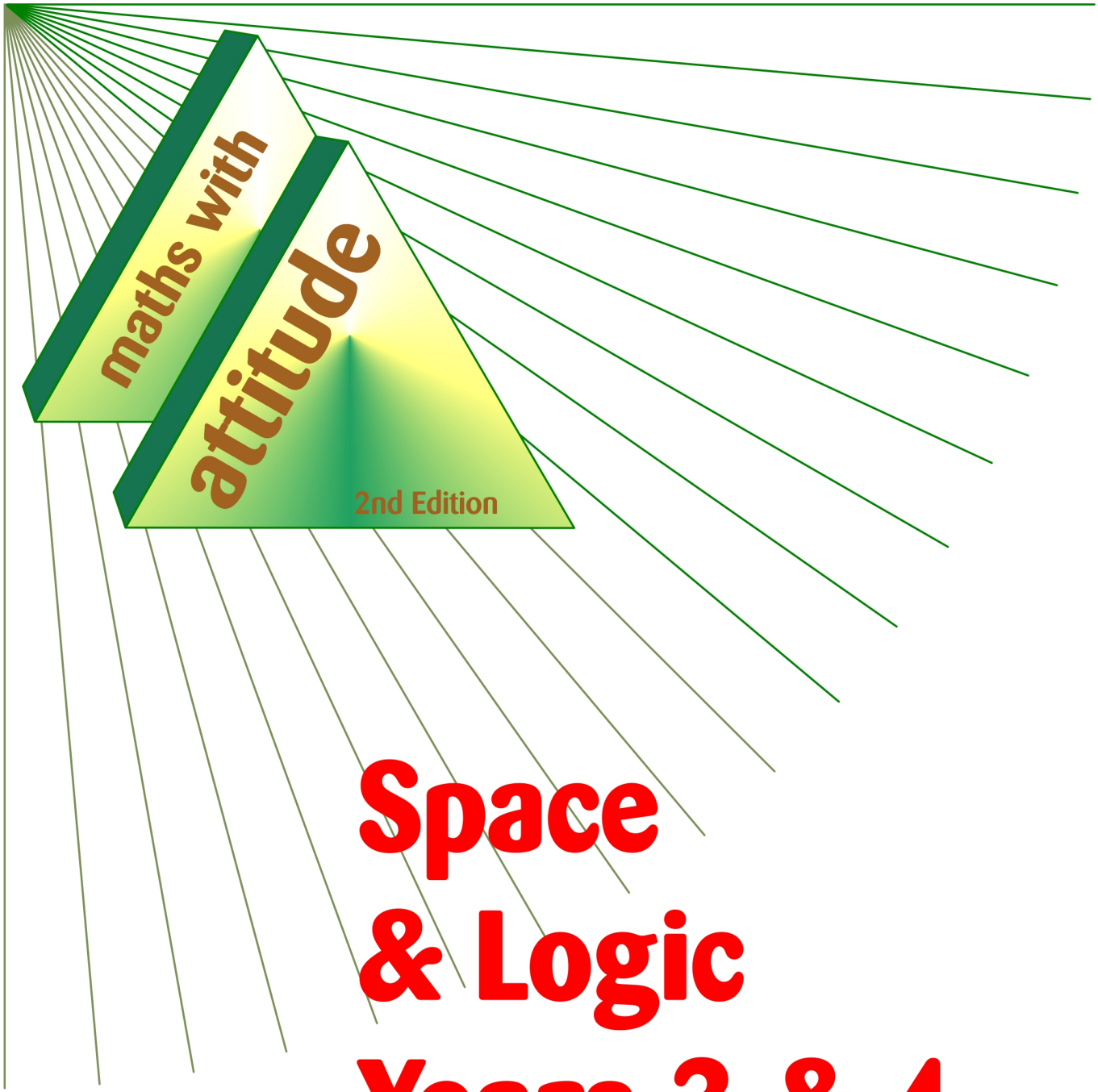

Space & Logic Years 3 & 4

Charles Lovitt
Doug Williams

Mathematics Task Centre & Maths300
helping to create happy healthy cheerful productive inspiring classrooms

Space & Logic

Years 3 & 4

In this kit:

- *Hands-on problem solving tasks*
- *Detailed curriculum planning*

Access from Maths300:

- *Extensive lesson plans*
- *Software*

Doug Williams
Charles Lovitt

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Maths With Attitude includes reference to hands-on problem solving tasks and Maths300 material.

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Our Objective

- To support teachers, schools and systems wanting to create:

  happy, healthy, cheerful, productive, inspiring classrooms

Our Attitude

- to learning:

  learning is a personal journey stimulated by achievable challenge
- to learners:

  stimulated students are creative and love to learn
- to pedagogy:

  the art of choosing teaching strategies to involve and interest all students
- to mathematics:

  mathematics is concrete, visual and makes sense
- to learning mathematics:

  all students can learn to work like a mathematician
- to teachers:

  the teacher is the most important resource in education
- to professional development:

  teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

Our Objective in Detail

What do we mean by creating:

happy, healthy, cheerful, productive, inspiring classrooms

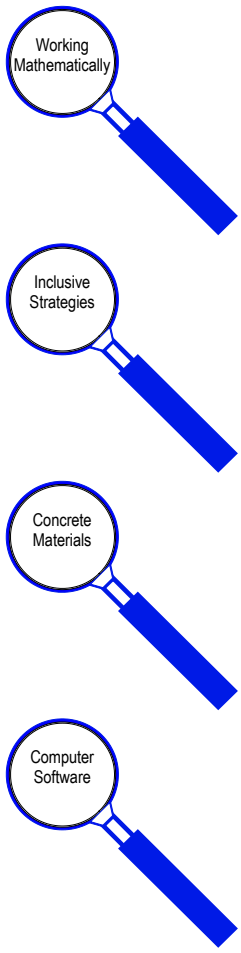
Happy…

means the elimination of the unnecessary fear of failure that hangs over so many students in their mathematics studies. Learning experiences *can* be structured so that all students see there is something in it for them and hence make a commitment to the learning. In so many 'threatening' situations, students see the impending failure and withhold their participation.

A phrase which describes the structure allowing all students to perceive something in it for them is *multiple entry points and multiple exit points*. That is, students can enter at a variety of levels, make progress and exit the problem having visibly achieved.

Healthy…

means *educationally healthy*. The learning environment should be a reflection of all that our community knows about how students learn. This translates into a rich array of teaching strategies that could and should be evident within the learning experience.



If we scrutinise the *exploration* through any lens, it should confirm to us that it is well structured or alert us to missed opportunities. For example, peering through a pedagogy lens we should see such features as:

- a story shell to embed the situation in a meaningful context
- significant active use of concrete materials
- a problem solving challenge which provides ownership for students
- small group work
- a strong visual component
- access to supportive software

Cheerful…

because we want 'happy' in the title twice!

Productive…

is the clear acknowledgment that students are working towards recognisable outcomes. They should know what these are and have guidelines to show they have either reached them or made progress. Teachers are accountable to these outcomes as well as to the quality of the learning environment.

Inspiring…

is about creating experiences that are uplifting or exalting; that actually *turn students on*. Experiences that make students feel great about themselves and empowered to act in meaningful ways.

Space & Logic Resources

To help you create

happy, healthy, cheerful, productive, inspiring classrooms

this kit contains

- 20 hands-on problem solving tasks from Mathematics Centre

and a Teachers' Manual which integrates the use of the tasks with

- 6 detailed lesson plans from Maths300

The kit offers **5 weeks** of Scope & Sequence planning in Space and Logic for *each* of Year 3 and Year 4. This is detailed in *Part 2: Planning Curriculum* which begins on Page 12. You are invited to map these weeks into your Year Planner. Together, the four kits available for these levels provide 25 weeks of core curriculum in Working Mathematically (working like a mathematician).

Note: Membership of Maths300 is assumed.
The kit will be useful without it, but it will be much more useful with it.

Tasks

- Back to Back Building
- Cross & Square
- Farmyard Friends
- Farmyard Race Day
- Farmyard Views
- Hearts & Loops
- In The Bag
- Leading The Blind
- Making Solids
- McMahon's Triangles 1
- McMahon's Triangles 2
- Mirror Patterns 1
- Police Line Up
- Racetrack
- Soma Cube 1
- Sphinx
- Symmetric Shapes
- Tangram Teasers
- Two Colours Game
- Who Lives Where?

Part 2 of this manual introduces each task. The latest information can be found at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm

Maths300 Lessons

- Farmyard Friends
- Knight's Tour
- Newspaper Shapes
- Police Line Up
- Spirolaterals
- String Shapes

Lessons with Software

- Knight's Tour
- Spirolaterals

Part 2 of this manual introduces each lesson. Full details can be found at:

- http://www.maths300.com

Working Like A Mathematician

Our attitude is:

all students can learn to work like a mathematician

What does a mathematician's work actually involve? Mathematicians have provided their answer on Page 8. In particular we are indebted to Dr. Derek Holton for the clarity of his contribution to this description.

Perhaps the most important aspect of Working Mathematically is the recognition that *knowledge is created by a community and becomes part of the fabric of that community*. Recognising, and engaging in, the process by which that knowledge is generated can help students to see themselves as able to work like a mathematician. Hence Working Mathematically is the framework of **Maths With Attitude**.

Skills, Strategies & Working Mathematically

A Working Mathematically curriculum places learning mathematical skills and problem solving strategies in their true context. Skills and strategies are the tools mathematicians employ in their struggle to solve problems. Lessons on skills or lessons on strategies are not an end in themselves.

- **Our skill toolbox** can be added to in the same way as the mechanic or carpenter adds tools to their toolbox. Equally, the addition of the tools is not for the sake of collecting them, but rather for the purpose of getting on with a job. A mathematician's job is to attempt to solve problems, not to collect tools that might one day help solve a problem.
- **Our strategy toolbox** has been provided through the collective wisdom of mathematicians from the past. All mathematical problems (and indeed life problems) that have ever been solved have been solved by the application of this concise set of strategies.

About Tasks

Our attitude is:

mathematics is concrete, visual and makes sense

Tasks are from Mathematics Task Centre. They are an invitation to two students to work like a mathematician (see Page 8).

The Task Centre concept began in Australia in the late 1970s as a collection of rich tasks housed in a special room, which came to be called a Task Centre. Since that time hundreds of Australian teachers, and, more recently, teachers from other countries, have adapted and modified the concept to work in their schools. For example, the special purpose room is no longer seen as an essential component, although many schools continue to opt for this facility.

A brief history of Task Centre development, considerable support for using tasks, for example Task Cameos, and a catalogue of all currently available tasks can be found at:

- http://www.mathematicscentre.com/taskcentre

Key principles are:

- A good task is the tip of an iceberg
- Each task has three lives
- Tasks involve students in the Working Mathematically process

The Task Centre Room or the Classroom?

There are good reasons for using the tasks in a special room which the students visit regularly. There are also different good reasons for keeping the tasks in classrooms. Either system can work well if staff are committed to a core curriculum built around learning to work like a mathematician.

- A task centre room creates a focus and presence for mathematics in the school. Tasks are often housed in clear plastic 'cake storer' type boxes. Display space can be more easily managed. The visual impact can be vibrant and purposeful.
- However, tasks can be more readily integrated into the curriculum if teachers have them at their finger tips in the classrooms. In this case tasks are often housed in press-seal plastic bags which take up less space and are more readily moved from classroom to classroom.

Tip of an Iceberg

The initial problem on the card can usually be solved in 10 to 20 minutes. The investigation iceberg which lies beneath may take many lessons (even a lifetime!). Tasks are designed so that the original problem reveals just the 'tip of the iceberg'. Task Cameos and Maths300 lessons help to dig deeper into the iceberg.

We are constantly surprised by the creative steps teachers and students take that lead us further into a task. No task is ever 'finished'.

> *Most tasks have many levels of entry and exit and therefore offer an on-going invitation to revisit them, and, importantly, multiple levels of success for students.*

Three Lives of a Task

This phrase, coined by a teacher, captures the full potential and flexibility of the tasks. Teachers say they like using them in three distinct ways:

1. As on the card, which is designed for two students.
2. As a whole class lesson involving all students, as supported by outlines in the Task Cameos and in detail through the Maths300 site.
3. Extended by an Investigation Guide (project), examples of which are included in both Task Cameos and Maths300.

The first life involves just the 'tip of the iceberg' of each task, but nonetheless provides a worthwhile problem solving challenge - one which 'demands' concrete materials in its solution. This is the invitation to work like a mathematician. Most students will experience some level of success and accomplishment in a short time.

The second life involves adapting the materials to involve the whole class in the investigation, in the first instance to model the work of a mathematician, but also to develop key outcomes or specific content knowledge. This involves choosing teaching craft to interest the students in the problem and then absorb them in it.

The third life challenges students to explore the 'rest of the iceberg' independently. Investigation Guides are used to probe aspects and extensions of the task and can be introduced into either the first or second life. Typically this involves providing suggestions for the direction the investigation might take. Students submit the 'story' of their work for 'portfolio assessment'. Typically a major criteria for assessment is application of the Working Mathematically process.

About Maths300

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

Maths300 is a subscription based web site. It is an attempt to collect and publish the 300 most 'interesting' maths lessons (K - 12).

- *Lessons have been successfully trialed in a range of classrooms.*
- *About one third of the lessons are supported by specially written software.*
- *Lessons are also supported by investigation sheets (with answers) and game boards where relevant.*
- *A 'living' Classroom Contributions section in each lesson includes the latest information from schools.*
- *The search engine allows teachers to find lessons by pedagogical feature, curriculum strand, content and year level.*
- *Lesson plans can be printed directly from the site.*
- *Each lesson supports teaches to model the Working Mathematically process.*

Modern internet facilities and computers allow teachers easy access to these lesson plans. Lesson plans need to be researched, reflected upon in the light of your own students and activated by collecting and organising materials as necessary.

Maths300 Software

Our attitude is:

> *stimulated students are creative and love to learn*

Pedagogically sound software is one feature likely to encourage enthusiastic learning and for that reason it has been included as an element in about one third of Maths300 lesson plans. The software is used to develop an investigation beyond its introduction and early exploration which is likely to include other pedagogical techniques such as concrete materials, physical involvement, estimation or mathematical conversation. The software is not the lesson plan. It is a feature of the lesson plan used at the teacher's discretion.

For school-wide use, the software needs to be downloaded from the site and installed in the school's network image. You will need to consult your IT Manager about these arrangements. It can also be downloaded to stand alone machines covered by the site licence, in particular a teacher's own laptop, from where it can be used with the whole class through a data projector.

Note:

- *Maths300 lessons and software may only be used by Maths300 members.*

Working Mathematically

First give me an interesting problem.

When mathematicians become interested in a problem they:

- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

Questions which help mathematicians learn more are:

- Can I check this another way?
- What happens if ...?
- How many solutions are there?
- How will I know when I have found them all?

When mathematicians have a problem they:

- Read & understand the problem.
- Plan a strategy to start the problem.
- Carry out their plan.
- Check the result.

A mathematician's strategy toolbox includes:

- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- Break a problem into smaller parts
- ...

If one way doesn't work, I just start again another way.

Professional Development Purpose

Our attitude is:

the teacher is the most important resource in education

> *We had our first study group on Monday. The session will be repeated again on Thursday. I had 15 teachers attend. We looked at the task Farmyard Friends (Task 129 from the Mathematics Task Centre). We extended it out like the questions from the companion Maths300 lesson suggested, and talked for quite a while about the concept of a factorial. This is exactly the type of dialog that I feel is essential for our elementary teachers to support the development of their math background. So anytime we can use the tasks to extend the teacher's math knowledge we are ahead of the game.*
> District Math Coordinator, Denver, Colorado

Research suggests that professional development most likely to succeed:

- is requested by the teachers
- takes place as close to the teacher's own working environment as possible
- takes place over an extended period of time
- provides opportunities for reflection and feedback
- enables participants to feel a substantial degree of ownership
- involves conscious commitment by the teacher
- involves groups of teachers rather than individuals from a school
- increases the participant's mathematical knowledge in some way
- uses the services of a consultant and/or critical friend

Maths With Attitude has been designed with these principles in mind. All the materials have been tried, tested and modified by teachers from a wide range of classrooms. We hope the resources will enable teacher groups to lead themselves further along the professional development road, and support systems to improve the learning outcomes for students K - 12.

With the support of Maths300 ETuTE, professional development can be a regular component of in-house professional development. See:

- http://www.mathematicscentre.com/taskcentre/resource.htm#etute

For external assistance with professional development, contact:

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M: 0401 177 775
E: doug@blackdouglas.com.au

Part 2: Planning Curriculum

Curriculum Planners

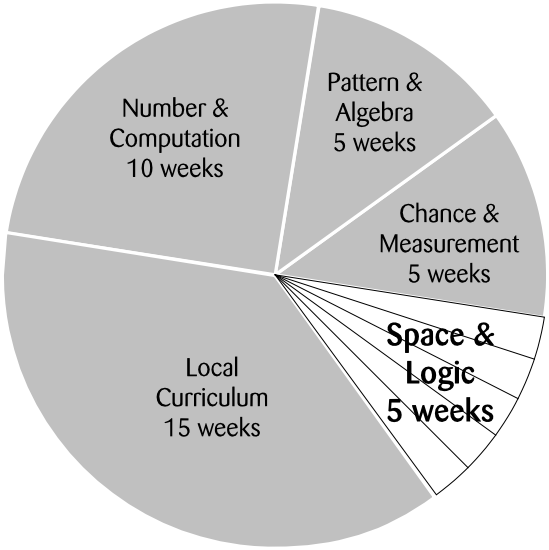
Our attitude is:

learning is a personal journey stimulated by achievable challenge

Curriculum Planners:

- show one way these resources can be integrated into your weekly planning
- provide a starting point for those new to these materials
- offer a flexible structure for those more experienced

You are invited to map Planner weeks into your school year planner as the core of the curriculum.



Planners:

- detail each week lesson by lesson
- offer structures for using tasks and lessons
- are sequenced from lesson to lesson, week to week and year to year to 'grow' learning

Teachers and schools will map the material in their own way, but all will be making use of extensively trialed materials and pedagogy.

Using Resources

- Your kit contains 20 hands-on problem solving tasks and reference to relevant Maths300 lessons.
- Tasks are introduced in this manual and supported by the Task Cameos at:
  http://www.mathematicscentre.com/taskcentre/iceberg.htm
- Maths300 lessons are introduced in this manual and supported by detailed lesson plans at:
  http://www.maths300.com

In your preparation, please note:

- Planners assume 4 lessons per week of about 1 hour each.
- Planners are *not* prescribing a continuous block of work.
- Weeks can be interspersed with other learning; perhaps a **Maths With Attitude** week from a different strand.
- Weeks can sometimes be interchanged within the planner.
- Lessons can sometimes be interchanged within weeks.
- The four **Maths With Attitude** kits available at each year level offer 25 weeks of a Working Mathematically core curriculum.

A Way to Begin

- Glance over the Planner for your class. Skim through the comments for each task and lesson as it is named. This will provide an overview of the kit.
- Task Comments begin after the Planners. Lesson Comments begin after Task Comments. The index will also lead you to any task or lesson comments.
- Select your preferred starting week - usually Week 1.
- Now plan in detail by researching the comments and web support. Enjoy!

Research, Reflect, Activate

Curriculum Planner
Space & Logic: Year 3

	Session 1	Session 2	Session 3	Session 4
Week 1	*Newspaper Shapes*: A great way to recycle newspapers and investigate lots of mathematics at the same time. The class begins by making their own construction materials, then the door is wide open to 2D and 3D problem solving. There is lots of room for student-led investigation and usually the teacher's biggest problem is to collect and record all the learning.			
Week 2	**Task Week 1**: Invite the students to work in pairs on the tasks. This is their opportunity to show how well they can 'work like a mathematician'. There are many ways to do this and some are explored in more depth in the Curriculum Planning Stories (Page 35). Perhaps the simplest way is to sort the class into two groups. While one group continues with text book type work the other works on the tasks. In essence, two sessions in the week on tasks and two on your choice of text-style work.			
Week 3	*Spirolaterals*: The first lesson in this sequence involves students physically in investigating a curious set of spatial patterns. There is lots to explore and predict and the investigation is supported by software that comes into its own later in the week.			
Week 4	**Task Week 2**: A second opportunity for students to practise working like a mathematician. Depending on the number of students in the class you could use the Logic tasks in Task Week 1 (above) and the Space tasks in this week. Ten tasks is usually sufficient for about 7 pairs of students.			
Week 5	*Farmyard Friends*: This could be a pencil and paper exercise but so much quality would be lost from the learning. Physically taking the places of the animals by 'wearing' an animal card appeals to this age. Then using the table top form of the cards encourages the students to work for longer, and brings more success when designing their own puzzles.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Curriculum Planner

Space & Logic: Year 4

	Session 1	Session 2	Session 3	Session 4
Week 1	*String Shapes*: Revise and extend knowledge of shapes using a large loop of string for each group. In particular investigate triangles and/or quadrilaterals. Lots of opportunity for group work, but the lesson does require outside area or a large inside hall. Check the weather forecast.			
Week 2	**Task Week 1**: Invite the students to work in pairs on the tasks. This is their opportunity to show how well they can 'work like a mathematician'. There are many ways to do this and some are explored in more depth in the Curriculum Planning Stories (Page 35). Perhaps the simplest way is to sort the class into two groups. While one group continues with text book type work the other works on the tasks. In essence, two sessions in the week on tasks and two on your choice of text-style work.			
Week 3	*Knight's Tour*: By this age most students have explored at least some of the moves of chess pieces. This lesson focuses on the Knight and the challenge is to find a way to move it around the chess board so that it visits every square once only. The lesson has an engaging quality and at this age it is usually enough just to allow the students to explore then report on their attempts and any developing strategies. Option 1 of the software allows the students to try the puzzle on screen.			
Week 4	**Task Week 2**: A second opportunity for students to practise working like a mathematician. Depending on the number of students in the class you could use the Logic tasks in Task Week 1 (above) and the Space tasks in this week. Ten tasks is usually sufficient for about 7 pairs of students.			
Weeks 5	*Police Line Up*: A language and logic puzzle that opens the door to a wealth of cross-curriculum activity. The focus is on mathematical reasoning and the lesson offers stimulating cartoon characters to grab student interest. In fact, this is several puzzles in one, so the teacher can choose from the clue sets to best suit student abilities. The lesson also allows students to show how much they know about mathematics. The Classroom Contributions section of this lesson is definitely worth a look.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Planning Notes

Enhancing Maths With Attitude

Resources to support learning to work like a mathematician are extensive and growing. There are more tasks and lessons available than have been included in this Space & Logic kit. You could use the following to enhance this kit.

Additional Tasks

- Task 12, Matching Cards
  The initial challenge is to correctly match given pieces, then to order the correct matches. But how many not-matches are possible? Originally designed as an example of what a task card for Infants (Years K - 2) might look like, it sits on an iceberg which offers plenty for Year 3 & 4.
- Task 50, Flight Departures
  An observer gives clues about the take off order of four planes at an airport. The challenge is to decide the actual order in which the planes took off.
- Task 170, Equilateral Triangles
  Designed to invite younger students to think 'outside the square' this task hints at both algebraic pattern and the idea that three dimensional objects are made from flat surfaces which have familiar names.

More information about these tasks may be available in the Task Cameo Library:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Additional Lessons

- Lesson 73, Halving Squares
  Two students each take a square of the same coloured paper (about 10 cm square), fold it and then cut it (or neatly tear it) in half. There are two main ways to do this. The students now have four pieces between them. Combine these using the rule matching edges have to be the same length. There are several ways to do this and they make an interesting variety of shapes. Challenge questions include: In how many ways can these shapes be combined? and How many different shapes could we make?.
- Lesson 134, Pentagon Triangles
  Take a regular pentagon and cut it into three triangles along its diagonals. Easy to state, easy to start and heaps of maths. At this level the three pieces can be used for creating spatial patterns and exploring shapes such as triangles, pentagons and decagons. The lesson develops in a problem posing and problem solving environment which reflects the work of a professional mathematician.

Keep in touch with new developments which enhance **Maths With Attitude** at:

- http://www.mathematicscentre.com/taskcentre/enhance.htm

Additional Materials

As stated, our attitude is that mathematics is concrete, visual and makes sense. We assume that all classrooms will have easy access to many materials beyond what we supply. For this unit you will need:

- Piles of old newspaper in both Tabloid and Broadsheet sizes if possible
- Loops of strong string about 20m in circumference
- Cubes that join in 3 dimensions (Multi-Link is one brand)

Special Comments Year 3

- Plan ahead to collect newspapers. Rather than stack them flat, some teachers build up to this session by arranging for student teams to make the rods and joiners described in the lesson on a daily basis as the papers come in. These can be readily stored. That way teachers also know when to tell students to stop bringing in the papers.

Special Comments Year 4

- If this is the first year that your students have experienced the **Maths With Attitude** curriculum materials, then you can choose to use any of the lessons. There is still lots of learning in the lessons listed for Year 3 that Year 4 students would find interesting and challenging.
- If you are 'looping' into Year 4 with the same student group you are in a position to plan your own way of extending the previous work and adding in the fresh material. Each Maths300 lesson has a depth that has probably not yet been fully explored in Year 3, so if 'looping' you are in a position to revisit earlier lessons. Also consider the Additional Lessons listed on the previous page.
- If a colleague has used **Maths With Attitude** in Year 3, then the planner offers fresh lessons and broad suggestions that will be enhanced by consultation with the previous teacher.
- In this kit, the same set of tasks is available both Years 3 & 4 students, so students may have attempted some of them before. However, you can be confident that (a) children can be encouraged to revisit an 'old friend' and (b) the Task Comments section on the next page, in conjunction with Task Cameos where referenced in those comments, will provide you with lots of ideas for extending the students.

Task Comments

- Tasks, lessons and unit plans prepare students for the more traditional skill practice lessons, which we invite you to weave into your curriculum. Teachers who have used practical, hands-on investigations as the focus of their curriculum, rather than focussing on the drill and practice diet of traditional mathematics, report success in referring to skill practice lessons as Toolbox Lessons. This links to the idea of a mathematician dipping into a toolbox to find and use skills to solve problems.

Back To Back Building

Student A makes a 3D object from a collection of linking cubes. Student B is sitting back to back with Student A and can't see what is made. Student B has the same colours as Student A and is challenged to recreate the original object by following the oral instructions of the first student. The objective is to become increasingly precise in using mathematical language. Students also experience transformations of objects in space which is informal groundwork for 3D geometry.

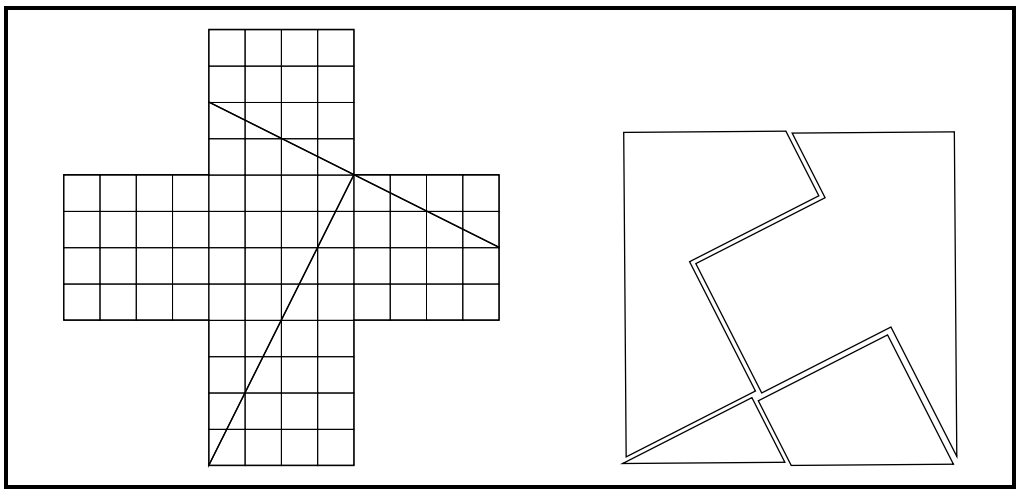
Is it possible to think of a system that would uniquely describe the position of each cube in space?

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Cross & Square

The first level of challenge is simply to make a cross and a square from the four given shapes. The solutions are...



... and at this level the task is included largely for the spatial perception involved in finding these solutions.

A language component can be added by placing the four pieces randomly on the table and challenging a child who thinks they know how to do both to keep their hands behind their back and instruct another child to make them. A discussion about the most useful language to use can follow.

The card also hints at another level of investigation. Clearly the same four pieces make both shapes, so the areas of both shapes should be the same. But are they? A grid has been added to the cross to show that it must have an area of 5 x 16 = 80 square units. This can also be measured and calculated. However, when the area of the square is measured and calculated it seems to be 81 square units. Oops!

It takes an application of Pythagoras' Theorem and Trigonometry to prove that the square is not actually 9 x 9, but at this level the question:

- *Where does the extra square come from?*

can still be asked. An approach to the answer might come from reasoning:

- The cross must be the starting point for making the puzzle because the cuts are simpler to do. So it's area must be the correct one.
- That means we have to check the area of the square. Let's put it on the grid paper. (The right size grid paper is supplied at the end of this manual.)

If this is done so that one corner of the square tucks into one corner of a grid square, it becomes clear that there is a thin strip around the two opposite sides that is just inside the 9 x 9 square. It's a good bet that the area of this strip is 1 square unit.

Farmyard Friends

Five animals have to be placed into five pens according to the restrictions on the card about the positions of the animals - which is next to which and so on. Students are enticed by toy animals and the apparent ease of the problem. However, there is more than one solution and that's where it becomes more interesting. Finding just one more solution is a challenge which relates to interpreting words such as 'beside' (Which side?) and 'after' (How many pens after?).

Students who accept the challenge of attempting to find all the solutions and explaining why this is all of them are further into the work of a young mathematician. Then there are several other questions that can be asked, for example *Can you create your own similar puzzle with the animals and pens?*

Find solutions and more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Farmyard Race Day

The task involves two separate puzzles. Each has a set of clues and a question. Students are expected to manipulate the six farmyard animals on the racetrack provided to answer the questions.

With the first set of clues, they find out the relative finishing positions of the animals when the winner crosses the line. The second set of clues establishes the finishing positions of the racing animals in relation to the position of the dog

watching at the side. In each case the emphasis is on the reasoning to find the solution and the justification and communication skills to explain it. Once solved, students could create similar puzzles of their own.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Farmyard Views

This task contributes to the development of spatial perception and includes a challenge to prove the result. The students are encouraged to look at a 2D representation of a farm paddock and imagine themselves in a 3D environment created from it. This skill is similar to that required when interpreting many text book illustrations.

In seeking a solution students will use lots of language and refine their concept of right and left. In being challenged to show that they have found the only solution, justification and communication skills come to the fore. Beyond these aspects, other challenges develop by changing the clues in the puzzle, or placing the objects on the grid in your own way and creating a set of clues.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Hearts & Loops

There is no need to ask about the solution of this task. Some day, someone will solve it and the problem will become how to prevent the successful person from telling everyone else. Of course, the problem isn't considered 'done' until the person can take it apart *and* put it back together.

Actually, in some ways, it doesn't seem to matter if the solution does travel the class 'grapevine'. Most students want to try for themselves, and, in fact, frequently revisit the puzzle just to make sure that they can still do it. Another challenge that can be added for those who do solve it is to *ask* the successful student to tell someone else how to do it *provided* the doer is the only one to hold the puzzle *and* the teller either sits on their own hands or keeps them behind their back.

Mathematically the task is developing spatial perception, and, through the extension, mathematical language.

A hint that sometimes helps when a student becomes too frustrated is to suggest they think in terms of turning a key in a lock.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

In The Bag

Putting shapes into a frame 'blind' because they are in a bag creates a fascinating series of spatial challenges that depend on the tactile/kinaesthetic sense. Students can exit this task after any question and feel successful, then re-enter it at another time and try a higher level of difficulty. The most difficult puzzle is hard to get into the frame even when the pieces can be seen. Any student who can do it in the bag has great command of their tactile skills and visual memory.

The task can be related to:

- some of the work done by astronauts in outer space
- operating 'robot arms' inside containers of radioactive material
- doctors instructing a robot to operate while viewing a monitor on the other side of the room.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Leading The Blind

Language, shape and space intersect again in this task. One player has to guide the other player, who has their eyes closed, to match a shape to its frame. The need for increasingly precise mathematical language becomes clear. There are many opportunities to introduce language that can help the students refine their efforts.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Making Solids

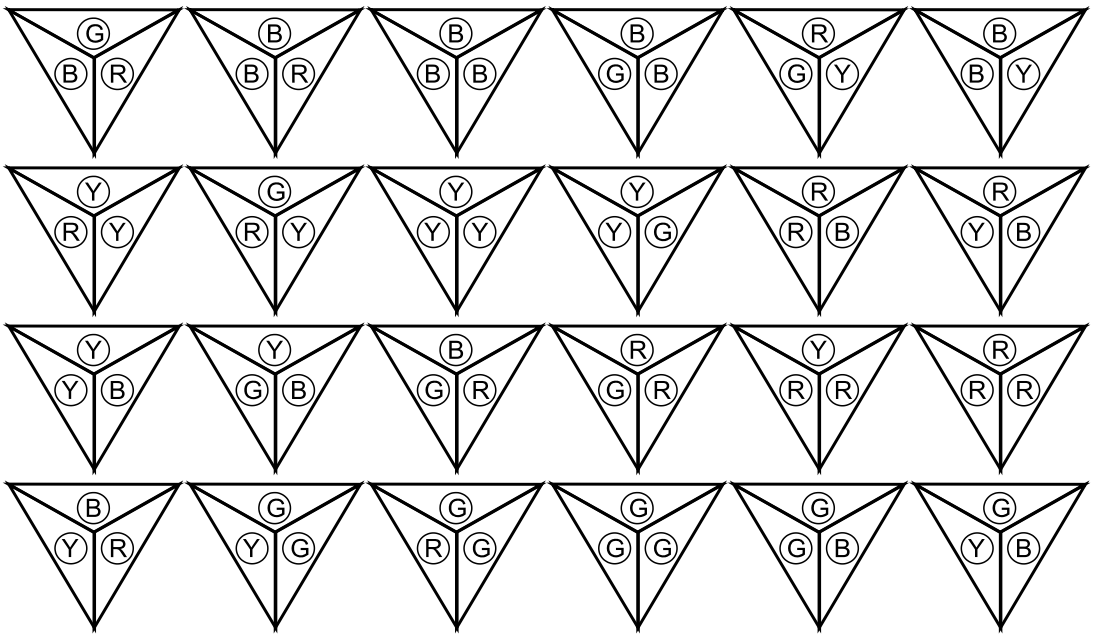
Colourful, tactile materials encourage students to tackle a collection of 3D spatial challenges presented as isometric drawings. They need not be done in a particular order. Students can come and go from the task and experience success each time. However, encourage students to record how the pieces fit to make the suggested object. This will involve using the isometric paper provided for the task at the end of this manual and using shading, or colours, on a reproduction of the task card drawing. The task also touches on the concept of volume as measured in unit cubes.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

McMahon's Triangles 1

Completing this task requires a high level of organised searching and teachers are

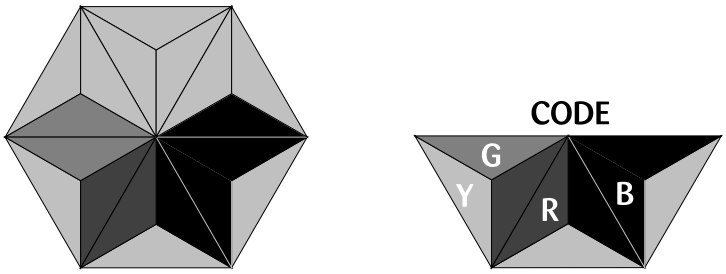


sometimes surprised by the students who achieve it. The 24 triangles represent every possible combination of mapping 4 colours into 3 positions. To find them and be sure that you have them all is a serious mathematician's challenge. The solution is then the source of the equipment for McMahon's Triangles 2.

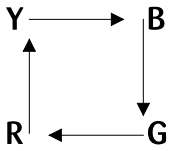
McMahon's Triangles 2

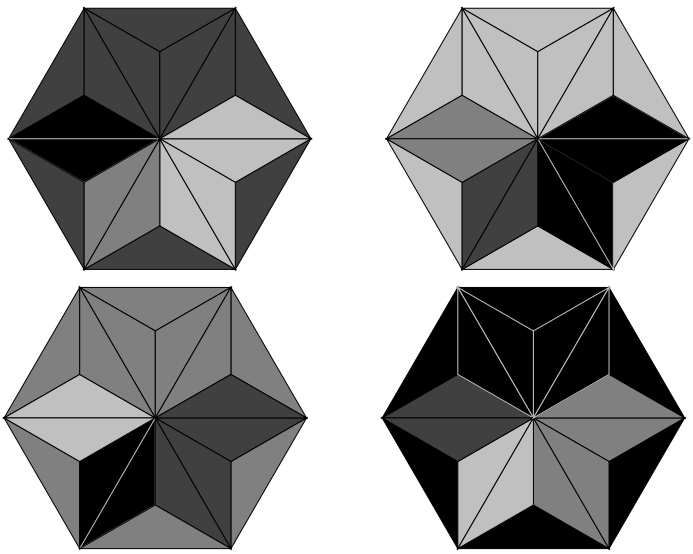
In this task, the 24 solutions of McMahon's Triangles 1 are put to use to encourage spatial perception and systematic thinking. The first question is accessible to most students, however, it is a tough challenge to complete the card and one that is likely to keep brighter students engaged for some time. Students find the colourful pieces attractive and even if they don't solve the puzzles readily, they often enjoy the patterns created while trying. Many are struck by the 3D illusion of stepped cubes which sometimes develops in the solutions. At times, small groups of students are likely to gather to try to work it out.

The first question on the task card has many answers because the colours aren't important. However, when the rules of Question 2 are applied it is not so easy to find a solution. One example is:

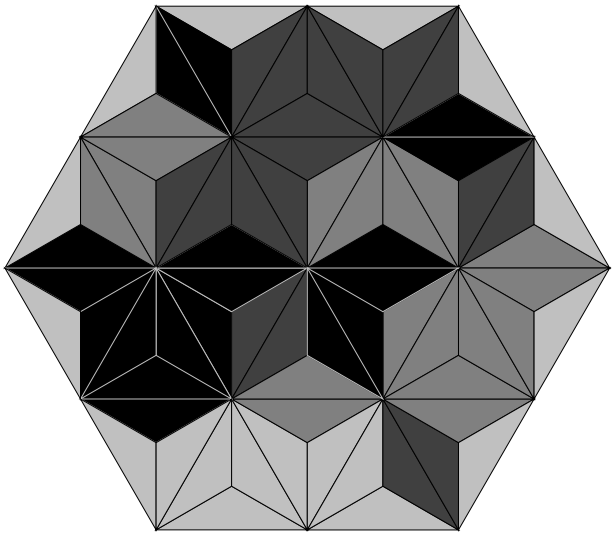


The other solutions can be found from this by systematically rotating the colours as shown. One set of four solutions is:





The final challenge on the card is to make one big hexagon with the same colour all around the edge *and* colours matching across all internal edges. One solution is shown, but there are others.



To find any of the solutions above requires considerable effort. Another way of using the pieces of the task is to challenge students to make and record as many shapes as possible which are symmetric either by rotation or reflection.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

where you will also be able to see (and print) solutions in colour.

Mirror Patterns 1

As engaging to most students as playing with a kaleidoscope. The task can open the door to other activities with mirrors. Lots of informal experience with the properties of reflection which will prepare the students for mathematics related to angles and their measurement.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Police Line Up

The focus of this task is mathematical reasoning. Students find the cartoon characters and the story context appealing and generally persist at the problem for some time. Clues are provided allowing the students to sort the characters into order by height. However, this language and logic puzzle opens the door to a wealth of cross-curriculum activity.

- *Is there only one solution?*
- *Can we make up more puzzles of our own?*
- *What if the suspects were holding numbers when they were photographed and these numbers were used as clues to finding the order of the line up?*

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Racetrack

This time reflective symmetry engages the body's kinaesthetic sense in an unexpected way. There are many levels of difficulty within the task so there is

something in the task for everyone. You will find students will often want to return to the challenge.

The worksheet you may need for this task is at the end of this manual.

It may be worth relating this task to the truck driver's challenge of reversing into a space using mirrors only. Students can come close to modelling this by holding the task mirror in front of them as if it were the central or side mirror of a vehicle, then, starting from the classroom doorway, trying to navigate themselves safely to their seat by looking in the mirror only.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Soma Cube 1

This puzzle is as famous in three dimensions as tangrams is in two. By taking students back to the design of the pieces themselves, the task opens the door to the many spatial challenges that have been constructed around it. Finding the pieces which Piet Hein used to design the puzzle is a significant mathematical challenge in itself. The card guides the students to find them all.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Sphinx

How can such an apparently simple four piece jigsaw prove so frustrating? Students will want to tell others when they have solved it, and, although it is better that they learn to give others the chance to also experience the thrill of discovery, such disclosure doesn't matter too much because the iceberg of this task is so deep. After all, once solved, you are proving that four Sphinxes make a Sphinx ... so four sphinxes of the new size will also make a Sphinx, and ...

However, if you want to put the 'classroom grapevine' to work, allow one student to talk another through the solution but only if they sit on their hands, or keep them behind their back.

Find more information about this task in the Task Cameo Library at:

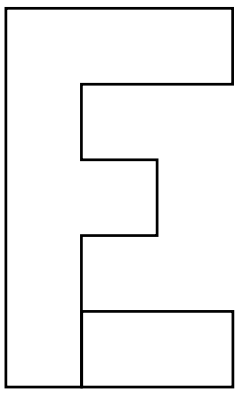
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Symmetric Shapes

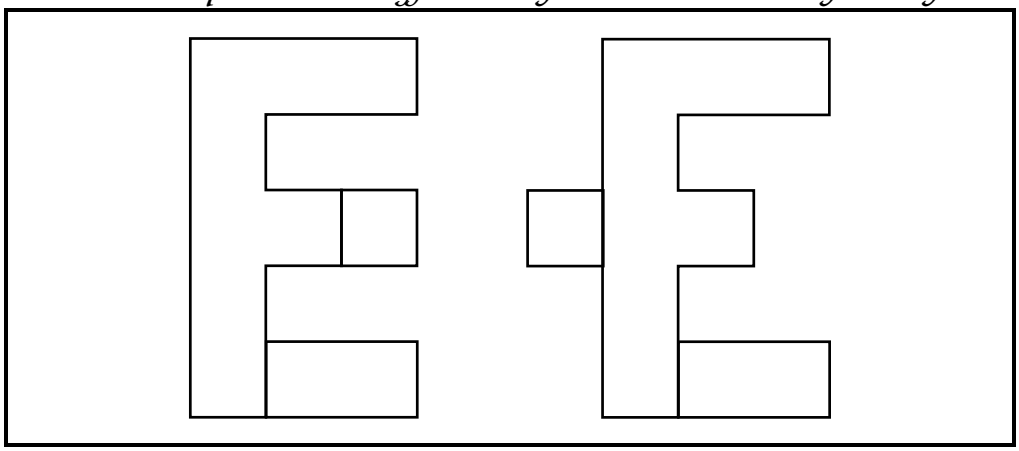
This task has its origins in the work of Geoff Giles and the DIME (Developments in Mathematics Education) Project. It offers problem solving in Line Symmetry which links with the other tasks involving mirrors.

It starts with challenges which are quite achievable, such as:

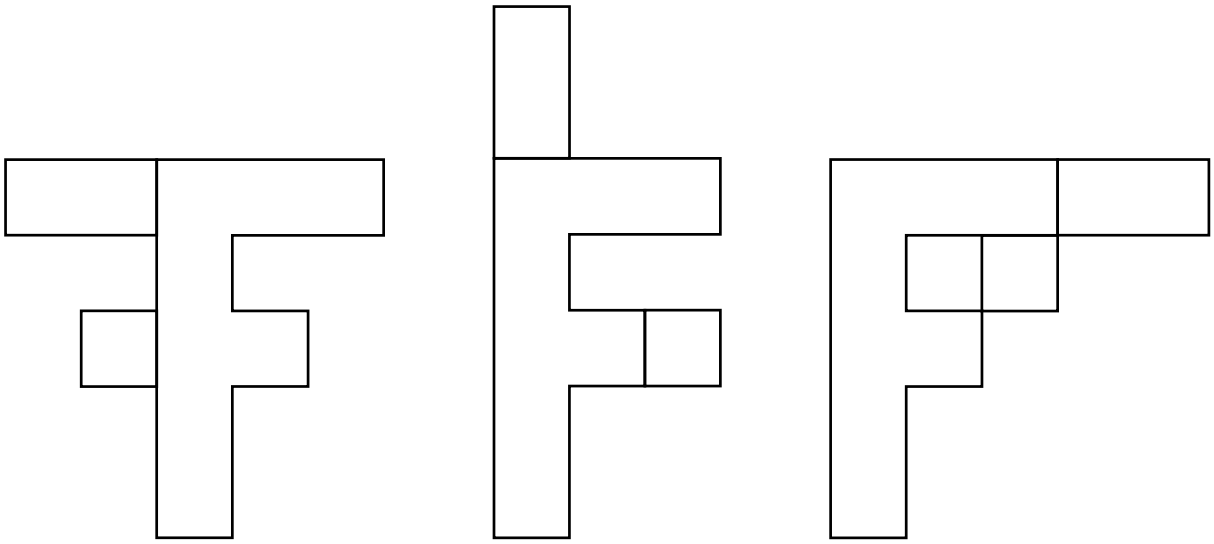
Make a line symmetric shape from the F and the rectangle.



Now add the square in two different ways to retain the line symmetry:



So far so good, but there are three more ways to combine the F, the rectangle and the square to make shapes with line symmetry. These are much more of a challenge, especially since we may have been led into expecting the line of symmetry to be horizontal or vertical. It takes a deal of thinking 'outside the square' to discover:



The worksheet you may need for this task is at the end of this manual.

If you would like to take the task a little further you might ask:

- How many rotationally symmetric shapes could you make with these three pieces?

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Tangram Teasers

The tangram is a classic problem and the challenges on this card merely open the door to its amazing depth. Students could research some of this in the school library and on the web. All the puzzles on the card can be done, but the rocket has been known to test the spatial abilities of adults let alone children.

To dear Mr Williams,
I am a student for Mrs Court's class and my name is Jessy. I am writting to you to say that I have been working on the Tangram Teasers and I have been on it for 2 days and can not do the Third shape at the bottom it is impossiple! and I think it can't be done.
Can you please write back and tell me how to do it PLEASE!
Yours Sincerly Jessy

Dear Jessy,
Thank you for writing to me. I don't have a lot of time to answer a million letters from children, but today you were the only one. Phew! Now that was one tough puzzle you sent me. I have been struggling with it for about half an hour. But at last I can tell you it is NOT impossible.
Do you want to keep trying or do you need a clue?
Say hello to Mrs. Court for me.
Keep smiling, Mr. Williams

Dear mr Williams
Thankyou for writting back please can i have a clue.
From Jessy

Hi Jessy,
The first clue is that when a mathematician finds that one strategy doesn't work, she simply tries a different strategy. When I looked at the picture my first strategy was that the biggest triangle should be the bottom piece and the straight side of the rocket should come up from there. But I was wrong! The biggest triangle is not in the base at all!
Keep smiling, Mr. Williams

Dear Mr Williams
Thanks for the clue. I can't look at it today because I have to go away for my holidays but I will try again when school comes back. I thought I had tried EVERYTHING!!
From Jessy

Dear Jessy,
Why don't you ask Mrs. Court if you can take the Tangram Teasers task home with you on holiday.
Keep smiling, Mr. Williams

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Two Colours Game

Played on a 5x5 grid this is an easy game to get started and it is fun to play, but it is also an invitation to work together to analyse the game. Take turns placing counters of different colours one at a time so that counters next to each other horizontally and vertically are not the same. Is there a winning strategy?

It usually takes some time before students understand that in looking for a winning strategy you must assume that your opponent plays the best possible game.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Who Lives Where?

Language and logic work together in the solution of this puzzle. When it is found, ask the students how they think the designer of the puzzle came up with it in the first place. It is most likely that they started from the answer and worked backwards. Realising this encourages students to make more of their own. In fact, this is an opportunity for the class to create their own set of such puzzles.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Lesson Comments

- These comments introduce you to each Maths300 lesson. The complete plan is easily accessed through the lesson library available to members at:
  http://www.maths300.com
  where they are listed alphabetically by lesson name.

Newspaper Shapes

Beginning by making shapes in small groups using all or part of the body, as in the photographs in the lesson plan seems to appeal to younger children - actually to adults as well. Visit:

- http://www.mathematicscentre.com/taskcentre/2012may.htm#bodies

for photos of trainee teachers at work on this activity.

The lesson expands from there and has a special appeal because students make the necessary equipment themselves from newspaper and small pieces of masking tape. Co-operative group work and learning in community are special features of this lesson. The size of the material almost demands this.

The lesson notes offer a very wide collection of problems from which you can select to build the week's work. Of course there are also the problems students will create themselves.

Spirolaterals

Try hard to resist the temptation to use the software too soon. The physical involvement in this lesson is vital to the quality of the learning and helps those students who are still unsure about left and right to gain further practical experience. You will need a reasonably-sized open space for this lesson. If you can't use an outside area, then an assembly hall or multi-purpose room will work.

Farmyard Friends

If you have access to toy farm animals you could make this into a whole class investigation with the same equipment as in the task. However, reproducible cards are available in the lesson as a substitute. You can also create one set of A4 size animal cards to hang around students' necks by enlarging on the photocopier.

Marking out pens on the floor, or behind the 'animals' on the display wall adds a new dimension to the problem that has proved successful for several teachers. In some ways it is surprising that such an apparently simple problem can lead to so many educational possibilities. The lesson plan identifies many of them.

String Shapes

The focus of this lesson is on the properties of polygons and it provides so many options that you may wish to select the types of polygons that students explore. The loops of string involve the students at a physical level in a sort of human geoboard. Teachers find the discussion that develops in this group situation is far richer and more purposeful than attempting to explore the properties of these shapes from a text book page. On the other hand the text book exercises make much more sense if they follow the String Shapes experience.

This is a lesson that requires lots of space. So if you are going outside, plan ahead for the weather. If not, you will need a gymnasium or multi-purpose room.

Knight's Tour

> *I was delighted to discover that the lessons from Maths300 work equally successfully with Scottish students as with Australian students and am still amazed at how quickly a disruptive class can be calmed by presenting them with the investigation 'Knight's Tour'!*

It is not necessary to be able to play chess to tackle this problem. The students only have to understand how a knight moves on the board. The main objective at this level is to engage the students in the problem and encourage them to begin forming strategies. Hands-on involvement is important to establish the problem, but the software is available to extend this engagement. If students develop strategies as represented in Option 2, then they also have the opportunity to test these many more times than is possible by hand in the real time of the classroom.

Police Line Up

This lesson can go in so many directions that you will have to choose what parts to use in the time available. Some teachers have reported being involved with the lesson in one way or another for as much as three weeks. There is considerable mathematical content in the lesson related to the consequences of choosing different clue sets, but the lesson also offers cross-curriculum connections. What's more, some teachers have used the 'design your own clues' option to assess the students' current understanding of mathematics.

All these opportunities are fully explained in the Extensions section of the Lesson Plan.

As with *Farmyard Friends*, the cartoon characters supplied can be enlarged on a copier to make a set large enough for students to wear around their necks. In fact doing this brings up to 100% of the class into the investigation, whereas, if the problem was just pencil and paper, as it would be in a text book, teachers report that perhaps only 25% of their students would be involved.

The Classroom Contributions section of the notes offers examples of how teachers have adapted the activity to be a learning source within Language Other Than English (L.O.T.E.) classes.

Part 3: Value Adding

The Poster Problem Clinic

Maths With Attitude kits offer several models for building a Working Mathematically curriculum around tasks. Each kit uses a different model, so across the range of 16 kits, teachers' professional learning continues and students experience variety. The Poster Problem Clinic is an additional model. It can be used to lead students into working with tasks, or it can be used in a briefer form as a opening component of each task session.

> *I was apprehensive about using tasks when it seemed such a different way of working. I felt my children had little or no experience of problem solving and I wanted to prepare them to think more deeply. The Clinic proved a perfect way in.*

Careful thought needs to be given to management in such lessons. One approach to getting the class started on the tasks and giving it a sense of direction and purpose is to start with a whole class problem. Usually this is displayed on a poster that all can see, perhaps in a Maths Corner. Another approach is to print a copy for each person. A Poster Problem Clinic fosters class discussion and thought about problem solving strategies.

Starting the lesson this way also means that just prior to liberating the students into the task session, they are all together to allow the teacher to make any short, general observations about classroom organisation, or to celebrate any problem solving ideas that have arisen.

One teacher describes the session like this:

> *I like starting with a class problem - for just a few minutes - it focuses the class attention, and often allows me to introduce a particular strategy that is new or needs emphasis.*

It only takes a short time to introduce a poster and get some initial ideas going. The class discussion develops a way of thinking. It allows class members to hear, and learn from their peers, about problem solving strategies that work for them.

> *If we don't collectively solve the problem in 5 minutes, I will leave the problem 'hanging' and it gives a purpose to the class review session at the end.*
> *Sometimes I require everyone to work out and write down their solution to the whole class problem. The staggered finishing time for this allows me to get organised and help students get started on tasks without being besieged.*
> *I try to never interrupt the task session, but all pupils know we have a five minute review session at the end to allow them to comment on such things as an activity they particularly liked. We often close then with an agreed answer to our whole class problem.*

A Clinic in Action

The aims of the regular clinic are:

- to provide children with the opportunity to learn a variety of strategies
- to familiarise children with a process for solving problems.

The following example illustrates a structure which many teachers have found successful when running a clinic.

Preparation

For each session teachers need:

- a Strategy Board as below
- a How To Solve A Problem chart as below
- to choose a suitable problem and prepare it as a poster
- to organise children into groups of two or three.

The Strategy Board can be prepared in advance as a reference for the children, or may be developed *with* the children as they explore problem solving and suggest their own versions of the strategies.

The problem can be chosen from

- a book
- the task collection
- prepared collections such as Professor Morris Puzzles which can be viewed at: http://www.mathematicscentre.com/taskcentre/resource.htm#profmorr

The example which follows is from the task collection. The teacher copied it onto a large sheet of paper and asked some children to illustrate it. *The teacher also changed the number of sheep to sixty* to make the poster a little different from the one in the task collection.

The Strategy Board and the How To Solve A Problem chart can be used in any maths activity and are frequently referred to in Maths300 lessons.

The Clinic

The poster used for this example session is:

> Eric the Sheep is lining up to be shorn before the hot summer ahead. There are sixty [60] sheep in front of him. Eric can't be bothered waiting in the queue properly, so he decides to sneak towards the front.
>
> Every time one [1] sheep is taken to be shorn, Eric then sneaks past two [2] sheep. How many sheep will be shorn before Eric?

This Poster Problem Clinic approach is also extensively explored in Maths300 Lesson 14, *The Farmer's Puzzle*.

Strategy Board

DO I KNOW A SIMILAR PROBLEM?	ACT IT OUT
GUESS, CHECK AND IMPROVE	DRAW A PICTURE OR GRAPH
TRY A SIMPLER PROBLEM	MAKE A MODEL
WRITE AN EQUATION	LOOK FOR A PATTERN
MAKE A LIST OR TABLE	TRY ALL POSSIBILITIES
WORK BACKWARDS	SEEK AN EXCEPTION
BREAK INTO SMALLER PARTS	...

How To Solve A Problem

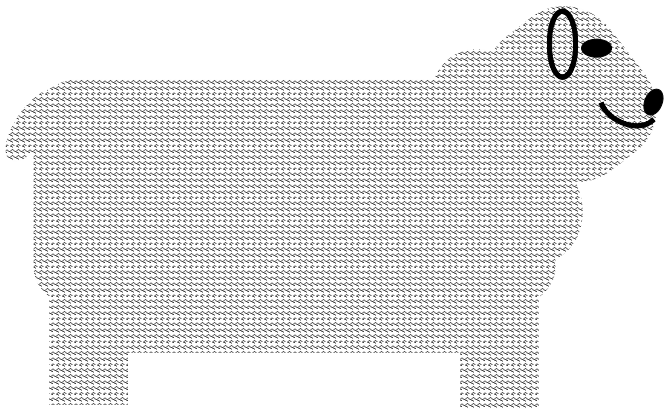
SEE & UNDERSTAND	Do I understand what the problem is asking? Discuss
PLANNING	Select a strategy from the board. Plan how you intend solving the problem.
DOING IT	Try out your idea.
CHECK IT	Did it work out? If so reflect on the activity. If not, go back to step one.

Step 1

- Tell the children that we are at Stage 1 of our four stage plan ... **See & Understand** ... Point to it! Read the problem with the class. Discuss the problem and clarify any misunderstandings.
- If children do not clearly understand what the problem is asking, they will not cope with the next stage. A good way of finding out if a child understands a problem is for her/him to retell it.
- Allow time for questions - approximately 3 to 5 minutes.

Step 2

- Tell the children that we are at Stage 2 of our four stage plan ... **Planning**. In their groups children select one or more strategies from the Strategy Board and discuss/organise how to go about solving the problem.
- Without guidance, children will often skip this step and go straight to Doing It. It is vital to emphasise that this stage is simply planning, not solving, the problem.
- After about 3 minutes, ask the children to share their plans.



Plan 1

Well we're drawing a picture and sort of making a model.

Can you give me more information please Brigid?

We're putting 60 crosses on our paper for sheep and the pen top will be Eric. Then Claire will circle one from that end, and I will pass two crosses with my pen top.

Plan 2

Our strategy is Guess and Check.

That's good Nick, but how are you going to check your guess?

Oh, we're making a model.

Go on ...

> *John's getting MAB smalls to be sheep and I'm getting a domino to be Eric and the chalk box to be the shed for shearing.*

Plan 3

> *We are doing it for 3 sheep then 4 sheep then 5 sheep and so on. Later we will look at 60.*

Great so you are going to try a simpler problem, make a table and look for a pattern.

This sharing of strategies is invaluable as it provides children who would normally feel lost in this type of activity with an opportunity to listen to their peers and make sense out of strategy selection. Note that such children are not given the answer. Rather they are assisted with understanding the power of selecting and applying strategies.

Step 3

- Tell the children that we are at Stage 3 of our four stage plan ... **Doing It**. Children collect what they need and carry out their plan.

Step 4

- Tell the children that we are at Stage 4 of our four stage plan ... **Check It**. Come together as a class for groups to share their findings. Again emphasis is on strategies.

> *We used the drawing strategy, but we changed while we were doing it because we saw a pattern.*

So Jake, you used the Look For A Pattern strategy. What was it?

> *We found that when Eric passed 10 sheep, 5 had been shorn, so 20 sheep meant 10 had been shorn ... and that means when Eric passes 40 sheep, 20 were shorn and that makes the 60 altogether.*

Great Jake. How would you work out the answer for 59 sheep or 62 sheep?

Sharing time is also a good opportunity to add in a strategy which no one may have used. For example:

> *Maybe we could've used the Number Sentence strategy, ie: 1 sheep goes to be shorn and Eric passes two sheep. That's 3 sheep, so perhaps, 60 divided into groups of 3, or 60 ÷ 3 gives the answer.*

Round off the lesson by referring to the Working Mathematically chart. There will be many opportunities to compliment the students on working like a mathematician.

Curriculum Planning Stories

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

In more than a decade of using tasks and many years of using the detailed whole class lessons of Maths300, teachers have developed several models for integrating tasks and whole class lessons. Some of those stories are retold here. Others can be found at:

- http://www.mathematicscentre.com/taskcentre/plans.htm

Story 1: Threading

> *Educational research caused me a dilemma. It tells us that students construct their own learning and that this process takes time. My understanding of the history of mathematics told me that certain concepts, such as place value and fractions, took thousands of years for mathematicians to understand. The dilemma was being faced with a textbook that expected students to 'get it' in a concentrated one, two or three week block of work and then usually not revisit the topic again until the next academic year.*

A Working Mathematically curriculum reflects the need to provide time to learn in a supportive, non-threatening environment and…

> *When I was involved in a Calculating Changes PD program I realised that:*
>
> - *choosing rich and revisitable activities, which are familiar in structure but fresh in challenge each time they are used, and*
> - *threading them through the curriculum over weeks for a small amount of time in each of several lessons per week*
>
> *resulted in deeper learning, especially when partnered with purposeful discussion and recording.*

Calculating Changes:

- http://www.mathematicscentre.com/calchange

Story 2: Your turn

Some teachers are making extensive use of a partnership between the whole class lessons of Maths300 and small group work with the tasks. Setting aside a lesson for using the tasks in the way they were originally designed now seems to have more meaning, as indicated by this teacher's story:

> *When I was thinking about helping students learn to work like a mathematician, my mind drifted to my daughter learning to drive. She*

needed me to model how to do it and then she needed lots of opportunity to try it for herself.

That's when the idea clicked of using the Maths300 lessons as a model and the tasks as a chance for the students to have their turn to be a mathematician.

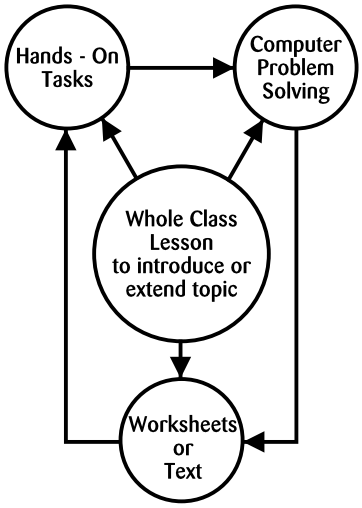
The Maths300 lessons illustrate how other teachers have modelled the process, so I felt I could do it too. Now the process is always on display on the wall or pasted inside the student's journal.

A session just using the tasks had seemed a bit like play time before this. Now I see it as an integral part of learning to work mathematically.

Story 3: Mixed Media

It was our staff discussion on Gardner's theory of Multiple Intelligences that led us into creating mixed media units. That and the access you have provided to tasks and Maths300 software.

We felt challenged to integrate these resources into our syllabus. There was really no excuse for a text book diet that favours the formal learners. We now often use four different modes of learning in the work station structure shown. It can be easily managed by one teacher, but it is better when we plan and execute it together.



Story 4: Replacement Unit

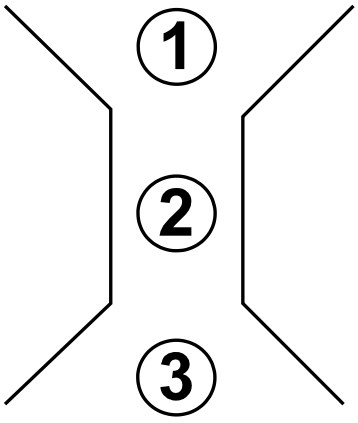
We started meeting with the secondary school maths teachers to try to make transition between systems easier for the students. After considerable discussion we contracted a consultant who suggested that school might look too much the same across the transition when the students were hoping for something new. On the other hand our experience suggested that there needed to be some consistency in the way teachers worked.

We decided to 'bite the bullet' and try a hands-on problem solving unit in one strand. We selected two menus of twenty hands-on tasks, one for the primary and one for the secondary, that became the core of the unit. We deliberately overlapped some tasks that we knew were very rich and added some new ones for the high school.

Class lessons and investigation sheets were used to extend the tasks, within a three week model.

It is important to note that although these teachers structured a 3 week unit for the students, they strongly advised an additional *Week Zero* for staff preparation. The units came to be called Replacement Units.

Week Zero - Planning



Staff familiarise themselves with the material and jointly plan the unit. This is not a model that can be 'planned on the way to class'.

> *Getting together turned out to be great professional development for our group.*

Week 1 - Introduction

Students explore the 20 tasks listed on a printed menu:

- students explore the tip of the task, as on the card
- students move from task to task following teacher questioning that suggests there is more to the task than the tip
- in discussion with students, teachers gather informal assessment information that guides lesson planning for the following week.

> *We gave the kids an 'encouragement talk' first about joining us in an experiment in ways of learning maths and then gave out the tasks. The response was intelligent and there was quite a buzz in the room.*

Week 2 - Formalisation

> *It was good for both us and the students that the lessons in this week were a bit more traditional. However, they weren't text book based. We used whole class lessons based on the tasks they had been exploring and taught the Working Mathematically process, content and report writing.*
>
> *Assessment was via standard teacher-designed tests, quizzes and homework.*

Week 3 - Investigations

> *We were most delighted with Week 3. Each student chose one task from the menu and carried out an in-depth investigation into the iceberg guided by an investigation sheet. They had to publish a report of their investigation and we were quite surprised at the outcomes. It was clear that the first two weeks had lifted the image of mathematics from 'boring repetition' to a higher level of intellectual activity.*

Story 5: Curriculum shift

> *I think our school was like many others. The syllabus pattern was 10 units of three weeks each through the year. We had drifted into that through a text book driven curriculum and we knew the students weren't responding.*

Our consultant suggested that there was sameness about the intellectual demands of this approach which gave the impression that maths was the pursuit of skills. We agreed to select two deeper investigations to add to each unit. It took some time and considerable commitment, but we know that we have now made a curriculum shift. We are more satisfied and so are the students.

The principles guiding this shift were:

- Agree

  The 20 particular investigations for the year are agreed to by all teachers. If, for example, Cube Nets is decided as one of these, then all the teachers are committed to present this within its unit.

- Publish

  The investigations are written into the published syllabus. Students and parents are made aware of their existence and expect them to occur.

- Commit

  Once agreed, teachers are required to present the chosen investigations. They are not a negotiable 'extra'.

- Value

  The investigations each illustrate an explicit form of the Working Mathematically process. This is promoted to students, constantly referenced and valued.

- Assess

  The process provides students with scaffolding for their written reports and is also known by them as the criteria for assessment. *(See next page.)*

- Report

  The assessment component features within the school reporting structure.

A Final Comment

Including investigations has become policy.

Why? Because to not do so is to offer a diminished learning experience.

The investigative process ranks equally with skill development and needs to be planned for, delivered, assessed and reported.

Perhaps most of all we are grateful to our consultant because he was prepared to begin where we were. We never felt as if we had to throw out the baby and the bath water.

Assessment

Our attitude is:

stimulated students are creative and love to learn

Regardless of the way you use your **Maths With Attitude** resource, a variety of procedures can be employed to assess this learning.

Where these assessment procedures are applied to task sessions and involve written responses from students, teachers will need to be careful that the writing does not become too onerous. Students who get bogged down in doing the writing may lose interest in doing the tasks.

In addition to the ideas below, useful references are:

- http://www.mathematicscentre.com/taskcentre/assess.htm
- http://www.mathematicscentre.com/taskcentre/report.htm

The first offers several methods of assessment with examples and the second is a detailed lesson plan to support students to prepare a Maths Report.

Journal Writing

Journal writing is a way of determining whether the task or lesson has been understood by the student. The pupil can comment on such things as:

- What I learned in this task.
- What strategies I/we tried (refer to the Strategy Board).
- What went wrong.
- How I/we fixed it.
- Jottings - ie: any special thoughts or observations

Some teachers may prefer to have the page folded vertically, so that children's reflective thoughts can be recorded adjacent to critical working.

Assessment Form

An assessment form uses questions to help students reflect upon specific issues related to a specific task.

Anecdotal Records

Some teachers keep ongoing records about how students are tackling the tasks. These include jottings on whether students were showing initiative, whether they were working co-operatively, whether they could explain ideas clearly, whether they showed perseverance.

Checklists

A simple approach is to create a checklist based on the Working Mathematically process. Teachers might fill it in following questioning of individuals, or the students may fill it in and add comments appropriately.

Pupil Self-Reflection

Many theorists value and promote metacognition, the notion that learning is more permanent if pupils deliberately and consciously analyse their own learning. The

deliberate teaching strategy of oral questioning and the way pupils record their work is an attempt to manifest this philosophy in action. The alternative is the tempting 'butterfly' approach which is to madly do as many activities as possible, mostly superficially, in the mistaken belief that quantity equates to quality.

> *I had to work quite hard to overcome previously entrenched habits of just getting the answer, any answer, and moving on to the next task.*

Thinking about *what* was learned *how* it was learned consolidates and adds to the learning.

When it follows an extensive whole class investigation, a reflection lesson such as this helps to shift entrenched approaches to mathematics learning. It is also an important component of the assessment process. On the one hand it gives you a lot of real data to assist your assessment. On the other it prepares the students for any formal assessment which you may choose to round off a unit.

Introduction

Ask students to recall what was done during the unit or lesson by asking a few individuals to say what *they* did, eg:

> *What did you do or learn that was new?*
> *What can you now do/understand that is new?*
> *What do you know now that you didn't know 1 (2, 3, ...) lesson ago?*

Continuing Discussion

Get a few ideas from the first students you ask, then:

- organise 5 -10 minute buzz groups of three or four students to chat together with one person to act as a recorder. These groups address the same questions as above.
- have a reporting session, with the recorder from each group telling the class about the group's ideas.

Student comments could be recorded on the board, perhaps in three groups.

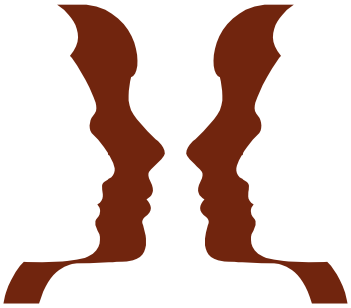
Ideas & Facts	Maths Skills	Process (learning) Skills

If you need more questions to probe deeper and encourage more thought about process, try the following:

> *What new things did you do that were part of* how *you learned?*
> *Who uses this kind of knowledge and skill in their work?*

Student Recording

Hand out the REFLECTION sheet (next page) and ask students to write their own reflection about what they did, based on the ideas shared by the class. Collect these for interest and, possibly, assessment information.



REFLECTION

me looking at me learning

NAME: CLASS:

Reproducible Page

Working With Parents

Balancing Problem Solving with Basic Skill Practice

Many schools find that parents respond well to an evening where they have an opportunity to work with the tasks and perhaps work a task together as a 'whole class'. Resourced by the materials in this kit, teachers often feel quite confident to run these practical sessions. Comments from parents like:

> *I wish I had learnt maths like this.*

are very supportive. Letting students 'host' the evening is an additional benefit to the home/school relationship.

The 4½ Minute Talk

Charles Lovitt has considerable experience working with parents and has developed a crisp, parent-friendly talk which he shares below. Many others have used it verbatim with great success.

Why the Four and a Half Minute Talk?

When talking with parents about Problem Solving or the meaning of the term Working Mathematically, I have often found myself in the position, after having promoted inquiry based or investigative learning, of the parents saying:

> *Well - that's all very well - BUT ...*

at which stage they often express their concern for basic (meaning arithmetic) skill development.

The weakness of my previous attempts has been that I have been unable to reassure parents that problem solving does not mean sacrificing our belief in the virtues of such basic skill development.

One of the unfortunate perceptions about problem solving is that if a student is engaged in it, then somehow they are not doing, or it may be at the expense of, important skill based work.

This Four and a Half Minute Talk to parents is an attempt to express my belief that basic skill practice and problem solving development can be closely intertwined and not seen as in some way mutually exclusive.

(I'm still somewhat uncomfortable using the expression 'basic skills' in the above way as I am certain that some thinking, reasoning, strategy and communication skills are also 'basic'.)

Another aspect of the following 'talk' is that, as teachers put more emphasis on including investigative problem solving into their courses, a question arises about the source of suitable tasks.

This talk argues that we can learn to create them for ourselves by 'tweaking' the closed tasks that heavily populate out existing text exercises, and hence not be dependent on external suppliers. (Even better if students begin to create such opportunities for themselves.)

The Talk

In preparation, write the following graphic on the board:

CLOSED	OPEN	EXTENDED INVESTIGATION
		How many solutions exist? How do you know you have found them all?

I would like to show you what teachers are beginning to do to achieve some of the thinking and reasoning and communication skills we hope students will develop. I would like to show you three examples.

Example One: *6 + 5 = ?*

I write this question under the 'closed' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 —		How many solutions exist? How do you know you have found them all?

And I ask:

What is the answer to this question?

I then explain that:

We often ask students many closed questions such as 6 + 5 = ?

The only response the students can tell us is "The answer is 11." … and as a reward for getting it correct we ask another twenty questions just like it.

What some teachers are doing is trying to *tweak* the question and ask it a different way, for example:

I have two counting numbers that add to 11. What might the numbers be?

[Counting numbers = positive whole numbers including zero]

I write this under the 'open' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 —	? + ? 11	How many solutions exist? How do you know you have found them all?

What is the answer to the question now?

At this stage it becomes apparent there are several solutions:

The question is now a bit more open than it was before, allowing students to tell you things like 8 + 3, or 10 + 1, or 11 + 0 etc.

Let's see what happens if the teacher 'tweaks' it even further with the investigative challenge *or* extended investigation question:

How many solutions are there altogether?

and more importantly, and with greater emphasis on the second question:

How could you convince someone else that you have found them all?

Now the original question is definitely different - it still involves the skills of addition but now also involves thinking, reasoning and problem solving skills, strategy development and particularly communication skills.

Young students will soon tell you the answer is 'six different ones', but they must also confront the communication and reasoning challenge of convincing you that there are only six and no more.

Example Two: *Finding Averages*

Again, as I go through this example, I write it into the diagram on the board in the relevant sections.

The CLOSED question is: *11, 12, 13 - find the average*

Tweaking this makes it an OPEN question and it becomes:

I have three counting numbers whose average is 12. What might the numbers be?

Students will often say:

10, 12, 14 ... or 9, 12, 15 ... or even 12, 12, 12

After realising there are many answers, you can tweak it some more and turn it into an EXTENDED INVESTIGATION:

How many solutions exist? ... AND ...

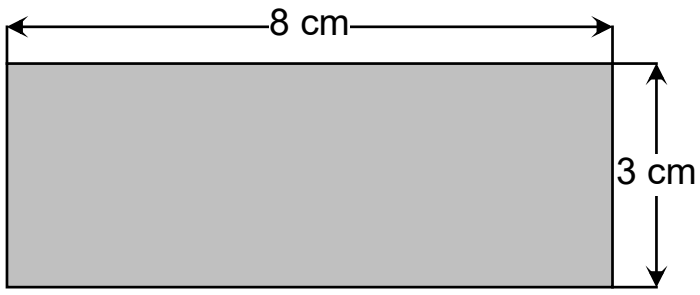
How do you know you have found them all?

Now the question is of a quite different nature. It still involves the arithmetic skill, but has something else as well - and that something else is the thinking, reasoning and communication skills necessary to find all of the combinations and convince someone else that you have done so.

By the time a student announces, with confidence, there are 127 different ways (which there are) that student will have engaged in all of these aspects, ie: the skill of calculating averages, (and some combination number theory) as well as significant strategy and reasoning experiences.

Example Three: *Finding the Area of a Rectangle*

A typical CLOSED question is:



Find the area. Find the perimeter.

The OPEN question is:

A rectangle has 24 squares inside:

What might its length and width be?

What might its perimeter be?

The EXTENDED INVESTIGATION version is:

Given they are whole number lengths, how many different rectangles are there? … AND …

How do you know you have found them all?

In summary, mathematics teachers are trying to convert *some* (not all) of the many closed questions that populate our courses and 'push' them towards the investigation direction. In doing so, we keep the skills we obviously value, but also activate the thinking, reasoning and justification skills we hope students will also develop.

This sequence of three examples hopefully shows two major features:

- That skills and problem solving can 'live alongside each other' and be developed concurrently.
- That the process of creating open-ended investigations can be done by anyone - just go to any source of closed questions and try 'tweaking' them as above. If it only worked for one question per page it would still provide a very large supply of investigations.

In terms of the effect of the talk on parents, I have usually found them to be reassured that we are not compromising important skill development (and nor do we want to). The only debate then becomes whether the additional skills of thinking, reasoning and communication are also desirable.

I've also been told that parents appreciate it because of the essential simplicity of the examples - no complicated theoretical jargon.

A Working Mathematically Curriculum

An Investigative Approach to Learning

The aim of a Working Mathematically curriculum is to help students learn to work like a mathematician. This process is detailed earlier (Page 8) in a one page document which becomes central to such a curriculum.

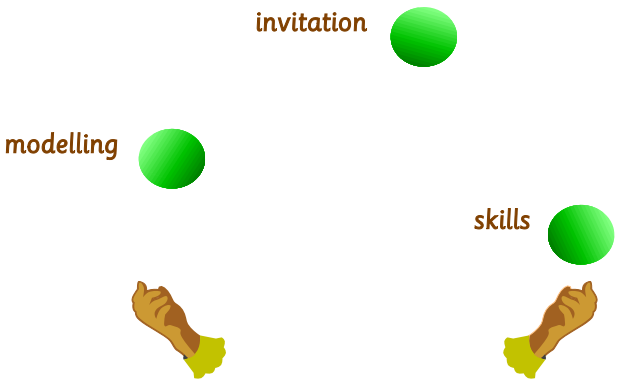
The change of emphasis brings a change of direction which *implies and requires* a balance between:

- the process of being a mathematician, and
- the development of skills needed to be a *successful* mathematician.

This journey is not two paths. It is one path made of two interwoven threads in the same way as DNA, the building block of life, is one compound made of two interwoven coils. To achieve a Working Mathematically curriculum teachers need to balance three components.

The task component of **Maths With Attitude** offers each pair of students an invitation to work like a mathematician.

The Maths300 component of **Maths With Attitude** assists teachers to model working like a mathematician.



Content skills are developed in context. They *are* important, but it is the application of skills within the process of Working Mathematically that has developed, and is developing, the human community's mathematical knowledge.

A focus for the Working Mathematically teacher is to help students develop mathematical skills in the context of problem posing and solving.

> *We are all 'born' with the same size mathematical toolbox, in the same way as I can own the same size toolbox as my motor mechanic. However, my motor mechanic has many more tools in her box than I and she has had more experience than I using them in context. Someone has helped her learn to use those tools while crawling under a car.*

Afzal Ahmed, Professor of Mathematics at Chichester, UK, once quipped:

> *If teachers of mathematics had to teach soccer, they would start off with a lesson on kicking the ball, follow it with lessons on trapping the ball and end with a lesson on heading the ball. At no time would they play a game of football.*

Such is not the case when teaching a Working Mathematically curriculum.

Elements of a Working Mathematically Curriculum

Working Mathematically is a K - 12 experience offering a balanced curriculum structured around the components below.

Hands-on Problem Solving Play

Mathematicians don't know the answer to a problem when they start it. If they did, it wouldn't be a problem. They have to play around with it. Each task invites students to play with mathematics 'like a mathematician'.

Skill Development

A mathematician needs skills to solve problems. Many teachers find it makes sense to students to place skill practice in the context of *Toolbox Lessons* which *help us better use the Working Mathematically Process* (Page 8).

Focus on Process

This is what mathematicians do; engage in the problem solving process.

Strategy Development

Mathematicians also make use of a strategy toolbox. These strategies are embedded in Maths300 lessons, but may also have a separate focus. Poster Problem Clinics are a useful way to approach this component.

Concept Development

A few major concepts in mathematics took centuries for the human race to develop and apply. Examples are place value, fractions and probability. In the past students have been expected to understand such concepts after having 'done' them for a two week slot. Typically they were not visited again until the next year. A Working Mathematically curriculum identifies these concepts and regularly 'threads' them through the curriculum.

Planning to Work Mathematically

The class, school or system that shifts towards a Working Mathematically curriculum will no longer use a curriculum document that looks like a list of content skills. The document would be clear in:

- choosing genuine problems to initiate investigation
- choosing a range of best practice teaching strategies to interest a wider range of students
- practising skills for the purpose of problem solving

Some teachers have found the planning template on the next page assists them to keep this framework at the forefront of their planning. It can be used to plan single lessons, or units built of several lessons. There are examples from schools in the Curriculum & Planning section of Maths300 and a Word document version of the template.

Unit Planning Page

Class

Topic

Pedagogy	Problem Solving In this topic how will I engage my students in the Working Mathematically process?	Skills
How do I create an environment where students know what they are doing and why they have accepted the challenge?		Does the challenge identify skills to practise? Are there other skills to practise in preparation for future problem solving?

Notes

As a general guide:

- Find a problem(s) to solve related to the topic.
- Choose the best teaching craft likely to engage the learners.
- Where possible link skill practice to the problem solving process.

More on Professional Development

For many teachers there will be new ideas within **Maths With Attitude**, such as unit structures, views of how students learn, teaching strategies, classroom organisation, assessment techniques and use of concrete materials. It is anticipated (and expected) that as teachers explore the material in their classrooms they will meet, experiment with and reflect upon these ideas with a view to long term implications for the school program and for their own personal teaching.

Being explored 'on-the-job' so to speak, in the teacher's own classroom, makes the professional development more meaningful and practical for the teacher. This is also a practical and economic alternative for a local authority.

Strategic Use by Systems

From Years 3 - 10, **Maths With Attitude** is designed as a professional development vehicle by schools or clusters or systems because it carries a variety of sound educational messages. They might choose **Maths With Attitude** because:

- It can be used to highlight how investigative approaches to mathematics can be built into balanced unit plans without compromising skill development and without being relegated to the margins of a syllabus as something to be done only after 'the real' content has been covered.
- It can be used to focus on how a balance of concept, skill and application work can all be achieved within the one manageable unit structure.
- It can be used to show how a variety of assessment practices can be used concurrently to build a picture of student progress.
- It can be used to focus on transition between primary and secondary school by moving towards harmony and consistency of approach.
- It can be used to raise and continue debate about the pedagogy (art of teaching) that supports deeper mathematical learning for a wider range of students.

Teachers in Years K - 2 are similarly encouraged in professional growth through **Working Mathematically with Infants**, which derives from Calculating Changes, a network of teachers enhancing children's number skills from Years K - 6.

In supporting its teachers by supplying these resources in conjunction with targeted professional development over time, a system can fuel and encourage classroom-based debate on improving outcomes. There is evidence that by exploring alternative teaching strategies and encouraging curriculum shift towards Working Mathematically, learners improve and teachers are more satisfied. For more detail visit Research & Stories at:

- http://www.mathematicscentre.com/taskcentre/do.htm

We would be happy to discuss professional development with system leaders.

Web Reference

The starting point for all aspects of learning to work like a mathematician, including Calculating Changes, and the teaching craft which encourages it is:

- http://www.mathematicscentre.com/mathematicscentre

Appendix: Recording Sheets

Cross & Square Area Sheet

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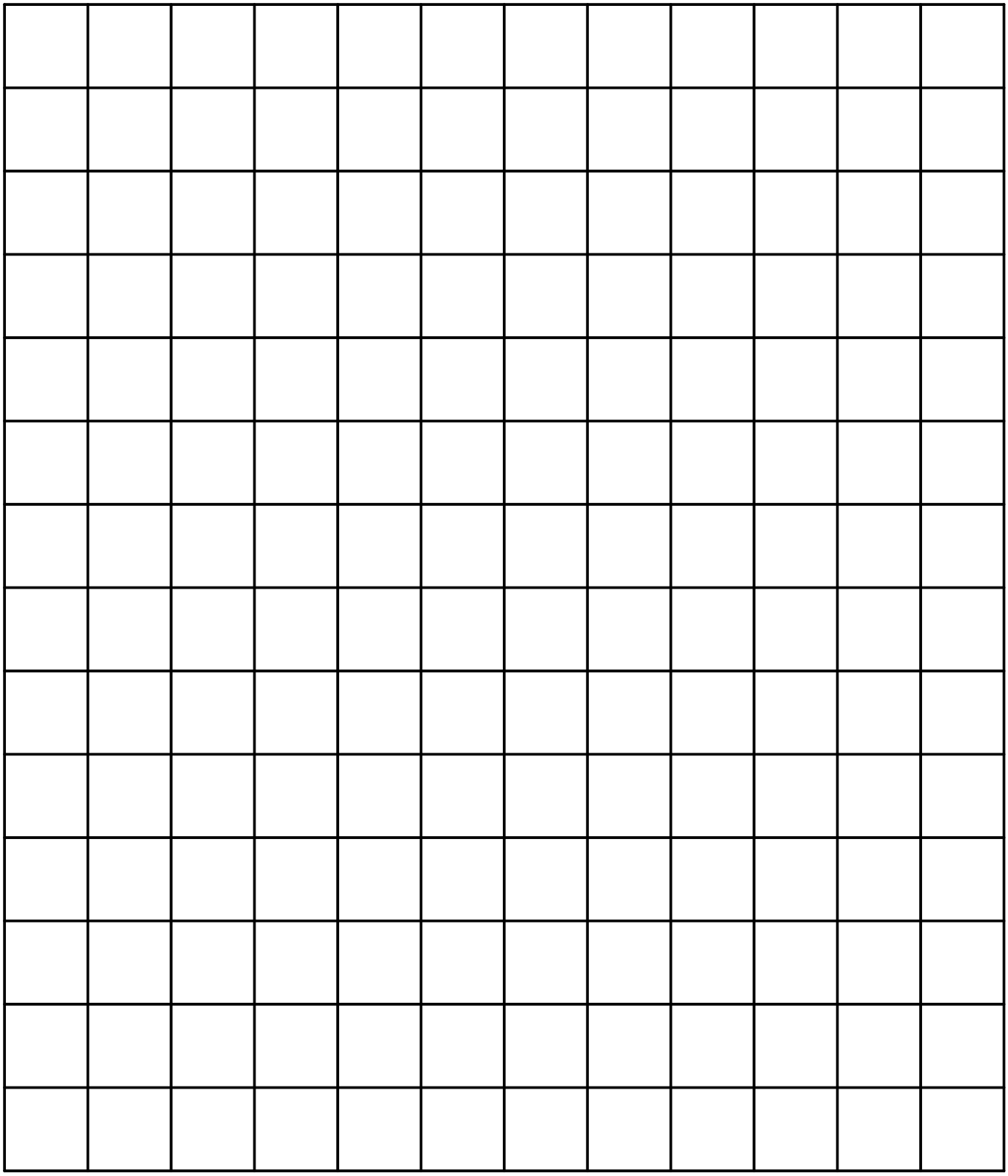

Making Solids Recording Sheet

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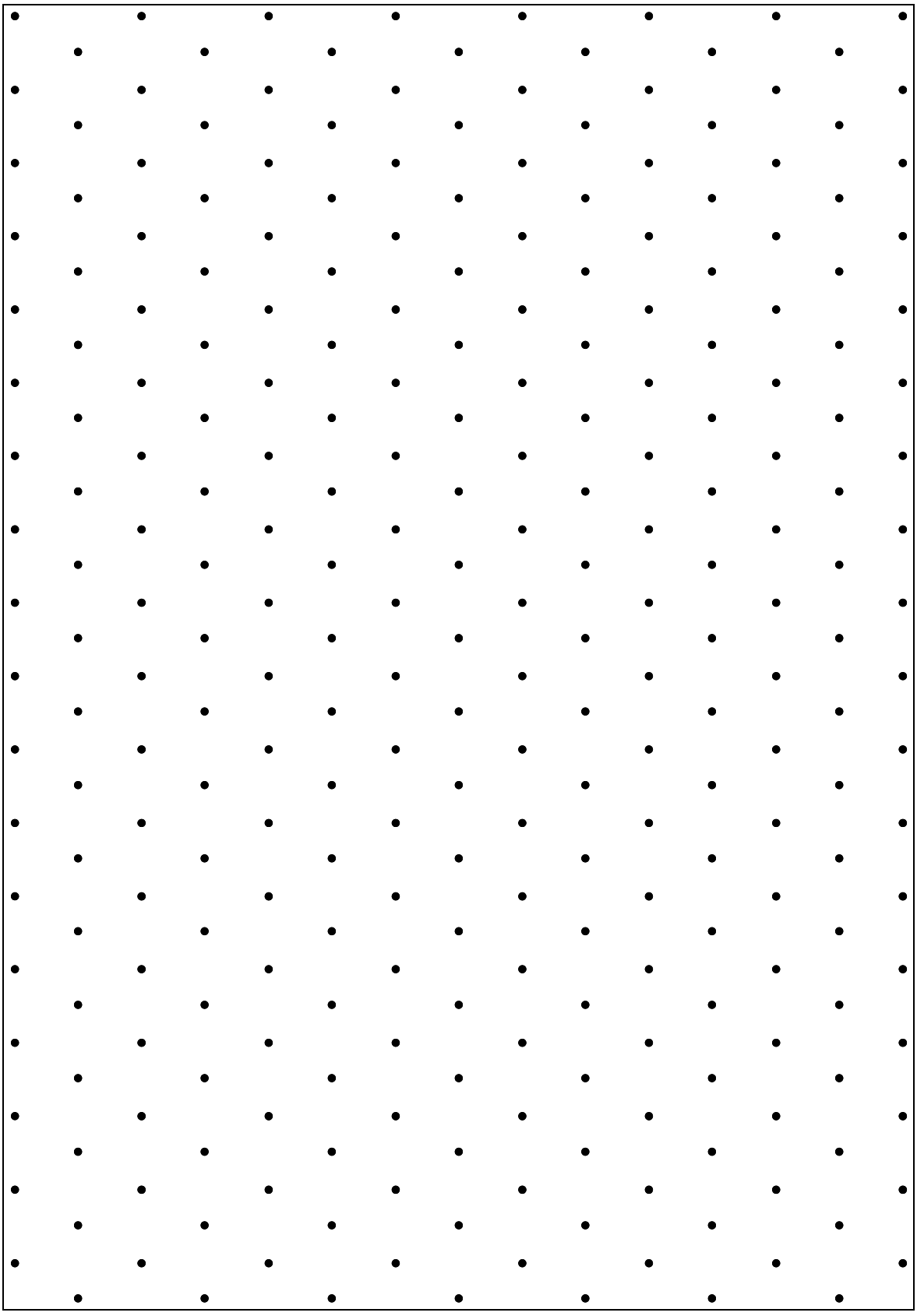

Racetrack Results

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Record your time in seconds for each track. You don't have to do them all today.

Racetrack One

	First Try	Second Try	Third Try
Jockey A			
Jockey B			

Racetrack Two

	First Try	Second Try	Third Try
Jockey A			
Jockey B			

Racetrack Three

	First Try	Second Try	Third Try
Jockey A			
Jockey B			

Symmetric Shapes Recording Sheet

Reproducible Page

Names: ..... Class: .....