then we hope ***WMI*** will enable teacher groups to lead themselves further along the professional development road, and will support systems to improve learning outcomes for children K-2.

For more professional development information see Page 86 and, for external assistance with professional development, contact:

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# *Part 2: Planning Curriculum*

# Curriculum Planners

Our attitude is:

learning is a personal journey stimulated by achievable challenge

Curriculum Planners on the following pages integrate the use of **Threaded Activities** and *Investigations* into 10 weeks of concrete, visual, open-ended, challenging number work per semester, for each semester in each year from K to 2.

- ◆ Typically Threaded Activities are used for a few minutes each day, several days per week over several weeks.
- ◆ Typically Investigations are used for a block of time - often much more time than is usually set aside for a maths period.

Planners provide a starting point for those new to these materials and a flexible structure for those more experienced. You will need to map the Planner weeks into your local curriculum document. Different teachers and schools will timetable the same planners in different ways, but all will be making use of the same extensively trialed materials, activities, investigations and pedagogy, which for others has produced:

happy, healthy, cheerful, productive, inspiring classrooms

*Threaded Activities are shown in bold.*

- ◆ The first time one is used it sometimes takes a substantial part of a session to introduce. Therefore they are shown as the only activity during this introduction session. You will also need other 'standby' activities for these sessions.
- ◆ Introduction sessions are highlighted with a background colour.
- ◆ Identifying the first use of a Threaded Activity in this way also serves to identify the introduction of new content in the development of the planning sequence.
- ◆ Threaded Activities can be used with the whole class as starter activities; and can be used as small group activities (including with your teaching group); and can be combined with any other best practice you already use. Their critical property is that, once learnt, they need to be used for 10-15 minutes per day, several days per week over several weeks. Planners have been structured in this way.

*Investigations are shown in italic.*

- ◆ Investigations frequently captivate children and need to be continued beyond a single time slot. Therefore they are shown in the first session of a week in the expectation that they will need to run over more sessions.
- ◆ Sometimes teachers continue an Investigation into the following time slots of the same day. Sometimes continuation is built in as group work in mathematics slots on following days.
- ◆ Investigations will teach, review and develop content but their main purpose is to absorb children in learning to work like a mathematician (see p. 11).



## Planning Points

- ◆ Planners assume four lessons per week of about 1 hour each.
- ◆ Planners *do not* prescribe a continuous block of work. Use them flexibly.
- ◆ Planners offer *only* 10 weeks per semester and those weeks are almost exclusively in number. They do not represent a full year's curriculum.
- ◆ Map these weeks into your program then complete your year as appropriate.
- ◆ There *is* a developmental flow when the planners are viewed as a whole from Year K to Year 2, however, there is plenty of room for flexibility within and across Year levels. Your children will guide you.

### Using The Planners

- ◆ Choose a week. It is most likely to include both Threaded Activities and an Investigation.
- ◆ Find out more about a Threaded Activity by:
  - (a) reading its introduction in the Threaded Activities section from Page 22.
  - (b) exploring its depth on site.
- ◆ Find out more about an Investigation by reading its description in the Investigations section from Page 37.
- ◆ Descriptions are all the detail you need to explore the problem. However, most Investigations can be enriched by reading the additional detail stored in the web links listed for each investigation. Task Cameo links are public access. Maths300 links require membership of that site. These are the only additional links used.
- ◆ Map the week into your syllabus.

### Questions

- ◆ Can I use Planner activities in weeks other than those ten?  
*Absolutely. Planners offer a starting point. There will be many times when you or your children will want to continue using what the Planners offer.*
- ◆ Can I use in my current year an activity or investigation listed for another year?  
*Certainly, provided you break from the planning sequence as part of a team planning approach from Years K - 2.*

Expect to be surprised by what your children can do!

## Additional Resources

For the most part Investigations use Poly Plug or calculators or other materials normally available in classrooms. However you will also need:

Dominoes - Year 1 - *Domino Trails*

Cuisenaire Rods - Year 2 - *Rod Mats*

Further, many Investigations require printing and preparing game boards or other resources. Plan well ahead to allow enough time for this preparation.

In many Investigations, you will notice reference to the children's mathematics journal. In the early years this might be made for them page by page as they record their understanding. By Year 2 however, a self-maintained journal is an expectation. As well as consolidating learning, journals offer significant assessment information.

# Curriculum Planner

## Year K ... Semester 1

	Session 1	Session 2	Session 3	Session 4
<b>Week 1</b>	<b>Free Play</b>	<b>Activities from FP</b>	<b>Flash &amp; Show</b>	<b>Flash &amp; Show Activities from FP</b>
<b>Week 2</b>	<i>Where Do We Sit?</i>	<b>Roll, Make &amp; Write (R, M &amp; W)</b>	<b>Roll, Make &amp; Write Flash &amp; Show Activities from FP</b>	<b>Roll, Make &amp; Write Flash &amp; Show Activities from FP</b>
<b>Week 3</b>	<b>Making Number Lines</b>	<b>Making No. Lines Roll, Make &amp; Write Flash &amp; Show</b>	<b>Taking The Train</b>	<b>Taking The Train Making No. Lines Roll, Make &amp; Write Flash &amp; Show</b>
<b>Week 4</b>	<i>Maths In Motion</i>	<b>Ten Friends</b>	<b>Ten Friends Taking The Train Making No. Lines</b>	<b>Ten Friends Taking The Train Making No. Lines</b>
<b>Week 5</b>	<b>Counting On</b>	<b>Counting On Ten Friends Taking The Train Making No. Lines</b>	<b>Counting On Ten Friends Taking The Train Making No. Lines</b>	<b>Counting On Ten Friends Taking The Train Making No. Lines</b>
<b>Week 6</b>	<i>Calculator Walk</i>	<b>R, M &amp; W (2 dice) Counting On Ten Friends</b>	<b>R, M &amp; W (2 dice) Counting On Ten Friends</b>	<b>R, M &amp; W (2 dice) Counting On Ten Friends</b>
<b>Week 7</b>	<b>Working With Books</b>	<b>Working With Books Counting On Ten Friends</b>	<b>Working With Books Counting On Ten Friends</b>	<b>Working With Books Counting On Ten Friends</b>
<b>Week 8</b>	<i>Playing With Patterns</i>	<b>Plug Lines</b>	<b>Plug Lines Working With Books Counting On</b>	<b>Plug Lines Working With Books Counting On</b>
<b>Week 9</b>	<b>Buttons</b>	<b>Buttons Plug Lines Working With Books</b>	<b>Buttons Plug Lines Working With Books</b>	<b>Buttons Plug Lines Working With Books</b>
<b>Week 10</b>	<i>Measuring Up</i>	<b>Number of the Day</b>	<b>Number of the Day Buttons Plug Lines</b>	<b>Number of the Day Buttons Plug Lines</b>

- ◆ Map these weeks into your curriculum:  
For example, will you map *Planner* Week 1 to *Your* Term 1 Week 2 or ...?
- ◆ **Threaded Activities** are in **bold**. Details from Page 22 linking to more on site.
- ◆ *Investigations* are in *italic*. Details from Page 37 linking to more on the web.

# Curriculum Planner

## Year K ... Semester 2

	Session 1	Session 2	Session 3	Session 4
<b>Week 1</b>	<i>Cars In A Garage</i>	<b>Secret Spaces</b>	<b>Secret Spaces</b> <b>Number of the Day</b> <b>Favourites - Sem.1</b>	<b>Secret Spaces</b> <b>Number of the Day</b> <b>Favourites - Sem.1</b>
<b>Week 2</b>	<b>Move Around</b>	<b>Move Around</b> <b>Secret Spaces</b> <b>Number of the Day</b>	<b>Flash &amp; Show (10s)</b>	<b>Flash &amp; Show (10s)</b> <b>Move Around</b> <b>Secret Spaces</b>
<b>Week 3</b>	<i>The Frog Pond</i>	<b>Flash &amp; Show (10s)</b> <b>Move Around</b> <b>Secret Spaces</b>	<b>Flash &amp; Show (10s)</b> <b>Move Around</b> <b>Secret Spaces</b>	<b>Flash &amp; Show (10s)</b> <b>Move Around</b> <b>Secret Spaces</b>
<b>Week 4</b>	<b>Six Plus</b>	<b>Six Plus</b> <b>Flash &amp; Show (10s)</b> <b>Move Around</b>	<b>Six Plus</b> <b>Flash &amp; Show (10s)</b> <b>Move Around</b>	<b>Six Plus</b> <b>Flash &amp; Show (10s)</b> <b>Move Around</b>
<b>Week 5</b>	<i>Body Counting</i>	<b>Bridging 10</b>	<b>Bridging 10</b> <b>Six Plus</b> <b>Flash &amp; Show (10s)</b>	<b>Bridging 10</b> <b>Six Plus</b> <b>Flash &amp; Show (10s)</b>
<b>Week 6</b>	<b>Plug Catcher</b>	<b>Plug Catcher</b> <b>Bridging 10</b> <b>Six Plus</b>	<b>Plug Catcher</b> <b>Bridging 10</b> <b>Six Plus</b>	<b>Plug Catcher</b> <b>Bridging 10</b> <b>Six Plus</b>
<b>Week 7</b>	<i>Crazy Animals</i>	<b>Two Dollar Shop</b>	<b>Two Dollar Shop</b> <b>Plug Catcher</b> <b>Bridging 10</b>	<b>Two Dollar Shop</b> <b>Plug Catcher</b> <b>Bridging 10</b>
<b>Week 8</b>	<b>Two Dollar Shop</b> <b>Plug Catcher</b> <b>Bridging 10</b>	<b>Two Dollar Shop</b> <b>Plug Catcher</b> <b>Bridging 10</b>	<b>Add Town</b>	<b>Add Town</b> <b>Two Dollar Shop</b> <b>Plug Catcher</b>
<b>Week 9</b>	<i>Party Time</i>	<b>Add Town</b> <b>Two Dollar Shop</b> <b>Plug Catcher</b>	<b>Add Town</b> <b>Two Dollar Shop</b> <b>Plug Catcher</b>	<b>Add Town</b> <b>Two Dollar Shop</b> <b>Plug Catcher</b>
<b>Week 10</b>	Review and revisit through this week as appropriate.			

- ◆ Map these weeks into your curriculum:  
For example, will you map *Planner* Week 1 to *Your* Term 1 Week 2 or ...?
- ◆ **Threaded Activities** are in **bold**. Details from Page 22 linking to more on site.
- ◆ *Investigations* are in *italic*. Details from Page 37 linking to more on the web.

# Curriculum Planner

## Year 1 ... Semester 1

	Session 1	Session 2	Session 3	Session 4
<b>Week 1</b>	<b>Free Play</b>	Activities from FP and/or activities from Year K, such as...	Move Around Bridging 10 Secret Spaces	Number of the Day Counting On Buttons
<b>Week 2</b>	<i>Around Our Neighbourhood</i>	<b>Rory's Pattern Game</b>	<b>Rory's Pattern Game</b> Move Around Bridging 10 Secret Spaces	<b>Rory's Pattern Game</b> Number of the Day Counting On Buttons
<b>Week 3</b>	<b>Cross Off</b>	<b>Cross Off</b> <b>Rory's Pattern Game</b> Move Around	<b>Cross Off</b> Bridging 10 Secret Spaces	<b>Cross Off</b> <b>Rory's Pattern Game</b> Number of the Day
<b>Week 4</b>	<i>Jumping Joey</i>	<b>Cross Off</b> Counting on Buttons	<b>Plug Catcher</b>	<b>Plug Catcher</b> <b>Cross Off</b> <b>Rory's Pattern Game</b>
<b>Week 5</b>	<b>Add Town</b>	<b>Add Town</b> <b>Plug Catcher</b> <b>Cross Off</b>	<b>Add Town</b> <b>Plug Catcher</b> <b>Cross Off</b>	<b>Add Town</b> <b>Plug Catcher</b> <b>Cross Off</b>
<b>Week 6</b>	<i>Lining Up</i>	<b>Ten Tens</b>	<b>Ten Tens</b> Add Town Plug Catcher	<b>Ten Tens</b> Add Town Plug Catcher
<b>Week 7</b>	<b>Number Shapes</b>	<b>Number Shapes</b> <b>Ten Tens</b> Add Town	<b>Number Shapes</b> <b>Ten Tens</b> Add Town	<b>Number Shapes</b> <b>Ten Tens</b> Add Town
<b>Week 8</b>	<i>Back To Back Building</i>	<b>Broken Calc Probs</b>	<b>Broken Calc Probs</b> Number Shapes Ten Tens	<b>Broken Calc Probs</b> Number Shapes Ten Tens
<b>Week 9</b>	<i>Visual = Number</i>		<b>Broken Calc Probs</b> Number Shapes Ten Tens	<b>Broken Calc Probs</b> Number Shapes Ten Tens
<b>Week 10</b>	<i>Domino Trails</i>		<b>Broken Calc Probs</b> Number Shapes Best of semester	<b>Broken Calc Probs</b> Number Shapes Best of semester

- ◆ Map these weeks into your curriculum:  
For example, will you map *Planner* Week 1 to *Your* Term 1 Week 2 or ...?
- ◆ **Threaded Activities** are in **bold**. Details from Page 22 linking to more on site.
- ◆ *Investigations* are in *italic*. Details from Page 37 linking to more on the web.

# Curriculum Planner

## Year 1 ... Semester 2

	Session 1	Session 2	Session 3	Session 4
<b>Week 1</b>	<i>Trial, Record &amp; Improve</i> <i>This investigation can be threaded.</i>	<b>Two Dollar Shop</b>	<b>Two Dollar Shop</b> <i>Trial, Rec. Improve</i> <b>Six Plus</b> <b>Other Sem 1 activities</b>	<b>Two Dollar Shop</b> <i>Trial, Rec. Improve</i> <b>Six Plus</b> <b>Other Sem 1 activities</b>
<b>Week 2</b>	<i>Reflections</i>	<b>Take A Trip</b>	<b>Take A Trip</b> <b>Two Dollar Shop</b> <i>Trial, Rec. Improve</i> <b>Six Plus</b>	<b>Take A Trip</b> <b>Two Dollar Shop</b> <i>Trial, Rec. Improve</i> <b>Six Plus</b>
<b>Week 3</b>	<i>Dice Differences</i>	<b>Plug Snakes</b>	<b>Plug Snakes</b> <b>Take A Trip</b> <b>Two Dollar Shop</b>	<b>Plug Snakes</b> <b>Take A Trip</b> <b>Two Dollar Shop</b>
<b>Week 4</b>	<i>Highest Number</i> <i>This investigation can be threaded.</i>	<i>Highest Number</i> <b>Plug Snakes</b> <b>Take A Trip</b>	<i>Highest Number</i> <b>Plug Snakes</b> <b>Take A Trip</b>	<i>Highest Number</i> <b>Plug Snakes</b> <b>Take A Trip</b>
<b>Week 5</b>	<i>Halving Squares</i>	<b>Number Slider</b>	<b>Number Slider</b> <i>Highest Number</i> <b>Plug Snakes</b>	<b>Number Slider</b> <i>Highest Number</i> <b>Plug Snakes</b>
<b>Week 6</b>	<i>Cat &amp; Mouse</i>	<b>Rows &amp; Straws</b>	<b>Rows &amp; Straws</b> <b>Number Slider</b> <i>Highest Number</i>	<b>Rows &amp; Straws</b> <b>Number Slider</b> <i>Highest Number</i>
<b>Week 7</b>	<i>Farmyard Friends</i>	<b>Counting Machines</b>	<b>Counting Machines</b> <b>Rows &amp; Straws</b> <b>Number Slider</b>	<b>Counting Machines</b> <b>Rows &amp; Straws</b> <b>Number Slider</b>
<b>Week 8</b>	<i>Eric The Sheep</i>	<b>Counting Machines</b> <b>Rows &amp; Straws</b> <b>Number Slider</b>	<b>Counting Machines</b> <b>Rows &amp; Straws</b> <b>Number Slider</b>	<b>Counting Machines</b> <b>Rows &amp; Straws</b> <b>Number Slider</b>
<b>Week 9</b>	<i>Crossing The River</i>	<b>Smarties</b>	<b>Smarties</b> <b>Counting Machines</b> <b>Rows &amp; Straws</b>	<b>Smarties</b> <b>Counting Machines</b> <b>Rows &amp; Straws</b>
<b>Week 10</b>	<i>12 Counters</i>	<b>Take A Score</b>	<b>Take A Score</b> <b>Smarties</b> <b>Counting Machines</b>	<b>Take A Score</b> <b>Smarties</b> <b>Counting Machines</b>

- ◆ Map these weeks into your curriculum:  
For example, will you map *Planner* Week 1 to *Your* Term 1 Week 2 or ...?
- ◆ **Threaded Activities** are in **bold**. Details from Page 22 linking to more on site.
- ◆ *Investigations* are in *italic*. Details from Page 37 linking to more on the web.

# Curriculum Planner

## Year 2 ... Semester 1

	Session 1	Session 2	Session 3	Session 4
<b>Week 1</b>	<b>Free Play Calculator Art</b>	<b>Predict A Count</b>	<i>Truth Tiles 2</i>	<b>Predict A Count Activities from FP</b>
<b>Week 2</b>	<i>13 Away</i>	<b>Win A Flat</b>	<b>Win A Flat Predict A Count Activities from FP</b>	<b>Win A Flat Predict A Count Activities from FP</b>
<b>Week 3</b>	<i>Cookie Count</i>	<b>Please May I Have</b>	<b>Please May I Have Win A Flat Predict A Count</b>	<b>Please May I Have Win A Flat Predict A Count</b>
<b>Week 4</b>	<i>Have A Hexagon</i>	<b>Please May I Have Win A Flat Predict A Count</b>	<b>Please May I Have Win A Flat Predict A Count</b>	<b>Please May I Have Win A Flat Predict A Count</b>
<b>Week 5</b>	<i>Police Line Up</i>	<b>Six Times</b>	<b>Six Times Please May I Have Win A Flat</b>	<b>Six Times Please May I Have Win A Flat</b>
<b>Week 6</b>	<i>Row Points</i>	<b>Calculator Go Froms</b>	<b>Calculator Go Froms Six Times Please May I Have Win A Flat</b>	<b>Calculator Go Froms Six Times Please May I Have Win A Flat</b>
<b>Week 7</b>	<i>Football Ladder</i>	<b>Poly Plug Values</b>	<b>Poly Plug Values Calculator Go Froms Six Times</b>	<b>Poly Plug Values Calculator Go Froms Six Times</b>
<b>Week 8</b>	<i>Spiders &amp; Ants</i>	<b>Luke's Fraction Game</b>	<b>Luke's Fraction Game Poly Plug Values Calculator Go Froms</b>	<b>Luke's Fraction Game Poly Plug Values Calculator Go Froms</b>
<b>Week 9</b>	<i>Counter Escape</i>	<b>Uncover Counting</b>	<b>Uncover Counting Luke's Fraction Game Poly Plug Values</b>	<b>Uncover Counting Luke's Fraction Game Poly Plug Values</b>
<b>Week 10</b>	<i>Make A Snake</i>	<b>What Can You Do...</b>	<b>What Can You Do... Uncover Counting Poly Plug Values</b>	<b>What Can You Do... Uncover Counting Poly Plug Values</b>

- ◆ Map these weeks into your curriculum:  
For example, will you map *Planner* Week 1 to *Your* Term 1 Week 2 or ...?
- ◆ **Threaded Activities** are in **bold**. Details from Page 22 linking to more on site.
- ◆ *Investigations* are in *italic*. Details from Page 37 linking to more on the web.

# Curriculum Planner

## Year 2 ... Semester 2

	Session 1	Session 2	Session 3	Session 4
<b>Week 1</b>	<i>Where Is The Rectangle?</i>	<i>Rod Mats</i> <i>This investigation can be threaded.</i>	<i>Fractions To Decimals</i>	<i>Rod Mats</i> <b>What Can You Do... Uncover Counting</b>
<b>Week 2</b>	<i>Bob's Buttons</i>	<b>Rows &amp; Straws</b>	<b>Rows &amp; Straws</b> <i>Rod Mats</i> <b>What Can You Do...</b>	<b>Rows &amp; Straws</b> <i>Rod Mats</i> <b>What Can You Do...</b>
<b>Week 3</b>	<i>Fill The Board</i>	<b>Number Shapes</b>	<b>Number Shapes</b> <b>Rows &amp; Straws</b> <i>Rod Mats</i>	<b>Number Shapes</b> <b>Rows &amp; Straws</b> <i>Rod Mats</i>
<b>Week 4</b>	<i>4 Arm Shapes</i>	<b>Maths Dictionary</b>	<b>Maths Dictionary</b> <b>Number Shapes</b> <b>Rows &amp; Straws</b> <i>Rod Mats</i>	<b>Maths Dictionary</b> <b>Number Shapes</b> <b>Rows &amp; Straws</b> <i>Rod Mats</i>
<b>Week 5</b>	<i>Take A Chance</i>	<b>Times Town</b>	<b>Times Town</b> <b>Maths Dictionary</b> <b>Number Shapes</b> <i>Rod Mats</i>	<b>Cross Off</b>
<b>Week 6</b>	<i>What's It Worth?</i>	<b>Exploring Times Tables</b>	<b>Exploring Times Tab.</b> <b>Cross Off</b> <b>Times Town</b>	<b>Exploring Times Tab.</b> <b>Cross Off</b> <b>Times Town</b>
<b>Week 7</b>	<i>Crossing The Desert</i>	<b>Times Tables Torture</b>	<b>Times Tables Torture</b> <b>Exploring Times Tab.</b> <b>Cross Off</b> <b>Times Town</b>	<b>Times Tables Torture</b> <b>Exploring Times Tab.</b> <b>Cross Off</b> <b>Times Town</b>
<b>Week 8</b>	<i>Which Floor</i>	<b>The Big One</b>	<b>The Big One</b> <b>Times Tables Torture</b> <b>Exploring Times Tab.</b>	<b>The Big One</b> <b>Times Tables Torture</b> <b>Exploring Times Tab.</b>
<b>Week 9</b>	<i>4 &amp; 20 Blackbirds</i>	<b>Columns, Rows &amp; Fractions</b>	<b>Columns, Rows, Fra.</b> <b>The Big One</b> <b>Times Tables Torture</b>	<b>Columns, Rows, Fra.</b> <b>The Big One</b> <b>Times Tables Torture</b>
<b>Week 10</b>	<i>Nim</i>	<b>Calculating Excursion</b>	<b>Calculating Excursion</b> <b>Columns, Rows, Fra.</b> <b>Times Tables Torture</b> <b>Best of Semester</b>	<b>Calculating Excursion</b> <b>Columns, Rows, Fra.</b> <b>Times Tables Torture</b> <b>Best of Semester</b>

- ◆ Map these weeks into your curriculum:  
For example, will you map *Planner* Week 1 to *Your* Term 1 Week 2 or ...?
- ◆ **Threaded Activities** are in **bold**. Details from Page 22 linking to more on site.
- ◆ *Investigations* are in *italic*. Details from Page 37 linking to more on the web.

# Threaded Activities

**Threaded Activities** are listed in alphabetical order by name, except for Free Play, which is the first activity in any year. Year K assumes children from 4·5 to 5·5 years old, as the first of seven primary school years.

Introductory information about each threaded activity is provided on these pages. **Full details are on the Calculating Changes site.**

## Free Play

Whether or not children have used Poly Plug and calculators previously it makes sense to ask them what they:

- ◆ can show you
- ◆ know already
- ◆ can do with

the resources. This activity offers many starting point ideas based on examples of children's first lesson with Poly Plug. Samples of children's first responses to using calculators are also given.

Things won't be the same in your class but these examples will prepare you to work creatively with whatever your children offer. The investigation *Calculator Walk* follows on well from **Free Play**, especially for Year K.

### Main Content

- ◆ Various - depends on children's responses & how teachers build on them.

## Add Town/Times Town

Several red boards are laid out in a grid to make the streets of Add Town. Start with a 2 x 2 town. Streets are named by numbers.

The people of Add Town (plugs) like to go for walks and try to get the total of people at every intersection to be the total of the two street names that intersect there.

It probably sounds more difficult in words than it is in actuality. Photographs in the activity make it quite clear that children half way through Year K handle the concept very well.

When the activity is used later in their school career, say Year 1, the problems become more challenging by introducing working backwards. Later still, say Year 2, **Add Town** becomes **Times Town** and the people of the town now line up in arrays at every intersection.

The site also explains how this activity leads into the Maths300 investigation *Chart Strategies*, which is supported by software.

### Main Content

- ◆ addition facts to 10
- ◆ complementary addition
- ◆ numeral recognition
- ◆ recording - written
- ◆ visual and kinaesthetic representation of number
- ◆ writing numerals



## Bridging 10

Each pair needs one red board with two ten frames pressed out and numeral or dot cards, or a ten face dice. Each player chooses one of the two ten frames. Players take turns to roll the dice (or take a card) and plug in their frame. The challenge is to work out the score they have made together.

They write an equation on their calculator to record. The aim is that they discover the 'easy' way of counting by transferring some plugs to count 10 and then add to that. After all, this is how the calculator shows the answer - one ten and ...

### Main Content

- ◆ addition facts beyond 10
- ◆ making groups of 10
- ◆ place value
- ◆ recording - calculator

## Broken Calculator Problems

What happens if one of the buttons on your calculator, say 9, doesn't work? How do you evaluate equations with a 9 in them?

The answer is that you find alternative forms of the broken number, which makes this activity an application of the mathematician's question:

- ◆ Can I check this another way?

This is one activity where the MathMaster calculator stands out, because it has a built in function that allows you to actually turn off a given key, say 9.

However the problems aren't just about pushing symbols around. Poly Plug is used to see ways to work around the broken number. There are several photographs in the activity that make this process clear.

### Main Content

- ◆ addition facts beyond 10
- ◆ addition facts to 10
- ◆ conservation of number
- ◆ operations - whole number
- ◆ problem solving
- ◆ recording - calculator
- ◆ visual and kinaesthetic representation of number

## Buttons

Children love the feel of plugs and this activity builds on that interest. An uncounted number of plugs is put in a container. Children run their hand through it and remove a handful. The challenge is to sort them into groups of various sizes. Lots of counting, recording and checking with a calculator. Eventually too the opportunity to ask challenging open-ended 'backwards' questions.

### Main Content

- ◆ group (or skip) counting
- ◆ multiples, factors & primes
- ◆ problem solving
- ◆ visual and kinaesthetic representation of number

## Calculator Art

This one-off activity is intended to develop familiarity with the way the calculator is laid out by developing an art activity around it. Suggestions in the activity include collage or construction in 2D or 3D. The activity also explores making calculator-like digits and the Braille alphabet with Poly Plug. This later activity integrates mathematics with social education.

### Main Content

- ◆ problem solving

## Calculating Excursions

The activity reminds us to be aware of situations in the children's real life that can be explored mathematically. Examples are given in the activity of children in one class calculating the cost of an excursion and another class calculating the amount of compost produced in the school. Look for similar opportunities in your situation.

### Main Content

- ◆ content varies widely in this activity depending on the source problem you (or the children) choose

## Calculator Go Froms

Number charts from 0 - 99 are the focus of this activity and reproducible versions are available from the site. The children circle one number each on the chart, then Player A marks a pathway of about five steps to join them. There are always many alternatives. The challenge for Player B is to teach the calculator to take the chosen path. Children come to learn how left and right movement on the chart corresponds to addition and subtraction of single digit numbers and up and down corresponds to addition and subtraction of tens numbers. Diagonal are combinations of these.

### Main Content

- ◆ addition facts to 10
- ◆ operations - whole number
- ◆ pattern recognition
- ◆ recording - calculator
- ◆ subtraction

The activity continues by writing the backwards equation equivalent to returning along the same path, and later involves working out the starting point, given the path and the end point.

## Columns, Rows & Fractions

The word column is not necessary when dealing with arrays where the smallest object has the value 1. The focus in that multiplication work is on rows (which go across your tummy), so the word column is potentially confusing. However, if a red board of gaps (or a masked section of it) has the value 1, that is, the red board (or rectangle section of it) is the whole, then it is partitioned into equal parts by its rows, columns and plugs.

### Main Content

- ◆ fractions as an array
- ◆ fractions as a partition of a whole
- ◆ visual representation of fractions

- ◆ Make the whole yellow.
- ◆ Now make some of the yellow plugs blue.

You will immediately know a fraction by the number of blue out of the number in total. There is another fraction too for the yellow plugs. But perhaps the plugs can be arranged so they are in rows or columns. Then you will know equivalent fractions for the blue and yellow.

The activity also explains how this brain picture can lead into confident addition and subtraction of fractions.

## Counting Machines

Many teachers have to see the slide show of Year 1 children's work included in this activity on site, before being convinced that it is a vital part of developing place value concepts. Selected children become cogs in a counting machine that is operated by the rest of the class taking turns to tell the person in the right hand column how many places to slide. Appropriate digit cards are used to generate random movement. The machine is deliberately, at least in the beginning, not a 0 to 9 machine. The activity is about abstracting the movement pattern of place value and later applying it to our particular choice of a Base 10 system.

The physical involvement and recording and publishing phases of the activity are confirmed when the same activity becomes a table top game for two using Poly Plug.

### Main Content

- ◆ place value
- ◆ visual and kinaesthetic representation of number

## Counting On

Players push out three rows of five red plugs to make their playing board. Two dice are rolled (perhaps one is the teacher's) and the children learn how to count on from the larger. Having decided the total of the dice they plug in with one colour. Before their next turn the player has to calculate the dice total that will see them complete their board - complementary addition and counting on wrapped up together.

The game is followed through alternating colours to keep a record of how the dice total worked out for each player. This becomes a picture/word record in the children's maths journal. Lots of photographs and variations in the activity.

### Main Content

- ◆ addition facts to 10
- ◆ complementary addition
- ◆ conservation of number
- ◆ counting
- ◆ visual and kinaesthetic representation of number

## Cross Off

This activity can be introduced through physical involvement of the whole class, or through the animation provided on site. The animation can be shown on your electronic whiteboard. Once the children have the idea of the game, they can play it in pairs. They need a piece of scrap paper and a calculator.

Children sketch their own rough number line and mark numbers in the range they are given. Player A writes an addition or subtraction which uses only numbers shown and crosses them off. Player B starts with the answer to this equation and writes another one, still restricted to the available numbers.

Play continues until one person can't write an equation and loses.

### Main Content

- ◆ addition facts beyond 10
- ◆ mathematical conversation
- ◆ number line - ordering, operations
- ◆ operations - whole number
- ◆ properties of zero
- ◆ recording - calculator
- ◆ recording - written
- ◆ subtraction

## Exploring Times Tables

Four yellow/blue boards placed together to make a square offer a picture of every times table from  $1 \times 1$  to  $10 \times 10$ . To see any particular table the children only need to place straws (make a fence) to isolate it, as shown in photographs in the activity. Most children soon learn their tables up to  $5 \times 5$ , and these can all be seen within one yellow/blue board.

Further, if you are only learning times tables as symbolic repetition of rules then every fact is something new. If however you are using four plug boards, every times table is not only represented but if its answer is larger than 25, the 25 can be seen within it and the other parts can be seen as more times tables up to  $5 \times 5$ . It becomes obvious that if you know your tables up to  $5 \times 5$  you can work out all the others with a brain picture. A great reproducible page which substitutes for the numeric tables charts often stuck to children's tables is supplied in the activity. The partitioning process in this activity is also equivalent to the partitions used to learn long multiplication in **Rows & Straws**.

## Flash & Show

Prepare a set of flash cards with pictures of dots or objects, sometimes neatly arranged, sometimes not. The activity can be used with the class, groups or individuals. The teacher flashes a card and the children respond by either writing a number on their calculator, or matching the picture with Poly Plug, or both. The calculator adds a dimension to this activity that seems to contribute to a non-threatening atmosphere and motivation. Discussion of children's ways of knowing is important.

Variations are suggested on site. When the activity is revisited later in the year, use cards that encourage subitising tens, for example, two groups of dice fives.

## Luke's Fraction Game

Luke was about six years old and in a gifted and talented program when he invented this game. At the time the teacher was discussing biscuits that can be purchased arranged in trays. The game involves asking others for plugs by using fractions.

The red board is arranged as an array of gaps (the tray) and this is the whole which has to be filled. Partners walk around exchanging plugs with other pairs by asking for, say, two fifths of a tray. Since all the trays were packed full with red biscuits before the activity started, it must eventually be that everyone finishes with a full tray again. That's the bones of the concept and there is more detail in the activity.

### Main Content

- ◆ group (or skip) counting
- ◆ multiplication - array model
- ◆ times tables
- ◆ visual and kinaesthetic representation of number

### Main Content

- ◆ 1:1 correspondence
- ◆ conservation of number
- ◆ estimating number
- ◆ mathematical conversation
- ◆ recording - calculator
- ◆ subitising
- ◆ visual and kinaesthetic representation of number

### Main Content

- ◆ fractions as an array
- ◆ fractions as a partition of a whole
- ◆ mathematical conversation
- ◆ visual representation of fractions

The game has lots of work on fraction language in context, visualising and naming fractions of a whole, and fractions which add to or subtract from 1. The social interaction in the activity is great too. Your children will respond well to knowing it was invented by someone their age.

## Making Number Lines

You will need a length of yarn and some digit cards to hang on it with pegs. The simple starting point is asking children to peg chosen numbers in order. The conversation that follows, as suggested in the activity, offers considerable insight into their understanding of number relationships.

There are many variations suggested, including ones which take this activity right through to the end of primary school.

## Maths Dictionary

Creating either a class, or personal, mathematics dictionary is a project applicable at any level. It is an opportunity to share, refine and display current mathematical knowledge and understanding. Text on site includes brilliant photographs of personal dictionaries made by Year 5/6 children. Showing these to Year 2 children will generate interest in producing their own. It will take time, but some of that time will be addressing literacy, creative arts and perhaps technology learning, so some of the work can be done in those class times too.

## Move Around

For this activity you will probably have to move outside or to a hall. Children enter a number into their calculator that is within a range you set. Then they arrange themselves in order. The challenge is in choosing the limits to be just outside the children's comfort zone. The activity involves heaps of peer tutoring and its description on site includes examples from one school where the activity was used at every year level.

## Number of the Day

The teacher sketches a large picture of a calculator and writes a number in the screen area. Children have to find all the ways they can to show this number on the screen. They use their calculator backed up with plug representations. The activity has photographs of children's work and quotes from the report of the Calculator Aware Number project. It is applicable to many year levels by choosing numbers appropriately and encouraging responses that *...show me all you know about mathematics*.

### Main Content

- ◆ mathematical conversation
- ◆ number line - ordering, operations
- ◆ visual and kinaesthetic representation of number

### Main Content

- ◆ mathematics & literacy partnership

### Main Content

- ◆ mathematical conversation
- ◆ number line - ordering, operations
- ◆ recording - calculator

### Main Content

- ◆ operations - whole number
- ◆ pattern generalisation
- ◆ pattern recognition

## Number Shapes

Easy to state and easy to start. Children draw an outline shape, say a square, and write a number inside it. With the help of their calculator they have to write a number at each corner so the total is the answer in the inside number. Remarkably simple and remarkable samples of young children's work are shown in the activity. Experience with **Number Shapes** is a perfect lead into a later years activity called **Box Hunt**.

## Number Slider

A Number Slider is a device invented by Alistair McIntosh for the Mental Computation Project in Tasmania, where it was called a Place Value Board. With Alistair's permission the board was adapted to use with Poly Plug to make the activity even more colourful and tactile.

If you have ever played billiards the device will remind you of a billiards scoreboard because, after initial activities plugs slide across a template showing lines of ones, tens, hundreds (and larger groups of ten if you want).

Number Sliders take a little while to copy and prepare (work for an aid or helping mum) but the time spent is absolutely worth it. There is so much exploring that can be done with a Number Slider. Reading the activity on site, which is full of challenges, is the best way to find your starting point.

## Please May I Have?

As you can tell from the title, this is a very polite game. Two children have a calculator each and enter a number in a given range. The aim is to make their calculator go over the high end of the range and win.

To do this players take turns to ask their partner for a digit. If the partner's screen shows that digit it must be given over, but it is given with its place value. For example, if my screen was 475 and I was asked for a 7 I would have to give 7 tens ... 70. That gift is subtracted from my screen and added to the other player's screen. Look for more details in the activity.

### Main Content

- ◆ operations - whole number
- ◆ problem solving
- ◆ recording - written

### Main Content

- ◆ addition facts beyond 10
- ◆ addition facts to 10
- ◆ place value
- ◆ subtraction
- ◆ visual and kinaesthetic representation of number

### Main Content

- ◆ addition facts beyond 10
- ◆ mathematical conversation
- ◆ numeral recognition
- ◆ operations - whole number
- ◆ place value
- ◆ subtraction

## Plug Catcher

Children begin by removing any number of plugs from a red board.

- ◆ *How many are there?*
- ◆ *Can you check it another way?*
- ◆ ...
- ◆ *Watch out, here comes the Plug Catcher!*

As Player A says these words and advances towards the plugs with a margarine container (or the like), Player B has to hide their eyes.

Player A then catches some of the red plugs with the Plug Catcher and keeps them hidden within it. When told to look, Player B has to work out the number of plugs that have been caught. An equation representing what has happened is written on the calculator and recorded in the children's maths journals.

### Main Content

- ◆ addition facts to 10
- ◆ complementary addition
- ◆ conservation of number
- ◆ recording - calculator
- ◆ subtraction
- ◆ visual and kinaesthetic representation of number

## Plug Lines

Each child pushes out the top row of plugs in their red board. Placing all these red boards side by side makes a number line of gaps. But where does it start and how 'big' are the 'jumps' between the gaps? This is an empty number line with many possibilities.

Patterns develop when the children plug in blue and yellow according to a rule like 3 yellow/2 blue. Also if teachers prepare 'sign post' number cards lots of number work can be explored.

- ◆ *What happens if 4 is here? ... What number is this gap?*

At this age the counting will be on the assumption of jumps of 1 between the gaps, but the same activity can be used with decimals in Year 6.

### Main Content

- ◆ counting
- ◆ group (or skip) counting
- ◆ number line - ordering, operations
- ◆ pattern generalisation
- ◆ pattern interpretation
- ◆ pattern recognition
- ◆ visual and kinaesthetic representation of number

## Plug Snakes

One child is yellow and one blue and they co-operate to make a wiggly wobbly snake in a red board of spaces. They take turns to roll a dice to add a segment of the snake in their colour.

When the snake is made, the children can see in it the record of the dice rolls and they record in their maths journal either their own part in the construction, or the total sequence.

The real objective though is to work with the snake in a mathematical discussion that reveals, manipulates and checks the amazing range of equations that each snake suggests. Photographs in the activity show young children at work on their snakes.

### Main Content

- ◆ 1:1 correspondence
- ◆ addition facts to 10
- ◆ counting
- ◆ mathematical conversation
- ◆ operations - whole number
- ◆ recording - calculator
- ◆ recording - written
- ◆ visual and kinaesthetic representation of number

## Poly Plug Values

As told on site, this activity was invented by a class. Their teacher wanted to use Poly Plug for counting experiences but found that if the numbers counted to were in the children's comfortable range they would always count by ones. Her decision to challenge counting to much larger numbers brought unexpected results.

The children decided to assign values to each of the three colours and count in groups accordingly. They changed the assignment of values to suit the total they were trying to reach. The teacher comments: *It was tremendously exciting to watch as the children's confidence and ingenuity increased each week.*

The activity has several photographs of children at work and examples of their publishing about their challenge for the day.

### Main Content

- ◆ addition facts beyond 10
- ◆ counting
- ◆ exploring large numbers
- ◆ group (or skip) counting
- ◆ mathematical conversation
- ◆ recording - written
- ◆ visual and kinaesthetic representation of number

## Predict A Count

Without doubt this is one of most powerful activities you can use. Some evidence is provided for this claim in the activity and more is on offer in our related professional development sessions.

The calculator is used to count from a chosen starting number in groups of a chosen size, which of course would be just useless button pushing without a tweak. The key is the child must record a guess at every step before they press the equal sign.

It may take a long-ish time to introduce the activity the first time, because there is a process to follow that is part of the teaching craft. But once learnt, **Predict A Count** is self-driven. There are several variations and extensions in the activity and one of the special ones is teachers asking children to review the pattern they have made and write about it.

### Main Content

- ◆ addition facts beyond 10
- ◆ estimating number
- ◆ group counting
- ◆ pattern interpretation
- ◆ pattern recognition
- ◆ recording - calculator
- ◆ recording - written
- ◆ subtraction

## Roll, Make & Write

Easy to state and easy to start, this is a great counting and recording activity. In its simplest variation the child rolls a dice, makes a copy of the dice dots with plugs and writes the numeral on their calculator.

Variations suggested on site are driven by the mathematician's question *Can you check it another way?* They include using more than one dice.

### Main Content

- ◆ 1:1 correspondence
- ◆ conservation of number
- ◆ counting
- ◆ numeral recognition
- ◆ recording - calculator
- ◆ visual and kinaesthetic representation of number
- ◆ writing numerals



## Rory's Pattern Game

Invented by a six year old, this game is like a relay race. Each team has a yellow/blue board and they turn over plugs in it to make a pattern or picture. This is their reference board and it is placed on a chair at the 'lining up' end of a run between two chairs.

At the other end there is another yellow/blue board which, at the start, has no plugs turned over. Players take turns to run to the other end and turn over one plug. The objective is to make the two boards show the same pattern.

When the teams have raced and finished, their boards become the centre of a class or group discussion about number stories that can be found in the board.

### Main Content

- ◆ pattern interpretation
- ◆ pattern recognition
- ◆ visual and kinaesthetic representation of number

## Rows & Straws (Multiplication Arrays)

This large suite of activities uses rows of Poly Plug partitioned with drinking straws to explore multiplication arrays. There is something in this activity for every primary classroom. The activity contains both a broad brush picture (The Big Idea), which will provide lots of thoughts for in-house professional learning, and a sequence of activities across year levels (Contributions).

### Main Content

- ◆ multiplication - array model
- ◆ multiplication
- ◆ properties of number
- ◆ visual and kinaesthetic representation of number

## Secret Spaces

This activity encourages visualisation of a hidden number. It can be introduced to the whole class and then played in pairs. Person A removes some plugs from the red board. The number removed might be agreed, or it might be a secret - two ways to play. Then Person A *secretly* plugs in *some* yellow/blue plugs and covers the board with a cloth.

Person B has to feel under the cloth - no peeking - and count the empty spaces. When they think they know, they write the number on their calculator. The cloth is removed and the answer checked.

### Main Content

- ◆ 1:1 correspondence
- ◆ complementary addition
- ◆ counting
- ◆ recording - calculator
- ◆ visual and kinaesthetic representation of number

## Six Plus/Six Times

Once again the calculator stimulates children to do confidently what in other circumstances they would often find difficult.

Exercises like  $6 + \square = 15$  presented in this symbolic form confuse many children, even if they have materials available. **Six Plus** is a calculator game equivalent to this type of equation, but, in contrast, children love it.

### Main Content

- ◆ addition facts beyond 10
- ◆ estimating number
- ◆ solving equations

The consequence is that addition facts are strengthened and the game provides a pathway to understanding and confidently solving the symbolic form. Later in the children's learning the same game becomes 99 Minus or **Six Times** or any other appropriate variation.

## Smarties

Smarties are candy-covered chocolate sweets that come in several colours in small packets of around 30. M & Ms are similar. The investigation begins by sorting into colours, counting and displaying the data for the packet. Packets may not all contain the same contents.

After this phase, Smarties are returned to the packet and children begin a game with the same number of points as there are Smarties in their packet. Each player in the pair guesses the colour of the Smartie that is drawn out next. A correct guess gains two points. An incorrect guess loses two points. Number lines and calculators are available to support keeping score.

Negative numbers are very likely to be part of the activity; they happen in a natural way in the same sense as children experience negative temperatures. There is support discussion within the activity about ways teachers have helped children visualise these numbers. There are also several variations and extensions, one of which is for the children to create their own pretend Smarties packets with coloured counters. With this resource the activity can be threaded because each container is rich in mathematics, is a familiar structure, but a new challenge.

### Main Content

- ◆ addition facts beyond 10
- ◆ addition facts to 10
- ◆ data: collecting, recording, displaying
- ◆ negative numbers
- ◆ subtraction

## Take A Score

When children have free access to calculators, as is the pedagogy of Calculating Changes, negative numbers turn up. This activity is one that allows that to happen. It comes directly from the Calculator Aware Number project and includes children's work from that project.

To play, children are given a dice and a score card as shown. Each player in a pair rolls the dice four times and enters results in the boxes.

### Main Content

- ◆ negative numbers
- ◆ number line - ordering, operations
- ◆ operations - whole number
- ◆ properties of number
- ◆ properties of zero

The person with the higher score wins the round, but a game might be best out of five rounds.

True, negative numbers might result, but in the words of one teacher: *Negative numbers don't worry my Year 1s; they just call them underground numbers.*

Part of the teaching challenge is helping children understand and makes use of interpretations like this. There are several supportive suggestions on site.

## Taking A Trip

The children pretend they are a teacher taking some children on an excursion. Yellow/blue Poly Plug are the people. Children know that whenever they are on a school trip teachers are always counting them. They are told the number of children going with them today and are asked to put out and count that many plugs.

- ◆ Can you check it another way?
- ◆ Can you arrange your children so I can walk past and just know how many there are just by looking?

There are several photographs in the activity that illustrate children's amazing ingenuity.

### Main Content

- ◆ counting
- ◆ group (or skip) counting

## Taking The Train

A train is provided on site and children use Poly Plugs as people. The activity encourages children to create number stories, act them out and record them with calculator and on paper. Every time the train pulls into a platform a new story emerges related to how many are on board, how many get on and how many get off.

As children's number sense expands it tends to show in the sophistication of their stories. In this way the activity has a built in assessment component. Also, when the activity is revisited later in the year, consider working backwards. For example:

- ◆ The train comes into an empty station.  
Five people get off. How many people might have been on the train?

### Main Content

- ◆ addition facts to 10
- ◆ recording - calculator
- ◆ recording - written
- ◆ subtraction

## Ten Friends

The red Poly Plug becomes a ten frame. Children play a game where one rolls a dice to plug in part of the frame in yellow and the other has to 'think really hard' to guess the number of blue plugs they will need to complete the frame. They follow up by telling each other how they have made the ten and recording it on a calculator.

But there is much more that teachers have developed from this

### Main Content

- ◆ 1:1 correspondence
- ◆ addition facts to 10
- ◆ complementary addition
- ◆ estimating number
- ◆ mathematical conversation

incredibly rich activity. See the activity for the Touch & Tell additions and lots of *What happens if...?* questions.

As children's number sense expands it tends to show in the sophistication of their stories. In this way the activity has a built in assessment component.

## Ten Tens

Place value is the focus of the game. Children fill a Poly Plug ten frame with one colour by rolling a dice. Then they record each ten that is made with the other colour in another part of the board.

The game can be played to ten tens, but that can take a long time so any other objective still makes the point.

The information on site includes photographs from a Year 1 involved in the activity.

- ◆ recording - calculator
- ◆ visual & kinaesthetic representation of number

### Main Content

- ◆ addition facts beyond 10
- ◆ addition facts to 10
- ◆ making groups of 10
- ◆ place value
- ◆ recording - calculator
- ◆ visual & kinaesthetic representation of number

## The Big One

This calculator game supports a growing understanding of the way division works. Its basis is knowing what has to be done to make a calculator screen show the number 1 as the answer to a division.

But if you do a division on the calculator and the answer is not 1, does that tell you anything? Yes it does.

- ◆ If the answer is bigger than 1 you know that the divisor is smaller than the dividend.
- ◆ If the answer is smaller than 1 you know that the divisor is bigger than the dividend.

But what do numbers bigger and smaller than 1 look like?

- ◆ They might have digits after the decimal point!

**The Big One** draws these few bits of knowledge into a secret number game that is explained in the activity. It also includes a variation that allows children to discover both times tables and patterns in the decimal form of numbers.

### Main Content

- ◆ decimal interpretation
- ◆ division
- ◆ estimating number
- ◆ pattern interpretation
- ◆ pattern recognition
- ◆ problem solving
- ◆ recording - calculator

## Times Tables Torture

Continuing the thrust of helping children build brain pictures to support their learning, this activity is a times table test unlike any other. Plug pictures of all times tables up to 10 x 10 are provided as downloadable images that can be arranged in a timed slide show. The challenge question for each picture is:

- ◆ How many yellow plugs can you see?

### Main Content

- ◆ data: collecting, recording, displaying
- ◆ data: describing & comparing with statistics
- ◆ data: interpretation

Following the test, the slides are replayed and discussed.

- ◆ How many yellow plugs on this slide?
- ◆ How do you know?
- ◆ Can you check it another way?

The whole activity is about extending your personal best. Using the test regularly and keeping class or personal records, even exploring how a spreadsheet can be used to keep such records, opens the door to heaps of work in data collection, display and interpretation that is based on meaningful personal data.

## Two Dollar Shop

This early version of the classroom shop has objects marked with whole dollar prices up to say \$10 each. Children visit the shop in threes. One person is the shopkeeper and the other pair go shopping together.

Shoppers use one of the plastic bags from the plugs and are limited to a certain amount of 'money'. Plugs are the money - red plugs are worth \$1 and yellow/blue plugs are worth \$2.

The activity suggests the variation of a regular market day where every table sets up a shop. Other variations are suggested too such as using the least number of coins in a transaction.

## Uncover Counting

This is a whole class activity for which you will need a cloth, such as a table cloth. The children need one yellow/blue Poly Plug between two. They are instructed to, for example:

- ◆ To make rows of three blues down the board. Everything else should be yellow.

Then the boards are arranged on the mat to turn them into rows of six blue. This is where discussion starts.

The first thing to investigate is that all children can run their finger along a row of six blue. (Remember 'rows go across your tummy' so this step is important depending on where children are standing.) Now you cover all the rows of six. They will be revealed row by row after children have predicted each time how many blue plugs will be seen in total. The next phase of the discussion is driven by the mathematician's question:

- ◆ Can you check it another way?

The information on site offers lots more for this activity.

- ◆ mathematical conversation
- ◆ multiplication - array model
- ◆ times tables

### Main Content

- ◆ addition facts to 10
- ◆ complementary addition
- ◆ making change
- ◆ mathematical conversation
- ◆ operations - money
- ◆ operations - whole number
- ◆ recording - calculator
- ◆ recording - written

### Main Content

- ◆ group (or skip) counting
- ◆ mathematical conversation
- ◆ multiplication - array model
- ◆ times tables
- ◆ visual and kinaesthetic representation of number

## What Can You Do With ... How Can You Show Me?

The teacher writes two numbers on the board and asks:

- ◆ What can you do with \_\_\_ and \_\_\_ on the calculator?

Children explore in pairs and the teacher collects and records ideas as part of a class discussion. Children record key elements of this discussion in their maths journal. There are several variations and extensions in the activity. A key one is choosing one or more of the many possible equations and demonstrating it with Poly Plug.

### Main Content

- ◆ operations - whole number

## Win/Lose A Flat

You need Base 10 Multi-base Arithmetic Blocks (MAB 10) for this activity and the board supplied on site. You may have played this trading game before, but there are important differences in this version that make it even more powerful. The activity differentiates for a wide range of ability and experience, while at the same time nurturing the development of almost every number skill from one to one correspondence to place value to children's algorithms for addition and subtraction.

As you prepare for Win/Lose A Flat, it would be worthwhile to visit the Story Telling link in the Members section of the Calculating Changes site to read the article *Davina & Friends Learn To Subtract*. You are bound to be surprised by children's insights described in the story, and you will certainly realise how Win/Lose A Flat, used as a threaded activity, contributed to the growth of those insights.

### Main Content

- ◆ addition facts beyond 10
- ◆ addition facts to 10
- ◆ making/recording groups of 10
- ◆ place value
- ◆ recording - calculator
- ◆ visual and kinaesthetic representation of number

## Working With Books: Put Me In The Zoo

There are many children's books that offer heaps of mathematical learning in partnership with literacy development. 'Put Me In The Zoo', is a rhyming text about a Spotty Creature who thinks it belongs in the zoo. However, despite all the clever things Spotty can do with his spots, he is rejected. There is of course a happy ending.

The activity supplies a master Spotty Creature and Poly Plug supplies the spots. The creature 'gives us permission' to add and subtract spots to the body and arrange the spots in patterns.

'Put Me In The Zoo' is rich with possibilities and the activity includes photographs of young children's work with the Spotty Creature. When you develop similar experiences **Working With Books**, we would love to hear about them for possible inclusion in this section of the Calculating Changes site.

### Main Content

- ◆ addition facts beyond 10
- ◆ addition facts to 10
- ◆ counting
- ◆ mathematical conversation
- ◆ mathematics & literacy partnership
- ◆ subtraction
- ◆ visual and kinaesthetic representation of number

# Investigations

*Investigations* are grouped by Year Level and listed in alphabetical order by name within those groups. Each investigation is listed only once in the planners from Years K - 2.

The information provided on these pages is sufficient for successfully running the investigation. However in most cases more detail is available on listed web sites. Some web links are public access but some are to Maths300. Your school will need Maths300 membership for these.

## Year K

### Body Counting

Children's bodies offer many counting opportunities. We have two eyes, ears, nostrils, arms, hands, legs, feet. We have five fingers or ten fingers or toes - some might argue that we actually have four or eight fingers. Using groups of children, the investigative questions are of this form:

- ◆ There are 6 children standing up today. How many ears are standing up altogether?
- ◆ Can we check it another way?
- ◆ Can we check it with our calculator?

and are clearly designed to encourage group counting.

Combine the idea with children's art work too. Perhaps each child can make a collage monster, but it must have three legs - two to walk with and one for stomping on houses. Displaying these monsters gives opportunity to encourage counting in threes.

Also, group counting can become musical. Divide the class into two groups, the soft group and the loud group.

If we are counting ears, soft and loud create a pattern like this

soft	1		3		5		7		9
<b>LOUD</b>		<b>2</b>		<b>4</b>		<b>6</b>		<b>8</b>	

If we are counting the legs of 3-legged monsters, soft and loud create a pattern like this

soft	1	2		4	5		7	8	
<b>LOUD</b>			<b>3</b>			<b>6</b>			<b>9</b>

The teacher records the **LOUD** responses and the appropriate group counting pattern develops revealing its connection to counting by ones.

A visual/kinaesthetic approach to counting patterns can be developed with Poly Plug. One group places red plugs on the floor or table to match the oral counting and the other places yellow ones. After each group, a new row is started by the red players.

For example:

RY ... (2)

RY ... (4)

RY ... (6)

or

RRY ... (3)

RRY ... (6)

RRY ... (9)

The more we demonstrate that we value counting in groups, the more the children will find groups to count. *Body Counting* uses the teaching craft of personal data, multiple intelligences and technology (calculators are a form of technology) to foster and develop an interest in, and facility with, group counting.

## Calculator Walk

Arrange some parents or Year 6 buddies to join the class so you can send the children into the school ground in groups of four or so with each leader. They take their calculator with them and hunt for numbers (numerals really) displayed around the school. At each stop the group records the number on their calculator and talks about its purpose.

- ♦ Can you think of anywhere else where numbers are used like this?

Eventually each child is asked to decide on the favourite number they have seen and records it on their calculator. You may have to revisit some spots before all the children in the group are able to bring their favourite back to the room on the calculator. Back in the room children draw their number on a piece of paper and then record (or have recorded for them) and explanation of their number, where they found it and what it was for:

- ♦ What can you tell me about your number?

The pages become a class book which becomes part of the literacy program.

To add to the investigation further when outside, leaders can be encouraged to look for opportunities to introduce and discuss location skills:

- ♦ Everyone point to the Principal's office.
- ♦ We are outside Miss Gray's room. Whose room is behind her room?

Also try to arrange at least one roaming leader with a digital camera. The photographs will add to the quality of the class book.

The investigation dips into many aspects of number work, calculator use and pattern. Physical involvement, the concreteness of the calculator, group work, mathematical conversation, student ownership and recording and publishing all contribute to the best practice nature of this investigation.

For more ideas and discussion about this investigation visit Maths300 Lesson 15, *Calculator Walk*.

## Cars In A Garage

This Semester 2 investigation has mathematical similarities to *Where Do We Sit?* which is explored in Semester 1. It will be interesting to see if your children make any connections. Of course, it is not essential at this level that they do.



Prepare three cards (about 20cm square), one shows a red car, one a blue car and one a yellow car. Choose three drivers for these cars and park them in garages side by side, three chairs works well. Record, in a way the class suggests, the order in which they are parked. On your signal they *Brrrm...* out of the garage and around the room and return to *any* garage.

- ◆ How many different parking arrangements are there?

and the deeper question, which may need to be adapted at this level:

- ◆ How do we know when we have found them all?

Explore and then convert the investigation to a table top activity in pairs. Encourage recording and make a class display. Red, yellow and blue Poly Plugs can be useful as pretend cars. One advantage is that each time an arrangement is found it can become part of the table display and three new plugs can be used for the next experiment.

- ◆ How many parking arrangements are there if the blue one must be on the end?
- ◆ What happens if there is also a white car and four garages?

Stimulating features of this activity are acting out, make believe ('pretends'), concrete materials, mathematical conversation, visual/tactile learning and inclusiveness. There is considerable counting content.

For more ideas and discussion about this investigation visit the cameo for Task 2, *Cars In A Garage* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 2 will be revisited in Year 3/4 Pattern & Algebra.

## Crazy Animals

One day a child in the first year of school brought along a book that allowed the reader to match the head of one animal with the body of another and the legs of a third. The teacher saw this as a great learning opportunity. After school she drew an animal, say a duck, and added two dotted lines - one 'cutting' between the head and body and one 'cutting' between the body and legs. Then she extended those lines sideways and drew a second and third animal, say a horse and a giraffe, so that the heads, bodies and legs were placed in the correct areas and were the same width as each other where they crossed the lines. The finished sheet was a master set of Crazy Animals.

The investigation is based around:

- ◆ How many crazy animals can be made?

and supplementary investigations such as:

- ◆ How many of these animals have two parts from the same animal?
- ◆ If we have one tub of heads, one of bodies and one of legs and we choose one piece from each tub without looking will we make a ...?

Concrete materials, the visual nature of the animals, kinaesthetic learning and group work are features of this investigation and its content includes counting, pattern, sorting and classifying and chance.

For more ideas and discussion about this investigation visit the cameo for Task 102, *Crazy Animals* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 102 and its companion Maths300 lesson will be revisited in Year 3/4 Pattern & Algebra.

## Maths In Motion

Prepare pairs of number cards from 0 to 10 that are about 20cm square and consider laminating them because they will be used a lot. Punch holes in the top of each pair and tie them with string so they can fit over a child's head and be worn like sports uniform numbers showing on the child's back and front.

There are now many activities that can be explored with these Number Children and, although the investigation is set down for just one week in the Planners, you will probably want to return to it on many occasions. Children who are not acting in the story today encourage and advise the actors.

Possible scenarios include:

1. The teacher is taking the children for a walk like little ducks following mother duck. The investigation question might be:

- ◆ How many number rules can we think of to tell the Number Children how to line up?

Some possibilities are ...in order from 1 to 10 ...in order from 10 to 1 ...odd numbers then even numbers ...

This activity might also include singing and acting out the song about 10 little ducks going out one day, over the hills and far away.

2. Exploring the 'right order' with a sequence (not necessarily 0 to 10 but perhaps 4 to 9 or 1 to 7) of the Number Children by first bunching them up or telling them to hide or spreading them around the space and on a signal asking them to 'stand along this line in the right order'. Student-generated discussion is at the heart of exploring the many examples of 'the right order'.

Later in the learning program a variation on this activity is to spread the children around and ask for 'the right ones' to run to the line to show two digit numbers such as 23 or 13.

3. Arranging children from 0 to 10 and exploring all the Number Children who are next to a given number ...one more than a given number ... less than a given number ...

*Maths In Motion* stimulates a wide range of content and is supported by teaching craft focusing on outdoor activity, physical involvement, mathematical discussion and recording. Learning can be consolidated further by returning to the classroom and encouraging the children to build 'table top' models of what was done outside using number tiles to represent the Number Children.

For more ideas and discussion about this investigation visit Maths300 Lesson 142, *Maths In Motion*.

## Measuring Up

There are any number of materials (large and small) in an infant classroom that can be used for informal measurement and every such experience can involve investigative questions that encourage children to count their units, compare, sort, classify and collect and record data. The work of a mathematician indeed. Encourage:

- ◆ building towers, pathways, ramps
- ◆ estimating before measuring
- ◆ measuring each other
- ◆ measuring self and comparing self with other objects

- ◆ measuring objects inside and outside the classroom
- ◆ comparing and combining measurements

Building on the children's personal experience, multiple entry and exit points, the outdoor component, differentiation for ability levels and skill development in context are all best practice features of this investigation.

For more ideas and discussion about this investigation visit Maths300 Lesson 92, *Measuring Up*.

## Party Time

This investigation could be generated by a child's show and tell about a birthday party or the upcoming class Christmas party. Parties have to be planned so there is enough food and drink for everyone. Set up a scenario where each group of three is responsible for planning a party and ask them first to plan a menu. You might need to limit the planning to choosing *...one special food or drink each that you want to have at the party*.

- ◆ What will your guests eat and drink?

Ask each group to draw, or make, or in some other way show the things that will be on the menu at their party.

The investigation begins when the children are asked to find the quantity of each item that will be needed.

- ◆ How many guests will you invite?
- ◆ How many of each item on your menu will you need?

Groups have to explain how many of each item and how they worked it out. Again a visual presentation of the calculations is encouraged. Some groups may be able to add a costing to their report.

Personal choices, multiple entry and exit points, open-ended nature, differentiation for varying abilities, group work and the story shell all contribute to the children's involvement in Party Time. The investigation brings together much of the counting work, group counting, calculation and recording that have been developing through the year.

## Playing With Patterns

Experience suggests that exploring patterns like the ones below every session for a week somehow validates that you think patterns are very important and children begin to recognise and want to analyse and explain patterns in the room, at home, or in books that you haven't thought of. You might like to prepare a Pattern Person award to encourage this very mathematical activity.

Mathematicians are always looking for patterns and that doesn't necessarily mean patterns in number. However, there are many times when a visual or sound or movement pattern can be described with numbers. For example we are all used to musical patterns being structured into bars that count 4 or 3 or 6 and so on. Explore all sorts of ways to make patterns such as:

- ◆ Body Patterns: Line up facing forward, backward, forward, backward etc. Jump, sit, stand, touch toes, jump, sit, stand, touch toes etc. Blink, touch nose, touch nose, frown, blink, touch nose, touch nose, frown...
- ◆ Sound Patterns with claps, stamps, dance steps, musical instruments, ostinato patterns...

- ◆ Drawing Patterns: Use felt tip markers to explore, and make repetitions of various hand movements such as strokes, swirls, twists, waves, wiggles, twists... Give each child a starting point drawing with a large oil pastel mark on it that you have drawn and ask them to make patterns grow from the mark.
- ◆ Take a pattern walk around the school.

All these situations are explored through the same investigative challenges:

- ◆ What comes next (...on this end ...on the other end). How do you know?
- ◆ Can you record the pattern?
- ◆ Can you create your own pattern like this one?
- ◆ Tell me about your pattern.

Digital cameras can be a very useful way to record much of the children's work.

The multiple intelligences approach is a key learning feature in this investigation and the content clearly delves into a broad spectrum of the pattern curriculum.

For more ideas and discussion about this investigation visit Maths300 Lesson 93, *Playing With Patterns*.

## The Frog Pond

For this game you need either special dice, or special rules for using cube dice. The special dice can be easily made from wooden cubes by a parent or Year 6 buddy. The faces of the dice are: 1 IN, 2 IN, 3 IN, 1 OUT, 2 OUT, 3 OUT

Alternatively make (and display) the rule that in this game:

1, 2, 3 means 1 IN, 2 IN, 3 IN and 4, 5, 6 means 1 OUT, 2 OUT, 3 OUT

Take the children outside to a suitable 'frog pond' - grassy patch, sand pit etc. - or mark one out with a piece of rope. Choose 5 frogs to go into the pond. Pass the teacher's large version of the dice around to the other children in turn. Each roll will determine how many frogs go in or out of the pond. The aim of the game is to empty the pond.

Return to the classroom and arrange for each pair to make a table top model of the game. A very easy way is to use paper plates as ponds and red Poly Plug as frogs.

Investigative questions could be:

- ◆ What is the most likely number of rolls to empty the pond?
- ◆ How many frogs should be in the pond to start with to make a good game?

but the truth is that children of this age tend not to need such questions. They simply become absorbed in the way the game:

- ◆ sometimes means that the pond empties quickly
- ◆ sometimes means that it takes a long time
- ◆ sometimes means that they get lots of frogs in their pond.

The mathematical content of counting, operating with numbers, exploring chance and the collecting and displaying data is stimulated, among other things, by the game context, its easy to state/easy to start nature, the outdoor component, mathematical conversation and the story shell.

Teachers sometimes press the advantage further by requiring:

- ♦ I always want you to arrange your frogs so I can know how many you have in the pond just by looking.

This encourages subitising. Encouraging reflecting on and recording how the game went and 'what you learnt' adds even more to the learning.

There is also the possibility of extending the investigation by asking:

- ♦ What happens if we change the dice to simpler dice rules such as 1, 2, 3 each mean 1 IN and 4, 5, 6 each mean 1 OUT.
- ♦ What happens if we change the dice to unequal probabilities such as 1 to 4 mean 1 to 4 IN and 5 & 6 mean 5 & 6 OUT?

For more ideas and discussion about this investigation visit the cameo for Task 13, *The Frog Pond* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 13 will be revisited in Year 3/4 Chance & Measurement.

## Where Do We Sit?

A story such as The Three Bears, or simply a made up story about three dolls, or the children's own behaviour when sitting on the mat for a story are possible starting points for this investigation. Three chairs, three different bears and all the children are asked to do is seat the bears in front of the fire. This investigation has been suggested for the beginning of the year because there is often a focus on where children sit in their new classroom.

The investigative question is:

- ♦ How many ways can the bears be arranged?

The question stimulates content related to counting, order, positional language and the use of symbols. The teaching craft possibilities include use of story shell, physical involvement, using concrete materials, student ownership, mathematical discussion, group work, cross-curricula connections and recording & publishing.

The investigation can be extended by asking:

- ♦ What happens if Goldilocks comes to visit?

Well firstly there needs to be another chair for her to sit on. Now investigate the number of ways the Bears and Goldilocks can be arranged on the four chairs.

There is no need to feel that the children have to find all the ways to arrange the 'people' - which is 6 for three people and 24 for four. Learning is generated by the interest in exploring the problem and the experimenting, discussion, justification and recording that develops.

For more ideas and discussion about this investigation visit Maths300 Lesson 8, *Where Do We Sit?* If your school uses Maths With Attitude, Lesson 8 will be revisited in Year 5/6 Space & Logic.

# Year 1

## Around Our Neighbourhood

This has proved a very successful theme study in many classrooms. We begin with school ground familiarity, then take an excursion into local streets and later extend to the whole area from which the children come.

The investigation of the neighbourhood could begin with a book reading, as there are many children's books that explore neighbourhoods. A special one is *Niki's Walk* (Jane Tanner, 1987, MacMillan, Southern Cross) which is both a Big Book and wordless. It is out of print but will be in many school libraries. However, teachers can create their own similar starting point by taking photographs around the neighbourhood from interesting camera angles and mounting them in a slide show for the electronic white board. This could be even more stimulating if a well known person such as the Principal is featured in the photographs.

- ◆ *Where is our principal going?*

Alternatively, arrange for a friend to photograph yourself on an excursion to buy sausages.

The investigation questions for the whole unit are:

- ◆ *What do we know about our neighbourhood?*
- ◆ *What are our ways of knowing about our neighbourhood?*
- ◆ *What else can we learn about our neighbourhood?*
- ◆ *How can we explain to others what we know about our neighbourhood?*

There is a huge amount of literacy, technology, art/craft/construction, social studies, health and science content that is possible in this investigation and the mathematical content includes considerable work in number, 2D and 3D space, pattern, location and mapping skills and measurement. Teaching craft includes student ownership, personal data, estimation, physical involvement, making models and visual learning.

For more ideas and discussion about this investigation visit Maths300 Lesson 91, *Around Our Neighbourhood*, which includes a contribution from a class that made a masking tape map of their neighbourhood on the carpet and marked where every child lived. Of course they had to 'sit on their home' when they were learning on the mat/map.

## Back to Back Building

Two children have a yellow/blue Poly Plug each and sit back to back. They both begin with the all the plugs yellow. Player A turns over some plugs to make them blue in any pattern or picture they like. Now, while still back to back, Player A has to instruct Player B which plugs to turn over to make exactly the same board as Player A is holding.

This is a great activity to begin in a 'fish bowl' with the teacher being Player A for one of the children. When the first turn is complete ask if Player B would like to be the 'maker and teller' for you. They can ask any of the other 'watching' children to help at any time.

Of course the children will want to try this in pairs many times. As appropriate, gather the class and discuss the language that can be used to give Player B the best instructions.

The investigation is driven by the question:

- ♦ What is the best information we can give our partner to help them get the picture correct?

The teacher being part of the activity, its visual/kinaesthetic nature, focus on language, mathematics in community and the mathematical conversation all contribute to the children's fascination with *Back to Back Building*. Number and space are brought together as children explore and develop concepts of left, right, top, bottom, up, down, vertical, horizontal, diagonal and use number to specify 'steps' from the edge of the board. Pattern creation and recognition are other likely content outcomes.

For more ideas and discussion about this investigation visit the cameo for Task 60, *Back To Back Building* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 60 will be revisited in Year 3/4 Space & Logic.

## Cat & Mouse

The game board in the appendix (p. 99) is, in the first instance, a template for using hoops, strips of card (or similar) and Odd and Even cards to mark out a Cat & Mouse playing field on the classroom floor. You will also need a large dice for this phase and a large picture of cheese - or perhaps add to the game by making it a plate of real cheese morsels.

The children are arranged in pairs - one is the cat and one the mouse. Make one set of appropriate signs to place around the players' necks.

In turn each pair is going to play one round of the game and a record will be kept of whether the mouse gets the cheese or the cat gets the mouse. The non-players take turns to roll the big dice and call out whether the result is odd or even.

Before you begin, take a vote to see which result most children expect. The investigative question that drives this investigation is:

- ♦ What do you think will happen most often? Will the mouse get the cheese or will the cat get the mouse?

When all partners have had one turn at the big board, discuss the data so far.

- ♦ Can we be sure yet which one happens most often?

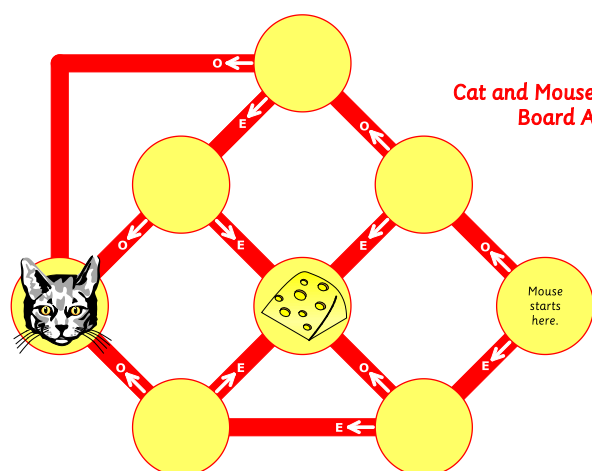
The children are certain to want to have another turn so run the game again with each pair swapping roles. Young children find it very hard to let go of their original prediction even in the face of growing data, so you might also run the game in table top version using copies of the board on Page 99. Continue to add to the class data as necessary.

Extend the investigation as appropriate with:

- ♦ What happens if we change ...the number of circles ... the directions of odd and even ...the number of roads ...the type of dice?

The more flexible the materials you have used to build the playing field, the easier it will be to make these changes.

Cat & Mouse encourages children to explore chance events while simultaneously practising their knowledge related to odds and evens. They become involved in so doing



because of the game context, the physical involvement, the predictive element, the openness, the concrete materials and the display of data, among other things, which weave the investigation together.

Based on Task 223, *Cat & Mouse*, which spawned this investigation, you can find more ideas and discussion about this investigation in Maths300 Lesson 155, *Cat & Mouse*, which also allows the children to explore the game as a software experience. If your school uses Maths With Attitude, Task 223 will be revisited in Year 5/6 Chance & Measurement.

## Crossing The River

Four children and two adults have to cross a river in a small canoe. The canoe can take the weight of:

- ◆ one adult, or
- ◆ one child, or
- ◆ two children

and everyone can row.

The investigation is driven by:

- ◆ Show me how they can all get across the river.
- ◆ How many crossings does it take?
- ◆ What happens if there is a different number of adults (or children)?

Act out the problem first with everyone being able to advise. When the children have seen that it can be done, challenge them to try it for themselves with the Poly Plug. The larger ones can be adults and the smaller ones can be children.

There are several ways to tackle the problem, but it is interesting that at this age children seem to think that the number of children over adults means *they just have to go first*.

Push the children's analysis of the problem a little further by asking:

- ◆ Why doesn't an adult go over first?

You are looking for an explanation along the lines of:

- ◆ They would only have to come back again and that would be a waste.

Before you move on with the problem take time to compare, discuss and refine the number of crossings decided by each group. They are unlikely to be the same and you are looking for the smallest. When that is settled, ask the children to publish (like mathematicians - see Page 11) all they know about the problem so far. Some will have started to 'feel' a pattern in the way the plugs are moved.

When appropriate, explore the same problem (4 children) for different numbers of adults. This time the collection of minimum crossing numbers will show a pattern.

*Crossing The River* differentiates for ability range in the class because some will be operating at the level of moving and counting. However, there will also be some who perceive that it takes four crossings to move each adult across the river and have the children back to the starting side, then five extra crossings to get all the children over at the end (or beginning).



This problem is a variation of one that children are likely to meet in their later years. That problem restricts the children to just two and varies the adults - well at least in the beginning. In the long run, perhaps in high school, the mathematician would be asking:

- ♦ If I tell you any number of adults and children, can you tell me the number of crossings?

Counting, pattern, problem solving skills and, most importantly the opportunity to illustrate the process of working like a mathematician, are the key content components of this investigation.

Physical involvement, kinaesthetic/visual learning, mathematical discussion, working as a mathematical community, the story shell and the non-threatening atmosphere are features that contribute to the quality of the learning.

Based on Task 173, *Crossing The River 1*, which spawned this investigation, you can find more ideas and discussion about this investigation in Maths300 Lesson 23, *Crossing The River*. If your school uses Maths With Attitude, Task 173 and its companion Maths300 lesson will be revisited in Year 3/4 Pattern & Algebra.

## Dice Differences

You will need two reasonable size dice for demonstration, two normal cube dice for each pair (different colours), two Poly Plug boards for each pair and one set of large cards numbered 0 to 5.

Sit the children on the mat and have your cards spread out where they can be seen. Perhaps even on various tables.

- ♦ Today we are going to roll two dice and build towers (or stacks) with our red plugs.

Demonstrate with two helpers. Person A rolls and builds a tower (it won't be taller than 6 plugs so it won't fall over). Person B does the same, then players compare their stacks and find out the difference in heights. Introduce the term difference number.

- ♦ Mary you are going to choose whether we all use yellow or blue to record our difference.
- ♦ Yellow. Okay. The difference number for your towers is written on a card over there. Put a yellow plug near that card.

Players A and B repeat this process two or three times before each pair is given their own dice and Poly Plug to continue the process of recording the difference number. It's very exciting when the towers are the same height.

When every pair has had the time to place three or four yellow plugs, pause the class to set the main challenge. Some children may already be able to work out the difference number without using the towers.

- ♦ Have you noticed that the cards don't all have the same number of plugs? What I am really interested in today is which difference number is going to win if we keep playing this game.

Ask the children to take a guess based on the data so far. A show of hands will give a quick record of predictions on the whiteboard. Your data is probably 'all over the place' because you have asked the children to put their plugs near the number card. Discuss

how the data could be arranged so it would be easy to see the winner. Some form of lining up is bound to be suggested.

Continue playing the game until the children come to an agreement about which difference number is the winner. It is probably worth mentioning that one Year 1 teacher, reporting on this investigation being used in a team with all the classes at the level, explained that the children worked on the investigation for two hours and the only reason they had to be stopped was because it was home time. It seems the children felt that if they kept playing, their prediction would eventually catch up and win.

The next day, the morning literacy session was based around reviewing and reporting on the Dice Differences investigation.

The game context in this investigation is very powerful and it is complemented by the concrete materials, group work, class discussion, prediction (estimation) and visual nature of the difference record.

This powerful investigation simultaneously links the development of the difference concept, subtraction by complementary addition or taking away, skill of calculating difference, exploration of chance and representation of data.

For more ideas and discussion about this investigation visit the cameo for Task 34, *Dice Differences* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 34 and its companion Maths300 lesson will be revisited in Year 3/4 Chance & Measurement.

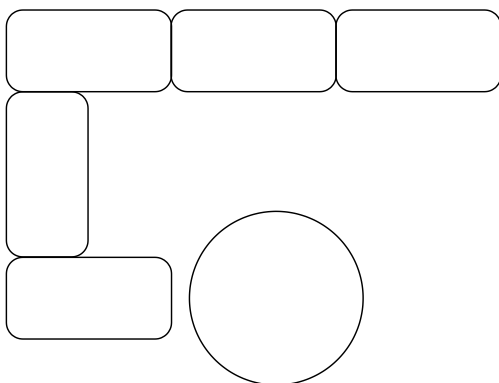
## Domino Trails

For this investigation you need sets of double six dominoes. There are 28 in a box. Also print the file of large dominoes (one per landscape page) linked to the cameo below and slip each one into a plastic envelope. In addition you will need a hoop or bin lid or other circle to use as the end of the trail.

Explore sorting, patterns and operations with the large dominoes and ask the children to record what is discovered, before beginning *Domino Trails*.

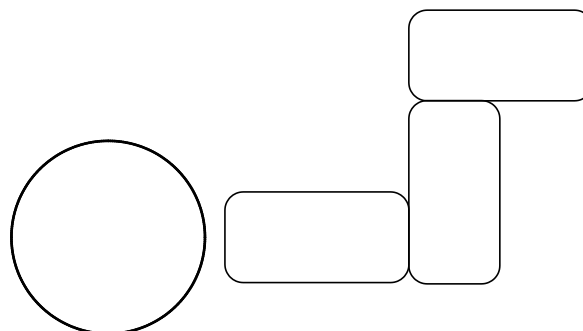
Then use the floor dominoes to explain how a domino trail is made. Perhaps begin with a three-domino trail. Ask the children to add the dots.

♦ Can you check it another way?



Write the total on a piece of paper and put it on the circle. Do this several times and ask the children to record. Then distribute the domino sets and the 3 domino boards copied from the master in the appendix of this manual (pages 100-102). Ask children to explore and record.

The investigation is initiated by working backwards. Use the floor dominoes again but begin with a total in the circle.



- ♦ If I tell you the total can you make a 3 domino trail with the correct number of dots?
- ♦ Can you do it so the dominoes match where they join?

Return to the table dominoes and ask the children to make up their own numbers for the circle.

- ♦ What would happen if you wrote 100 in the circle?

Continue the investigation using a 5 domino trail.

The visual nature of the floor dominoes, the kinaesthetic nature of the table dominoes and the challenge in the non-threatening team environment captivate and absorb children in this investigation. There is also lots of opportunity to differentiate for the experience of various children. Heaps of number work is involved including 1:1 correspondence, counting and addition.

For more ideas and discussion about this investigation visit the cameo for Task 15, *Domino Trails* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm> which includes a master for floor dominoes. If your school uses Maths With Attitude, Task 15 and its companion Maths300 lesson will be revisited in Year 3/4 Number & Computation.

## Eric The Sheep

Eric is a sheep at the end of a line of sheep waiting to be shorn. Naughty Eric doesn't want to wait his turn, so every time the shearer takes one sheep from the front of the line and turns his/her back to shear it, Eric sneaks past two sheep in the line. A perfect story shell for acting out.

The investigation question is:

- ♦ If I tell you the number of sheep in front of Eric can you tell me the number that will be shorn before Eric gets to the front?

This is the general question that drives this investigation, but at this level we adapt it to special cases. Act it out first with 2 and 10 and 15 sheep in front of Eric. Make a point of keeping the shorn sheep in a 'holding paddock' so the focus is on the ones shorn before Eric gets to the front.

When the children have the idea of the problem, set a specific challenge that is not possible to act out with the number of children in your class such as:

- ♦ Suppose there are fifty sheep in front of Eric. Can we work out the number that will be shorn before he gets to the front?

This is where your Poly Plug becomes useful - one colour for Eric and one colour for the others. It is likely that not all groups will agree on the answer (which is 17), so this is a great opportunity to encourage these young mathematicians to justify their solution to each other. Lots of opportunity for personal recording and classroom display too as children are challenged to explain the problem to 'a visitor'.

If you push the investigation further by using other specific numbers of sheep, the children may discover that there is an underlying pattern.

For more ideas and discussion about this investigation visit the cameo for Task 45, *Eric The Sheep* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm> which includes fabulous reports, with photographs of children's journal work, from both a regular and an

accelerated class at an infant school in USA. If your school uses Maths With Attitude, Task 45 and its companion Maths300 lesson will be revisited in Year 3/4 Number & Computation.

*My Year 1 children loved working with the plugs on this problem. There were 13 or 14 pairs around the room all trying it out. Plugs were everywhere. When it came to pack up time I was amazed the way they realised that if there was a gap there must be a plug somewhere in the room to fill it. When everyone had scampered everywhere, one little girl sat almost in tears holding up a board with one gap. The others looked at her, looked at each other and then dived under every table, chair and locker trolley looking for it. Someone found it in under a minute. It was a great feeling for everyone.*

## Farmyard Friends

There are five pens side by side and five animals. Use chairs as pens and make large cards for the animals so the children can hang them around their neck. Clues are given as to which animal is in which pen. For example:

- ◆ The cow is beside the chicken.
- ◆ The horse is in the pen at the end.
- ◆ The pig comes after the cow.
- ◆ The goat is beside the chicken.



If the class is divided into five groups the activity becomes even more involving. One group is the five animals. The remainder of the class is separated into four groups and each group is 'in charge' of one clue card, that you have prepared in advance.

The investigation questions are:

- ◆ How were the animals arranged?
- ◆ How many solutions are there?
- ◆ How do we know when we have found them all?

Each card group is responsible for making sure their clue is properly 'obeyed'.

The range of solutions (there are 2, 4, 8 or 16) depends on the interpretation of particular words and the context of the problem.

- ◆ If 'end' is strictly interpreted in a left/right reading manner, the children may see only the right hand pen as the end. This interpretation will also affect how the word 'after' is interpreted.
- ◆ The word 'after' also has to be explored. Does it mean immediately after (ie: beside) or can it mean several pens after?

Once this level of investigation has been suitably explored and recorded, there are many other problems that can develop from it.

- ◆ What happens if we change one clue?

For example if the fourth clue above becomes: *The goat is NOT beside the chicken?*

- ◆ What happens if there is only one clue?

For example: The horse is in the pen at the end.

- ♦ What happens if we use four animals and four pens and make up our own clues?

At this point you might want to make worksheets of pens and animals printed on cards so the children can experiment on their table top. The most efficient way to do this is to print the sheet from Page 103 onto card. The top row stays as the pens and bottom row is cut into separate cards. Each child in a group of four draws their own animal on their card. Then the investigation begins.

Concurrent teaching of language and mathematical thinking is one of the strong features of this investigation. This is supported by physical involvement, hands-on materials (including materials children make themselves), the open-ended nature of the investigation and the non-threatening environment. These features encourage the development of mathematical language and problem solving involving tallying and counting.

Based on Task 129, *Farmyard Friends*, which spawned this investigation, you can find more ideas and discussion about this investigation in Maths300 Lesson 47, *Farmyard Friends*. If your school uses Maths With Attitude, Task 129 and its companion Maths300 lesson will be revisited in Year 3/4 Space & Logic.

## Halving Squares

Squares of coloured paper about 10cm square (Kindergarten Squares) are common in schools. To begin you will need one square per child - everyone should have the same colour. The investigation is about folding and cutting these squares into halves. There are only two ways to do this:



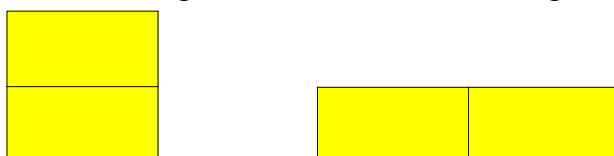
The investigative question is:

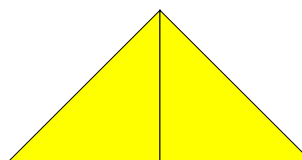
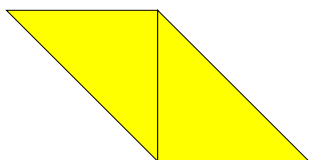
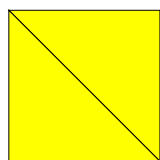
- ♦ How many ways can we put these pieces together?
- ♦ How do we know that we have found them all?

The only condition is that when edges meet they must match. Explore for the rectangle cut and the triangle cut separately. One child in a pair makes the rectangle cut and one makes the triangle cut. Then they explore together how each can be joined again.

- ♦ I found it useful to have larger squares for demonstration so we could gather on the floor to share our ways of doing it. I like to keep a reference square too so we always have a picture of the whole.

There are two ways for the rectangle cut and three for the triangle.





Partners now have two rectangles and two triangles between them. The investigation questions above are now applied to the four pieces. Perhaps you will want to make a class display at this point. Each child could mount their favourite four piece shape to a background paper.

The investigation continues by using just two colours through the class and each child has one of each. Again one person does the rectangle halves and one does the triangle. Now there are eight pieces for each pair and two colours. The number of possible ways to put them together becomes astounding and offers wonderful display possibilities.

Colour, kinaesthetic learning, the open-ended investigation and group work all encourage the children to be fully involved in this investigation. Content includes development of the idea of half, shape language development, concept of different when applied to shapes and a sense of a growth pattern (ie: the number of basic pieces doesn't change much but the number of possible results from them grows much more). Most importantly the whole investigation allows the teacher to set the context and language of the lesson within the objective of learning to work like a mathematician.

For more ideas and discussion about this investigation visit Maths300 Lesson 73, *Halving Squares*, which includes several photographs of halving squares displays from teachers in training.

## Highest Number

You will need two sets of cards numbered 1 - 6 (playing cards for example) and two boards drawn up as shown for each pair. The space below each heading should fit one card. Each pair will also need one dice.

HUNDREDS	TENS	ONES

If you wish, the board could just be Tens & Ones.

The initial game is straightforward. Children take turns to roll a dice. They have to decide whether the result will represent hundreds, tens or ones. They place the appropriate card in the appropriate column to record their choice.

- ◆ Each card can only be used once and can't be moved once placed.
- ◆ If a player rolls a number they have already used, they roll again.

The aim of this introductory game is for one player to make a higher number than the other and score a point.

The challenge comes when the children are familiar with the game. The teacher declares an interest not in who wins, but rather, in the total score each player gets in, say, three rounds.

- ♦ *I want you to play only three times and keep your score each time whether you win or lose.*

Set the children to work and move around making sure each pair is clear about the game and their recording. When all have the three scores, introduce the next challenge.

- ♦ *Now I want you to find the total of your three scores. You can use anything in the room to help you. And I want you to help each other.*

This is a magic time as children think, choose, try and compare. When they believe they have their total, ask the mathematician's question:

- ♦ *Can you check that another way?*

It's great if children work out a total on a calculator first and then, for example, use MAB or Poly Plug to check. It's equally great if someone does it the other way and uses rulers or a number line or a number chart first and then checks on the calculator.

When the children all have their result, declare an interest in the highest and the lowest totals.

- ♦ *What I really want to know today is the highest total we got and the lowest total. How can we find that out?*

This question opens up opportunities for displaying data. For example, children could write their total on a piece of paper and hold it in front of them. Then everyone shuffles around to get into order. There may be some with the same total, which will encourage further discussion of where these 'repeats' should stand. Record the outcome and encourage the children to write and draw about their experiment.

If you want to extend further, a starting point question could be:

- ♦ *If we did the three round game again and found our totals, do you think we would get the same highest and lowest?*

You could also play the game with a ten-sided dice and 0 - 9 cards.

The game context, the developing level of challenge, the mathematical discussion, the support of the calculator, the freedom to choose and group work all help to fascinate, captivate and absorb children in this activity. In the process they are exploring place value, addition, chance and data representation.

Based on Task 127, *Highest Number 1*, which spawned this investigation, you can find more ideas and discussion about this investigation in Maths300 Lesson 26, *Highest Number*, which includes software that allows children to play the game against the computer. If your school uses Maths With Attitude, Task 127 and its companion Maths300 lesson will be revisited in Year 3/4 Number & Computation.

## Jumping Joey

Children love a story with repetition and in this investigation that fascination is enlisted to build a revealing number activity. The storyshell is about Jumping Joey who spends much of his time 'boinging' about the bush getting into trouble. You can use the story as written (see p. 106) or make it your own by telling it, rather than reading it, and beginning by asking the children to tell you the bush animals they want you to include in the story. It's a good idea to read it through now.

There is a great deal of literacy, art and community relations work that can come from the story, but that comes after the maths. The investigative question that intrigues children is:

- ◆ How many animals were chasing Jumping Joey altogether? You can use anything in the room to help you work it out.

If you say this and really mean it you will find some who draw, some who make models with plasticine or the like, some who use various blocks and equipment (often in diverse and surprising ways), some who use a calculator, some who use their body parts and some who use scales such as rulers.

However the investigation is attempted, try to answer the question "Am I right Miss?" with the mathematician's question:

- ◆ Can you check it another way?
- rather than with yes or no.

As the buzz about finding the solution builds up share, discuss and expand the children's responses and look for opportunities to *publish like mathematicians*.

The main content focus in this investigation is on number, counting, addition and pattern, with extensions into location, mapping and distance and other measurements. The teaching craft techniques that fascinate, captivate and absorb the children include story shell, ownership, concrete materials, mathematical conversation, self-directed investigation, kinaesthetic learning and multiple entry and exit points.

For more ideas and discussion about this investigation visit Maths300 Lesson 141, *Jumping Joey*, which includes samples of the broad range of responses from one Year 1 class and many possible extensions.

## Lining Up

Standing in lines is a common experience among young children. This investigation involves finding how many are in the line given you know your position from each end.

Begin by acting out with a small group of children.

- ◆ Please stand in a line here.
- ◆ How many people are in our line?
- ◆ James, what is your position in the line from the whiteboard end?
- ◆ ...
- ◆ Can everyone help me count to check James' answer.
- ◆ But James, what is your position from the other end?
- ◆ ...
- ◆ Can everyone help me count to check James' answer.



Try this several times with different numbers of people in the line and record data each time.

Now encourage the children to make a table top model of the situation with their Poly Plug.

- ◆ Line up your plugs. You can choose any number, but make them all blue.
- ◆ Now turn one over and make it yellow. Any one, anywhere.
- ◆ Can you find out which position it is in from each end?

Ask children to draw and record their plug people and the position of the yellow one.

When the children are experienced, it's time to turn the investigation around. Stand one person in a 'line' by themselves.

- ◆ Emilie your line isn't there yet, but I know that you are seventh from each end. I want you to choose children and tell them where to stand so seventh from each end is true.
- ◆ ...
- ◆ So, how many people in your line Emilie?

Again act out with different children and different challenges before returning to the table top model.

The background investigative questions are:

- ◆ If I say you are \_\_th from each end, can you tell me the number of children in the line?
- ◆ Tell me how you worked this out?
- ◆ Can you check it another way?
- ◆ Can you explain that to someone with a drawing?
- ◆ What happens if I tell you a different position from each end?

However, start with specific examples and modify the questions as appropriate.

The teaching craft featuring in this investigation is the basis in children's experience, physical involvement, mathematics in community, group work with concrete materials, visualising mathematics, recording and explaining to others. Content involves cardinal and ordinal number, position and location in a line, pattern, operations with number, problem solving, and the encouragement of brain pictures to represent mathematical situations.

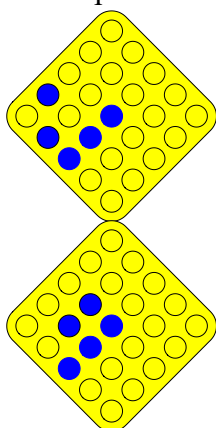
For more ideas and discussion about this investigation visit the cameo for Task 11, *Lining Up* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 11 and its companion Maths300 lesson will be revisited in Year 3/4 Pattern & Algebra.

## Reflections

Many patterns and pictures children make with their yellow/blue plugs display line symmetry. Begin the investigation by asking children to turn over any plugs they like to make a picture or pattern. Hold up a symmetric result and declare an interest in this family:

- ◆ Today our investigation begins by finding all the different symmetric plug pictures we can.

Record these on Poly Plug Paper or Poly Plug Frame which are in the appendix of this manual, Pages 112 and 113. There will be many discussions about whether various pictures are the same. (Note: Try to encourage children to record by using a cross in the circles that represent blue plugs. It's quick and easy to do this and doesn't lead to 'hours' of non-productive colouring in.)



An extension of the investigation is for children to work as partners with two boards. The boards are side by side with the junction being the line of symmetry. Player A turns over some yellow plugs to make them blue. Player B has to turn the ones that represent the reflection. A digital camera can be used to capture final images and display them through your data projector.

A slightly trickier extension uses two boards corner to corner so that the two diagonals are a long line of symmetry. In the example, Player B will have to turn over plugs to complete a picture that is symmetric about the long diagonal. Sometimes children like to make their reflections even more interesting by taking out some yellow/blue

plugs altogether and replacing them with red.

For more ideas and discussion about this investigation visit the cameo for Task 95, *Reflections* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 95 will be revisited in Year 5/6 Space & Logic.

## Trial, Record & Improve

This investigation is calculator based and backs up many of the threaded activities, for example **Six Plus**. It can also be threaded. It begins with the children creating equations with materials. Plugs can be used of course, but it doesn't matter which materials.

A way to begin is to write a different number for each group of 4 on a large piece of scrap paper. Make it a challenging number. Two digit numbers work well.

- ◆ This is your answer. Use your materials to make equations with this answer. Check with your calculator. Each person has to make a different equation, but you can help each other.

(There is an example of this on the Free Tour Activities index page of the Calculating Changes site at: [http://www.mathematicscentre.com/calchange/cch\\_act.htm](http://www.mathematicscentre.com/calchange/cch_act.htm)). Ask each child to record their result on scrap paper.

Gather the class and, using some of the children's examples, show how a number in their equation can be hidden. So, if a child has written  $12 + 13 = 25$  show that a number could be hidden (sticky tape a flap over it) and recorded as:

$$\square + 13 = 25 \text{ or } 12 + \square = 25$$

Demonstrate how to record this on a strip of card and...

- ◆ So we don't forget we write the hidden number on this other card too.

Send the children back to their tables where each person prepares two cards. Now the game starts.

The four group members lay their cards face down on the table and mix them up. Then they take turns to pick up two, looking for an answer card that matches an equation. The game is called Concentration or Memory. After a while swap the cards between groups.

The deeper investigation begins when the teacher, on a regular basis, writes a random equation on the board, for example:

$$2 + \square - 7 = 15$$

- ◆ You can use anything you like to help you find my hidden number.

As children ask if they have the right answer, respond with the mathematician's question:

- ◆ Can you check it another way?

Children use many ways to solve this equation, but the focus of the investigation is on supporting the solution with a trial, record and improve strategy using a calculator. For example:

<u>Try</u>	<u>Result</u>	<u>Comment</u>
8	3	Not 15 - too small
25	20	Not 15 - too big
20	15	Yeh!!

The investigation becomes much more interesting when two numbers are hidden. For example:

$$\square + \square - 6 = 30 \quad \text{or} \quad \bigcirc + \square = 79$$

where you agree that if the 'flap' is the same shape the hidden numbers are the same and if the 'flaps' are different the hidden numbers can be different or the same.

The game-like feel of this investigation, the technological support of the calculator, working in groups, recording and the openness of many of the solutions all contribute to the non-threatening environment that encourages children to be involved. From a content point of view, children apply and extend their number knowledge and number sense in significant ways.

For more ideas and discussion about this investigation visit *Trial, Record & Improve* on Calculating Changes or Maths300 Lesson 94, *Trial, Record & Improve*. If your school uses Maths With Attitude, Lesson 94 will be revisited in Year 3/4 Number & Computation.

## Twelve Counters

This investigation is another opportunity for the children to explore their understanding of chance events while practising skills in context. In this case, number facts to 12. To introduce the game to the class, you need twelve large cards (say 20cm square) numbered from 1 to 12. You also need two large dice, preferably different colours. First the game is played with the whole class, then it is converted to a table top game.

Spread your large cards around where the children can see them. They don't have to be in order or in a line, but the children are going to gather around them, so allow room. Perhaps they can be table numbers.

- ◆ We are going to play a new game. I will explain it in a minute. Then I am going to ask you to guess something before we start to play.

Explain that each child in turn will roll the two dice and add the numbers. Then they will sit at the table with that total.

- ◆ What I want you to guess is which table will have the most children and which table will have the least.

On the board write the headings Most and Least and under each write the table numbers from 1 to 12. Ask the children to vote and record the results.

- ◆ Now let's play the game.

As you play, opportunities will arise to encourage children to count on from the larger number. Once all the plug children have 'been sent' to their table, record the data and compare it to the guesses. Discuss as appropriate.

- ◆ If we played again, do you think it would be the same?

Play again as many times as the children's enthusiasm suggests before converting to a table top game. Provide cards and dice for each pair and use the Poly Plug as children. (See Page 108 in the appendix for a card master.)

- ◆ You have fifty children in your Poly Plug set. Tell each other your guess first then play the game until all fifty are at their table.

No doubt the children will want to tell you what happened in their game, so encourage them to draw, write or cut and paste to explain

The next phase of the investigation is to put to work their experience with the chances explored so far.

- ◆ In the next game you still have the tables 1 to 12. But this time each of you has only 12 plugs. One of you is yellow and one is blue. And you are going to take them away from the tables instead of sitting them down at the tables.

Explain that the new game begins with each player placing their plugs at the 'tables' as they wish. They can have as many players on the same table as they want.

- ◆ Your aim is to arrange your plugs at the tables so that you are first to take all your plugs away.

As the children play encourage them to realise that every time they take away a plug they are actually practising a skill such as  $10 - 1 = 9$ . When the children are experienced discuss the children's thoughts about where their plugs should be placed.

- ◆ Which tables are not good ones to use? Why?
- ◆ Which ones do you think are the best? Why?

Your class is likely to be involved in this investigation for several sessions. The game context, physical involvement, prediction, open-ended nature of the investigation, concrete materials and the kinaesthetic/visual components of the activity are some of the features that contribute to this involvement.

For more ideas and discussion about this investigation visit the cameo for Task 117, *12 Counters* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 117 will be revisited in Year 3/4 Chance & Measurement.

## Visual = Number

Wherever there is a visual pattern there is a corresponding number pattern and vice versa. That's the dictum that drives this investigation described by a teacher as it happened in her Year 1 class. It is a Poly Plug investigation from Calculating Changes so you will find it fully detailed on Calculating Changes.

Perhaps the most important thing about this investigation, which as described is certainly early algebra, is that it teaches us that young children can show remarkable mathematical insight when we leave the learning door open. We are alerted to look for and build on children's observations and actions. There are many similar opportunities that arise in a Calculating Changes classroom because children are fascinated by Poly Plug and regularly make visual patterns that can become number patterns.

Odd and Even numbers, Square Numbers, multiplication and heaps of pattern work are the main content thrusts of the investigation. Challenge in a non-threatening, visual, kinaesthetic context with mathematical conversation encouraged between members of a learning community (including the teacher) encourage fascination with the problem.

# Year 2

## 4 & 20 Blackbirds

Some children today don't know much about nursery rhymes, so here is an opportunity to find out what is known. Start with *'Sing a song of sixpence a pocketful of rye. Four and ...'* and see what response you get. If someone completes the line *'...twenty blackbirds baked in a pie'* you can launch directly into the storyshell. If not, then wing it by finishing the line yourself and launching off with something like *'Today I am going to tell you a story about a king and a queen...'*

When the birds popped out of the pie and started singing (*'When the pie was opened the birds began to sing...'*) the queen loved their songs so much that she asked the king to find a way to make sure that they sang for her every morning.

The king talked with the smartest people in the kingdom and they all agreed that the best thing to do was to build some feeding platforms in the royal courtyard, just like in the picture.

The next morning the queen heard the birds singing and popped her head out of the castle window to look. Wow! All the twenty-four blackbirds were on the feeding platforms. She was very excited. And then she noticed something so special she had to tell the king.

There were nine blackbirds on each line of platforms ... twenty-four blackbirds altogether, with nine along the platforms at the top, nine along the platforms at the bottom, nine down the left side and nine down the right side.

### ♦ How were they arranged?

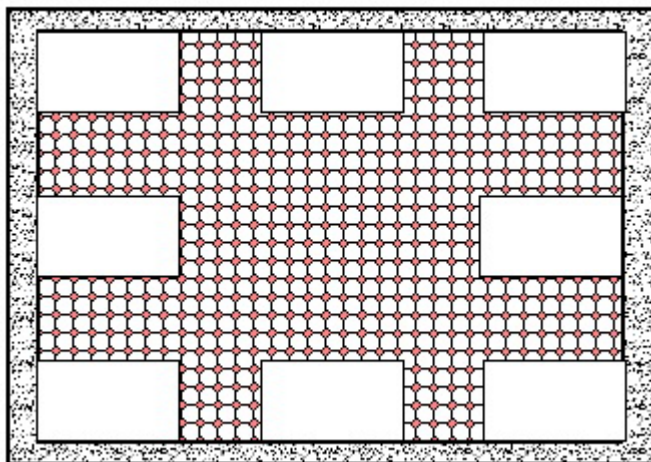
On the carpet, pieces of paper are easy to lay out as feeding platforms. You will also need 24 blackbirds (that's about one for each child). You can find a print master for these on Page 109 and a Royal Garden master on Page 110.

Let the children explore and experiment until a solution is found. Record the solution then announce:

- ♦ That's fantastic! It works. There are only twenty-four birds and you have arranged them with nine on each side. Now the next day when the queen looked out of her window the twenty-four birds were still there AND there were still nine on each side BUT they were arranged a different way. How were they arranged?

Explore, experiment and record again.

- ♦ Whenever a mathematician discovers more than one answer to a question, they get very excited. Now they want to know how many ways the twenty-four birds can be arranged so there are nine on each side of the courtyard.



Show the children how to use the Royal Courtyard and red plugs as red birds (*'oops, blackbirds'*) to explore in pairs and find more solutions. When the children have found all the solutions they can, use a creative art session to make a Four & Twenty Blackbirds wall display to explain the problem and show the solutions.

This investigation includes a great deal of arithmetic skill practice in context and it can lead into exploring pattern, but its most important feature is the opportunity to model what it means to work like a mathematician.

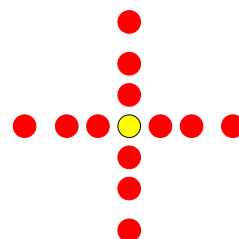
However, the problem could have been presented as a written word problem in a book. There is a considerable difference between that presentation and the outline above. So, the story shell, community floorboard, whole class discussion, a large blackbird for each child, concrete materials, group work and so on are deliberate teaching craft choices to reflect that a mathematician wants to first be *interested* in a problem (see Page 11).

For more ideas and discussion about this investigation visit the cameo for Task 62, 4 & 20 Blackbirds at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 62 and its companion Maths300 lesson will be revisited in Year 5/6 Number & Computation.

## 4 Arm Shapes

Demonstrate, perhaps with circular margarine lids on the carpet how a path is built leading up to a monument in the park.

- ◆ Now I want you to build in the same way on your table with plugs. A yellow one is the monument. Red ones are the stepping stones. Your first challenge is to find out how many plugs you need if I tell you each arm has ten stepping stones.



Gather the children to discuss their answers. The discussion is driven by the mathematician's question:

- ◆ Can I check this another way?

Perhaps children will answer:

- ◆ *We built it and counted.*
- ◆ *If there's 10 in an arm, that's 4 times 10, equals 40, and one for the monument.*
- ◆ *Ten in an arm means 21 across and 20 down, so that's 41.*

Celebrate all the answers and ask set challenges for other length arms. Then investigate further by asking backwards questions such as:

- ◆ If I tell you there are seventy-three plugs used, can you tell me the length of each arm?

Explore the ways children work this out and set additional similar challenges.

Review the way 4 Arm Shapes grows. The form of the picture stays the same - a centre and four arms - and it just gets bigger. A further extension is to invite children to create their own centre and their own way to make things coming out of it in a pattern. Then they investigate their own problem in a similar way.

Discuss with the children how they have worked like mathematicians (see Page 11) and set the final challenge to make a poster explaining to someone else all they know about 4 Arm Shapes.

For more ideas and discussion about this investigation visit the cameo for Task 154, *4 Arm Shapes* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 154 and its companion Maths300 lesson will be revisited in Year 3/4 Pattern & Algebra.

## 13 Away

This investigation is easy to state, easy to start and has an underlying strategy. Begin with 13 red Poly Plug in a pile. Players take turns to remove 1 or 2 or 3 plugs at a time. The player who removes the last one loses.

The investigative question is:

- ◆ Can you find a way to always win?

Start the game in a 'fish bowl' situation with two children demonstrating as you explain the rules. Then set partners to work exploring the problem. Every now and then pause the class and ask what has been discovered so far. Build a whiteboard list of ideas, hypotheses, questions and explanations. As children become confident, set up challenges - pairs can play pairs so that individuals feel supported.

Gradually share the winning strategy with everyone so that each child feels successful. Model how to prepare a report (written, poster, slide show, oral presentation, ...) to explain to someone else how the game is played and what has been learnt.

Suggest that if children are really confident they could test themselves by playing their parents for additional pocket money.

The investigation continues when we ask:

- ◆ What happens if the person who takes the last counter is the winner?
- ◆ What happens if we change the number of plugs?
- ◆ What happens if we change the number that can be removed?
- ◆ What happens if we start with 21 on a calculator and we take turns to take away 1 or 2 or 3?

This game situation, its concrete materials, its simplicity and underlying pattern all help to involve the children. From the content viewpoint children apply basic number skills, discover patterns and reason like a mathematician.

For more ideas and discussion about this investigation visit the cameo for Task 59, *13 Away* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>.



## Bob's Buttons

Take the children outside or to a gymnasium space to play the game. Begin by appointing one of the children as your secretary. That person doesn't take part in the game. Also make sure you count the number of players and that everyone knows that number.

Children have to walk around in a defined area singing to themselves - or all singing the same song - but listening for your voice. Every now and then you will call out a number and children instantly have to clump into groups of that size. Record the result, along the lines of the diagram.

No one is out in this game. The number playing stays the same. So once the record is made, the children start walking and singing again until the next number is called. However, this time as soon as you call the number also start calling out to the children to predict what's going to happen. When they are settled in their groups ask some children what they predicted. Then count, check and record the result.

When you return to the classroom the secretary transfers the data so far to the whiteboard while everyone is settling and getting out their journal and Poly Plug. The children's first job is to copy the diagram data so far.

Suggest that they are now going to make a table top model of the game. Poly Plug will be the people.

- ◆ One of you becomes the teacher and the other moves the people and sings. I want you to find what happens for all the other numbers the teacher could have called today and write them on your diagram.

Over time, the investigation is extended by the question:

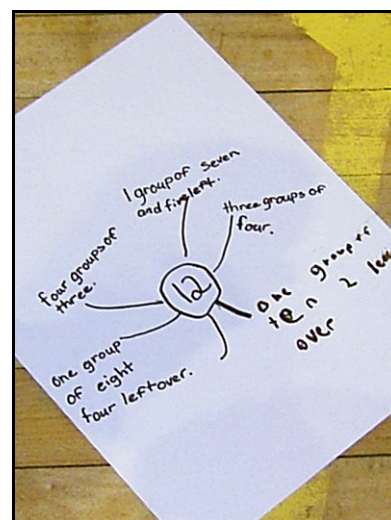
- ◆ What happens if we change the number of children?

Then later by asking backwards questions such as:

- ◆ One of the children in another school couldn't remember the number of children playing the game. But they could remember that when the teacher called out groups of four, there were two leftover. And when the teacher called out groups of five there was one left over. Can you work out how many children were in the game that day?
- ◆ How many solutions are there?

*Bob's Buttons* is closely related to the threaded activity **Buttons** that was introduced in Year 1. In this variation the multiple intelligences approach, outdoor activity, ownership of the problem and emphasis on recording are key features of the investigation. The investigation builds concepts related to multiplication and division through the sharing concept and develops calculation skills in context.

For more ideas and discussion about this investigation visit the cameo for Task 123, *Bob's Buttons* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 123 and its companion Maths300 lesson will be revisited in Year 5/6 Number & Computation.



## Cookie Count

If possible, grow this investigation from a reading of the story *The Doorbell Rang* by Pat Hutchins. It's a favourite and is likely to be in the school library. If not, you will find it through the web. Each pair needs a paper plate and at least one set of Poly Plug between them. The plugs are cookies.

With the class gathered in a circle, or around a central table, present them with a plate of red Poly Plug cookies. Something like 48 is a good number and easy to extract from two boards.

- ♦ How can I check that there are 48 cookies on this plate?
- ♦ Can we check it another way?

Don't forget the two left unused in the red boards can also be used to check.

- ♦ There are three people at a party. (Choose three to act the part.) How many cookies would each person get if they were shared equally?

Ask the class to show/explain at least two ways to work this out. Record the answer and all the methods. Put all the cookies back on the plate.

- ♦ Before the cookies could be shared, one more person came to the party. Now there are four people. How many cookies would they each get?

Apply the various methods to work out the answer. Put all the cookies back on the plate.

- ♦ Before the cookies could be shared, one more person came to the party. Now there are five people. How many cookies would they each get?

This example introduces the situation of 'left-overs'. Discuss what could be done with them. Depending on your children, you might want to just deal with groups and leftovers. However, children tend not to like leftovers - they would rather see the cookies shared down to the last crumb - so it is likely they will suggest interesting ways of sharing these few and that will introduce fractions.

- ♦ Could we use our calculator to show how you shared those leftover cookies?

*Cookie Count* can be visited again and again because the number of cookies can change and the people at the party can change and every change is a new challenge. If you don't want to thread the investigation in this way, then ask each pair to investigate their own number of cookies and prepare a poster to explain what they found.

The investigation connects content in counting strategies, whole number operations, multiples, fair shares, division and fractions. Imagine it was presented as a text exercise in which the children were only able to draw or calculate. Would it be as inclusive, involving and extensive? Not likely - the story shell, the connected literacy experiences, the concrete materials, working in pairs and owning the particular problem all contribute to the children wanting to learn and apply the content.

For more ideas and discussion about this investigation visit the cameo for Task 19, *Cookie Count* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>.

## Counter Escape

For this investigation you will use six hula hoops, which you will be able to get from the physical education teacher, and Poly Plug. You will also need one dice for each pair and a large one for class demonstration. Label the hoops as two sets of A, B C.

Set up a story about three Intergalactic Good Guys who have to go to another universe to save the world. There are three interstellar transport tubes they can use. The Tubes are controlled by a computer. The computer rolls a dice. If it is:

- ◆ 1, a person from Tube A is transported.
- ◆ 2 or 3, a person from Tube B is transported
- ◆ 4 or 5 or 6, a person from Tube C is transported.
- ◆ The computer keeps rolling until everyone is transported.

(Of course the computer does it this way so the warriors from the Evil Empire will never be able to guess which Tube is being used each time, therefore they won't be able to catch our Good Guys.)

- ◆ Let's see how this works.

Ask for three volunteers and let them decide which Tubes to stand in. They can have any number they like in each Tube. Record their decision by showing the occupied Tubes. For example, AAA means three Good Guys in A and ABC means one good guy in each of the hoops.

The other children are the computer. Pass the dice around, play the game and keep a record of the number of rolls.

- ◆ We know this is only one way of standing in the Tubes. My first question today is how many ways are there to stand in the Tubes?

Send the children back to their tables in pairs to quickly draw three tubes and take out three red Poly Plug. As they find a new way of placing the three Good Guys they write it on the board. Some will arrange randomly, but others will begin an organised search something like:

Three in one  
AAA, BBB, CCC

Two in one  
AAB, AAC, ABB, BBC, ACC, BCC

One in each  
ABC

Take a moment to discuss that searching in an orderly way such as this is how mathematicians can be sure that they have all the possibilities.

Gather children on the mat again.

- ◆ It took \_\_\_ rolls to send the Good Guys to the other universe when we used this way of standing in the tubes. Do you think it would be the same number for a different way of standing? How can we find out?

Set up an experiment with two sets of hoops running simultaneously, one of which is the first strategy, and find out the number of rolls for each.

- ◆ What I didn't tell you was that the Good Guys have to get to the other universe as quickly as possible to save the world. Our challenge

is to find which of these placement strategies will send the Good Guys away in the least number of rolls.

Discuss and develop an investigation plan that is led by the children's suggestions. You might like to include an experiment using 30 hoops and 30 children to set up a whole class experiment using all the strategies at once. Carry out the children's plans, however they develop, and publish the results in the Intergalactic Good Guys Transportation Handbook.

- ◆ What happens if we reprogram the computer with different dice rules?
- ◆ What happens if we change the number of Transportation Tubes?
- ◆ What happens if we change the number of Good Guys?
- ◆ What happens if we change the type of dice (eg: to 10-sided)?

What fascinates, captivates and absorbs children in this investigation? Could it be:

- ◆ Story Shell?
- ◆ Non-threatening environment?
- ◆ Physical involvement?
- ◆ Personal and class data?
- ◆ Challenge?
- ◆ Inclusivity?
- ◆ Concrete materials?
- ◆ Game context?
- ◆ Openness?
- ◆ Reporting to others?

And what content do you value most:

- ◆ Organised counting?
- ◆ Tallying?
- ◆ Experiencing chance?
- ◆ Problem solving strategies such as breaking the problem into parts?
- ◆ Collecting, displaying and comparing data?
- ◆ Modelling of the process of working like a mathematician?

For more ideas and discussion about this investigation visit the cameo for Task 6, *Counter Escape* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 6 will be revisited in Year 3/4 Chance & Measurement.

## Crossing The Desert

Two travellers set out on a 9 day journey across a desert to deliver a message and return with the answer. Each person can only carry enough food for 12 days. How can the message be delivered and the answer returned?

Using Poly Plug as the food - one red plug per person per day - and the board supplied on Page 111, children explore this easy to state and easy to start problem. However they soon realise that although 24 days food seems enough, the travellers will die of hunger on the way back if they both try to do the full journey. One of them has to be the Messenger and the other has to be the Food Manager, who buries food on the way and goes back to the start. The food is recovered by the Messenger on the return journey.

The investigative questions are:

- ◆ On which day should the food be buried?
- ◆ How many portions should be buried?
- ◆ How many solutions are there?
- ◆ How do we know when we have found them all?

The mathematics in this problem includes counting, problem solving strategies and exploring ways to break up numbers. The story shell, concrete materials and mathematical conversation generated by working in groups seem to be factors that encourage involvement.

For more ideas and discussion about this investigation visit the cameo for Task 94, *Crossing The Desert* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 94 will be revisited in Year 5/6 Space & Logic.

## Fill The Board

This is an intensely involving investigation that is easy to state and easy to start. All you need is a Poly Plug set and a dice for each pair.

Beginning with an empty red board, pairs take turns to roll a dice and plug in until the board is filled. The investigative question is:

- ◆ What is the most likely number of rolls to fill the board?

This is an investigation from Calculating Changes. It is fully explained in on site, including reports and comments from teachers who have used the activity at all school levels.

At one level the investigation involves number work such as 1:1 correspondence between the dice dots and plugs, and counting. At another it is designed to experience matters of chance such as like and less likely events. At still another the investigation involves skills related to data collection, tallying, data display and data interpretation. Thus concurrent teaching of topics and skill development in context are key pedagogical features of the investigation. The multiple exit and entry points also make it perfect for differentiating for different experience levels.

Other features are the colourful, tactile nature of the game situation and the extensive mathematical conversation.

## Football Ladder

Each football code has its own way of ranking teams on a ladder as the season progresses. Choose the code most appropriate for your class and find out the rules for its ladder. It is likely there is already someone in your class who knows. In the early weeks of the season keep a weekly record of how the teams are positioned and how their points are calculated. This helps to develop interest in football ladders.

The focus of the investigation is essentially the strategy of working backwards. That is, given the names of the teams and some clues, the children have to construct the way the ladder looks. At this age you might begin with a mythical small rural league, or local school league. For example:

*There are five teams in this league: Sunshine, Darebin, Altona, Werribee, Footscray. At the end of one week of the competition these things were true:*

***Werribee is in the middle.***

***Darebin is not in the top three.***

***Sunshine is in fourth position.***

***Footscray is above Altona.***

*Find out the order of the teams on the ladder.*

Clues can also relate to the points score for each team that decides its place on the ladder.

With a starting example such as this, refer to the mathematician's toolbox of strategies (p. 11) and before beginning discuss the strategies that might be used. One that seems generally applicable is tearing a piece of paper for each team name. That is, making our own concrete materials. But some children might like to draw diagrams and some might like to try every possible way to arrange the teams and check each time until they find an arrangement that fits the clues. It is also often worth checking whether there is more than one solution.

The next phase of the investigation can be for children to choose their own team names and ranking and write and test their own clues. Groups swap with each other to try them out.

- ◆ What happens if there are 6 teams, 7 teams, ...?

Of course the big challenge is to work out a puzzle for the teams in the actual football code of the class. This can be a project for small groups, partners or individuals. Part of the assessment process can be peer assessment. After all, the puzzle has to work for, and be interesting to, the intended audience.

The integration of literacy and mathematics is a powerful component of this investigation. Its content is firmly grounded in the process of working like a mathematician (p. 11) and can be modified to include arithmetic calculations. With some children there is also room to explore combination theory (or clever counting) through questions like:

- ◆ If there are only 4 teams in the competition, how many ways could they be arranged on a ladder?

For more ideas and discussion about this investigation visit the cameo for Task 122, *Football Ladder* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 122 and its companion Maths300 lesson will be revisited in Year 5/6 Space & Logic.

## Fractions To Decimals

This is a calculator investigation largely taken from the Calculator Aware Number project report. It is fully detailed on site and includes work from seven and eight year old children.

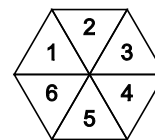
The activity is essentially driven by the questions:

- ◆ How could we show fractions on the calculator?
- ◆ Can we check that another way?

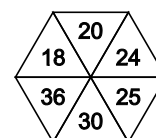
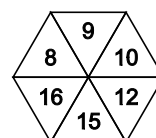
It encompasses content related to division, decimal representation of fractions, interpretation of decimals and recognition and interpretation of pattern. Children become involved through the non-threatening, exploratory environment created with the calculator and the open-ended nature of the investigation.

## Have A Hexagon

This game for two players is designed to encourage investigation of chance while simultaneously practising times tables. The playing board with three hexagons numbered as shown can be found in the appendix on Page 104.



Introduce the game using two volunteers. The rest of the class gathers around to see how the game is played. Explain the rules, which are:



- ◆ Players take turns to roll two numeral (not spot) dice. (It is useful if each pair has two different coloured dice.)
- ◆ The numbers showing are multiplied. (The answer will be on the hexagons.)
- ◆ A red plug is placed beside the answer number.
- ◆ Play continues until one hexagon has at least one plug beside every number.
- ◆ If an answer number is repeated the children stack red plugs beside it.
- ◆ But before you start you must write down your guess about which one will win - top, left or right.

Ask the class to vote before the demonstrators play. Then follow through one round to check the result against class predictions.

- ◆ *The first thing I am interested in today is which hexagon is most likely to be finished first. How could we find that out?*

Allow the class to suggest that everyone should play one game and a record should be kept. Follow the experiment through until the children think there is enough data to suggest which one is finished first. It turns out to be the top one. Discuss why this might be. The children's stacks of red counters will give a clue. Some numbers, such as 4, can be made more ways than other numbers such as 3, so they probably have higher stacks.

- ◆ *Okay, so we think the top one will be finished first. Now I am interested in how many turns it takes to finish a round, even if it isn't the top one that wins.*

Again ask the children to guess first. They have already played some games so will have personal data on which to base their judgement. Playing the game again, as many times as is appropriate will generate class data that is similar to that which a mathematician often faces - a spread of information but some results appearing more often than others. This is not the time to be formally teaching statistics, but it is a perfect opportunity, based on the class's personal data, to ask:

- ◆ *If someone else didn't want to do all these experiments and asked us how many turns it took to finish a round, what could we tell them?*

Encourage the class to look for more than one way to explain. They might choose the:

- ◆ range from smallest to biggest.
- ◆ section where most results seem to fall.
- ◆ result that occurs most often.
- ◆ result that is in the middle when they are arranged in order.

The investigation can be extended further with the blank hexagons provided on Page 105. Children place the number in their own way on the hexagons with the aim of trying to make each hexagon have a nearly equal chance of finishing first.

The game context, estimation/prediction and personal data gathered in the context of a challenge are three important contributors to the children's involvement in this investigation. Concurrent teaching of chance and data concepts and times tables is a strong content component.

For more ideas and discussion about this investigation visit the cameo for Task 53, *Have A Hexagon* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 53 will be revisited in Year 3/4 Chance & Measurement and its companion Maths300 lesson will be revisited in Year 5/6 Chance & Measurement.

## Make A Snake

Introduce the children to Mungo the Maths Snake who grows in a special way. When she is born her body is only one yellow segment long. Each season Maths Snakes grow a little longer by adding segments to their body. Mungo adds segments in a special pattern. Each season each yellow segment she has changes to yellow/blue/yellow. Blue segments don't change; they are just pushed along her body as the yellow segments grow.

- ♦ Find out all you can about how Mungo grows season after season.

Poly Plug are perfect for this exploration. It helps to control the plugs a little if each row in a red board is considered as a season. Then Mungo's different length in each season can be clearly seen.

The exploration is deliberately open-ended but the children are sure to find patterns in the yellow and the blue and perhaps the total of segments. They may also be able to use the patterns to predict blue and yellow for say the tenth season. There is also the opportunity to ask the mathematician's question:

- ♦ What happens if we change the growing rule?

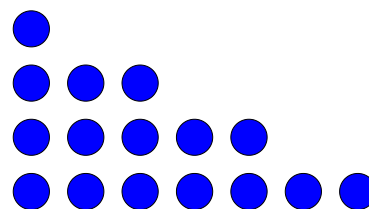
The story shell and its connection to nature knowledge about snakes shedding skin, the visual and concrete presentation, the open-ended challenge and sharing discoveries in a community of mathematicians all help to interest and absorb children in this problem. This could be a text book problem. Would children be as likely to become involved with it if it were presented in this way?

For more ideas and discussion about this investigation visit the cameo for Task 5, *Make A Snake* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 5 will be revisited in Year 3/4 Pattern & Algebra.



## Nim

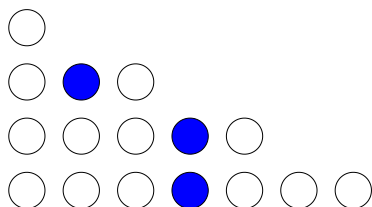
This is a strategy game for two players which is easily introduced in a 'fish bowl' situation with two children playing to your instructions. Plugs are arranged as shown in rows of 1, 3, 5 & 7. Children take turns to remove one or more counters from *just one row*. They can even remove a whole row! The person who takes the last plug loses.



The investigative question is:

- ♦ What is a winning strategy for this game?

After the demonstration, invite children to play the game a few times. Discuss anything they notice and suggest that you want their help to find a way to be sure of winning. Have there been any times when you knew you were going to win? How did you know? Encourage further investigation with a focus on knowing when you are going to win (or lose). It should start to become clear that you can win if you leave your opponent facing an odd number of moves to the end of the game, for example, in this situation, where there are only three left in three rows...



...the person about to move must lose. They take one plug, their opponent takes one plug and then they must take the last plug.

The investigation now becomes a search for winning positions like this. Make a classroom display of the winning strategies as they are found.

Further thoughts for investigation are:

- ♦ What happens if we arrange the plugs in rows of 1, 3, & 5 or 1, 3, 5, 7 & 9?

And for a little extra, notice the total of 1, 3, 5, & 7. It is 16, a square number.

- ♦ What is the smallest number of slides you could do to change the arrangement above into a square number?
- ♦ What about 1, 3, 5, 7, & 9?
- ♦ What happens if the plugs are actually arranged in a square instead of odd rows, but the rules are the same?

The game situation with its simple starting point, concrete materials, visual & tactile nature and mathematical discussion in community is the motivation for becoming involved in an interesting problem just like a mathematician. Working like a mathematician is therefore the core content of the investigation, but along the way children are reminded about odd and even numbers and square numbers and are challenged to develop their spatial perception.

For more ideas and discussion about this investigation visit the cameo for Task 120, *Nim* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 120 will be revisited in Year 3/4 Number & Computation and its companion Maths300 lesson will be revisited in Year 5/6 Space & Logic.

## Police Line Up

Photograph each of your children and print off the photographs as large as possible with names below. These photographs will become the suspects in a police investigation. (It is just as much fun, perhaps more, to use photographs of the teachers.)

Place three of the photographs on the floor and explain that these people are in a line of suspects from shortest to tallest. (Mention that you are pretending about the heights, that is, you are not using the real heights of the children or teachers.) Build a story shell around a visiting mathematician who sees the line up and later makes a clue puzzle about the order of the suspects. The idea is that someone else should be able to work out the order of the line up from the clues.

- ◆ Tell me some clues that could be in the puzzle. Use the names on the photographs.

Record all the suggestions, for example, using Principal, Art Teacher, My Teacher:

- ◆ The Principal is in the middle.
- ◆ My Teacher is to the left of the Principal.
- ◆ Art Teacher is taller than My Teacher.
- ◆ ...

Mess up the line up of photographs.

- ◆ Now suppose you were making the puzzle and you ONLY gave the clue that the Principal is in the middle. Show me how the line could be.

There are two solutions: MT/P/AT or AT/P/MT

- ◆ So the person who did your puzzle would need at least one more clue to decide which way is correct. What extra clue or clues would you give?

Explore possibilities.

- ◆ Today we are going to design puzzles like this and see if other groups can solve them. We are going to start with four photographs.

Add another photo to the collection but make sure they are not arranged in a line. Send the children back to their tables of four and give each one a piece of paper.

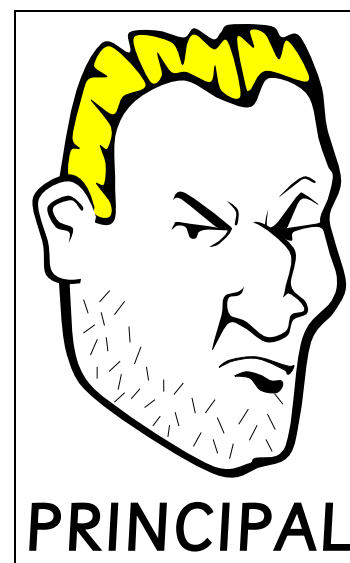
- ◆ First I want you to draw one of these teachers and write their name underneath.

You will probably have to institute a 'finish off your drawing in free time' arrangement because children of this age can be quite particular about their drawings.

- ◆ Now I want you to arrange your suspects in a line from shortest to tallest. It can be any way you like - forget their real height.

The challenge now is to decide and record some clues as group, test them and then challenge another group to solve the puzzle.

- ◆ What is the least number of clues you can write so your puzzle works?
- ◆ Could there be more than one answer to your puzzle?



### ◆ What happens if we use more suspects?

The investigation revises spatial relationships such as left, right next to, beside, to the right of and so on, but its primary purpose is the application of reasoning and communication skills. Teachers have added a number component to the investigation by assigning numbers to the suspects (like the Beagle Boys have numbers in Donald Duck comics) and you can see examples of children's work in this and other directions in the link below.

Some features that encourage children to be absorbed in this investigation are:

- ◆ connections to personal experience
- ◆ story shell
- ◆ multiple intelligences approach
- ◆ children's ownership of the materials
- ◆ mixed ability application - inclusivity

For more ideas and discussion about this investigation visit the cameo for Task 119, *Police Line Up* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 119 and its companion Maths300 lesson will be revisited in Year 3/4 Space & Logic. The Maths300 lesson includes clue sets for a range of age levels and examples of children creating puzzles based on number clues.

## Rod Mats

The picture shows a rod mat, made from Cuisenaire Rods, with Orange as the whole. Each of the other rows in the mat is made entirely from multiples of rods which exactly match the length of the whole.

Rods only have value because of their relationship to the whole. Change the whole, make a new mat, and you change the relationships.

ORANGE									
YELLOW						YELLOW			
RED		RED		RED		RED		RED	
W	W	W	W	W	W	W	W	W	W

So in this case Yellows are halves, Reds are fifths and whites are tenths. But we don't tell the children that. We ask questions designed to help them:

- ◆ Learn to recognise the conditions which determine the use of fraction language.
- ◆ Help them to investigate the particular rod mat.
- ◆ Orange is our whole today. It is worth one. Can anyone tell me the fraction name for the Yellow?

Some kindergarten children can say that is one half, so it is rare for that response not to appear in a Year 2 where this investigation is introduced. So now we probe more deeply.

- ◆ How do you know it's one half?

The first part of the response has to be (or has to become over time):

*Because I know what the whole is...*

Fractions make no sense without the whole. Before fraction language can be used meaningfully, the whole must be apparent.

The next part of the response has to be (or rather has to become over time):

*Yellow rods divide the whole into two parts and the parts are equal.*

It is not until those three conditions are checked:

- ◆ whole
- ◆ parts
- ◆ equality

that we can 'look in the fraction dictionary' and use the word half.

*There are two Yellows and we have chosen one of them, so one Yellow is one half.*

- ◆ Of...?

*The whole*

- ◆ Great! Count with me by halves...

*One half, two halves.*

- ◆ Which is also...?

*One whole.*

Add another Yellow to the row, and another, and another...

- ◆ Keep counting.

One whole and one half, one whole and two halves ... that's two ... two wholes and one half...

- ◆ Great! Now can anyone tell me the fraction name for the Red?...
- ◆ How do you know it's one fifth?

*Because we know what the whole is. Red rods divide the whole into five parts and the parts are equal.*

- ◆ whole? ...checked
- ◆ parts? ... checked
- ◆ equality? ... checked

The conditions allow us to use the word fifth. There are five fifths and we have chosen one of them.

♦ Great! Count with me by fifths...

And so on with tenths. By now children are bursting to tell you equivalent fractions:

*two tenths equals one fifth, ten tenths equals two halves ...*

even additions such as:

*one half equals two fifths plus one tenth ...*

and subtractions such as:

*one whole take away one fifth equals eight fifths.*

Record these *in words*. Encourage the children to make and trace the rod mat for today and record some of their own fraction equations *in words*.

Insist on words ... until they ask about using symbols. Then allow symbols *if* the children can consistently explain what the symbols mean.

These rod mat chats can be (must be) threaded into the curriculum in the same sense that learning 'language' has been since the children were born. More importantly the same language-based approach can be use for any fraction representation.

Contextualised, consistent use of language over time, matched with a concrete, visual reference point for discussion, in a mathematical community of peers sharing a growing understanding, is paramount to learning in this investigation.

Also, consistent with the growth of fraction concepts and skills throughout mathematical history, symbolic representation is the end point, not the starting point.

For more ideas and discussion about this investigation visit the cameo for Tasks 202 & 203, *Rod Mats & Make A Whole* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Tasks 202 & 203 will be revisited in Year 3/4 Number & Computation and their companion Maths300 lessons will be revisited in Year 5/6 Number & Computation.

## Row Points

Children have one yellow/blue Poly Plug board each. They are asked to turn over 13 plugs so that blue contrasts with yellow.

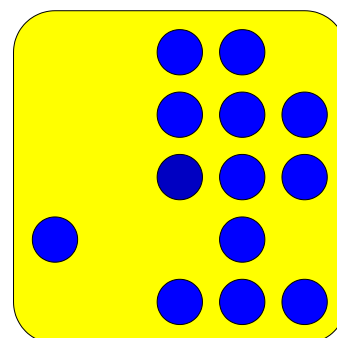
- ♦ Convince your partner that you have turned over 13?
- ♦ Now convince them in another way?

Ask a few children to demonstrate how they did their counting. Encourage responses that demonstrate visualising number rather than securely counting by ones. For example, in the board shown a child could see:

... 6 (like a dice) + 2 + 4 + 1

...  $3 \times 3 + 2 + 1 + 1$

... 5 (down) + 4 + 3 + 1



- ◆ Now I am going to tell you how you get points from your board. You are not allowed to change your board after I tell you.

Children score:

5 points for each five in a row

4 points for each four in a row

3 points for each three in a row

Rows can be horizontal, vertical or in either of the diagonal directions. So in the case of the board above:

Horizontal: 3 lots of 3 = 9

Vertical: 5 + 3 = 8

Diagonal falling right: 2 lots of 3 = 6

Diagonal falling left: 1 lot of 3 = 3

Total = 26

The teacher does *not* check these totals. It is the children's responsibility to convince their neighbour. While this is happening, the teacher writes rows of numbers across the board from 0 to 50.

- ◆ When you are sure of your total, come up and put your initials under it. Then record your picture on Poly Plug paper (see Page 112).

The data shows there are many answers and comparing plug boards shows that there can be several ways of getting the same total.

- ◆ No one has made zero yet. Let's make that our first challenge. Turn over 13 plugs but get a score of zero.

There are actually several ways to do this and some show symmetry - even rotational symmetry. Photograph the results and make a slide show of them for your electronic whiteboard.

- ◆ I can see plenty of other gaps in the list. Let's see how many totals we can find. Remember, write your initials up when you find one and record it on your Poly Plug paper.

Children will soon be able to explain why totals of 1 and 2 can't be found. Then the race is on to find all the others. So far, no one we know has found a score bigger than forty.

Among other things, children are captivated by the visual/kinaesthetic nature of the game, its openness, the number work in context, the incidental/informal teaching of number skills and ownership of the class data. These teaching craft features encourage the development and application of calculating skills and spatial reasoning in a problem solving context.

For more ideas and discussion about this investigation visit the cameo for Task 9, *Row Points* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 9 will be revisited in Year 3/4 Number & Computation.

## Spiders & Ants

This is a good investigation to link to a Creepy Crawlies theme. And it is a great one for giving you the chance to see what terrific mathematicians the children are.

Discuss the number of legs on an ant and how they are arranged. Look for ways to use objects in the classroom to arrange legs that way. Try out questions like:

- ◆ So if there are 4 ants, how many legs are there?
- ◆ Show me how you worked that out?
- ◆ Can you check it another way?
- ◆ Could you use your calculator to help you check?

Then go through the same process with spiders. The emphasis is on comparing and celebrating all the ways children can think of to answer these questions. The teacher will never have to say right or wrong. Children will convince each other.

The deeper investigation begins with a question like:

- ◆ I had a dream about spiders last night. But when I woke up I could only remember that there were 48 legs. Can you help me work out how many spiders there were? You can use anything in the room to help you.

*Grant decided to use the classroom clock to solve the 56 legs, how many spiders problem. He began with both hands on 12 and wound the big hand around to 56 minutes. Then he counted back 8 minute marks and put a stroke on a piece of paper. Then another 8, followed by another and so on until the minute hand was back at 12.*

*After we discussed all the approaches the children had used, and I had emphasised that they were great mathematicians because they were now able to check it another way, I asked 78 legs, how many ants? This time it was Carey and Nina who surprised me. Carey used a book from the shelf. He counted pages saying 1, 2, 3, 4, 5, 6. Nina put up a finger. Carey started at the first page again and counted 7, 8, 9, 10, 11, 12. Nina put up another finger. They kept this up until Carey reached 78 and Nina had used 13 fingers. Carey knew when he had counted each group of 6 pages because he always came to the one with the same picture.*

Features that fascinate and absorb children in this problem are the connection to knowledge outside maths, challenge, ownership of the approach to tackling the challenge, mathematics in community and multiple entry and exit points. The content involves counting, basic number calculations and concept development related to grouping, arrays, multiplication and division.

For more ideas and discussion about this investigation visit Maths300 Lesson 37, *Spiders & Ants*. If your school uses Maths With Attitude, Maths300 Lesson 37 will be revisited in Year 3/4 Number & Computation.

## Take A Chance

This is a card game that requires only one pack, which is for the teacher. The children use red Poly Plug as betting tokens. The investigation is intended to 'bring out' children's intuitive probability concepts and try to determine clues that guide them in making chance choices.

The children play in pairs so they have someone with whom to discuss decisions. They have one Poly Plug set between them. Children begin by removing 10 red plugs from their board. This is their kitty. Then the teacher shuffles the deck and places two cards on the table. Partners now decide whether to risk 0, 1, 2, or 3 red plugs on the chance of the next card being between the two on the table.

When everyone has placed their bets, the teacher turns over the next card.

- ◆ If it is between, partners keep the red plugs they risked in their kitty AND take the same number again out of the board to add to the kitty.
- ◆ If the next card is not between (ends are not between) the risked plugs have to go back into the board.

When the first round is completed, the teacher shuffles the deck again and play continues in this way until either one team collects 20 plugs in their kitty and wins or has all their plugs back in their board and loses.

It helps children to understand these elements if you play one game part way through in a fish bowl situation before starting the 'proper' game in pairs against the deck.

Discuss particular betweenes and the clues children use to make risk decisions. For example if the first two cards are A (=1) and 2, how many tokens would they risk? What about A and 3, A and 4, ...?

Once the game is learnt and some understanding of the chances has been developed an investigation question is:

- ◆ What is the best strategy to use to decide how many red plugs to risk?

One way to explore this is to count up the total number of plugs in the class pot when the game starts. So if there are ten pairs in the class, that total is 100. Then play the game through until one team wins or loses. Now total all the red plugs still in play. The children's strategies would be good if the end total was significantly more than the start total and not so good if the end total was significantly less than the start total.

Discuss possible strategies with a view to each pair adopting better strategies. As these better strategies are decided, the difference between the start pot and the end pot will be more and more positive. The overall focus becomes making the best choices so the class difference gets as high as possible.

The game context, small group and whole class mathematical discussion, the inclusive nature of the investigation and its connection to real life, all serve to captivate and absorb children in the investigation. The key concept of recognising that it is possible to use mathematics to make better choices about chance events is sound in both mathematics and social education.

For more ideas and discussion about this investigation visit the cameo for Task 49, *Take A Chance* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 49 will be revisited in Year 5/6 Chance & Measurement.



## Truth Tiles 2

The initial investigation is based around five consecutive numbers (3, 4, 5, 6, 7) and the equation:

$$\square + \square - \square = \square$$

In this case the boxes are simply places to put the number tiles and it is not the case that they all have to be the same. Interpret the equation as *...something plus something take away something is equal to something else, but we are only allowed to use 3, 4, 5, 6, 7.*

The investigative questions are:

- ◆ How many solutions are there to this equation using these numbers?
- ◆ How do we know we have found them all?

To start the investigation you will need 4 pieces of paper on the floor with +, -, = between them and some large digit cards. You also need another 'special testing place' for the children to try out their equations with plugs. Some will 'see' whether an equation works just from the numbers, but others will need this more concrete experience.

Exploring the problem together to find and record the first solutions has several benefits:

- ◆ Clarifying the problem in a non-threatening environment.
- ◆ Establishing a community of learners who celebrate ideas shared.
- ◆ Modelling how to keep a journal of an investigation.
- ◆ Confirming that any equation can be explored in a number of ways, for example, with materials, calculator or previous number bond knowledge.

Use number tiles to continue the investigation in pairs. If you don't have them, ask children to fold and tear a piece of scrap paper to quickly make substitute tiles.

The twelve basic solutions are listed in the link below, but there will be discussion about what makes solutions different. For example will your class decide that  $6 + 3 - 5 = 4$  is different from  $3 + 6 - 5 = 4$ ?

The investigation continues - with considerable depth if appropriate - when you explore what could be changed.

- ◆ What happens if we include one more consecutive number? ...two more? ...one less?
- ◆ What happens if we choose a different set of five consecutive numbers?

This investigation is easy to state and easy to start, it includes visual, kinaesthetic and conversational components and it models the work of a mathematician (see Page 11). Such features encourage children to become fascinated, captivated and absorbed and the open-ended nature of the problem, with its multiple solutions, means everyone can find success at their level and be challenged to step a little further.

The investigation revises and extends numbers bonds, uncovers properties of numbers, reveals patterns and encourages the use of problem solving strategies.

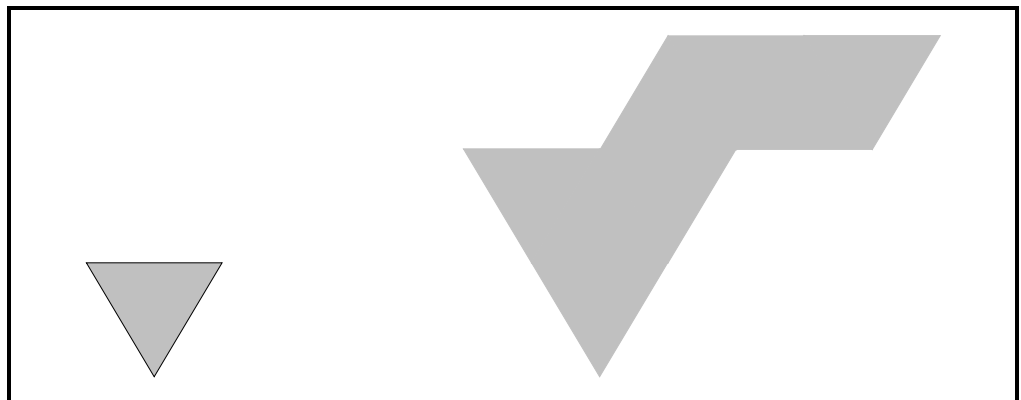
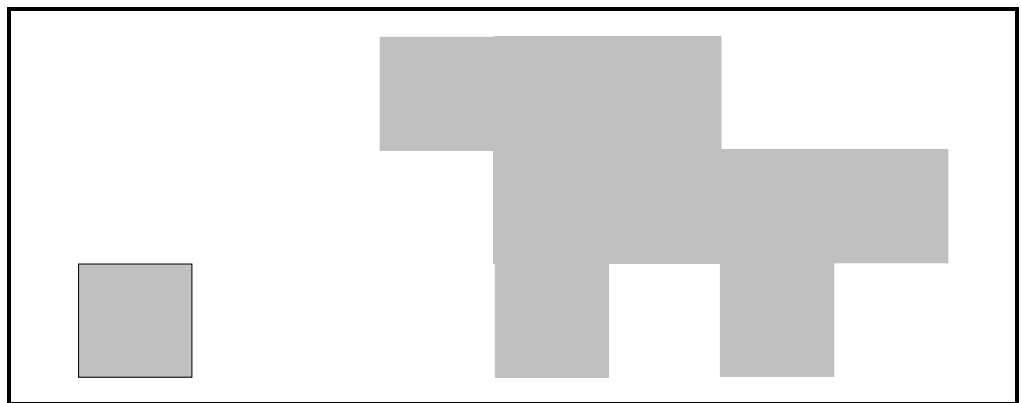
For more ideas and discussion about this investigation visit the cameo for Task 17, *Truth Tiles 2* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 17 will be revisited in Year 3/4 Number & Computation.

## What's It Worth?

The threaded activity **Poly Plug Values** assigns value to plugs according to colour. This investigation is similar except that the value is assigned to a shape and the challenge can be either about whole numbers or fractions depending on how the value is assigned.

Using tiles, pattern blocks, rods and similar items from the classroom store, you will need to create puzzle cards like those shown. Tracing onto black paper and pasting onto a lighter background card works well. In the large shape, only the outline is shown.

Children can make these for you if their tracing and cutting skills are sufficiently good. If you take this approach, you will have plenty of puzzle pages very quickly. Examples:



Next you produce a set of cards like this:

The small shape is worth 1.  
What is the big shape worth?

The big shape is worth 1.  
What is the small shape worth?

The small shape is worth 4.  
What is the big shape worth?

The big shape is worth 4?  
What is the small shape worth?

When all the materials are ready - and helping mums or older children can make this happen quite quickly - the children select one puzzle card and one question card to take to their table.

Firstly they have to cover the outline with the small shape. This is very important because the concrete materials often help the children calculate in the next section.

Secondly they try to work out the *What's It Worth?* question. Because they are learning to work like mathematicians they must ask:

- ◆ Can I check it another way?

When children are convinced of their answer, they make a record in their maths journal.

Try out the four cards above with the two puzzle cards above. Doing so will make clear the range of possible mathematics and will help you decide how to administer your card sets to differentiate for the range of experience in your class.

Ownership of the materials is a strong motivating factor in this investigation. This learning feature is supported by the concrete materials, calculator use, visual presentation and the range of achievable challenges.

The content involved includes spatial perception, whole/part and part/whole relationships and calculations with whole numbers and fractions.

For more ideas and discussion about this investigation visit the cameo for Task 75, *What's It Worth* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 75 will be revisited in Year 3/4 Number & Computation and its companion Maths300 lesson will be revisited in Year 5/6 Number & Computation.

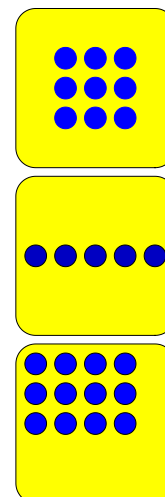
## Where Is The Rectangle?

In this investigation the aim is to identify the four (sometimes two) corner plugs of a rectangle that one player has hidden in a Poly Plug board, as in the examples, and to do so in the least number of guesses.

The game involves a Hider and a Finder. You will need to introduce the convention for identifying the position of a point in two dimensions. To do so we need two numbers - one counting horizontally and one counting vertically. We also need to know the starting point for the counting.

Hold up a board and point to the bottom left plug (from the children's view).

- ◆ If we say this plug is at zero in this direction (horizontal) and zero in this direction (vertical) what is the position of this plug (...point to one).



Work with the children to identify each position. There are 25 plugs in the board so just about one for each child. If you press them up from behind as you ask for the position it helps the children see which one you mean and its connection to the others around it.

Now play the game against one person, with the help of the rest of the class, to demonstrate how you use a Poly Plug Frame (Page 113) to help you find the rectangle. An overhead transparency of the Frame, or an equivalent on the electronic whiteboard, will be useful in this phase. The rules are:

- ◆ Hider secretly makes equal rows of plugs and tells Finder the **total number of plugs** turned over.
- ◆ Finder writes this total above the Poly Plug Frame and guesses a pair of numbers - by convention, horizontal comes first.
- ◆ Hider tells whether this plug is on the edge, inside or outside the rectangle of equal rows.
- ◆ Finder records this answer and continues guessing until the four (or in some cases two) corners can be identified. *Your corners are...*
- ◆ Hider must not rotate their board once the game has started.

*Where Is The Rectangle?* richly interconnects space and number. Perhaps the spatial component is obvious, but the rectangles are arrays and open up opportunities for learning about factors, prime numbers, and times tables.

The children will want to play more than once, so use Poly Plug Paper (p. 112) to record. Five turns each and keeping a running total of guesses is a good game.

To extend the investigation switch to square line paper (p. 114). Players first agree on the largest size rectangle within which they will play and both draw this as a border. Then Hider marks a rectangle inside this border - use crosses - and the game continues as before.

Learning is fostered through this investigation by the game context, visual/kinaesthetic learning, mathematical conversation, concurrent teaching of topics and connection to real life map reading experiences.

For more ideas and discussion about this investigation visit the cameo for Task 114, *Where Is The Rectangle?* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>.

## Which Floor

You will need large digit cards to use outside or in a hall. The story shell is about being in a building with a lift (elevator). A person uses the lift to do an up/down journey of several steps. You know the height of the building and the floor the person finishes. The objective is to work out the floor on which the person starts. For example:

*Rebecca works in a ten storey building. She enters the lift and goes down five floors, up six floors and down seven floors. She gets out on the second floor. Which floor did she start her journey?*

Lay out the cards 0 to 10. Zero is the ground floor. Present the problem (perhaps you have written it out already on a chart) and ask the children what to do. Encourage discussion and whole body experiment. Working backwards, guess and check and even writing some form of equation are all possible approaches to finding the answer 8.

Divide the class into teams and ask each to think up a similar puzzle for other teams to try. Each team needs to work a little bit in secret so their plans are not overheard. They can see, but not walk on the building. This phase helps to develop ownership of the type of problem.

Return to the original Rebecca problem and ask:

- ◆ If Rebecca did the same journey - down 5, up 6, down 7- in the same building, but did NOT finish on the second floor which floor did she start on?

Now the problem is beginning to open up. There is more than one solution.

- ◆ How many solutions are there?
- ◆ How do we know we have found them all?

The investigation extends further when we discuss what could be changed in these problems:

- ◆ the height of the building
- ◆ the journey
- ◆ the finishing floor

Now *Which Floor* becomes a choose your own adventure problem. Children can select, explore and publish findings for their own problem - just like mathematicians. Poly Plug can be used to make table top versions of the problem. Red boards with a column of gaps can be the buildings as high as necessary and a yellow/blue plug can be used as a person taking a journey in the lift.

Problem solving strategies feature in this investigation and encourage skills such as counting, addition and subtraction and even a little equation writing.

The story shell, outdoor learning, physical involvement, group work, open-ended challenge and concrete materials all help the children to become fascinated, captivated and absorbed.

For more ideas and discussion about this investigation visit the cameo for Task 52, *Which Floor* at <http://www.mathematicscentre.com/taskcentre/iceberg.htm>. If your school uses Maths With Attitude, Task 52 will be revisited in Year 3/4 Number & Computation.



# *Part 3: Value Adding*

# More on Professional Development

For many teachers there will be new ideas within *Working Mathematically with Infants*, such as: Threading, emphasis on mathematical discussion and articulating learning and Investigations with young children. It is anticipated (and expected) that as teachers explore the material in their classrooms they will meet, experiment with and reflect upon these ideas with a view to long term implications for the school program and for their own personal teaching.

Exploring new ideas in the teacher's own classroom, makes the professional development more meaningful and practical for the teacher. Our six day model sandwiches classroom trialling between intense out of school sessions creating a practical and economic approach for a local authority.

## Strategic Use by Systems

*Working Mathematically with Infants* is designed to be used by schools or clusters or systems because they see it as carrying appropriate messages to their teachers. It could be used to:

- ◆ introduce Threading as a planning technique encouraging and supporting children to construct their own learning.
- ◆ raise or continue debate about pedagogy (art of teaching) that supports deeper mathematical learning for a wider range of children.
- ◆ highlight how Investigations can be built into planning without compromising skill development and without being relegated to the margins of a syllabus to be done only after 'the real' content has been covered.
- ◆ focus on how a balance of concept, skill and application work can all be achieved within the one manageable unit structure.
- ◆ develop a variety of assessment practices.

In supporting its teachers by supplying *Working Mathematically with Infants* in conjunction with professional development meetings which explore its use, a system can fuel and encourage classroom-based debate on any of these features.

Such professional debate has been shown, over time, to change teaching practice and improve learning outcomes. Examples based on our 6 day course *Engineering 'aha' Moments K-8* are given below. We would be happy to discuss this course with system leaders, or assist in developing others to meet your needs.

We also offer courses specifically in *Working Mathematically with Infants* and two hour and one day programs to involve the whole school in *Calculating Changes*.

## Engineering 'aha' Moments in Number K-8

The course consists of 3 sets of two day workshops with school trialling between:

### Days 1 & 2

- ◆ Background
- ◆ Number Bonds & Tables
- ◆ Place Value
- ◆ Fractions & Challenges

### Days 3 & 4

- ◆ Refresh, Report & Multiply
- ◆ Investigating Tasks
- ◆ Tasks & Poly Plug
- ◆ Task Based Units

### Days 5 & 6

- ◆ Refresh, Report, Reflect
- ◆ Engineering More Moments
- ◆ Building More Structures
- ◆ Leading The Way



At the end of the first two days teachers are challenged to select an activity, thread it into their curriculum and report. Teachers at one of these courses offered the following reflections at the beginning of Days 3 & 4. They indicate the influence the program can have on teachers' practice.

### **Aaron and Stephanie: Years 1/2 & 7**

Our Buddy system has proved to be a very effective learning strategy.

#### **Background**

Last year, the current Year 7 students were split into two classes - segregated girls Year 6, boys Year 6 part way through the year. This resulted in a more stable learning environment and was continued with this group of students for Year 7.

The boy's class in particular has benefited from the segregation. Interestingly many 'behaviour problem' boys take the mentoring of their Year 1 buddy very seriously and are enjoying the novel situation of being looked up to and depended upon by someone much younger.

#### **Example: Prediction Game**

We adapted the Calculating Changes activities titled Plug Snakes (p. 29) and Fill The Board (p. 67) to the Buddy structure. The Prediction Game was the latter part of the unit and is an adaptation of Fill The Board.

The Year 1 students were given the task of deciding/'guessing' how many throws of the dice it would take to fill the Poly Plug frame. Initial guesses ranged from 4 - 20. After playing the game 4 times the prediction had narrowed to a much smaller range.

Interesting proof that statistics don't give all the story...

#### *Case Study 1*

*Chelsey (Year 1) predicted firstly:*

*10 ...because my sister Chloe is 10, later*

*6 ...because I am 6,*

*9 ...because I like to write it and*

*8 ...because Brock is 8 (her brother).*

*All of these predictions were reasonable in terms of the problem but the numbers chosen had nothing to do with playing the game and watching the results.*

#### *Case Study 2*

*Teagan (Year 1) initially chose:*

*6 ...too small,*

*10 ...too big, then chose*

*8 ...still too big, and lastly*

*7 ...just right!*

*and articulated clearly why she did it.*

Reports from other participants in this course were very interesting and gave many ideas on how to use/extend materials and games presented in the first two days of the course. One thread running through the reports was the value of getting students to articulate what they were doing and why. So often, correct results are achieved but for the wrong

reason - this came up a lot. The activities and materials we are using give great opportunity to monitor and assess individual students accurately but in a non-threatening and non-obtrusive way.

Maths can, and should be, achievable and fun.

### **Amy: Years 1/2**

What inspired me most this morning (in the reflection session)?

#### **Working Mathematically with a Buddy Class**

Being from a community school we are constantly looking for ways to connect the different classrooms to be a community. My children are aged 7-8 years old therefore I could connect the children quite easily with younger and older children using the Poly Plugs and calculator games we have been given.

#### **Articulating Maths**

Allowing time for your class to articulate their maths strategies is something that should be encouraged. It gives you the ability to understand what they know and find out if they are working mathematically or not. Through the use of Poly Plugs and the calculators the children are relaxed and happy so this articulation seems to occur with ease.

### **Claire: Year 2**

The activity I chose to thread in my Year 2 classroom was **Predict A Count**. All students began the activity counting by ones from 10. Although setting the students up for the activity took a while I really liked the way the students took to the task and were able to extend themselves, varying their starting numbers and what they counted by. Some students were counting by sevens, starting at a number between 20 and 100.

I like that I am able to support my students who have difficulty following the 1 - 9 counting pattern by allowing them to work at their own pace/level. They are counting by ones from their chosen starting point. I thoroughly enjoy watching the students pump up their arm saying "Yes" when their predictions are confirmed correctly. Some great 'aha' moments.

It was great hearing the enthusiasm from my colleagues in relation to their implementing the activities within their classrooms. The video footage clearly showed the students' enjoyment. I am constantly trying to think of ways to improve my teaching. The importance of getting students to verbalise and explain how they are making their predictions came up during the morning session. This is something I am realising more and more is vital to the students' learning. Allowing time and opportunities for reflection!

And from a different course, a similar story:

### **Meg: Year 2/3**

Lots of great ideas - makes maths fun. New ways of teaching old skills, processes, concepts. Challenging activities/games that help children internalise their learning. Using the calculator - a valuable tool to support learning. Allows children to get involved in the game/concept without becoming 'bogged down' in the calculations.

To find out more about *Engineering 'aha' Moments in Number K-8* visit:

[http://www.mathematicscentre.com/taskcentre/prog\\_lib.htm#ahaprimary](http://www.mathematicscentre.com/taskcentre/prog_lib.htm#ahaprimary)

# Maths Task Centre

In teaching from the point of view that all children can learn to work like a mathematician, we juggle our curriculum to:

- ◆ model what it means to work like a mathematician
- ◆ invite children to 'be' independent mathematicians
- ◆ practise the skills of a mathematician

invitation



modelling



skills



These elements are built into *Working Mathematically with Infants*, but we can add more resources designed to further encourage independent mathematicians. These resources are hands-on mathematics tasks from the Mathematics Task Centre.

Several Investigations in *Working Mathematically with Infants* are built on the whole class life of these tasks. But they can also be offered to pairs (or small groups) of children as invitations to work independently. There is a certain level of literacy competence required to use these tasks 'completely' independently, but many can be tackled successfully by Year 2 children.

Younger children, or those with less developed literacy can still be encouraged by using adult helpers in the class, or by developing a buddy system with older children. In many ways using these task cards can be thought of as practical situations encouraging the application and development of literacy. The card with its equipment, in a sense, makes reading concrete.

Many teachers in many schools have used tasks in many ways, another example being:



*One school we visited many years ago, which had developed its own infant task set, used large poster size problem cards. The students could gather around them on the floor. They were hand written on coloured card and laminated. The storing process was clever too. The cards were pegged to coat hangers hung from a rolling home-laundry drying rack.*

To find out more about Maths Tasks visit the Mathematics Task Centre:

<http://www.mathematicscentre.com/taskcentre>

# Beyond Year 2

## Calculating Changes

Your Calculating Changes membership, which is a once only fee, provides access to the site for all levels of the school, regardless of the number of children or staff. Teachers are also welcome to access from home.

It is only the activities suitable for Years K-2 that have been extracted for *Working Mathematically with Infants*. Many of those, you will soon realise, can be used at other levels simply by changing the challenge. **Predict A Count**, for example, can be used at any level because counting can be by 1 or 10 or 0.1 or  $\frac{1}{10}$  or ... There are also many activities specifically for classes beyond Year 2. A Year Level Finder and a Content Finder on site in the Activities link will support your colleagues in selecting appropriate activities.

Access to the Members section of the site is from:

<http://www.mathematicscentre.com/calchange>

## Maths With Attitude

Maths With Attitude eManuals are designed to help teachers of Years 3-10 create:

happy, healthy, cheerful, productive, inspiring classrooms

in which children learn to work like a mathematician.

They offer a real alternative to text-based mathematics learning and support the objectives of any official curriculum document. In each eManual

- ◆ Hands-on tasks
- ◆ Whole class investigations
- ◆ Skill development in context, and
- ◆ Integrated software (if the school is a Maths300 member)

are drawn together in a detailed week by week schedule that:

- ◆ supports all strands of the curriculum
- ◆ dovetails with textbook use
- ◆ encourages inclusion of the teacher's personal best practice

MWA provides 25 weeks of hands-on, small group and whole class investigative curriculum for each and every year from Year 3 to Year 10. It picks up from the 10 weeks per semester planning offered in *Working Mathematically with Infants* by offering 10 investigative weeks of Number & Computation in Years 3 & 4 and adding 15 more weeks of investigative work in other strands. This planning pattern continues into Years 5 & 6 with 8 weeks in each year of Number & Computation and 17 weeks spread across other strands.

### Features of MWA eManuals

- ◆ Published for your school and delivered by download at a very reasonable price.
- ◆ Each manual is specific to year level and curriculum strand.
- ◆ It is assumed that schools have a set of eTasks, or a set of the earlier ready-made tasks, to invite children to work like a mathematician for themselves.

- ◆ Whole class lesson outlines to model working like a mathematician.
- ◆ Each manual includes a week by week plan for integrating the tasks and lessons into a Working Mathematically curriculum.
- ◆ Schools with Maths300 membership will have access to more detailed lesson plans and software to extend some of the investigations.

Trial schools have shown that a curriculum integrating both can lead to substantial success for children. Self-directed professional development is supported and external professional development is also available.

As with *Working Mathematically with Infants*, these manuals are convenient, compact and complete. One teacher remarked:

*This is brilliant. I have been trying to use a combination of tasks and whole class investigations, but I have always felt it was a bit piecemeal. Now I can organise a much better problem solving curriculum.*

For further information about Maths With Attitude visit:

<http://www.mathematicscentre.com/taskcentre/mwa.htm>

## The Poster Problem Clinic

One approach to getting the class started on using tasks and giving their use a sense of direction and purpose other than as 'a game' is to start with a whole class problem. The problem may be a task, as below with *Eric The Sheep*, or any other suitable problem. One such set is the Professor Morris Puzzles:

<http://www.mathematicscentre.com/taskcentre/resource.htm#profmorr>

which have been successfully used with children from Year 2 to Year 8. Whatever the source of the problem the concept of a clinic is just as it is in sport - a training time for the big game.

*I was apprehensive about using tasks when it seemed such a different way of working. I felt my children had little or no experience of problem solving and I wanted to prepare them to think more deeply. The Clinic proved a perfect way in.*

Usually the chosen problem (the starting point for the work of a mathematician) is displayed on a poster that all can see, perhaps in a Maths Corner. Another approach is to print a copy for each person. A Poster Problem Clinic fosters class discussion and thought about problem solving strategies.

One teacher describes the session like this:

*I like starting with a class problem - for just a few minutes - it focuses the class attention, and often allows me to introduce a particular strategy that is new or needs emphasis.*

The problem doesn't have to be completed now. In fact, it is better if children are encouraged to take it home and work with their parents in a 'maths around the kitchen table' context. The return to school and whisper to their teacher, or post in a special box, discoveries their family has made. At the end of the week the teacher organises a plenary session to gather and share all the contributions.

It only takes a short time to introduce a poster and get some initial ideas going. The class discussion develops a way of thinking. It allows class members to hear and learn from their peers about problem solving strategies that work for them.

*If we don't collectively solve the problem in 5 minutes, I will leave the problem 'hanging' and it gives a purpose to the class review session at the end.*

*Sometimes I require everyone to work out and write down their solution to the whole class problem. The staggered finishing time for this allows me to get organised and help students get started on other activities without being besieged.*

## A Clinic in Action

The aims of the regular clinic are:

- ◆ to provide children with the opportunity to learn a variety of strategies
- ◆ to familiarise children with a process for solving problems

The following example illustrates a structure which many teachers have found successful when running a clinic.

## Preparation

For each session teachers need:

- ◆ a Strategy Board created from Working Mathematically (p. 11)
- ◆ a How To Solve A Problem chart created from Working Mathematically (p. 11)
- ◆ to choose a suitable problem and prepare it as a poster
- ◆ to organise children into groups of two or three

The Strategy Board can be prepared in advance as a reference for the children, or may be developed *with* the children as they explore problem solving and suggest their own versions of the strategies.

The example which follows is from the task collection. The teacher copied it onto a large sheet of paper and asked some children to illustrate it. *The teacher also changed the number of sheep to sixty* to make the poster a little different from the one in the task collection.

# The Clinic

The poster used for this example session is:

Eric the Sheep is lining up to be shorn before the hot summer ahead. There are sixty [60] sheep in front of him. Eric can't be bothered waiting in the queue properly, so he decides to sneak towards the front.

Every time one [1] sheep is taken to be shorn, Eric then sneaks past two [2] sheep. How many sheep will be shorn before Eric?

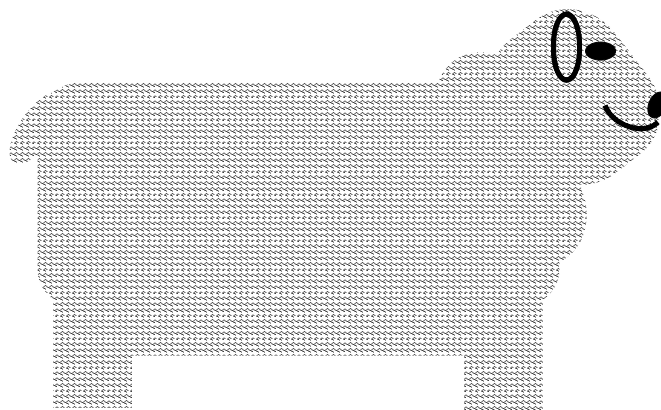
This Poster Problem Clinic approach is also extensively explored in Maths300 Lesson 14, *The Farmer's Puzzle*.

## Step 1

- ◆ Tell the children that we are at Stage 1 of our four stage plan ... **See & Understand** ... Point to it! Read the problem to/with the class. Discuss the problem and clarify any misunderstandings.
- ◆ If children do not clearly understand what the problem is asking, they will not cope with the next stage. A good way of finding out if a child understands a problem is for her/him to retell it.
- ◆ Allow time for questions - approximately 3 to 5 minutes.

## Step 2

- ◆ Tell the children that we are at Stage 2 of our four stage plan ... **Planning**. In their groups children select one or more strategies from the Strategy Board and discuss/organise how to go about solving the problem.
- ◆ Without guidance, children will often skip this step and go straight to Doing It. It is vital to emphasise that this stage is simply planning, not solving, the problem.
- ◆ After about 3 minutes, ask the children to share their plans.



## Plan 1

*Well we're drawing a picture and sort of making a model.*

Can you give me more information please Brigid?

# Strategy Board

DO I KNOW A SIMILAR PROBLEM?

GUESS, CHECK AND IMPROVE

TRY A SIMPLER PROBLEM

WRITE AN EQUATION

MAKE A LIST OR TABLE

WORK BACKWARDS

ACT IT OUT

DRAW A PICTURE OR GRAPH

MAKE A MODEL

LOOK FOR A PATTERN

TRY ALL POSSIBILITIES

BREAK THE PROBLEM INTO PARTS

SEEK AN EXCEPTION

...

...

## How To Solve A Problem

SEE & UNDERSTAND

Do I understand what the problem is asking? Discuss

PLANNING

Select a strategy from the board. Plan how you intend solving the problem.

DOING IT

Try out your idea.

CHECK IT

Did it work out? If so reflect on the activity. If not, go back to step one.



*We're putting 60 crosses on our paper for sheep and the pen top will be Eric. Then Claire will circle one from that end, and I will pass two crosses with my pen top.*

## Plan 2

*Our strategy is Guess and Check.*

That's good Nick, but how are you going to check your guess?

*Oh, we're making a model.*

Go on ...

*John's getting MAB smalls to be sheep and I'm getting a domino to be Eric and the chalk box to be the shed for shearing.*

## Plan 3

*We are doing it for 3 sheep then 4 sheep then 5 sheep and so on. Later we will look at 60.*

Great so you are going to try a simpler problem, make a table and look for a pattern.

This sharing of strategies is invaluable as it provides children who would normally feel lost in this type of activity with an opportunity to listen to their peers and make sense out of strategy selection. Note that such children are not given the answer. Rather they are assisted with understanding the power of selecting and applying strategies.

## Step 3

- ◆ Tell the children that we are at Stage 3 of our four stage plan ... **Doing It.** Children collect what they need and carry out their plan.

## Step 4

- ◆ Tell the children that we are at Stage 4 of our four stage plan ... **Check It.** Come together as a class for groups to share their findings. Again emphasis is on strategies.

*We used the drawing strategy, but we changed while we were doing it because we saw a pattern.*

So Jake, you used the Look For A Pattern strategy. What was it?

*We found that when Eric passed 10 sheep, 5 had been shorn, so 20 sheep meant 10 had been shorn ... and that means when Eric passes 40 sheep, 20 were shorn and that makes the 60 altogether.*

Great Jake. How would you work out the answer for 59 sheep or 62 sheep?

Sharing time is also a good opportunity to add in a strategy which no one may have used. For example:

*Maybe we could've used the Number Sentence strategy, ie: 1 sheep goes to be shorn and Eric passes two sheep. That's 3 sheep, so perhaps, 60 divided into groups of 3, or  $60 \div 3$  gives the answer.*

Round off the lesson by referring to the Working Mathematically chart. There will be many opportunities to compliment the children on working like a mathematician.

# A Working Mathematically Curriculum

## An Investigative Approach to Learning

The aim of a Working Mathematically curriculum is to help children learn to work like a mathematician. This process is detailed earlier (p. 11) in a one page document which becomes central to such a curriculum.

The change of emphasis brings a change of direction which *implies and requires* a balance between:

- ◆ the process of being a mathematician, and
- ◆ the development of skills needed to be a *successful* mathematician.

This journey is not two paths. It is one path made of two interwoven threads in the same way as DNA, the building block of life, is one compound made of two interwoven coils.

When ***Working Mathematically with Infants*** and ***Maths With Attitude*** are used as an integrated platform through primary school, the Working Mathematically process is central to and grows from a child's first school day. Secondary teachers can then pick up on this progress by using the Maths With Attitude eManuals developed for their classrooms. A key focus for the Working Mathematically teacher at every level is to help children develop mathematical skills in the context of problem posing and solving.

*We are all 'born' with the same size mathematical toolbox, in the same way as I can own the same size toolbox as my motor mechanic. However, my motor mechanic has many more tools in her box than I and she has had more experience than I using them in context. Someone has helped her learn to use those tools while crawling under a car.*

Referring to traditional mathematics education, Afzal Ahmed, Professor of Mathematics at Chichester, UK, once quipped:

*If teachers of mathematics had to teach soccer, they would start off with a lesson on kicking the ball, follow it with lessons on trapping the ball and end with a lesson on heading the ball. At no time would they play a game of football.*

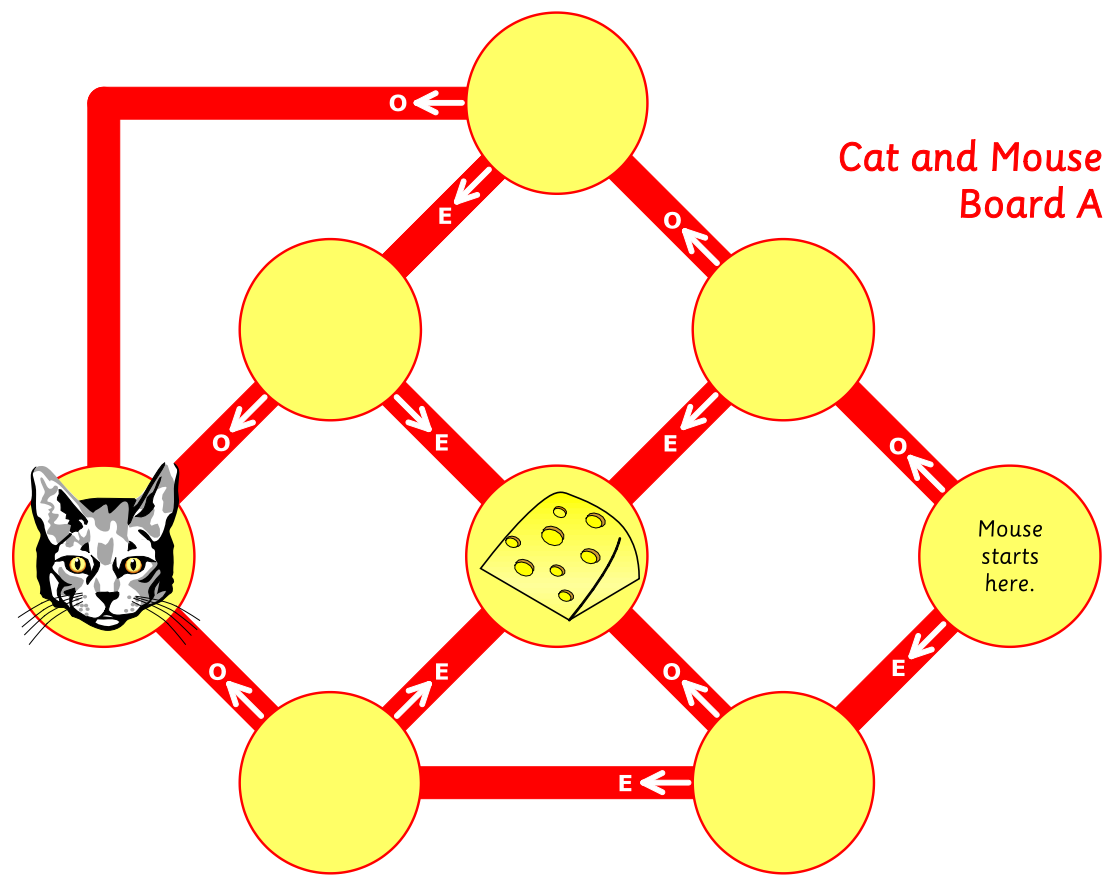
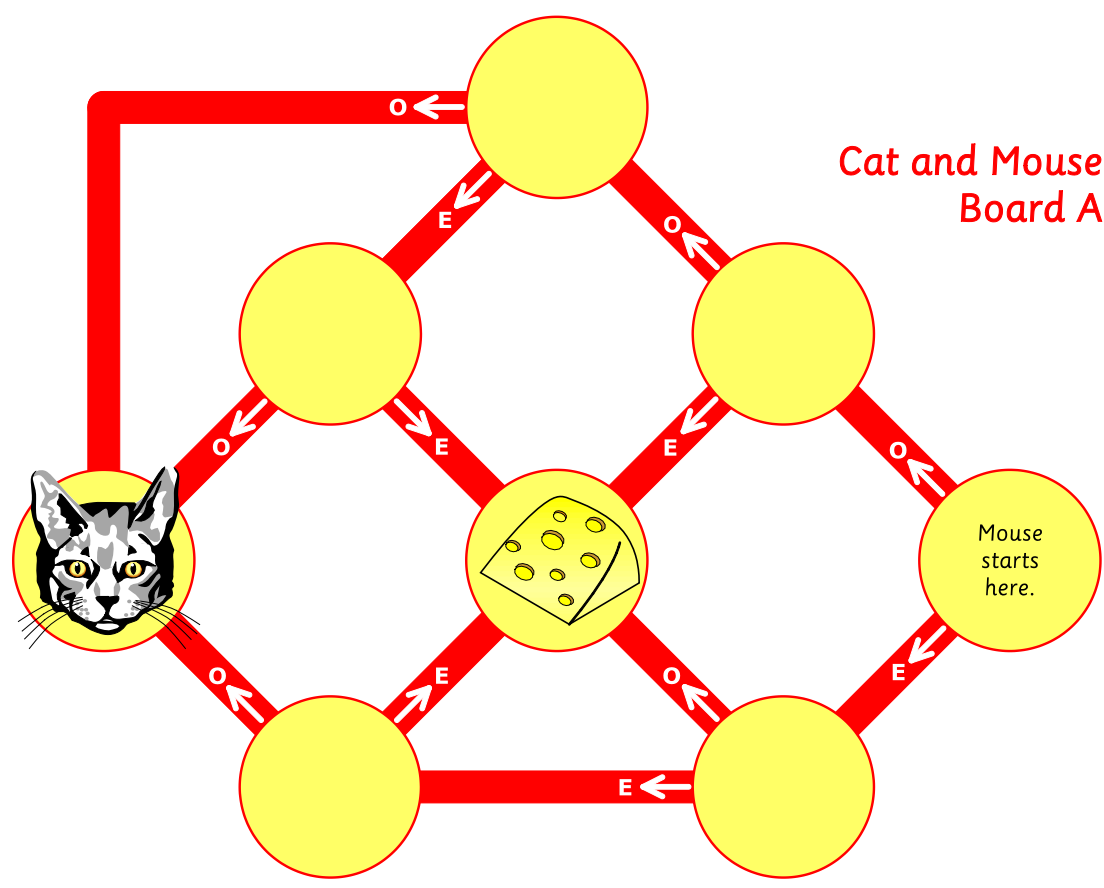
Such is not the case when teaching a Working Mathematically curriculum.

# ***Appendix: Recording Sheets***

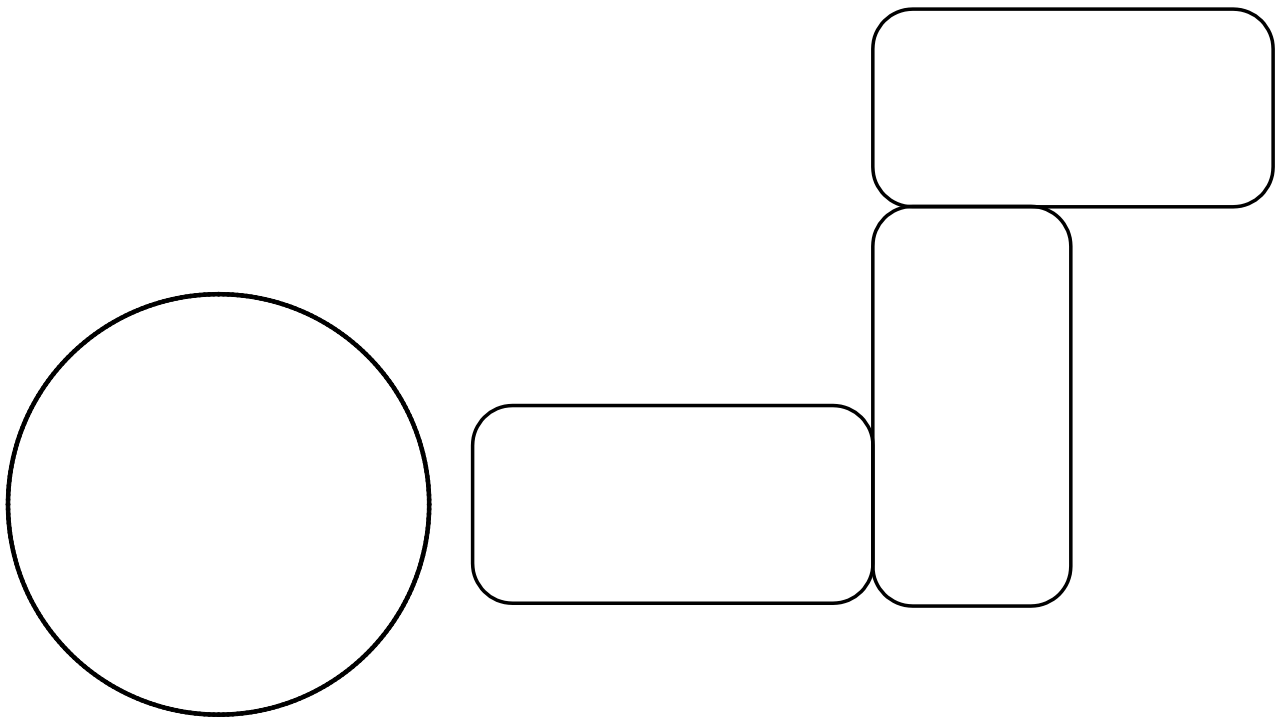
# Curriculum Planner

Year \_\_\_\_ ... Semester \_\_\_\_

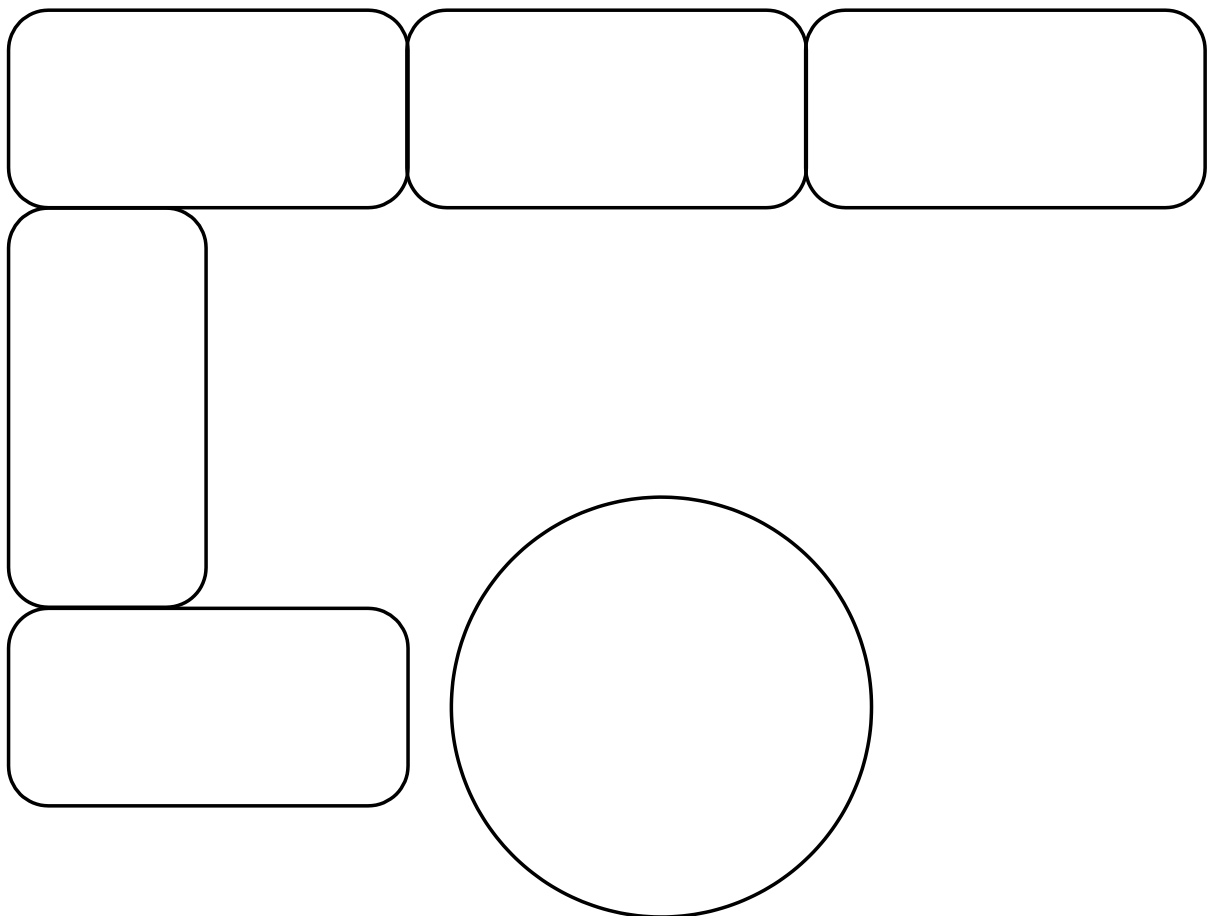
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<b>Week 2</b>				
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<b>Week 10</b>				



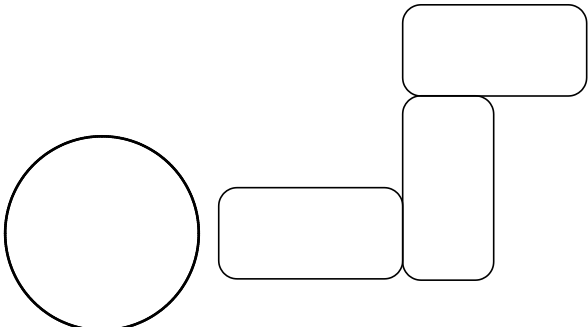
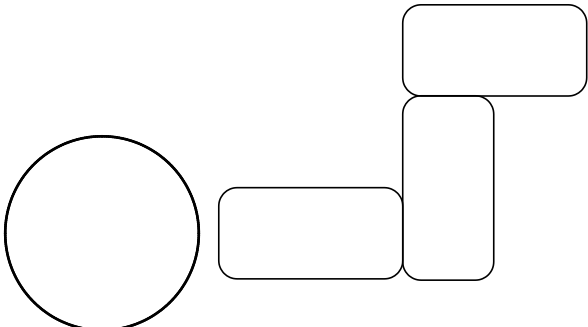
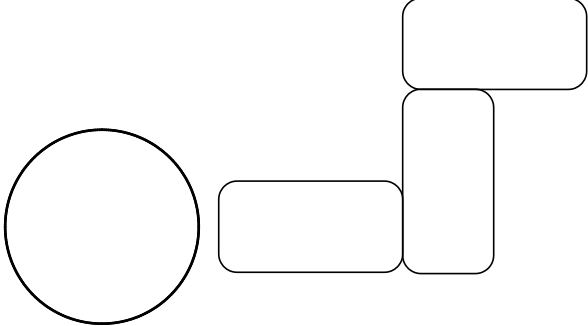
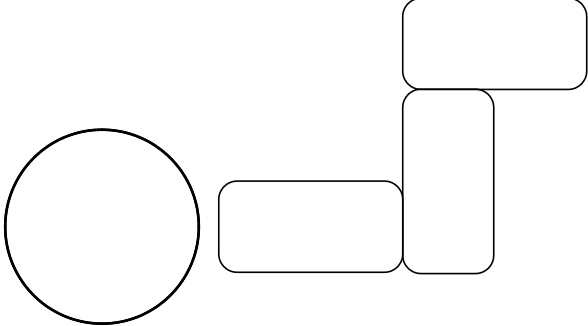
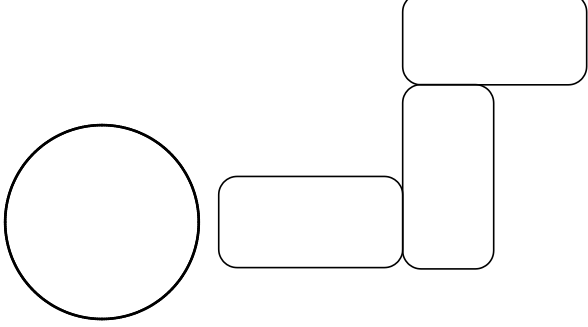
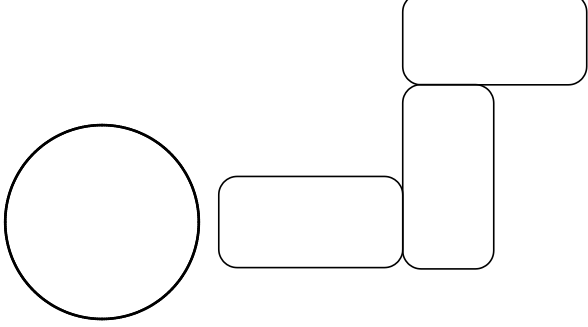
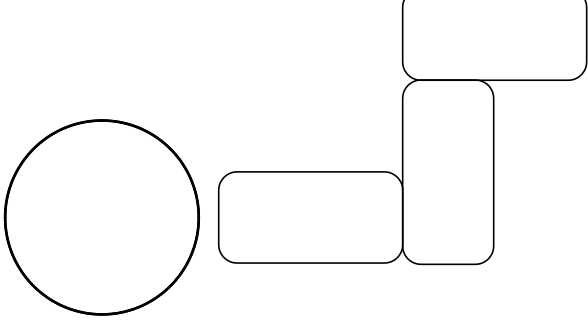
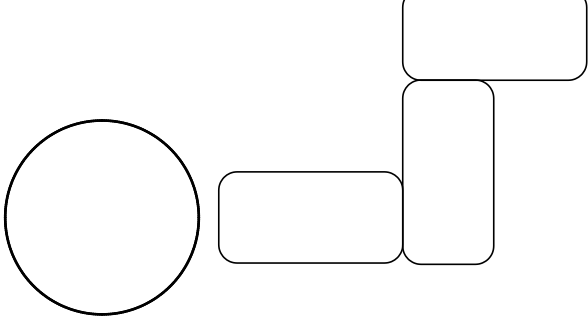
### 3 Domino Trail



### 5 Domino Trail



# 3 Domino Trails

# 5 Domino Trails

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# Farmyard Friends

## Animal Pens

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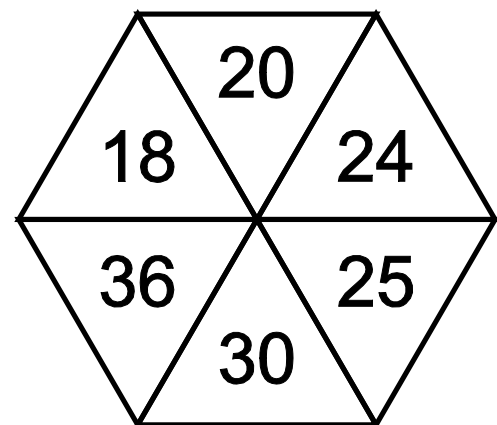
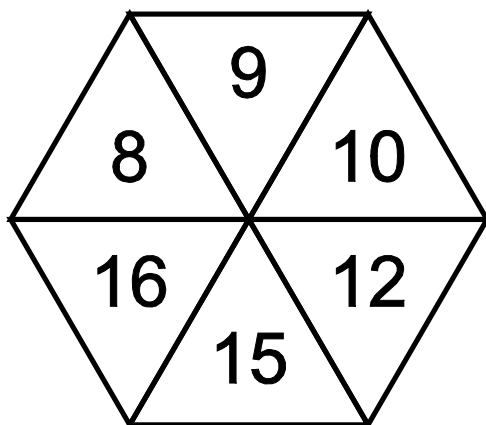
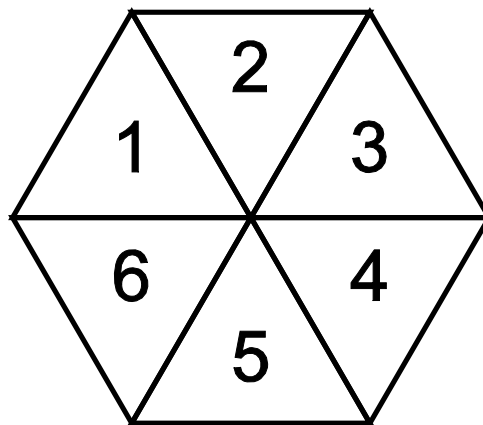
Cut out the cards and draw one animal on each one.

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# Have A Hexagon

Which hexagon will be finished first?

Top One? ... Left One? ... Right One?

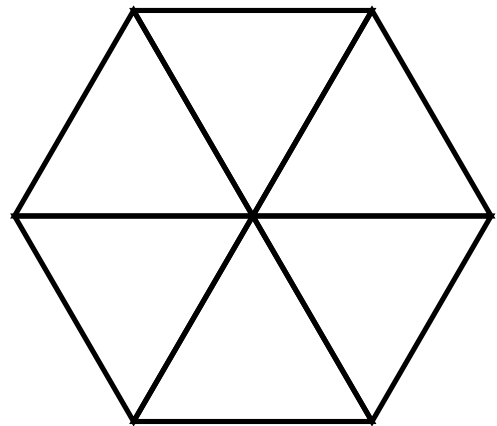
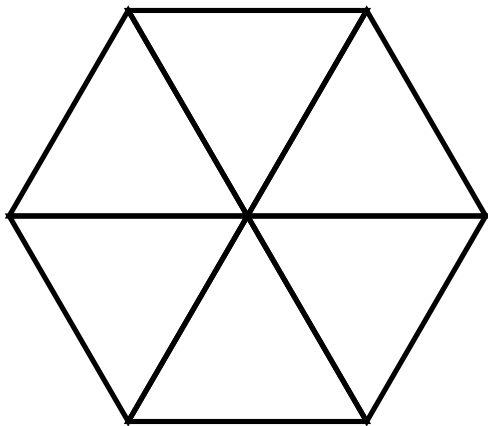
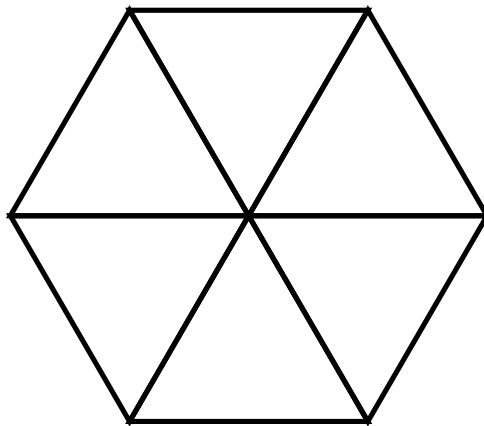


- ♦ Take turns to roll two dice and multiply the numbers.
- ♦ Place a red plug beside the answer - don't cover the answer.
- ♦ Stop playing when one hexagon has a plug next to every number.
- ♦ If an answer is repeated you can stack plugs beside the number.

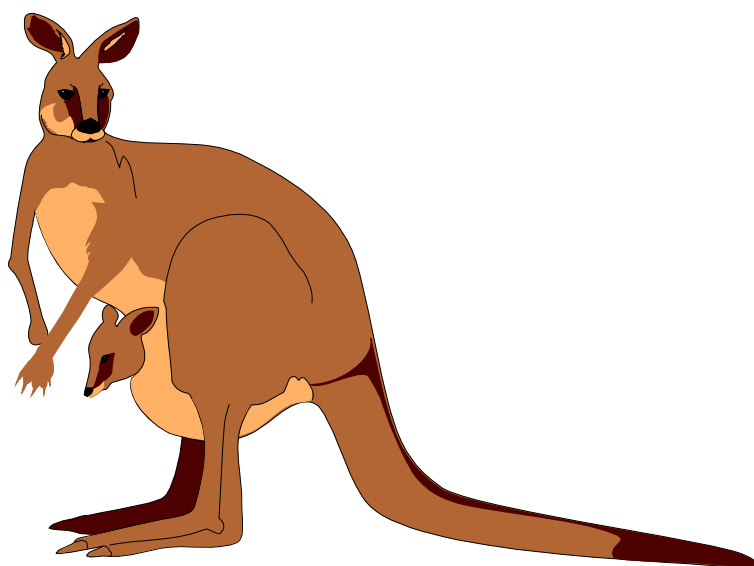
# Have A Hexagon

Which hexagon will be finished first?

Top One? ... Left One? ... Right One?



- ◆ Arrange the numbers your own way on the hexagons
- ◆ Try to do it so each hexagon has an equal chance.
- ◆ Play the game a few times to test your hypothesis.



## JUMPING JOEY

Once upon a time, in the Australian bush, there lived a young kangaroo named Jumping Joey. Jumping Joey thought that he could jump over anything and spent most of his time boinging around the country looking for new things to leap over.

One particularly fine day he was boinging and bouncing through the bush when he spied Mrs. Koala up ahead hanging out the washing.

"I could jump that clothesline easily," he thought and without even stopping to stretch his legs he bounded up and over both the clothesline and Mrs. Koala.

Mrs. Koala was not pleased. Jumping Joey had given her such a fright.

"Can't catch me," J.J. shouted as he boinged off down the track.

"Can't I?" grumbled Mrs. Koala and she puffed and panted and waddled after him.

J.J. found that the track led to a country town and he actually stopped in the middle of a very bouncy boing when he saw that there was a circus camped on the edge of the town. He had read about circuses in the books his mum kept for him to read at night when he was curled up in her pouch, but he had never actually seen one. He had read about elephants too and now he could see not one, but two.

"I could jump two elephants easily," he thought and without even stopping to stretch his legs he bounded up and over both those elephants.

The two elephants were not pleased. "Who does that little fellow think he is?" they trumpeted.

"Can't catch me," J.J. shouted as he boinged back into the bush with two elephants thundering after him and Mrs. Koala puffing and panting and waddling along behind.

J.J. came to a billabong. He was a long way ahead of the animals that were chasing him so he stopped for a drink.

"I could jump across this billabong easily," he thought, "but this time I will need to take a run up." So he boinged back along the track a bit, stretched his legs and bounced faster and faster towards the bank of the billabong. It was his best jump yet. But, unfortunately, just as he was leaping above the middle of the billabong, three parrots came flitting and fluttering across his flight path.

The three parrots were not pleased. They did not expect to see a flying kangaroo.

"Can't catch me," chuckled J.J. as he landed lightly on the other side of the billabong with a tiny tail feather tucked behind his ear.

"We'll see about that," squawked the three parrots and they flitted and fluttered after him, followed by the two thundering elephants and one Mrs. Koala puffing and panting and waddling along behind.

J.J. could see the farmer's fence ahead. J.J. could also see the farmer's four horses on the other side of the fence. Can you guess what Jumping Joey decided to do this time?

"Can't catch me," J.J. called as he sailed over the backs of the four horses. Now J.J. had four horses galloping after him, three parrots flitting and fluttering after him, two elephants thundering after him and one Mrs. Koala puffing and panting and waddling along behind.

Mr. and Mrs. Rabbit and their three babies had just decided to pop out of their burrow to nibble the grass when J.J. boinged out of the bush and bounced into the middle of their lunch.

"Can't catch me," they heard as he trampled their grass.

J.J. looked over his shoulder and decided to head for home. After all, now there were five unhappy rabbits hopping after him, four horses galloping after him, three parrots flitting and fluttering after him, two elephants thundering after him and one Mrs. Koala puffing and panting and waddling along behind.

"Open the pouch Mum, I'm coming in," J.J. called out when he got near home. And with one last boing he bounced inside. Only a tiny piece of tail, just like a twig, could be seen sticking out of his mother's pouch.

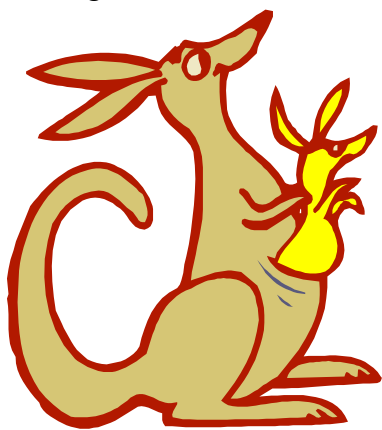
Five unhappy rabbits came hopping up to Mrs. Kangaroo. "Have you seen Jumping Joey?" they asked. Mrs. Kangaroo shook her head. The rabbits kept on hopping down the track.

Four horses came galloping up to Mrs. Kangaroo. "Have you seen Jumping Joey?" they asked. Mrs. Kangaroo shook her head. The horses kept on galloping down the track.

Three parrots came flitting and fluttering up to Mrs. Kangaroo. "Have you seen Jumping Joey?" they asked. Mrs. Kangaroo shook her head. The parrots kept flittering and fluttering down the track.

Two elephants came thundering up to Mrs. Kangaroo. "Have you seen Jumping Joey?" they asked. Mrs. Kangaroo shook her head. The elephants kept on thundering down the track.

One Mrs. Koala came waddling up to Mrs. Kangaroo. "Have you seen Jumping Joey?" she puffed and panted. Mrs. Kangaroo shook her head. Mrs. Koala decided she had to finish hanging out the washing, so she turned around and went home.



"Now," said Mrs. Kangaroo as she dragged the twig of a tail from her pouch and held her son up where she could look him in the eye. "What's all this about?"

J.J. told her everything. His mum explained to him that he had a special talent for jumping but he shouldn't use it to annoy people. He should try to use it to help people instead. So the next day Mrs. Kangaroo took J.J. to visit all the animals. Jumping Joey said he was sorry for annoying them and they all talked about ways that he could use his special talent for jumping to help make the bush a better place.

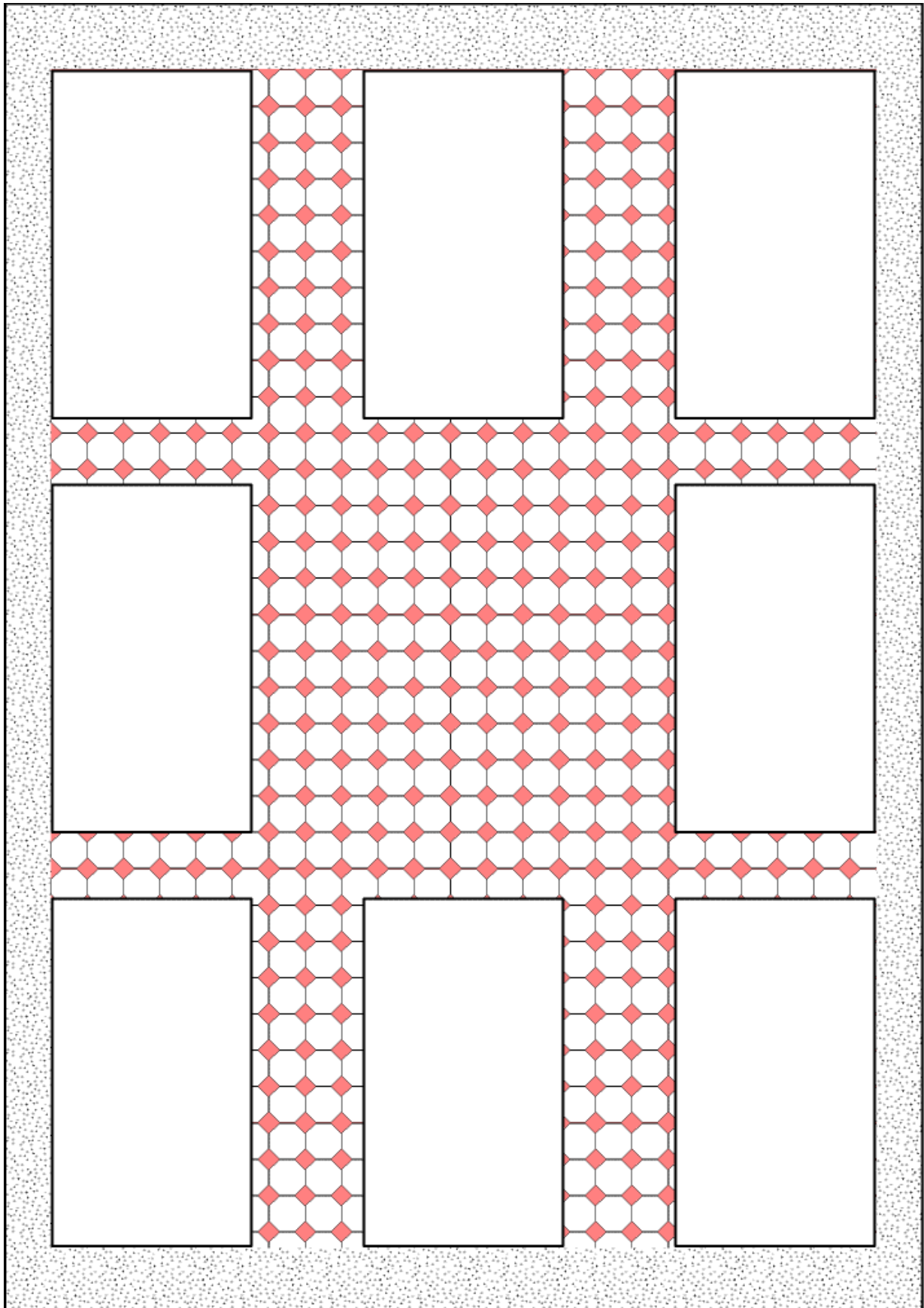
## Twelve Counters

1	2	3
4	5	6
7	8	9
10	11	12

## 4 & 20 Blackbirds - Big Birds



# 4 & 20 Blackbirds - Royal Garden

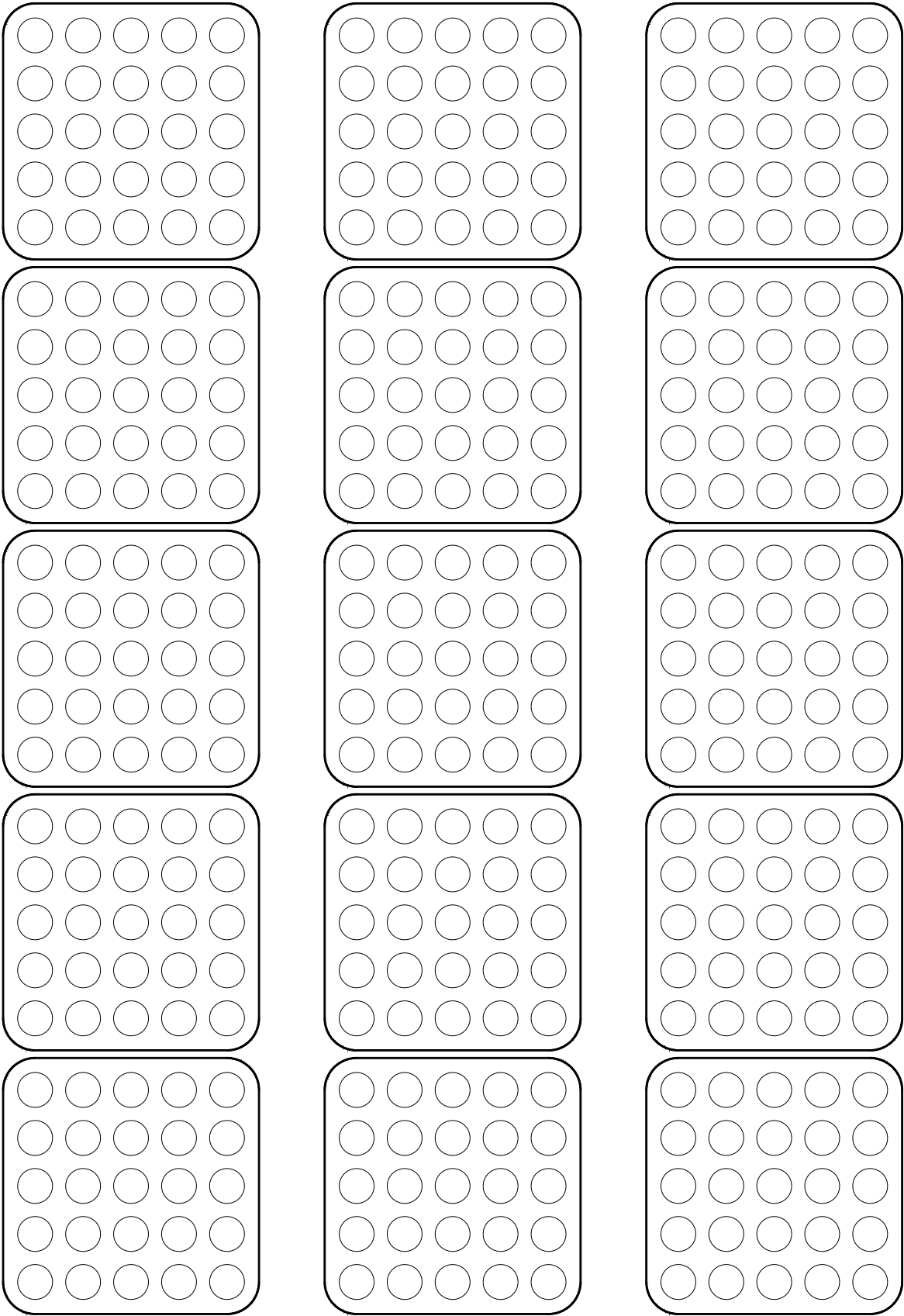




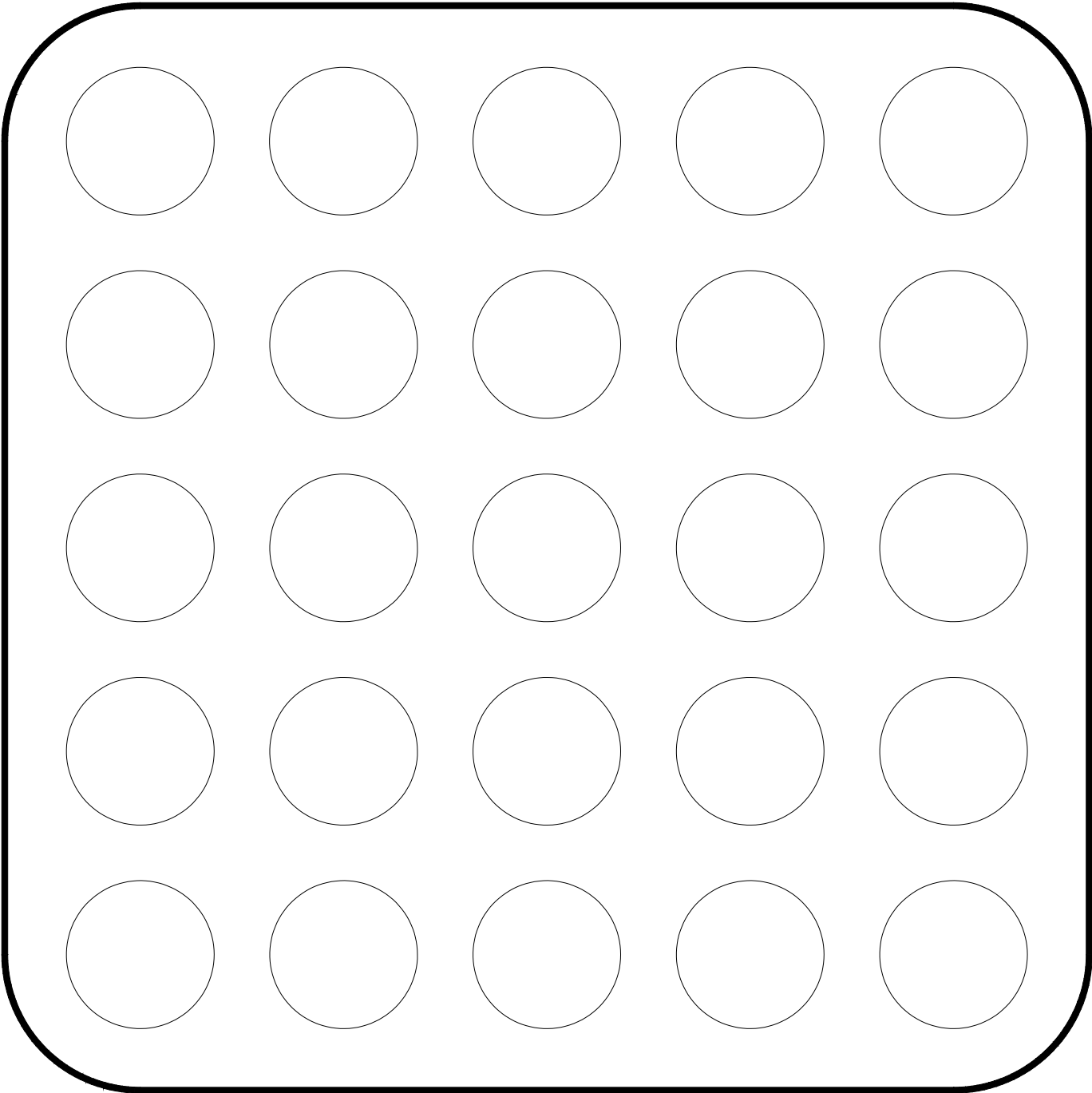
# Crossing The Desert

Desert View Caravan Park & Kiosk		
Day 1	<b>Start</b> 	Day 1
Day 2		Day 2
Day 3		Day 3
Day 4		Day 4
Day 5		Day 5
Day 6		Day 6
Day 7		Day 7
Day 8		Day 8
Day 9	 <b>Oasis</b>	Day 9

# Poly Plug Paper



# Poly Plug Frame



# Square Line Paper

