maths with attitude

2nd Edition

Pattern & Algebra Years 5 & 6

Charles Lovitt
Doug Williams

Mathematics Task Centre & Maths300
helping to create happy healthy cheerful productive inspiring classrooms

maths with attitude

2nd Edition

Pattern & Algebra

Years 5 & 6

In this kit:

- *Hands-on problem solving tasks*
- *Detailed curriculum planning*

Access from Maths300:

- *Extensive lesson plans*
- *Software*

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The **Maths With Attitude** series has been developed by The Task Centre Collective and is published by Black Douglas Professional Education Services.

Design and Desktop Publishing:
Black Douglas Professional Education Services

ISBN: 0-975-01910-4 ... 978-0-975-01910-8
First published 2002
2nd Edition 2012
2016: Designed for web delivery as a computer-based file

Maths With Attitude includes reference to hands-on problem solving tasks and Maths300 material.

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Part 1: Preparing To Teach

Our Objective

- To support teachers, schools and systems wanting to create:

  happy, healthy, cheerful, productive, inspiring classrooms

Our Attitude

- to learning:

  learning is a personal journey stimulated by achievable challenge
- to learners:

  stimulated students are creative and love to learn
- to pedagogy:

  the art of choosing teaching strategies to involve and interest all students
- to mathematics:

  mathematics is concrete, visual and makes sense
- to learning mathematics:

  all students can learn to work like a mathematician
- to teachers:

  the teacher is the most important resource in education
- to professional development:

  teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

Our Objective in Detail

What do we mean by creating:

happy, healthy, cheerful, productive, inspiring classrooms

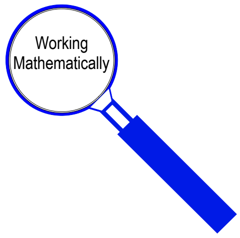
Happy…

means the elimination of the unnecessary fear of failure that hangs over so many students in their mathematics studies. Learning experiences *can* be structured so that all students see there is something in it for them and hence make a commitment to the learning. In so many 'threatening' situations, students see the impending failure and withhold their participation.

A phrase which describes the structure allowing all students to perceive something in it for them is *multiple entry points and multiple exit points*. That is, students can enter at a variety of levels, make progress and exit the problem having visibly achieved.

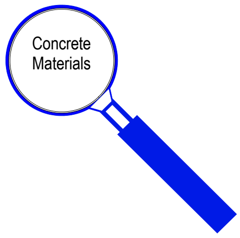
Healthy…

means *educationally healthy*. The learning environment should be a reflection of all that our community knows about how students learn. This translates into a rich array of teaching strategies that could and should be evident within the learning experience.



If we scrutinise the *exploration* through any lens, it should confirm to us that it is well structured or alert us to missed opportunities. For example, peering through a pedagogy lens we should see such features as:

- a story shell to embed the situation in a meaningful context
- significant active use of concrete materials
- a problem solving challenge which provides ownership for students
- small group work
- a strong visual component
- access to supportive software



Cheerful…

because we want 'happy' in the title twice!

Productive…

is the clear acknowledgment that students are working towards recognisable outcomes. They should know what these are and have guidelines to show they have either reached them or made progress. Teachers are accountable to these outcomes as well as to the quality of the learning environment.

Inspiring…

is about creating experiences that are uplifting or exalting; that actually *turn students on*. Experiences that make students feel great about themselves and empowered to act in meaningful ways.

Pattern & Algebra Resources

To help you create

happy, healthy, cheerful, productive, inspiring classrooms

this kit contains

- 20 hands-on problem solving tasks from Mathematics Centre

and a Teachers' Manual which integrates the use of the tasks with

- 12 detailed lesson plans from Maths300

The kit offers **7 weeks** of Scope & Sequence planning in Pattern and Algebra for *each* of Year 5 and Year 6. This is detailed in *Part 2: Planning Curriculum* which begins on Page 12. You are invited to map these weeks into your Year Planner. Together, the four kits available for these levels provide 25 weeks of core curriculum in Working Mathematically (working like a mathematician).

Note: Membership of Maths300 is assumed.
The kit will be useful without it, but it will be much more useful with it.

Tasks

- Arithmagons 1
- A Stacking Problem
- Can Stack
- Garden Beds
- How Many Triangles?
- Jumping Kangaroos
- Latin Squares
- Mirror Patterns 2
- Painted Cubes
- Pizza Toppings
- Plate Triangles
- Pointy Fences
- Shape Algebra
- Smooth Edge Tiles
- Snail Trail
- The Mushroom Hunt
- Thirty-one
- Time For Tiling
- Tower of Hanoi
- Triangles & Colours

Part 2 of this manual introduces each task. The latest information can be found at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm

Maths300 Lessons

- Arithmagons
- Back Tracking
- Billiard Ball Bounces
- Cracked Tiles
- Game of 31
- Garden Beds
- Jumping Kangaroos
- Painted Rods
- Simple, Elegant, Elusive
- Snail Trail
- Staircases
- Walking With Children

Lessons with Software

- Billiard Ball Bounces
- Cracked Tiles
- Game of 31
- Garden Beds
- Simple, Elegant, Elusive

Part 2 of this manual introduces each lesson. Full details can be found at:

- http://www.maths300.com

Working Like A Mathematician

Our attitude is:

all students can learn to work like a mathematician

What does a mathematician's work actually involve? Mathematicians have provided their answer on Page 8. In particular we are indebted to Dr. Derek Holton for the clarity of his contribution to this description.

Perhaps the most important aspect of Working Mathematically is the recognition that *knowledge is created by a community and becomes part of the fabric of that community*. Recognising, and engaging in, the process by which that knowledge is generated can help students to see themselves as able to work like a mathematician. Hence Working Mathematically is the framework of **Maths With Attitude.**

Skills, Strategies & Working Mathematically

A Working Mathematically curriculum places learning mathematical skills and problem solving strategies in their true context. Skills and strategies are the tools mathematicians employ in their struggle to solve problems. Lessons on skills or lessons on strategies are not an end in themselves.

- **Our skill toolbox** can be added to in the same way as the mechanic or carpenter adds tools to their toolbox. Equally, the addition of the tools is not for the sake of collecting them, but rather for the purpose of getting on with a job. A mathematician's job is to attempt to solve problems, not to collect tools that might one day help solve a problem.
- **Our strategy toolbox** has been provided through the collective wisdom of mathematicians from the past. All mathematical problems (and indeed life problems) that have ever been solved have been solved by the application of this concise set of strategies.

About Tasks

Our attitude is:

mathematics is concrete, visual and makes sense

Tasks are from Mathematics Task Centre. They are an invitation to two students to work like a mathematician (see Page 8).

The Task Centre concept began in Australia in the late 1970s as a collection of rich tasks housed in a special room, which came to be called a Task Centre. Since that time hundreds of Australian teachers, and, more recently, teachers from other countries, have adapted and modified the concept to work in their schools. For example, the special purpose room is no longer seen as an essential component, although many schools continue to opt for this facility.

A brief history of Task Centre development, considerable support for using tasks, for example Task Cameos, and a catalogue of all currently available tasks can be found at:

- http://www.mathematicscentre.com/taskcentre

Key principles are:

- A good task is the tip of an iceberg
- Each task has three lives
- Tasks involve students in the Working Mathematically process

The Task Centre Room or the Classroom?

There are good reasons for using the tasks in a special room which the students visit regularly. There are also different good reasons for keeping the tasks in classrooms. Either system can work well if staff are committed to a core curriculum built around learning to work like a mathematician.

- A task centre room creates a focus and presence for mathematics in the school. Tasks are often housed in clear plastic 'cake storer' type boxes. Display space can be more easily managed. The visual impact can be vibrant and purposeful.
- However, tasks can be more readily integrated into the curriculum if teachers have them at their finger tips in the classrooms. In this case tasks are often housed in press-seal plastic bags which take up less space and are more readily moved from classroom to classroom.

Tip of an Iceberg

The initial problem on the card can usually be solved in 10 to 20 minutes. The investigation iceberg which lies beneath may take many lessons (even a lifetime!). Tasks are designed so that the original problem reveals just the 'tip of the iceberg'. Task Cameos and Maths300 lessons help to dig deeper into the iceberg.

We are constantly surprised by the creative steps teachers and students take that lead us further into a task. No task is ever 'finished'.

> *Most tasks have many levels of entry and exit and therefore offer an on-going invitation to revisit them, and, importantly, multiple levels of success for students.*

Three Lives of a Task

This phrase, coined by a teacher, captures the full potential and flexibility of the tasks. Teachers say they like using them in three distinct ways:

1. As on the card, which is designed for two students.
2. As a whole class lesson involving all students, as supported by outlines in the Task Cameos and in detail through the Maths300 site.
3. Extended by an Investigation Guide (project), examples of which are included in both Task Cameos and Maths300.

The first life involves just the 'tip of the iceberg' of each task, but nonetheless provides a worthwhile problem solving challenge - one which 'demands' concrete materials in its solution. This is the invitation to work like a mathematician. Most students will experience some level of success and accomplishment in a short time.

The second life involves adapting the materials to involve the whole class in the investigation, in the first instance to model the work of a mathematician, but also to develop key outcomes or specific content knowledge. This involves choosing teaching craft to interest the students in the problem and then absorb them in it.

The third life challenges students to explore the 'rest of the iceberg' independently. Investigation Guides are used to probe aspects and extensions of the task and can be introduced into either the first or second life. Typically this involves providing suggestions for the direction the investigation might take. Students submit the 'story' of their work for 'portfolio assessment'. Typically a major criteria for assessment is application of the Working Mathematically process.

About Maths300

Our attitude is:

teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

Maths300 is a subscription based web site. It is an attempt to collect and publish the 300 most 'interesting' maths lessons (K - 12).

- *Lessons have been successfully trialed in a range of classrooms.*
- *About one third of the lessons are supported by specially written software.*
- *Lessons are also supported by investigation sheets (with answers) and game boards where relevant.*
- *A 'living' Classroom Contributions section in each lesson includes the latest information from schools.*
- *The search engine allows teachers to find lessons by pedagogical feature, curriculum strand, content and year level.*
- *Lesson plans can be printed directly from the site.*
- *Each lesson supports teaches to model the Working Mathematically process.*

Modern internet facilities and computers allow teachers easy access to these lesson plans. Lesson plans need to be researched, reflected upon in the light of your own students and activated by collecting and organising materials as necessary.

Maths300 Software

Our attitude is:

stimulated students are creative and love to learn

Pedagogically sound software is one feature likely to encourage enthusiastic learning and for that reason it has been included as an element in about one third of Maths300 lesson plans. The software is used to develop an investigation beyond its introduction and early exploration which is likely to include other pedagogical techniques such as concrete materials, physical involvement, estimation or mathematical conversation. The software is not the lesson plan. It is a feature of the lesson plan used at the teacher's discretion.

For school-wide use, the software needs to be downloaded from the site and installed in the school's network image. You will need to consult your IT Manager about these arrangements. It can also be downloaded to stand alone machines covered by the site licence, in particular a teacher's own laptop, from where it can be used with the whole class through a data projector.

Note:

- *Maths300 lessons and software may only be used by Maths300 members.*

Working Mathematically

First give me an interesting problem.

When mathematicians become interested in a problem they:

- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

Questions which help mathematicians learn more are:

- Can I check this another way?
- What happens if ...?
- How many solutions are there?
- How will I know when I have found them all?

When mathematicians have a problem they:

- Read & understand the problem.
- Plan a strategy to start the problem.
- Carry out their plan.
- Check the result.

A mathematician's strategy toolbox includes:

- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- Break a problem into smaller parts
- ...

If one way doesn't work, I just start again another way.

Professional Development Purpose

Our attitude is:

the teacher is the most important resource in education

> *We had our first study group on Monday. The session will be repeated again on Thursday. I had 15 teachers attend. We looked at the task Farmyard Friends (Task 129 from the Mathematics Task Centre). We extended it out like the questions from the companion Maths300 lesson suggested, and talked for quite a while about the concept of a factorial. This is exactly the type of dialog that I feel is essential for our elementary teachers to support the development of their math background. So anytime we can use the tasks to extend the teacher's math knowledge we are ahead of the game.*
> District Math Coordinator, Denver, Colorado

Research suggests that professional development most likely to succeed:

- is requested by the teachers
- takes place as close to the teacher's own working environment as possible
- takes place over an extended period of time
- provides opportunities for reflection and feedback
- enables participants to feel a substantial degree of ownership
- involves conscious commitment by the teacher
- involves groups of teachers rather than individuals from a school
- increases the participant's mathematical knowledge in some way
- uses the services of a consultant and/or critical friend

Maths With Attitude has been designed with these principles in mind. All the materials have been tried, tested and modified by teachers from a wide range of classrooms. We hope the resources will enable teacher groups to lead themselves further along the professional development road, and support systems to improve the learning outcomes for students K - 12.

With the support of Maths300 ETuTE, professional development can be a regular component of in-house professional development. See:

- http://www.mathematicscentre.com/taskcentre/resource.htm#etute

For external assistance with professional development, contact:

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Part 2: Planning Curriculum

Curriculum Planners

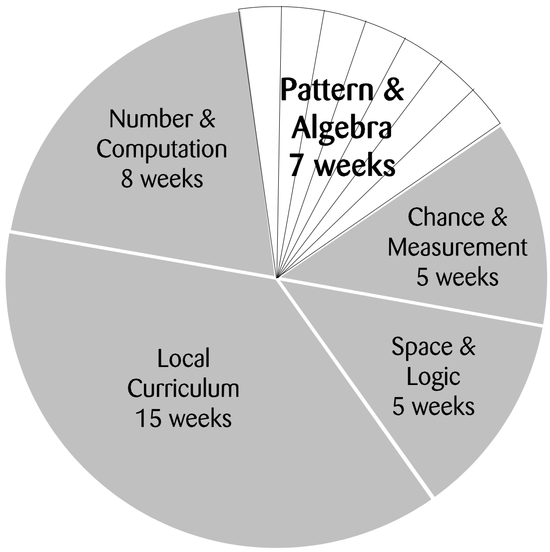
Our attitude is:

learning is a personal journey stimulated by achievable challenge

Curriculum Planners:

- show one way these resources can be integrated into your weekly planning
- provide a starting point for those new to these materials
- offer a flexible structure for those more experienced

You are invited to map Planner weeks into your school year planner as the core of the curriculum.



Planners:

- detail each week lesson by lesson
- offer structures for using tasks and lessons
- are sequenced from lesson to lesson, week to week and year to year to 'grow' learning

Teachers and schools will map the material in their own way, but all will be making use of extensively trialed materials and pedagogy.

Using Resources

- Your kit contains 20 hands-on problem solving tasks and reference to relevant Maths300 lessons.
- Tasks are introduced in this manual and supported by the Task Cameos at:
  http://www.mathematicscentre.com/taskcentre/iceberg.htm
- Maths300 lessons are introduced in this manual and supported by detailed lesson plans at:
  http://www.maths300.com

In your preparation, please note:

- Planners assume 4 lessons per week of about 1 hour each.
- Planners are *not* prescribing a continuous block of work.
- Weeks can be interspersed with other learning; perhaps a **Maths With Attitude** week from a different strand.
- Weeks can sometimes be interchanged within the planner.
- Lessons can sometimes be interchanged within weeks.
- The four **Maths With Attitude** kits available at each year level offer 25 weeks of a Working Mathematically core curriculum.

A Way to Begin

- Glance over the Planner for your class. Skim through the comments for each task and lesson as it is named. This will provide an overview of the kit.
- Task Comments begin after the Planners. Lesson Comments begin after Task Comments. The index will also lead you to any task or lesson comments.
- Select your preferred starting week - usually Week 1.
- Now plan in detail by researching the comments and web support. Enjoy!

Research, Reflect, Activate

Curriculum Planner
Pattern & Algebra: Year 5

	Session 1	Session 2	Session 3	Session 4
Week 1	**Whole Class Investigation:** *Snail Trail* is a well known problem that lends itself to acting out. Using this kinaesthetic approach as an introduction opens the learning door for students who may be less motivated by a printed presentation of the problem. The question *What happens if...?* introduces many extensions which can be approached as a class, or through small group or individual investigation.			
Week 2	**Whole Class Investigation:** *Billiard Ball Bounces* includes patterns that reinforce number work in multiples, factors and primes. The investigation also has a strong visual component and makes links to geometry. The investigation is easily introduced with a pencil and paper approach, although it becomes more flexible if this is combined with the software.			
Weeks 3 - 5	**Mixed Media Unit 1:** For two weeks the class adopts this work station approach as described on Page 16. The first lesson in the week is with the whole class and for the remaining three lessons the class is divided into three groups. One group uses software, one group uses tasks and one group uses local text material. All groups work within the pattern and algebra theme. The third week is used to bring the threads together and assess development. Use *Cracked Tiles* and *Simple, Elegant, Elusive* as the whole class lessons and use their software (and that from *Billiard Ball Bounces* if you wish) for the software work station.			
Week 6	**Whole Class Investigation:** *Painted Rods* works best with access to Cuisenaire Rods or the like. However, many students will soon be able to visualise the physical situation. There are several extensions in the Lesson Plan that could provide up to four sessions of involvement. Many of these are included on the companion Investigation Sheet. The extension that involves graphing student data leads well into Week 7.			
Week 7	**Whole Class Investigation:** In *Walking With Children* the students become data points in a large scale graph, then translate their experience to a table top model created with materials of their own choice. The Lesson Plan contains many examples of student work, most of it created in groups, and has much more to offer than can fit in the four sessions of this week.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Curriculum Planner

Pattern & Algebra: Year 6

	Session 1	Session 2	Session 3	Session 4
Week 1	**Skill Development:** *Back Tracking* will set students up well for textbook work on solving equations. It begins with a whole class approach and uses a pictorial model. Problem solving strategies are emphasised. After just one session most students are solving equations far more complex than texts usually offer. Use local text material to practise the new skills.			
Weeks 2 - 4	**Mixed Media Unit 2:** For two weeks the class adopts this work station approach as described on Page 16. The first lesson in the week is with the whole class and for the remaining three lessons the class is divided into three groups. One group uses software, one group uses tasks and one group uses local text material. All groups work within the pattern and algebra theme. The third week is used to bring the threads together and assess development. Use *Garden Beds* and *Game of 31* as the whole class lessons and use their software for the software work station.			
Week 5	**Whole Class Investigation:** *Arithmagons* is easy to manage because it only requires the Investigation Sheets and Boards supplied and scrap paper. However the lesson is based on a large family of puzzles which provide many extensions. The emphasis is on Working Mathematically.			
Week 6	**Whole Class Investigation:** *Jumping Kangaroos* is a well known problem that comes in many guises. Students act out the problem and then create a table top model to represent it. The underlying number pattern is a little more complex than usually experienced at this age, however many Year 6 students have been able to make the generalisation. The emphasis is on illustrating the Working Mathematically process. The question *Can I check this another way?* is a feature. There are many extensions.			
Week 7	**Whole Class Investigation:** *Staircases* draws together all the threads the students have been experiencing in the Pattern & Algebra kit throughout Years 5 & 6. It features: concrete materials as the inspiration for the algebra; representation of number patterns by graphs; and the process of working like a mathematician. The project suggested as a conclusion to the lesson will allow students to apply what they have been learning and will provide extensive assessment information.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Planning Notes

Enhancing Maths With Attitude

Resources to support learning to work like a mathematician are extensive and growing. There are more tasks and lessons available than have been included in this Pattern & Algebra kit. You could use the following to enhance this kit.

Additional Tasks

- Task 238, Growing Trisquares

  The crux of this problem is that 4 Trisquares can be joined to make a new, scaled up, Trisquare. This provides a 'template' for constructing the next size, and the next, and the next... The visual pattern can also be represented as a number pattern and this leads to graphing, equation work and scale factors.

More information about these tasks may be available in the Task Cameo Library:
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Additional Lessons

- Lesson 167, Twelve Days of Christmas

  Almost everyone knows that on the first day of Christmas my true love gave me a partridge in a pear tree and on and on for twelve days building up a musical pattern. Mathematicians seek and find patterns and teachers seek and find multiple points to encourage students to enter mathematical investigations. This lesson offers students with logical/mathematical, musical, visual/spatial, verbal, kinaesthetic and interpersonal intelligences opportunity to engage in a problem which can focus only on number patterns, or extend into complex quadratic algebra.

Keep in touch with new developments which enhance **Maths With Attitude** at:
- http://www.mathematicscentre.com/taskcentre/enhance.htm

Additional Materials

As stated, our attitude is that mathematics is concrete, visual and makes sense. We assume that all classrooms will have easy access to many materials beyond what we supply. For this unit you will need:
- Linking cubes like Unifix or Multilink
- Square tiles
- Cuisenaire Rods
- Packs of playing cards

Special Comments Year 5

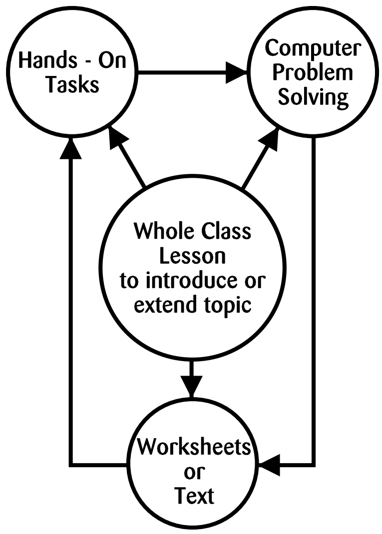
- Look ahead to Planner Week 6. You will need Cuisenaire Rods, and it could also be useful to make your own larger demonstration set as described in the notes. Cuisenaire Rods were once common in schools, but this may no longer be the case, so you may have to look around. An alternative is for students to build rods with Unifix or Multilink.

Special Comments Year 5 & 6

- The Mixed Media model may be a new unit structure for you so look ahead and plan carefully. Trial teachers suggest the planning is more effective and efficient when carried out as a team. The idea is predicated on enough computers for one third of the class to use them at one work station. On the basis of two students per computer - which frequently promotes significant mathematical conversation - and thirty students in the class you will need 5 machines. Many teachers use school notebook machines for this purpose. If this level of equipment is not available, for example, if your school only has a computer laboratory, you will need to restructure the work for these weeks.

Mixed Media Unit

Mixed Media Mathematics has been created as *one* structure which allows teachers to integrate problem solving tasks into the curriculum.



The design incorporates four different modes of learning into a structure which can be readily managed by one teacher, but which is enhanced when prepared and executed by a team.

A three week Mixed Media Unit includes:

- whole class lessons
- hands-on problem solving
- problem solving software
- skill practice worksheets (or text material)
- time to reflect on learning
- assessment opportunities

If this is the first time such a structure has been used in your classroom, it is a good idea to prepare the students in a manner which 'brings them into the experiment'.

A vital element of the process is to reflect on *what* is learned and *how* it is learned *before* the final assessment of the learning. Guidance with respect to assessment is also provided in this manual. In particular, the Pupil Self-Reflection information in the Assessment section Part 3 was designed by teachers who trialed the original Mixed Media units. The tasks suggested for these units are:

Mixed Media Unit 1

- Arithmagons 1
- Garden Beds
- How Many Triangles?
- Latin Squares
- Pizza Toppings
- Plate Triangles
- Pointy Fences
- Smooth Edge Tiles
- Snail Trail
- Triangles & Colours

Mixed Media Unit 2

- A Stacking Problem
- Can Stack
- Jumping Kangaroos
- Mirror Patterns 2
- Painted Cubes
- Shape Algebra
- The Mushroom Hunt
- Thirty-one
- Time For Tiling
- Tower of Hanoi

Task Comments

- Tasks, lessons and unit plans prepare students for the more traditional skill practice lessons, which we invite you to weave into your curriculum. Teachers who have used practical, hands-on investigations as the focus of their curriculum, rather than focussing on the drill and practice diet of traditional mathematics, report success in referring to skill practice lessons as Toolbox Lessons. This links to the idea of a mathematician dipping into a toolbox to find and use skills to solve problems.

Arithmagons 1

Although simple to start, this task requires the students to keep two conditions in mind simultaneously. For many that may be challenge enough, and they may well treat this task as a number puzzle and not see any pattern. However, the companion Maths300 whole class investigation brings out the algebraic links by challenging the students to explain to someone else how to always be able to find the circle number. In other words to make a generalisation.

A Stacking Problem

This is a tough problem for most students. It certainly doesn't need to be solved in one sitting and, equally, we need not offer hints too soon. It relates to Tower of Hanoi, but it is not identical. When the minimum number of moves is eventually counted (the answer being 60 moves), the challenge and the patterns in the problem may seem surprising for a problem so easy to begin.

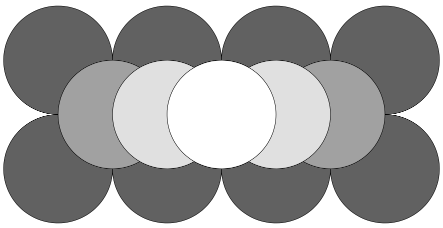
The turning point of the problem is when Block 6 is the only one left on Square A, and Blocks 1 through 5 are in order from 1 down to 5 on Square B. Even when students have reached this stage and continued to the solution, it may be necessary to repeat the solution several times to realise the pattern in the movements. As with Arithmagons, it is the description of how to solve it so that someone else might repeat their success which encourages the use of algebraic notation.

The problem encourages the mathematician's strategy of breaking a problem into smaller parts. Further investigation develops by asking whether similar problems (3, 9, 12, 15... blocks) on three squares, can be solved, and, if so, what is the minimum number of moves.

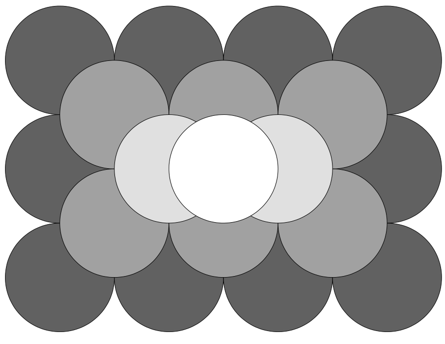
Can Stack

There are at least two ways to interpret and extend the diagram on the card.

One Direction - Top View



Two Directions - Top View



In each case the visual pattern has two parts, so the related number pattern will also have two parts.

One Direction

Each can stands on two others, except for those in the second last layer from the top. The last layer has twice the expected number of cans so each can in the second last layer stands on four below. From the top the building pattern is:

- 1 + 2 + 3 + 4 + 5 + ... and at the nth layer, an extra n cans are added.

This visual understanding of the problem provides an approach to finding both the number in any layer (the last will always be an even number) and the total of cans needed to build to any layer. To find this total students will need to apply what has been learnt from the lesson *Gauss Beats The Teacher*, which is included as a Maths300 lesson in **Number & Computation Years 5 & 6**.

Two Directions

Only the top can stands on two cans beneath. All others stand on four. The single top can is added last and since it is not part of the visual pattern, it becomes one extra added at the end of the number pattern investigation. Excluding this Layer 1 can for the moment, it can be seen that the building pattern for Layer 2 onwards is:

- (2 x 1) + (3 x 2) + (4 x 3) + (5 x 4) + ...

Again the visual understanding of the problem provides an approach to finding the number in any layer, ie: $n(n - 1)$ for $n > 1$. This approach also provides a way of finding the total number of cans needed to build to the 10th layer, as asked on the card, but generalising this to the nth layer is a more difficult challenge.

An extension to the task is to ask students to design their own can stacking system and investigate the number patterns that result.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Garden Beds

In Question 1, students might make a table to find the answers and hence reveal a number pattern. This is one of the strategies from the mathematician's toolbox. The pattern can be used to tackle the Challenge, but it is also important to encourage students to see and express one or more visual ways of building the tiles around the garden bed. The double line on the card after Question 2 can be used as a signal to students that they are expected to communicate with the teacher at this stage. This procedure provides opportunity for teachers to ask questions which bring out the students' visual generalisation of the problem, and support them to continue. There are several ways of 'seeing' the pattern other than the 'double and add six' approach that tends to develop from the table.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

How Many Triangles?

In some ways this task is a bit back to front. The generalisation involved in the first question is quite difficult and consequently not asked for on the card. On the other hand the pattern in the second part of the card is based on square numbers and can be readily generalised. The first part of the card is intended as a visual challenge related to organising a hunt for all the triangles of a particular size. Organising searches to be sure of finding every possibility is an important tool for a mathematician.

Question 1

In this search students will have to look for both point up and point down triangles. They will also have to find a starting triangle for each orientation and imagine it translated step by step to find all possibilities. These are the spatial challenges. For example, in the Size 2 triangle, there are 3 point up triangles and 1 point down. The triangle dot paper supplied at the end of this manual may assist with the search and record process.

The following table gives results for sizes of triangle beyond what is asked for on the card. As well as providing confirmation of students results, studying the diagonal lines of the table provides the first evidence that it is possible to predict the table for any size triangle. However this prediction is likely to be related to solving the sub-problem of determining the number of point up and point down triangles for each size.

	Number of each size within each triangle								
Size	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Tot.**
1	1								1
2	4	1							5
3	9	3	1						13
4	16	7	3	1					27
5	25	13	6	3	1				48
6	26	21	11	6	3	1			78
7	49	31	18	10	6	3	1		118
8	64	43	27	16	10	6	3	1	170

Questions 2 & 3

Question 2 involves counting the unit triangles within any size of triangle. The table above suggests that this will be a square number. Question 3 applies this to the next spatial challenge of creating a tetrahedron from any size equilateral triangle. A tetrahedron has four faces, so, if T is the total of unit triangles and S is the size, then $T = 4S^2$, which also measures the surface area of the tetrahedron (in unit triangles instead of unit squares).

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Jumping Kangaroos

Most students find this problem quite engaging, and although the pattern is of the second degree (what is referred to in high school as a quadratic equation), many Year 6 students have discovered and understood it. In essence the only skill they need to achieve this is proficiency with times tables. It is *not* necessary to 'do' quadratic equations before tackling this problem.

A clue to working out the challenge of solving the case for three kangaroos each side is to check the options at each move and look ahead to the consequences of each. The ‘critical’ moves (in other words the moves that many people get wrong) are moves 3, 6 and 10 which involve bringing up one of the end kangaroos to establish the alternating pattern of positions.

The problem also resolves when the mathematician's strategy of *Try a simpler case* is applied. There is more than one way to predict the number of moves given the number of kangaroos on each side. In addition the problem has many extensions.

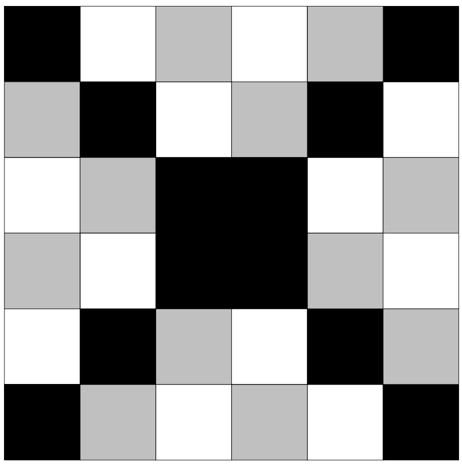
Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Latin Squares

In the first instance, students are likely to tackle this problem by guess and check. That will bring a solution for the 4 by 4 square, but it becomes a less successful strategy for 5 by 5 and beyond. Encourage them to look for a pattern by comparing the 3 by 3 solution on the card and their own 4 by 4 solution. The pattern is a movement based one. The next row is formed from the current row by shifting the right hand block to the left hand end and moving the others one space to the right. A visual check resulting from this movement pattern is to look at the top left to bottom right diagonals. The colours along each should be the same.

An extension of the task is to use more cubes of your own and create repeats of the Latin Square which are joined to the original unit by reflection or rotation. For example using the 3 by 3 and rotation creates:

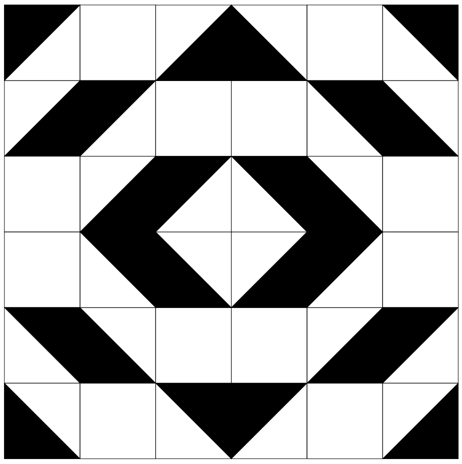


Think of the original unit as being in the bottom left quarter and the rotation as being 90° anti-clockwise.

If you don't have sufficient cubes, graph paper can be used either by colouring the 'cubes', or by assigning a piece of coloured design to each square in the order of a Latin Square. For example, assigning designs as follows:

Black above = , White above = , Grey above = 

and reflecting a 3 by 3 gives:



Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Mirror Patterns 2

This intriguing task makes links between geometry, number, pattern and algebra. It takes the students past the fun of creating mirror patterns by requiring them to measure the angle between the mirrors. Recording the measured angles in the table, as suggested, leads to the hypothesis that the angle $A = 360 \div n$, where n is the number of sides of the polygon.

The first part of the task explores regular polygons whose side numbers are a factor of 360. The task then suggests working backwards and asks about angles like 100° that would imply a shape with 3.6 sides. How is this to be interpreted?

Painted Cubes

This task is highly visual and generates a range of algebraic relationships. The cubes strongly support the investigation and can be extended by including a large MAB cube from the classroom resources.

Students will quickly see that the number of cubes with three painted faces is constant and represents the eight corner cubes. They may also notice that the other values 'grow' at different rates. In fact, one is linear, one is quadratic and one is cubic, and the experience of these now is good preparation for exploring these functions in future years. There is an algebraic connection between the size of the cube and the unit cubes with a particular number of faces painted. It is:

- 3 Painted Faces: 8
- 2 Painted Faces: $(n - 2) \times 12$
- 1 Painted Face: $(n - 2)^2 \times 6$
- 0 Painted Faces: $(n - 2)^3$

However, the emphasis in the task is on *seeing why* these must be the relationships. Where is the $(n - 2)$ on the cube in each case? The ability to be visually algebraic in this way can lead to the solution of a problem more quickly than trying to manipulate the numerical data recorded in a table.

It is *not necessary* to have studied square and cubic functions in order to be able to use the geometric properties of squares and cubes to count the units and identify and generalise a pattern.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Pizza Toppings

This task highlights the mathematicians' questions:

- How many solutions are there?
- How do you know you have found them all?

Students will have to decide some of the conditions of the problem for themselves. For example the answers to the above questions will be different depending on whether the students

- restrict the problem to using ingredients either zero or one times on a pizza, or
- whether they also investigate all the possibilities with double, triple or quadruple ingredients.

The first option is simpler and leads to:

- 1 pizza with no toppings
- 4 pizzas with just one topping - Y, G, W, R
- 6 pizzas with two toppings - YG, YW, YR, GW, GR, WR
- 4 pizzas with three toppings - YGW, YGR, YWR, GWR
- 1 pizza with the lot

If you have used **Number & Computation Years 5 & 6**, students may recognise a line of Pascal's Triangle in this solution. Could it be that if there were five ingredients rather than four, the next line of Pascal's Triangle would be give the number of pizzas with various toppings?

Plate Triangles

Plate Triangles needs plenty of table or floor space. Generally students respond well to this large scale. It seems the tactile nature encourages the search to begin. It is also an extensive task with several investigations lurking within it.

The first part of the task explores triangle numbers as they actually result from making triangles. The pattern 1, 3, 6, 10, 15, ... (which is the total of plates after each new row is added) is called the Triangle Numbers *because* these numbers can be represented in triangles. The task also mirrors the link between natural and triangle numbers. To make the next plate triangle you add the next natural number.

When the counters are placed on the plates, the numbers at the right hand end of each row provide a symbolic record of the Triangle Number pattern. They also open up the search for other patterns. Students will find many. The sheet at the end of this manual may be useful for recording them.

In addition students may find links in the task to **How Many Triangles?** and **Arithmagons 1**.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Pointy Fences

This task can be thought of as similar to **Garden Beds**. The plants have become pyramids and each pyramid has four triangular faces, but the tiles surround the pyramids in the same way as they surround the plants in **Garden Beds**. It is not necessary to try **Garden Beds** before **Pointy Fences**, however, the cameo for *Garden Beds* will support teachers with ideas for this task too.

Shape Algebra

Shape Algebra blows away the traditional opening line of an algebra lesson, ie: *Let x stand for any number*. In this task *x* is a shape! Well actually it is the number that stands for the area of the shape, but the exciting feature of the task is that the algebra becomes geometry. The task focuses on creating and manipulating algebraic terms, but all stages of the symbol manipulation are grounded in the concrete material. Students are often heard making statements like: *Shape D must be 4x - y because there were four x pieces there and a y bit has been cut out of it*. The recording sheet at the end of the manual may help with exploration.

You might like to turn the lesson into a whole class investigation by scanning or tracing the pieces and printing a copy for each pair. Students can carefully cut out their own sets. What other shapes can they make and record by combining the shapes in the set? What is the algebraic representation of each shape? Can the students transfer their experience to text book work on algebraic expressions and like and unlike terms?

Smooth Edge Tiles

The type of spatial thinking involved in this task is similar to that in **Painted Cubes**.

- The L-shape tiles are always in the corner, so there will always be four of them.
- Further, because the L-shape tiles take up the corners there will be two less single smooth edge tiles on each length and width.
- The tiles with zero smooth edges will always form a rectangle in the middle that is 2 less than each of the length and the width of the patio.

So, for an M by N patio:

- 2 smooth edges: 4
- 1 smooth edge: 2(M - 2) + 2(N - 2)
- 0 smooth edges: (M - 2) x (N - 2)

However, the development of this symbolic notation is not the objective of the task. The objective is to learn to:

- 'see' how the problem is constructed
- 'see' the construction in alternative ways if possible
- describe the general principles of construction to someone else in natural language

Then, when symbolic notation is used, to be able to give meaning to the symbols and operations in terms of the physical problem.

To extend the task, consider asking backwards questions like:

- *If Janine had only 14 single smooth edge tiles and all the other tiles she needed, what size patios could she build?*

Snail Trail

Any proposed solution the students think of when they first read the card can be readily checked with the equipment. This is important because it is often the case in this problem that first responses are inaccurate. The double line drawn across the card after Question 1 is to remind students to check with the teacher after this first step. Simply asking them to show you their solution with the wood is enough to confirm or deny their proposal.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

The Mushroom Hunt

Perhaps the first challenge in this task is to be clear about how the problem works. Two conditions have to be satisfied simultaneously. The total number of mushrooms must be sixty-three *and* combining and re-combining baskets must produce every number from 1 through to 63. The example makes this clear. The total of the baskets *is* 63. Some required totals can be made, including those shown. But how, for example, can the number 1 be made with the baskets shown? It can't be, and this realisation begins the search for the solution. There has to be a basket with just one mushroom in it.

Note that mushrooms are not shifted from basket to basket to make totals. Each calculation is based on *If we combined these baskets, what would the total be?*.

The pattern which develops in the solution is based on powers of 2, or as the students may perceive it, continuous doubling. The solution is 1, 2, 4, 8, 16, 32.

Extend the task by asking:

- *Suppose the Big Bad Wolf also went mushrooming with the Bears and Pigs. If the pattern continued, how many mushrooms would you expect in her basket and up to what number could totals now be made?*
- *The total number of mushrooms is 2048. If they were collected in baskets in this pattern, how many baskets would be needed and what number would be in each basket?*

The task also offers a way to introduce the Binary Number system. If a 1 means *use one of these baskets* and a 0 means *use none of these baskets* then the table shows the binary equivalent of each decimal number listed down the side.

	Basket Containing					
	32	16	8	4	2	1
1						1
2					1	0
3					1	1
4				1	0	0
5				1	0	1
...						
14			1	1	1	0
...						
38	1	0	0	1	1	0
...						

Thirty-One

Students generally find this game quite absorbing. At first they will play it in a random fashion, but in doing so they are taking the first step in working like a mathematician. The data they collect in their mind as they play the game will lead to realising that in order to take the total to 31 first you must be the person who reaches 24 first. But to be at 24 first you must be at 17 first, and so on (backwards). Once this version of the game is conquered, the *What if...?* questions extend the problem.

- What happens if we change the winning total?
- What happens if we change the range of cards used?

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Time For Tiling

As with **Smooth Edge Tiles**, the objective of this task is to:

- 'see' how the problem is constructed
- 'see' the construction in alternative ways if possible
- describe the general principles of construction to someone else in natural language

Then, when symbolic notation is used, to be able to give meaning to the symbols and operations in terms of the physical problem.

Odd Squares 1

The construction could be seen as the number of dark tiles being twice the side length less 1 because the diagonals are the same number of tiles as the side and counting both diagonals counts the centre tile twice. Once the dark tiles are known the light tiles can be found by subtraction from the number of tiles in the square.

Odd Squares 2

The light tiles are in four clusters. The longest line of the cluster is two less than the side of the square. Each successive row of the cluster is two less than the previous row because there are two dark tiles in each row - one from each diagonal. This pattern continues down to a one tile row. Adding this sequence for one cluster and multiplying by four gives the total of white tiles. Subtraction from the total number of tiles in the square give the total of dark tiles.

No doubt there are also other ways of seeing this construction. The more the merrier. One of the mathematician's questions is *Can I check this another way?*. Once students learn that the search for other ways is valued, they have less need to ask *Am I right Miss?*. Teachers meet this question with *Can YOU check it another way?* and thereby encourage students to be more responsible for their own learning.

Even Squares 1

Focussing on the dark tiles we might see a central square of four with four shortened diagonals leading out to the corners. Each of these short pieces is one less than one half of the diagonal length, which, of course, is the same length as the side of the square. Having calculated the dark tiles in this way, the light tiles can be found by subtraction.

Even Squares 2

The light tiles are in the same four clusters as above and the successive row lengths are shortened by two each time for the same reason as above. The difference from the odd case is that the sequence counts down to a row of two tiles, rather than a row of one.

Again, there will be other ways to view these constructions.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Tower of Hanoi

This is a little like **Jumping Kangaroos** in that rather than rushing into a move, it is better to consider the options at each stage. Careful examination of each step may lead to a solution. On the other hand, the legend told on the card suggests that other numbers of discs could be used, so why not apply the problem solving strategy of *Try a simpler problem* to see if the key movement patterns can be discovered and then applied to the problem on the card. One student described such a pattern as:

> *If you have an odd number you put the piece first where you want it. Where you don't want the pile you put it if there is an even number.*

This insight makes perfect sense once you have personal experience with attempting to solve the puzzle.

The number pattern that results from trying the simpler cases is 3, 7, 15, 31, 63 and so on. This will remind students of the numbers in **The Mushroom Hunt**. The other experience resulting from this approach may be to see that the moves for any tower relate to the moves for the previous tower as follows:

- Assume the previous tower has been solved.
- The next tower is the previous one atop a new base disc.
- Moving the discs above the base disc is the same as moving the previous tower.
- This reveals the new base which can be shifted in one move.
- Then the previous tower can be shifted onto the translated base disc in the same number of moves as before.

Consequently the students might interpret the sequence above as twice the previous number of moves plus one. This way of thinking is directly related to the sophisticated method of mathematical proof known as Mathematical Induction.

Of course the pattern could also be interpreted as $2^n - 1$. What would be the physical explanation from which this formula evolved?

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Triangles & Colours

The large scale colourful layout of triangles on each table makes this task very appealing. Students may only complete the questions on the card, the answers to which are 1, 4, 10, 20 & 35, but the card begs the question:

- *If I told you any number of colours could you tell me how many triangles?*

There is an algebraic generalisation that answers this question but it is not necessary for the children to chase it at this level. It is sufficient for them to solve all or part of the card, which requires organised pattern-based searching, and realise that the task can be revisited at a later date. However, if you have students who are ready to explore further there are extensive notes about the task on Maths300.

Lesson Comments

- These comments introduce you to each Maths300 lesson. The complete plan is easily accessed through the lesson library available to members at:
  http://www.maths300.com
  where they are listed alphabetically by lesson name.

Arithmagons

Although these delightful puzzles require only the ability to add whole numbers to 20, they are non-routine because the student needs to hold two conditions true simultaneously. The simple content level invites a wide range of students to participate and therefore become engaged in the process of Working Mathematically. The problems also succumb to algebraic representation and analysis. The only equipment needed is scrap paper and the recording sheets that can be printed from the site.

> *Just about finished Arithmagons lesson with my Year 7 class. Has gone very well. Much impressed with the lesson notes. Kids enjoyed it a lot. The LOTE teacher complained that the kids were drawing those so and so triangle things in Japanese. Kids currently finishing off by producing an explanatory poster, which must include some Arithmagons to be completed by another person such as their Mum or Dad.*

Back Tracking

After only one lesson students are able to use a pictorial method, combined with the problem solving strategies of working backwards and breaking a problem into smaller parts, to solve equations so complex that the text book examples seem like child's play. Some experience with algebraic symbols and facility with conventions for order of operations is required, but beyond that the lesson is open to students with a wide range of abilities. The lesson is supported by an Investigation Sheet which is much 'tougher' than the material offered in most texts. However, the experience of many teachers is that students 'love' the work. A great lesson to use before beginning the textbook chapter on equations, and plenty of room in the week to insert any relevant local material.

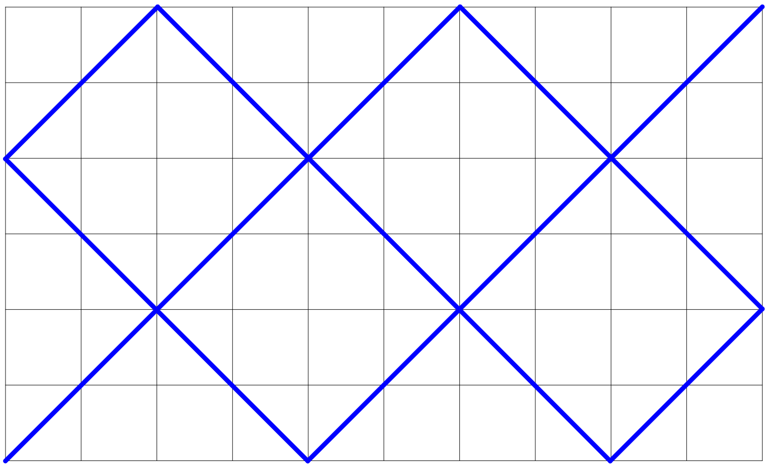
All the evidence is that the lesson ‘succeeds’ - so the question becomes *Why does it succeed?*, when so many text lessons on this topic fail. The answer has much to do with the effort that goes into attaching meaning to the equations rather than into instrumental techniques for their solution.

Billiard Ball Bounces

This lesson is one of several in the kit where geometric patterns lead to algebraic investigations. Two others in this genre are *Cracked Tiles* and *Simple, Elegant, Elusive*. Together the three have been integrated into the Planners so that the geometry/pattern/algebra theme is introduced in Year 5 through a Mixed Media Unit with these lessons and extended in Year 6 through another Mixed Media Unit.

Students have a 'billiard ball' table of size, say, 10 by 6. There are 'pockets' in all 4 corners. A ball is hit from the lower left corner at 45 degrees. How many bounces will it make until it goes into a pocket? And into which pocket will it go?

Drawing the pattern on grid paper produces:



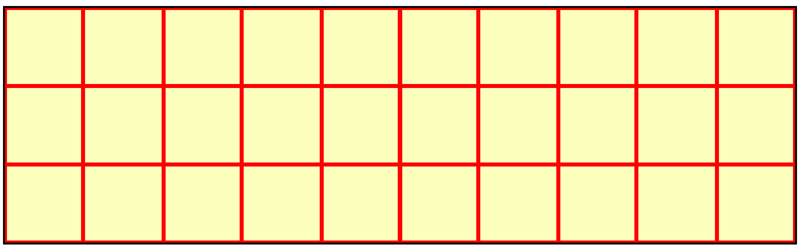
It takes 6 bounces and goes into the top right pocket. But what if the size of the table is changed to 10 by 7, or 8 by 5, or 100 by 47? Is it possible to predict in advance the number of bounces and final pocket?

Exploring this question converts the activity into an extended investigation involving many number patterns. A computer simulation allows theories to be developed and tested. The lesson nicely connects geometry patterns, symmetry, number theory (factors and ratios) and algebra (the generalisation of the patterns).

Cracked Tiles

Cracked Tiles and *Simple, Elegant, Elusive* are integrated into the kit in two ways. The first is an easy to state, easy to start whole class session which finishes by opening the door to continuing individual or small group investigation. The second is through the companion software which continues the investigation at one work station in the Year 5 Mixed Media Unit.

The story shell behind the lesson is that the builder has just finished laying a 10x3 tiled floor:



The electrician arrives and declares a (thin) cable has to be buried diagonally across the room, so some tiles have to be dug up (cracked) and then replaced. The builder asks:

- How many tiles have to be replaced?

This (fantasy?) story line sets the scene for an investigation that leads to algebraic descriptions of how many tiles need to be replaced for any sized floor. It involves drawing, data collecting and looking for patterns and their underlying relationships.

Game of 31

It is easy to start this simple card game and the playing rules are straightforward enough to allow the mind to begin thinking about strategy. The game begins as simple arithmetic, but can lead as deeply as you wish into the world of algebra.

As the game continues the basic arithmetic practice gives way to consideration of the logical strategies needed to win. These expose patterns which eventually make the game as much about patterns and algebra as about number skills.

It is also extremely flexible, in that it can be as simple or complex as desired. Consequently it has been successfully used from Year 2 to Year 10. The companion software allows students to test their emerging theories and understandings as they try to beat the computer.

Garden Beds

This Lesson Plan (and that for *Game of 31*) has been structured using teaching techniques developed by English as a Second Language (ESL) teachers. The devices within it that build mathematical conversation and recording/publishing of the investigation provide support for all students and can be abstracted for use with any lesson.

Garden Beds is a very rich context from which many mathematical concepts can be explored. The story of building a set of tiles around a garden captures students' interest. The mathematics of counting, area and perimeter, pattern and algebra are all evident. Within algebra the lesson covers several ideas concurrently, namely, the concepts of variable and function, substitution, solving equations, equivalence, and domain and range.

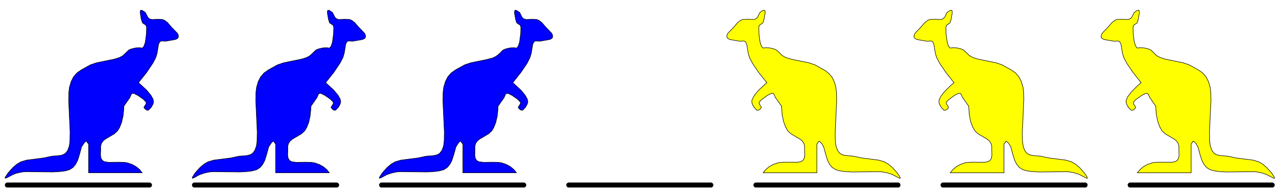
Garden Beds is a concrete and active lesson which suits many levels of ability. It provides opportunity for algebra to 'sneak up' on the students because the structure of the lesson encourages constructing and drawing, which leads to number patterns which the students naturally (and usually quite quickly) generalise. However, it is most often the case that within any class there is more than one way of seeing the generalisation. All are valid and the algebra makes sense because it can be related to the physical context. The investigation expands to seek:

- All the different ways we can predict the number of tiles for any garden.

These rules can be expressed first in natural language, then meaningfully converted to symbols. There is also ample opportunity in the lesson to link with graphing.

Jumping Kangaroos

Two groups of 'kangaroos' are facing each other on a narrow mountain trail. There are three going in each direction.



Their challenge is to swap places given that:

- one kangaroo can move at a time,
- it must either hop into an adjacent vacant place, or
- jump over an on-coming kangaroo into a vacant place.

This is a widely known puzzle that exists in many variations in many countries. (eg: 'Frogs on lily pads', 'Mountain goats on a trail', etc). To solve it requires a systematic approach to the movement of the 'kangaroos', and the patterns of moves yields a fascinating quadratic relationship. Students are encouraged to work out the number of moves required for different numbers of kangaroos and to look for patterns as the number of kangaroos increases. This allows them to make predictions for even higher numbers of 'kangaroos' on either side.

This is a great puzzle because, although it can be solved by logic, it becomes mathematically richer when the underlying algebra can be explicitly recognised and explored.

- *I used this lesson with some trepidation in a Grade 6. I had no worries about them being able to solve the problem, because it is essentially an application of logic, and I must admit the kinaesthetic introduction of the lesson appeals. However, I was concerned about the students' ability to deal with a quadratic relationship. What a waste of energy that concern was! It didn't come immediately, but the students did find me three different ways of predicting from the pattern. I realised that the fancy mathematical name 'quadratic' really relates to multiplying two linked numbers; and that's the pattern the students saw, even thought they didn't have a name for it.*

Painted Rods

Students who have used the **Pattern & Algebra Years 3 & 4** kit will have experienced this lesson as a task. If it is to be available to as wide a selection of learning styles as possible, the lesson is dependent upon equipment. It can also help if you create your own larger size demonstration set as suggested in the Lesson Plan.

The story shell surrounding the lesson is that a person decides to paint the outside surface of a series of rods of various lengths.

- *How can you predict the number of unit squares of area that will be painted for any length of rod?*

Students use Cuisenaire rods to model simple cases, collect data from these and generalise the results into a rule. The first challenge is to work out the painted area for a rod 100 units long.

A central aspect of the lesson is explaining and justifying the rule so that others can see how it is done. The collection of data points created as the problem is solved (ie: Size 1 rod = 6 units, Size 2 rod = 10 units, etc.) provides opportunity for graphing and therefore reinforcing with the students that whenever there is a visual pattern there will be an underlying number pattern and vice versa.

Simple, Elegant, Elusive

Students have a circle with 10 points numbered from 1 to 10. (The lesson includes a master document with circles already prepared). A pattern is generated by using a multiplication rule such as 'Times 3'. Starting at 1, 1 times 3 = 3, so draw a line from 1 to 3, then 2 times 3 = 6 so draw a line from 2 to 6. Continuing this for all other numbers from 1 to 10 produces an elegant and interesting pattern. The pattern is symmetric and has two separate sets of interlocking triangles.

- But what if the rule is changed to 'Times 4', or 'Times 7'. What happens to the pattern?
- And what if we change the number of points in the circle to 6, or 11, or any other number?

Exploring these questions converts the activity into an extended investigation involving many number patterns. The openness of the investigation invites many levels of challenge and offers opportunity for students to plan their own investigation.

The main challenge is the generalisation: *Predict the pattern for any number of points on the circle (P) and any rule (times T)*.

The companion software allows theories about the shapes and the relationship between P and T to be developed and tested. This lesson elegantly links geometry patterns, number theory (factors and ratios) and algebra (the generalisation of the patterns). It is also an excellent example of creating a learning environment to generate an 'aha' experience for students. The teacher cannot create the 'aha' moment - this occurs at different times for different students - but the environment can be orchestrated to increase the chances of it.

> Using the times table on the circle points I have had success with a disenfranchised girl who has done nothing for years ... she loved it! Take care - I love the lessons - even if I am so hassled that I can't remember their names! The kids do...

Snail Trail

A snail determined to climb out of a well sets out at a steady speed, but needs to rest after a given time. During the rest period it slips back a given amount. The challenge is to decide when it will escape. This is a well known problem that has appeared in many guises in many countries. For example, the snail is often a frog. The puzzle can help students develop logical skills, however a world of algebra opens up by exploring the effect of changing the many variables involved.

The lesson is perfect for a kinaesthetic start by taking the students outside and quickly marking some chalk lines on the asphalt or marking positions on the grass with twigs. Once begun it can be continued inside with the game board supplied in the lesson.

It is also a great lesson for conversation and ‘argument’. Students often get different answers and then argue among themselves in order to justify both their logic and calculations.

Staircases

Seeing visual patterns that lead to number patterns and the opportunity to generalise these and create algebra is a theme running through this kit and its Years 3 & 4 equivalent. Staircases provides opportunity for students to visualise in a 3D situation and because the focus is on the number pattern in the spatial and concrete context, rather than the number pattern as a printed list in the way it might appear in a text, students can achieve success at a range of levels, even though the activity might be considered sophisticated.

The Lesson Plan can be largely self-directed for the students using an investigation sheet that leads them into the iceberg of the initial task. You will need linking cubes such as Multi-Link or Unifix, or wooden cubes, or square tiles to build staircases. The visual pattern of the steps in the staircase is a hint that there is an associated number/algebra pattern. The discovery of that pattern opens the door to further algebra and to a visual representation of the pattern in graphical form.

The lesson offers many opportunities to model working like a mathematician and its pedagogy links well with *Garden Beds*, *Painted Cubes* and *Painted Rods* in this kit, and *4 Arm Shapes*, *Lining Up* and *Unseen Triangles* from the Years 3 & 4 kit.

Walking With Children

The authenticity of this lesson is that it has clearly come from a particular colleague's classroom. Inspired by the lesson *Algebra Walk* (Lesson 22, Maths300), the teacher in this story adapted the idea to use with younger children. The content was limited to straight line graphs, but the learning environment was open ended. The lesson contains examples of children's work which demonstrate that Year 5 children are capable of understanding far more sophisticated mathematical concepts and deeper mathematical thought than is usually plumbed.

Several times the teacher comments that he did not have a pre-determined outcome in mind for the students. On these occasions the 'next lesson' was planned only after the students had 'shown their stuff' in the current lesson. Teacher and students alike were prepared to follow where the intellectual challenge might lead. Intrinsic in this form of presentation is the notion that students can be trusted to be equal partners in the learning. This approach has been applied more widely in **Maths With Attitude** in the **Number & Computation Years 5 & 6** kit, in the units titled 'Self-directed Maths Journey'.

Taking a journey like this with a class is a risk, but clearly, for this teacher, walking with children in this way was very rewarding.

Part 3:
Value
Adding

The Poster Problem Clinic

Maths With Attitude kits offer several models for building a Working Mathematically curriculum around tasks. Each kit uses a different model, so across the range of 16 kits, teachers' professional learning continues and students experience variety. The Poster Problem Clinic is an additional model. It can be used to lead students into working with tasks, or it can be used in a briefer form as a opening component of each task session.

> *I was apprehensive about using tasks when it seemed such a different way of working. I felt my children had little or no experience of problem solving and I wanted to prepare them to think more deeply. The Clinic proved a perfect way in.*

Careful thought needs to be given to management in such lessons. One approach to getting the class started on the tasks and giving it a sense of direction and purpose is to start with a whole class problem. Usually this is displayed on a poster that all can see, perhaps in a Maths Corner. Another approach is to print a copy for each person. A Poster Problem Clinic fosters class discussion and thought about problem solving strategies.

Starting the lesson this way also means that just prior to liberating the students into the task session, they are all together to allow the teacher to make any short, general observations about classroom organisation, or to celebrate any problem solving ideas that have arisen.

One teacher describes the session like this:

> *I like starting with a class problem - for just a few minutes - it focuses the class attention, and often allows me to introduce a particular strategy that is new or needs emphasis.*

It only takes a short time to introduce a poster and get some initial ideas going. The class discussion develops a way of thinking. It allows class members to hear, and learn from their peers, about problem solving strategies that work for them.

> *If we don't collectively solve the problem in 5 minutes, I will leave the problem 'hanging' and it gives a purpose to the class review session at the end.*
> *Sometimes I require everyone to work out and write down their solution to the whole class problem. The staggered finishing time for this allows me to get organised and help students get started on tasks without being besieged.*
> *I try to never interrupt the task session, but all pupils know we have a five minute review session at the end to allow them to comment on such things as an activity they particularly liked. We often close then with an agreed answer to our whole class problem.*

A Clinic in Action

The aims of the regular clinic are:

- to provide children with the opportunity to learn a variety of strategies
- to familiarise children with a process for solving problems.

The following example illustrates a structure which many teachers have found successful when running a clinic.

Preparation

For each session teachers need:

- a Strategy Board as below
- a How To Solve A Problem chart as below
- to choose a suitable problem and prepare it as a poster
- to organise children into groups of two or three.

The Strategy Board can be prepared in advance as a reference for the children, or may be developed *with* the children as they explore problem solving and suggest their own versions of the strategies.

The problem can be chosen from

- a book
- the task collection
- prepared collections such as Professor Morris Puzzles which can be viewed at: http://www.mathematicscentre.com/taskcentre/resource.htm#profmorr

The example which follows is from the task collection. The teacher copied it onto a large sheet of paper and asked some children to illustrate it. *The teacher also changed the number of sheep to sixty* to make the poster a little different from the one in the task collection.

The Strategy Board and the How To Solve A Problem chart can be used in any maths activity and are frequently referred to in Maths300 lessons.

The Clinic

The poster used for this example session is:

> Eric the Sheep is lining up to be shorn before the hot summer ahead. There are sixty [60] sheep in front of him. Eric can't be bothered waiting in the queue properly, so he decides to sneak towards the front.
>
> Every time one [1] sheep is taken to be shorn, Eric <u>then</u> sneaks past two [2] sheep. How many sheep will be shorn <u>before</u> Eric?

This Poster Problem Clinic approach is also extensively explored in Maths300 Lesson 14, *The Farmer's Puzzle*.

Strategy Board

DO I KNOW A SIMILAR PROBLEM?	ACT IT OUT
GUESS, CHECK AND IMPROVE	DRAW A PICTURE OR GRAPH
TRY A SIMPLER PROBLEM	MAKE A MODEL
WRITE AN EQUATION	LOOK FOR A PATTERN
MAKE A LIST OR TABLE	TRY ALL POSSIBILITIES
WORK BACKWARDS	SEEK AN EXCEPTION
BREAK INTO SMALLER PARTS	...

How To Solve A Problem

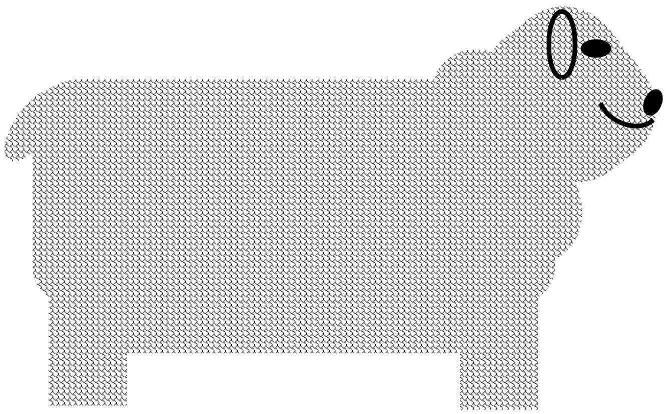
SEE & UNDERSTAND	Do I understand what the problem is asking? Discuss
PLANNING	Select a strategy from the board. Plan how you intend solving the problem.
DOING IT	Try out your idea.
CHECK IT	Did it work out? If so reflect on the activity. If not, go back to step one.

Step 1

- Tell the children that we are at Stage 1 of our four stage plan ... **See & Understand** ... Point to it! Read the problem with the class. Discuss the problem and clarify any misunderstandings.
- If children do not clearly understand what the problem is asking, they will not cope with the next stage. A good way of finding out if a child understands a problem is for her/him to retell it.
- Allow time for questions - approximately 3 to 5 minutes.

Step 2

- Tell the children that we are at Stage 2 of our four stage plan ... **Planning**. In their groups children select one or more strategies from the Strategy Board and discuss/organise how to go about solving the problem.
- Without guidance, children will often skip this step and go straight to Doing It. It is vital to emphasise that this stage is simply planning, not solving, the problem.
- After about 3 minutes, ask the children to share their plans.



Plan 1

Well we're drawing a picture and sort of making a model.

Can you give me more information please Brigid?

We're putting 60 crosses on our paper for sheep and the pen top will be Eric. Then Claire will circle one from that end, and I will pass two crosses with my pen top.

Plan 2

Our strategy is Guess and Check.

That's good Nick, but how are you going to check your guess?

Oh, we're making a model.

Go on ...

> *John's getting MAB smalls to be sheep and I'm getting a domino to be Eric and the chalk box to be the shed for shearing.*

Plan 3

> *We are doing it for 3 sheep then 4 sheep then 5 sheep and so on. Later we will look at 60.*

Great so you are going to try a simpler problem, make a table and look for a pattern.

This sharing of strategies is invaluable as it provides children who would normally feel lost in this type of activity with an opportunity to listen to their peers and make sense out of strategy selection. Note that such children are not given the answer. Rather they are assisted with understanding the power of selecting and applying strategies.

Step 3

- Tell the children that we are at Stage 3 of our four stage plan ... **Doing It**. Children collect what they need and carry out their plan.

Step 4

- Tell the children that we are at Stage 4 of our four stage plan ... **Check It**. Come together as a class for groups to share their findings. Again emphasis is on strategies.

> *We used the drawing strategy, but we changed while we were doing it because we saw a pattern.*

So Jake, you used the Look For A Pattern strategy. What was it?

> *We found that when Eric passed 10 sheep, 5 had been shorn, so 20 sheep meant 10 had been shorn ... and that means when Eric passes 40 sheep, 20 were shorn and that makes the 60 altogether.*

Great Jake. How would you work out the answer for 59 sheep or 62 sheep?

Sharing time is also a good opportunity to add in a strategy which no one may have used. For example:

> *Maybe we could've used the Number Sentence strategy, ie: 1 sheep goes to be shorn and Eric passes two sheep. That's 3 sheep, so perhaps, 60 divided into groups of 3, or 60 ÷ 3 gives the answer.*

Round off the lesson by referring to the Working Mathematically chart. There will be many opportunities to compliment the students on working like a mathematician.

Curriculum Planning Stories

Our attitude is:

teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

In more than a decade of using tasks and many years of using the detailed whole class lessons of Maths300, teachers have developed several models for integrating tasks and whole class lessons. Some of those stories are retold here. Others can be found at:

- http://www.mathematicscentre.com/taskcentre/plans.htm

Story 1: Threading

Educational research caused me a dilemma. It tells us that students construct their own learning and that this process takes time. My understanding of the history of mathematics told me that certain concepts, such as place value and fractions, took thousands of years for mathematicians to understand. The dilemma was being faced with a textbook that expected students to 'get it' in a concentrated one, two or three week block of work and then usually not revisit the topic again until the next academic year.

A Working Mathematically curriculum reflects the need to provide time to learn in a supportive, non-threatening environment and...

When I was involved in a Calculating Changes PD program I realised that:

- *choosing rich and revisitable activities, which are familiar in structure but fresh in challenge each time they are used, and*
- *threading them through the curriculum over weeks for a small amount of time in each of several lessons per week*

resulted in deeper learning, especially when partnered with purposeful discussion and recording.

Calculating Changes:

- http://www.mathematicscentre.com/calchange

Story 2: Your turn

Some teachers are making extensive use of a partnership between the whole class lessons of Maths300 and small group work with the tasks. Setting aside a lesson for using the tasks in the way they were originally designed now seems to have more meaning, as indicated by this teacher's story:

When I was thinking about helping students learn to work like a mathematician, my mind drifted to my daughter learning to drive. She *needed me to model how to do it and then she needed lots of opportunity to try it for herself.*

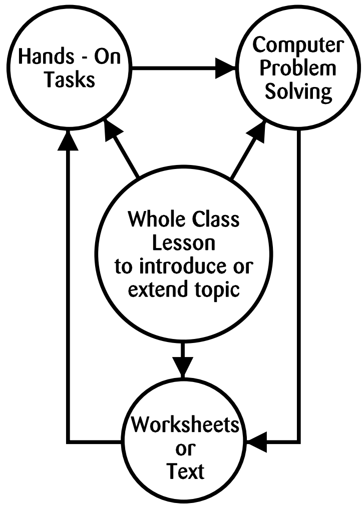
That's when the idea clicked of using the Maths300 lessons as a model and the tasks as a chance for the students to have their turn to be a mathematician.

The Maths300 lessons illustrate how other teachers have modelled the process, so I felt I could do it too. Now the process is always on display on the wall or pasted inside the student's journal.

A session just using the tasks had seemed a bit like play time before this. Now I see it as an integral part of learning to work mathematically.

Story 3: Mixed Media

It was our staff discussion on Gardner's theory of Multiple Intelligences that led us into creating mixed media units. That and the access you have provided to tasks and Maths300 software.



We felt challenged to integrate these resources into our syllabus. There was really no excuse for a text book diet that favours the formal learners. We now often use four different modes of learning in the work station structure shown. It can be easily managed by one teacher, but it is better when we plan and execute it together.

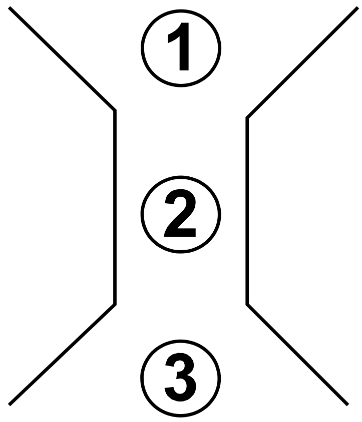
Story 4: Replacement Unit

We started meeting with the secondary school maths teachers to try to make transition between systems easier for the students. After considerable discussion we contracted a consultant who suggested that school might look too much the same across the transition when the students were hoping for something new. On the other hand our experience suggested that there needed to be some consistency in the way teachers worked.

We decided to 'bite the bullet' and try a hands-on problem solving unit in one strand. We selected two menus of twenty hands-on tasks, one for the primary and one for the secondary, that became the core of the unit. We deliberately overlapped some tasks that we knew were very rich and added some new ones for the high school.

Class lessons and investigation sheets were used to extend the tasks, within a three week model.

It is important to note that although these teachers structured a 3 week unit for the students, they strongly advised an additional *Week Zero* for staff preparation. The units came to be called Replacement Units.



Week Zero - Planning

Staff familiarise themselves with the material and jointly plan the unit. This is not a model that can be 'planned on the way to class'.

> *Getting together turned out to be great professional development for our group.*

Week 1 - Introduction

Students explore the 20 tasks listed on a printed menu:

- students explore the tip of the task, as on the card
- students move from task to task following teacher questioning that suggests there is more to the task than the tip
- in discussion with students, teachers gather informal assessment information that guides lesson planning for the following week.

> *We gave the kids an 'encouragement talk' first about joining us in an experiment in ways of learning maths and then gave out the tasks. The response was intelligent and there was quite a buzz in the room.*

Week 2 - Formalisation

> *It was good for both us and the students that the lessons in this week were a bit more traditional. However, they weren't text book based. We used whole class lessons based on the tasks they had been exploring and taught the Working Mathematically process, content and report writing.*
>
> *Assessment was via standard teacher-designed tests, quizzes and homework.*

Week 3 - Investigations

> *We were most delighted with Week 3. Each student chose one task from the menu and carried out an in-depth investigation into the iceberg guided by an investigation sheet. They had to publish a report of their investigation and we were quite surprised at the outcomes. It was clear that the first two weeks had lifted the image of mathematics from 'boring repetition' to a higher level of intellectual activity.*

Story 5: Curriculum shift

> *I think our school was like many others. The syllabus pattern was 10 units of three weeks each through the year. We had drifted into that through a text book driven curriculum and we knew the students weren't responding.*

Our consultant suggested that there was sameness about the intellectual demands of this approach which gave the impression that maths was the pursuit of skills. We agreed to select two deeper investigations to add to each unit. It took some time and considerable commitment, but we know that we have now made a curriculum shift. We are more satisfied and so are the students.

The principles guiding this shift were:

- Agree

  The 20 particular investigations for the year are agreed to by all teachers. If, for example, Cube Nets *is decided as one of these, then all the teachers are committed to present this within its unit.*

- Publish

  The investigations are written into the published syllabus. Students and parents are made aware of their existence and expect them to occur.

- Commit

  Once agreed, teachers are required to present the chosen investigations. They are not a negotiable 'extra'.

- Value

  The investigations each illustrate an explicit form of the Working Mathematically process. This is promoted to students, constantly referenced and valued.

- Assess

  The process provides students with scaffolding for their written reports and is also known by them as the criteria for assessment. *(See next page.)*

- Report

  The assessment component features within the school reporting structure.

A Final Comment

Including investigations has become policy.

Why? Because to not do so is to offer a diminished learning experience.

The investigative process ranks equally with skill development and needs to be planned for, delivered, assessed and reported.

Perhaps most of all we are grateful to our consultant because he was prepared to begin where we were. We never felt as if we had to throw out the baby and the bath water.

Assessment

Our attitude is:

stimulated students are creative and love to learn

Regardless of the way you use your **Maths With Attitude** resource, a variety of procedures can be employed to assess this learning.

Where these assessment procedures are applied to task sessions and involve written responses from students, teachers will need to be careful that the writing does not become too onerous. Students who get bogged down in doing the writing may lose interest in doing the tasks.

In addition to the ideas below, useful references are:

- http://www.mathematicscentre.com/taskcentre/assess.htm
- http://www.mathematicscentre.com/taskcentre/report.htm

The first offers several methods of assessment with examples and the second is a detailed lesson plan to support students to prepare a Maths Report.

Journal Writing

Journal writing is a way of determining whether the task or lesson has been understood by the student. The pupil can comment on such things as:

- What I learned in this task.
- What strategies I/we tried (refer to the Strategy Board).
- What went wrong.
- How I/we fixed it.
- Jottings - ie: any special thoughts or observations

Some teachers may prefer to have the page folded vertically, so that children's reflective thoughts can be recorded adjacent to critical working.

Assessment Form

An assessment form uses questions to help students reflect upon specific issues related to a specific task.

Anecdotal Records

Some teachers keep ongoing records about how students are tackling the tasks. These include jottings on whether students were showing initiative, whether they were working co-operatively, whether they could explain ideas clearly, whether they showed perseverance.

Checklists

A simple approach is to create a checklist based on the Working Mathematically process. Teachers might fill it in following questioning of individuals, or the students may fill it in and add comments appropriately.

Pupil Self-Reflection

Many theorists value and promote metacognition, the notion that learning is more permanent if pupils deliberately and consciously analyse their own learning. The deliberate teaching strategy of oral questioning and the way pupils record their work is an attempt to manifest this philosophy in action. The alternative is the tempting 'butterfly' approach which is to madly do as many activities as possible, mostly superficially, in the mistaken belief that quantity equates to quality.

> *I had to work quite hard to overcome previously entrenched habits of just getting the answer, any answer, and moving on to the next task.*

Thinking about *what* was learned *how* it was learned consolidates and adds to the learning.

When it follows an extensive whole class investigation, a reflection lesson such as this helps to shift entrenched approaches to mathematics learning. It is also an important component of the assessment process. On the one hand it gives you a lot of real data to assist your assessment. On the other it prepares the students for any formal assessment which you may choose to round off a unit.

Introduction

Ask students to recall what was done during the unit or lesson by asking a few individuals to say what *they* did, eg:

> *What did you do or learn that was new?*
> *What can you now do/understand that is new?*
> *What do you know now that you didn't know 1 (2, 3, ...) lesson ago?*

Continuing Discussion

Get a few ideas from the first students you ask, then:

- organise 5 -10 minute buzz groups of three or four students to chat together with one person to act as a recorder. These groups address the same questions as above.
- have a reporting session, with the recorder from each group telling the class about the group's ideas.

Student comments could be recorded on the board, perhaps in three groups.

<u>Ideas & Facts</u> <u>Maths Skills</u> <u>Process (learning) Skills</u>

If you need more questions to probe deeper and encourage more thought about process, try the following:

> *What new things did you do that were part of* how *you learned?*
> *Who uses this kind of knowledge and skill in their work?*

Student Recording

Hand out the REFLECTION sheet (next page) and ask students to write their own reflection about what they did, based on the ideas shared by the class. Collect these for interest and, possibly, assessment information.

REFLECTION

me looking at me learning

NAME: CLASS:

Working With Parents

Balancing Problem Solving with Basic Skill Practice

Many schools find that parents respond well to an evening where they have an opportunity to work with the tasks and perhaps work a task together as a 'whole class'. Resourced by the materials in this kit, teachers often feel quite confident to run these practical sessions. Comments from parents like:

> *I wish I had learnt maths like this.*

are very supportive. Letting students 'host' the evening is an additional benefit to the home/school relationship.

The 4½ Minute Talk

Charles Lovitt has considerable experience working with parents and has developed a crisp, parent-friendly talk which he shares below. Many others have used it verbatim with great success.

Why the Four and a Half Minute Talk?

When talking with parents about Problem Solving or the meaning of the term Working Mathematically, I have often found myself in the position, after having promoted inquiry based or investigative learning, of the parents saying:

> *Well - that's all very well - BUT ...*

at which stage they often express their concern for basic (meaning arithmetic) skill development.

The weakness of my previous attempts has been that I have been unable to reassure parents that problem solving does not mean sacrificing our belief in the virtues of such basic skill development.

One of the unfortunate perceptions about problem solving is that if a student is engaged in it, then somehow they are not doing, or it may be at the expense of, important skill based work.

This Four and a Half Minute Talk to parents is an attempt to express my belief that basic skill practice and problem solving development can be closely intertwined and not seen as in some way mutually exclusive.

(I'm still somewhat uncomfortable using the expression 'basic skills' in the above way as I am certain that some thinking, reasoning, strategy and communication skills are also 'basic'.)

Another aspect of the following ‘talk’ is that, as teachers put more emphasis on including investigative problem solving into their courses, a question arises about the source of suitable tasks.

This talk argues that we can learn to create them for ourselves by 'tweaking' the closed tasks that heavily populate out existing text exercises, and hence not be dependent on external suppliers. (Even better if students begin to create such opportunities for themselves.)

The Talk

In preparation, write the following graphic on the board:

CLOSED	OPEN	EXTENDED INVESTIGATION
		How many solutions exist? How do you know you have found them all?

I would like to show you what teachers are beginning to do to achieve some of the thinking and reasoning and communication skills we hope students will develop. I would like to show you three examples.

Example One: *6 + 5 = ?*

I write this question under the ‘closed’ label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 ___		How many solutions exist? How do you know you have found them all?

And I ask:

What is the answer to this question?

I then explain that:

We often ask students many closed questions such as 6 + 5 = ?

The only response the students can tell us is "The answer is 11." … and as a reward for getting it correct we ask another twenty questions just like it.

What some teachers are doing is trying to *tweak* the question and ask it a different way, for example:

I have two counting numbers that add to 11. What might the numbers be?

[Counting numbers = positive whole numbers including zero]

I write this under the 'open' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 ___	? + ? 11	How many solutions exist? How do you know you have found them all?

What is the answer to the question now?

At this stage it becomes apparent there are several solutions:

The question is now a bit more open than it was before, allowing students to tell you things like 8 + 3, or 10 + 1, or 11 + 0 etc.

Let's see what happens if the teacher 'tweaks' it even further with the investigative challenge *or* extended investigation question:

How many solutions are there altogether?

and more importantly, and with greater emphasis on the second question:

How could you convince someone else that you have found them all?

Now the original question is definitely different - it still involves the skills of addition but now also involves thinking, reasoning and problem solving skills, strategy development and particularly communication skills.

Young students will soon tell you the answer is 'six different ones', but they must also confront the communication and reasoning challenge of convincing you that there are only six and no more.

Example Two: *Finding Averages*

Again, as I go through this example, I write it into the diagram on the board in the relevant sections.

The CLOSED question is: *11, 12, 13 - find the average*

Tweaking this makes it an OPEN question and it becomes:

I have three counting numbers whose average is 12. What might the numbers be?

Students will often say:

10, 12, 14 … or 9, 12, 15 … or even 12, 12, 12

After realising there are many answers, you can tweak it some more and turn it into an EXTENDED INVESTIGATION:

How many solutions exist? … AND …

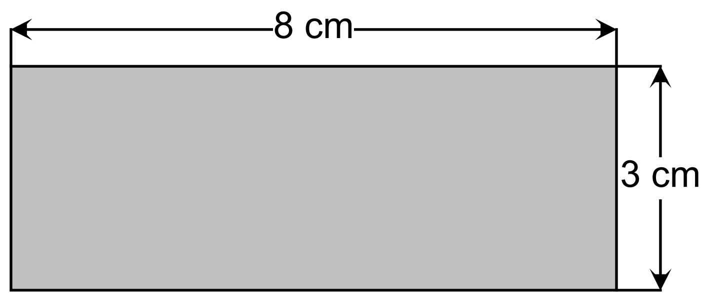
How do you know you have found them all?

Now the question is of a quite different nature. It still involves the arithmetic skill, but has something else as well - and that something else is the thinking, reasoning and communication skills necessary to find all of the combinations and convince someone else that you have done so.

By the time a student announces, with confidence, there are 127 different ways (which there are) that student will have engaged in all of these aspects, ie: the skill of calculating averages, (and some combination number theory) as well as significant strategy and reasoning experiences.

Example Three: *Finding the Area of a Rectangle*

A typical CLOSED question is:



Find the area. Find the perimeter.

The OPEN question is:

A rectangle has 24 squares inside:

What might its length and width be?

What might its perimeter be?

The EXTENDED INVESTIGATION version is:

Given they are whole number lengths, how many different rectangles are there? … AND …

How do you know you have found them all?

In summary, mathematics teachers are trying to convert *some* (not all) of the many closed questions that populate our courses and 'push' them towards the investigation direction. In doing so, we keep the skills we obviously value, but also activate the thinking, reasoning and justification skills we hope students will also develop.

This sequence of three examples hopefully shows two major features:

- That skills and problem solving can 'live alongside each other' and be developed concurrently.
- That the process of creating open-ended investigations can be done by anyone - just go to any source of closed questions and try 'tweaking' them as above. If it only worked for one question per page it would still provide a very large supply of investigations.

In terms of the effect of the talk on parents, I have usually found them to be reassured that we are not compromising important skill development (and nor do we want to). The only debate then becomes whether the additional skills of thinking, reasoning and communication are also desirable.

I've also been told that parents appreciate it because of the essential simplicity of the examples - no complicated theoretical jargon.

A Working Mathematically Curriculum

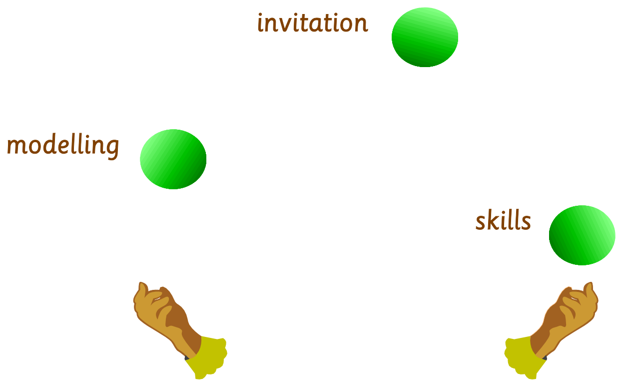
An Investigative Approach to Learning

The aim of a Working Mathematically curriculum is to help students learn to work like a mathematician. This process is detailed earlier (Page 8) in a one page document which becomes central to such a curriculum.

The change of emphasis brings a change of direction which *implies and requires* a balance between:

- the process of being a mathematician, and
- the development of skills needed to be a *successful* mathematician.

This journey is not two paths. It is one path made of two interwoven threads in the same way as DNA, the building block of life, is one compound made of two interwoven coils. To achieve a Working Mathematically curriculum teachers need to balance three components.



The task component of **Maths With Attitude** offers each pair of students an invitation to work like a mathematician.

The Maths300 component of **Maths With Attitude** assists teachers to model working like a mathematician.

Content skills are developed in context. They *are* important, but it is the application of skills within the process of Working Mathematically that has developed, and is developing, the human community's mathematical knowledge.

A focus for the Working Mathematically teacher is to help students develop mathematical skills in the context of problem posing and solving.

> *We are all 'born' with the same size mathematical toolbox, in the same way as I can own the same size toolbox as my motor mechanic. However, my motor mechanic has many more tools in her box than I and she has had more experience than I using them in context. Someone has helped her learn to use those tools while crawling under a car.*

Afzal Ahmed, Professor of Mathematics at Chichester, UK, once quipped:

> *If teachers of mathematics had to teach soccer, they would start off with a lesson on kicking the ball, follow it with lessons on trapping the ball and end with a lesson on heading the ball. At no time would they play a game of football.*

Such is not the case when teaching a Working Mathematically curriculum.

Elements of a Working Mathematically Curriculum

Working Mathematically is a K - 12 experience offering a balanced curriculum structured around the components below.

Hands-on Problem Solving Play

Mathematicians don't know the answer to a problem when they start it. If they did, it wouldn't be a problem. They have to play around with it. Each task invites students to play with mathematics 'like a mathematician'.

Skill Development

A mathematician needs skills to solve problems. Many teachers find it makes sense to students to place skill practice in the context of *Toolbox Lessons* which *help us better use the Working Mathematically Process* (Page 8).

Focus on Process

This is what mathematicians do; engage in the problem solving process.

Strategy Development

Mathematicians also make use of a strategy toolbox. These strategies are embedded in Maths300 lessons, but may also have a separate focus. Poster Problem Clinics are a useful way to approach this component.

Concept Development

A few major concepts in mathematics took centuries for the human race to develop and apply. Examples are place value, fractions and probability. In the past students have been expected to understand such concepts after having 'done' them for a two week slot. Typically they were not visited again until the next year. A Working Mathematically curriculum identifies these concepts and regularly 'threads' them through the curriculum.

Planning to Work Mathematically

The class, school or system that shifts towards a Working Mathematically curriculum will no longer use a curriculum document that looks like a list of content skills. The document would be clear in:

- choosing genuine problems to initiate investigation
- choosing a range of best practice teaching strategies to interest a wider range of students
- practising skills for the purpose of problem solving

Some teachers have found the planning template on the next page assists them to keep this framework at the forefront of their planning. It can be used to plan single lessons, or units built of several lessons. There are examples from schools in the Curriculum & Planning section of Maths300 and a Word document version of the template.

Unit Planning Page

Class

Topic

Pedagogy	Problem Solving	Skills
	In this topic how will I engage my students in the Working Mathematically process?	
How do I create an environment where students know what they are doing and why they have accepted the challenge?		Does the challenge identify skills to practise? Are there other skills to practise in preparation for future problem solving?

Notes

As a general guide:

- Find a problem(s) to solve related to the topic.
- Choose the best teaching craft likely to engage the learners.
- Where possible link skill practice to the problem solving process.

More on Professional Development

For many teachers there will be new ideas within **Maths With Attitude**, such as unit structures, views of how students learn, teaching strategies, classroom organisation, assessment techniques and use of concrete materials. It is anticipated (and expected) that as teachers explore the material in their classrooms they will meet, experiment with and reflect upon these ideas with a view to long term implications for the school program and for their own personal teaching.

Being explored 'on-the-job' so to speak, in the teacher's own classroom, makes the professional development more meaningful and practical for the teacher. This is also a practical and economic alternative for a local authority.

Strategic Use by Systems

From Years 3 - 10, **Maths With Attitude** is designed as a professional development vehicle by schools or clusters or systems because it carries a variety of sound educational messages. They might choose **Maths With Attitude** because:

- It can be used to highlight how investigative approaches to mathematics can be built into balanced unit plans without compromising skill development and without being relegated to the margins of a syllabus as something to be done only after 'the real' content has been covered.
- It can be used to focus on how a balance of concept, skill and application work can all be achieved within the one manageable unit structure.
- It can be used to show how a variety of assessment practices can be used concurrently to build a picture of student progress.
- It can be used to focus on transition between primary and secondary school by moving towards harmony and consistency of approach.
- It can be used to raise and continue debate about the pedagogy (art of teaching) that supports deeper mathematical learning for a wider range of students.

Teachers in Years K - 2 are similarly encouraged in professional growth through **Working Mathematically with Infants**, which derives from Calculating Changes, a network of teachers enhancing children's number skills from Years K - 6.

In supporting its teachers by supplying these resources in conjunction with targeted professional development over time, a system can fuel and encourage classroom-based debate on improving outcomes. There is evidence that by exploring alternative teaching strategies and encouraging curriculum shift towards Working Mathematically, learners improve and teachers are more satisfied. For more detail visit Research & Stories at:

- http://www.mathematicscentre.com/taskcentre/do.htm

We would be happy to discuss professional development with system leaders.

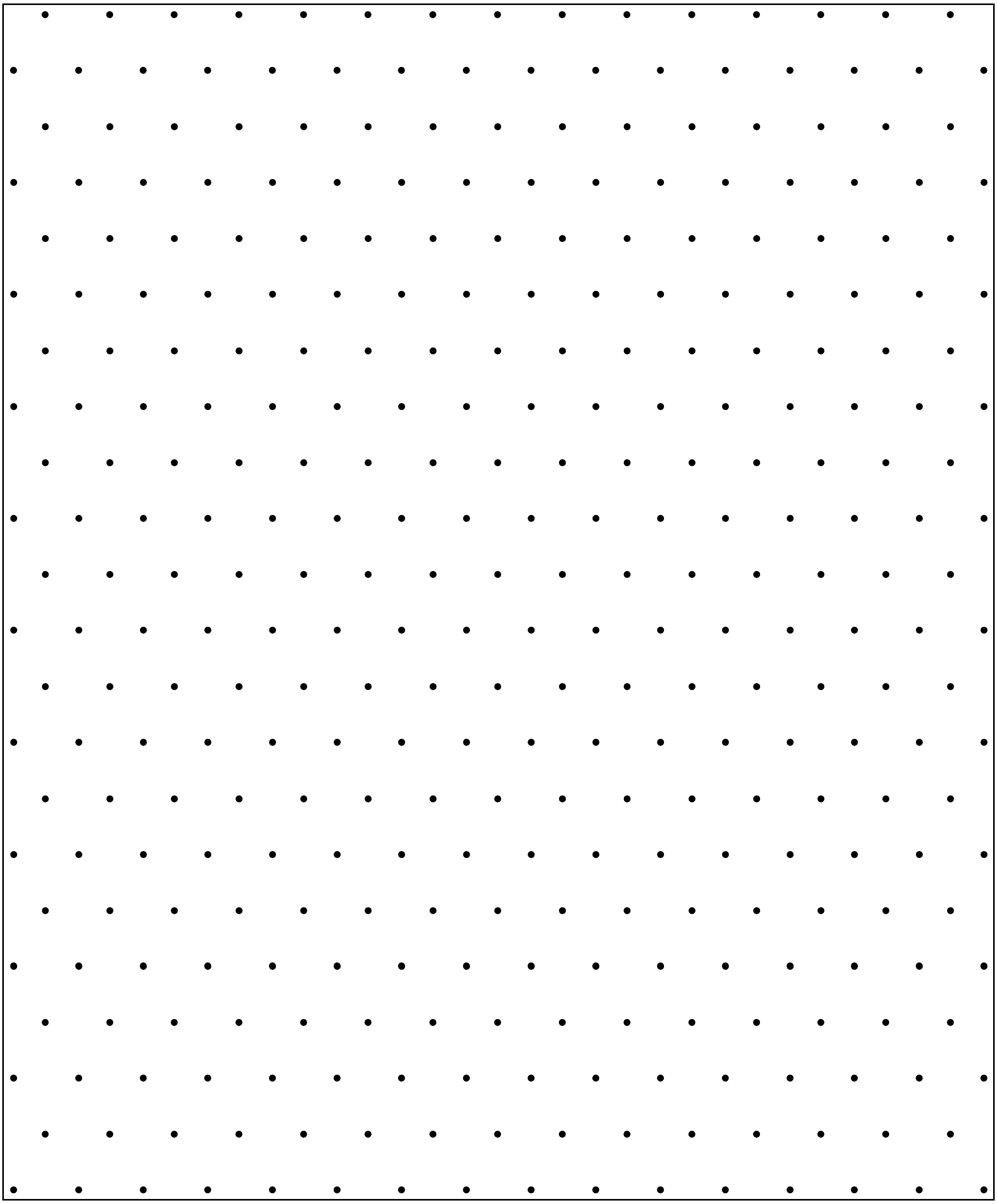
Web Reference

The starting point for all aspects of learning to work like a mathematician, including Calculating Changes, and the teaching craft which encourages it is:

- http://www.mathematicscentre.com/mathematicscentre

Appendix: Recording Sheets

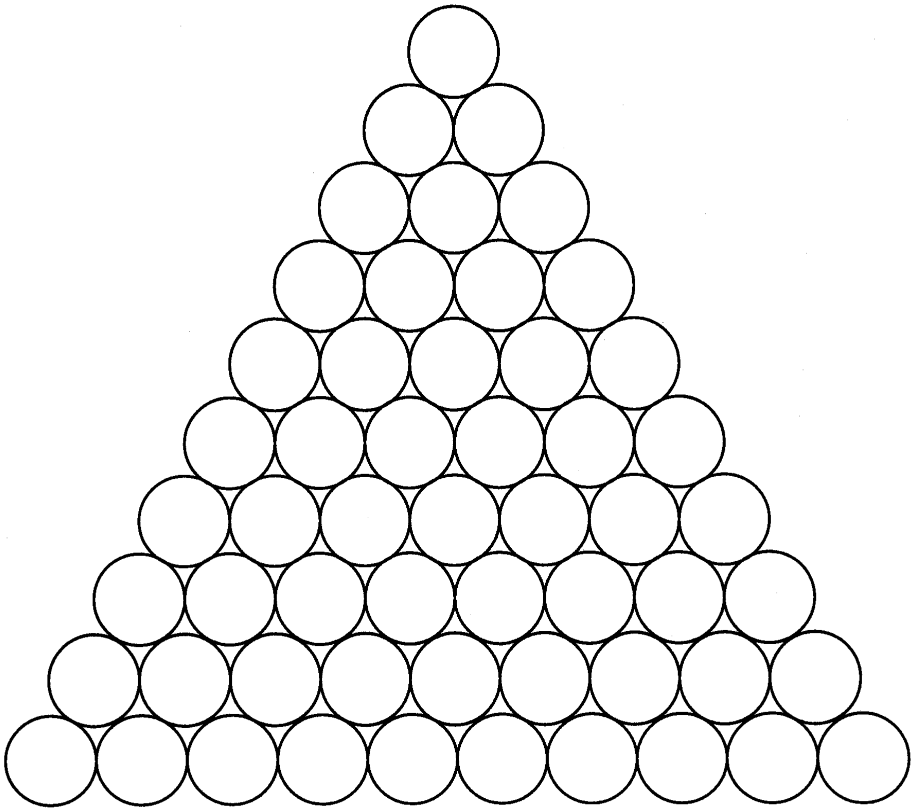
How Many Triangles?

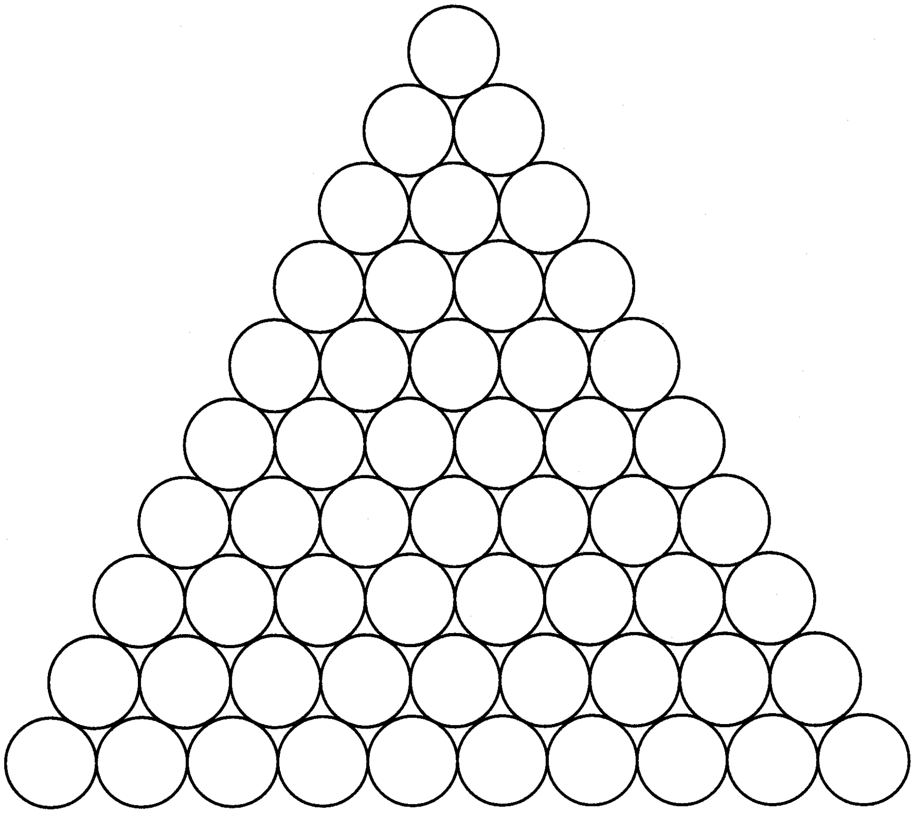


Names: .... Class: ....

Plate Triangles

Names: .... Class: ....

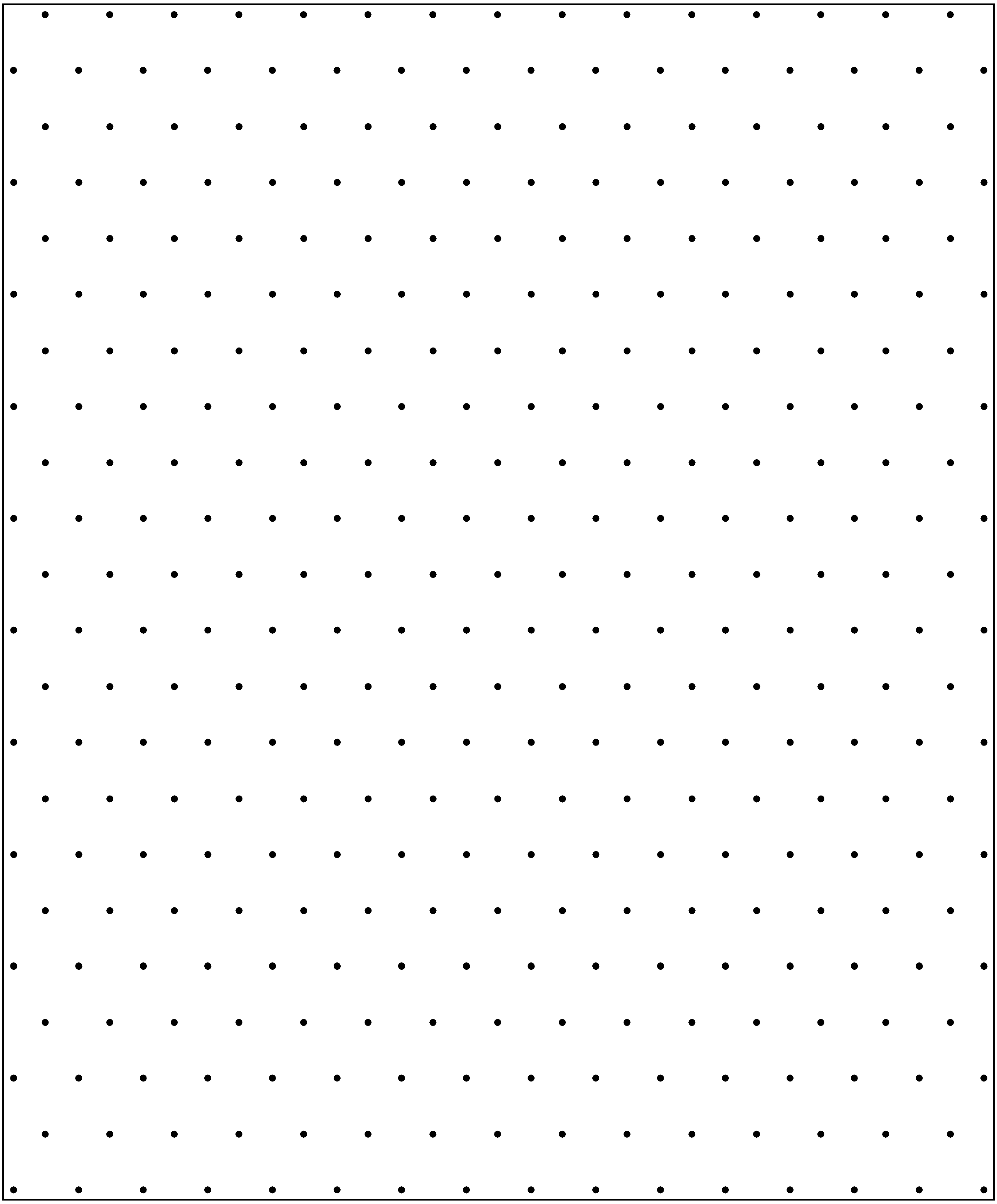




Reproducible Page

Pointy Fences

Reproducible Page



Names: .... Class: ....

Shape Algebra

Each square is the same size as the shape named x.
The sheet can help you work out the area of each shape.
You might trace the shapes and then remove them to work out the area.

Names: .... Class: ....