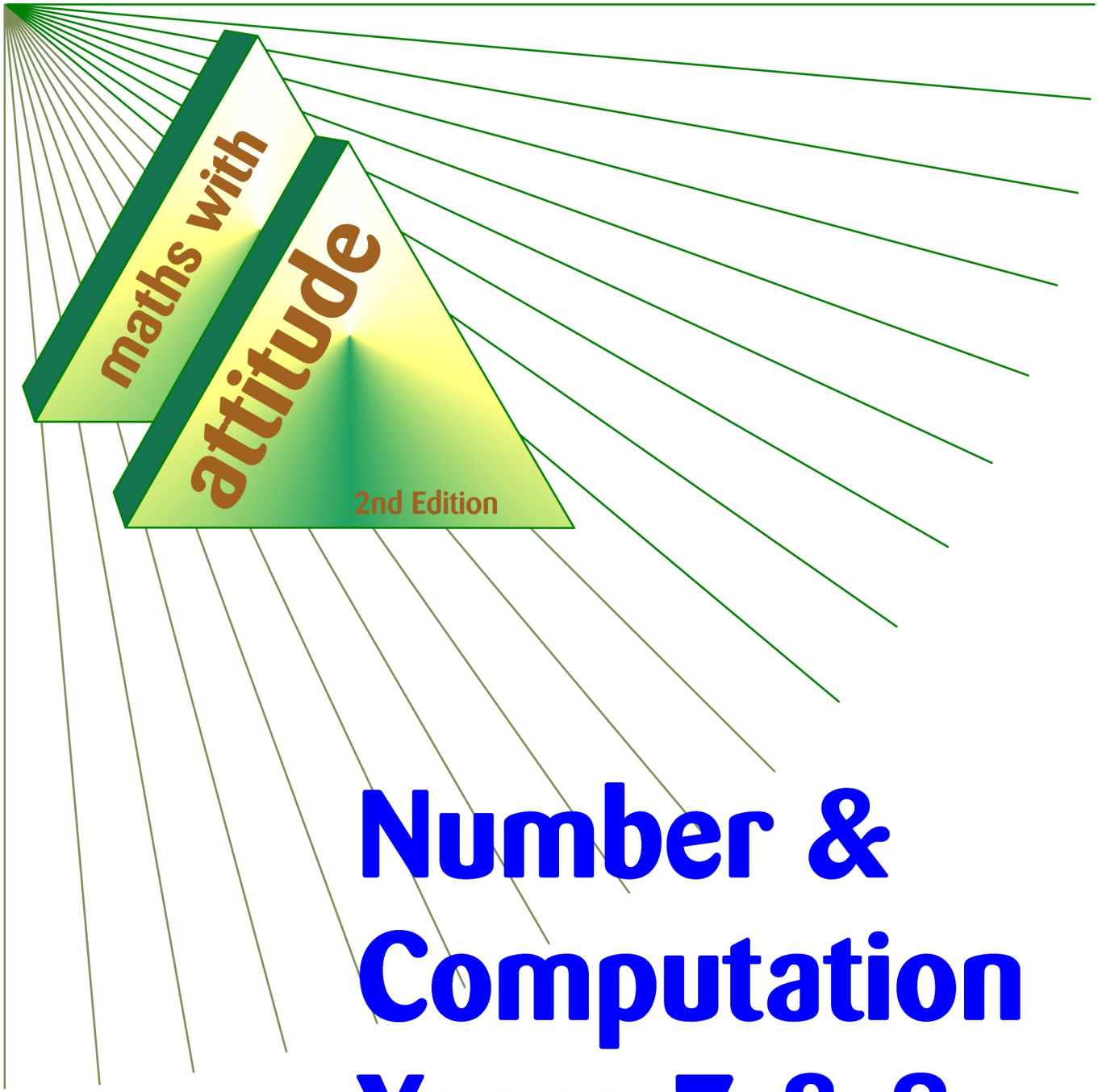


# Number & Computation Years 7 & 8

Charles Lovitt
Doug Williams

Mathematics Task Centre & Maths300
helping to create happy healthy cheerful productive inspiring classrooms


# Number & Computation

# Years 7 & 8

**In this kit:**

- *Hands-on problem solving tasks*
- *Detailed curriculum planning*

**Access from Maths300:**

- *Extensive lesson plans*
- *Software*

**Doug Williams**
**Charles Lovitt**


The **Maths With Attitude** series has been developed by The Task Centre Collective and is published by Black Douglas Professional Education Services.


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**Maths With Attitude** includes reference to hands-on problem solving tasks and Maths300 material.



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# Our Objective

- To support teachers, schools and systems wanting to create:

  *happy, healthy, cheerful, productive, inspiring classrooms*

# Our Attitude

- to learning:

  *learning is a personal journey stimulated by achievable challenge*
- to learners:

  *stimulated students are creative and love to learn*
- to pedagogy:

  *the art of choosing teaching strategies to involve and interest all students*
- to mathematics:

  *mathematics is concrete, visual and makes sense*
- to learning mathematics:

  *all students can learn to work like a mathematician*
- to teachers:

  *the teacher is the most important resource in education*
- to professional development:

  *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

# Our Objective in Detail

What do we mean by creating:

*happy, healthy, cheerful, productive, inspiring classrooms*

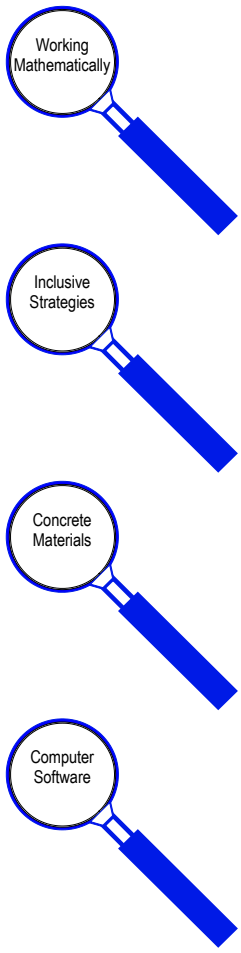
## Happy…

means the elimination of the unnecessary fear of failure that hangs over so many students in their mathematics studies. Learning experiences *can* be structured so that all students see there is something in it for them and hence make a commitment to the learning. In so many 'threatening' situations, students see the impending failure and withhold their participation.

A phrase which describes the structure allowing all students to perceive something in it for them is *multiple entry points and multiple exit points*. That is, students can enter at a variety of levels, make progress and exit the problem having visibly achieved.

## Healthy…

means *educationally healthy*. The learning environment should be a reflection of all that our community knows about how students learn. This translates into a rich array of teaching strategies that could and should be evident within the learning experience.




If we scrutinise the *exploration* through any lens, it should confirm to us that it is well structured or alert us to missed opportunities. For example, peering through a pedagogy lens we should see such features as:

- a story shell to embed the situation in a meaningful context
- significant active use of concrete materials
- a problem solving challenge which provides ownership for students
- small group work
- a strong visual component
- access to supportive software

## Cheerful…

because we want 'happy' in the title twice!

## Productive…

is the clear acknowledgment that students are working towards recognisable outcomes. They should know what these are and have guidelines to show they have either reached them or made progress. Teachers are accountable to these outcomes as well as to the quality of the learning environment.

## Inspiring…

is about creating experiences that are uplifting or exalting; that actually *turn students on*. Experiences that make students feel great about themselves and empowered to act in meaningful ways.

# Number & Computation Resources

To help you create

*happy, healthy, cheerful, productive, inspiring classrooms*

this kit contains

- 20 hands-on problem solving tasks from Mathematics Centre

and a Teachers' Manual which integrates the use of the tasks with

- 20 detailed lesson plans from Maths300

The kit offers **8 weeks** of Scope & Sequence planning in Number and Computation for *each* of Year 7 and Year 8. This is detailed in *Part 2: Planning Curriculum* which begins on Page 12. You are invited to map these weeks into your Year Planner.

Together, the four kits available for these levels provide 25 weeks of core curriculum in Working Mathematically (working like a mathematician).

**Note:** Membership of Maths300 is assumed.
The kit will be useful without it, but it will be much more useful with it.

## Tasks

- Add The Pack
- Arithmagons 2
- Consecutive Sums
- Dominoes
- Doug's Tablecloth
- Find My Pattern
- Four Bead Mix
- Fraction Magic Square
- Guessing Colours Game
- Make The Whole
- Making Fractions 2
- Making Fractions 3
- Number Game
- Peg & Tape Fractions
- Protons & Anti-Protons
- Rectangle Fractions
- Red/Black Card Game
- Rod Mats
- Row Points
- What's It Worth?

Part 2 of this manual introduces each task. The latest information can be found at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm

## Maths300 Lessons

- Arithmagons
- Chocolate Cake
- Dominoes
- First Principles Percent
- Fraction Estimation
- Fraction Magic Square
- Gauss Beats The Teacher
- Goldbach's Conjecture
- Multo
- Number Charts
- Odds & Evens
- Palindromes
- Protons & Anti-Protons
- Rectangle Fractions
- Rod Mats
- Row Points
- Simple, Elegant, Elusive
- Truth Tiles
- Walk The Plank
- What's It Worth?

## Lessons with Software

- Fraction Estimation
- Goldbach's Conjecture
- Multo
- Number Charts
- Odds & Evens
- Palindromes
- Protons & Anti-Protons
- Rectangle Fractions
- Simple, Elegant, Elusive
- Truth Tiles
- Walk The Plank
- What's It Worth?

Part 2 of this manual introduces each lesson. Full details can be found at:

- http://www.maths300.com

# Working Like A Mathematician

Our attitude is:

*all students can learn to work like a mathematician*

What does a mathematician's work actually involve? Mathematicians have provided their answer on Page 8. In particular we are indebted to Dr. Derek Holton for the clarity of his contribution to this description.

Perhaps the most important aspect of Working Mathematically is the recognition that *knowledge is created by a community and becomes part of the fabric of that community*. Recognising, and engaging in, the process by which that knowledge is generated can help students to see themselves as able to work like a mathematician. Hence Working Mathematically is the framework of **Maths With Attitude**.

### Skills, Strategies & Working Mathematically

A Working Mathematically curriculum places learning mathematical skills and problem solving strategies in their true context. Skills and strategies are the tools mathematicians employ in their struggle to solve problems. Lessons on skills or lessons on strategies are not an end in themselves.

- **Our skill toolbox** can be added to in the same way as the mechanic or carpenter adds tools to their toolbox. Equally, the addition of the tools is not for the sake of collecting them, but rather for the purpose of getting on with a job. A mathematician's job is to attempt to solve problems, not to collect tools that might one day help solve a problem.
- **Our strategy toolbox** has been provided through the collective wisdom of mathematicians from the past. All mathematical problems (and indeed life problems) that have ever been solved have been solved by the application of this concise set of strategies.

## About Tasks

Our attitude is:

*mathematics is concrete, visual and makes sense*

Tasks are from Mathematics Task Centre. They are an invitation to two students to work like a mathematician (see Page 8).

The Task Centre concept began in Australia in the late 1970s as a collection of rich tasks housed in a special room, which came to be called a Task Centre. Since that time hundreds of Australian teachers, and, more recently, teachers from other countries, have adapted and modified the concept to work in their schools. For example, the special purpose room is no longer seen as an essential component, although many schools continue to opt for this facility.

A brief history of Task Centre development, considerable support for using tasks, for example Task Cameos, and a catalogue of all currently available tasks can be found at:

- http://www.mathematicscentre.com/taskcentre

Key principles are:

- A good task is the tip of an iceberg
- Each task has three lives
- Tasks involve students in the Working Mathematically process

## The Task Centre Room or the Classroom?

There are good reasons for using the tasks in a special room which the students visit regularly. There are also different good reasons for keeping the tasks in classrooms. Either system can work well if staff are committed to a core curriculum built around learning to work like a mathematician.

- A task centre room creates a focus and presence for mathematics in the school. Tasks are often housed in clear plastic 'cake storer' type boxes. Display space can be more easily managed. The visual impact can be vibrant and purposeful.
- However, tasks can be more readily integrated into the curriculum if teachers have them at their finger tips in the classrooms. In this case tasks are often housed in press-seal plastic bags which take up less space and are more readily moved from classroom to classroom.

## Tip of an Iceberg

The initial problem on the card can usually be solved in 10 to 20 minutes. The investigation iceberg which lies beneath may take many lessons (even a lifetime!). Tasks are designed so that the original problem reveals just the 'tip of the iceberg'. Task Cameos and Maths300 lessons help to dig deeper into the iceberg.

We are constantly surprised by the creative steps teachers and students take that lead us further into a task. No task is ever 'finished'.

> *Most tasks have many levels of entry and exit and therefore offer an on-going invitation to revisit them, and, importantly, multiple levels of success for students.*

## Three Lives of a Task

This phrase, coined by a teacher, captures the full potential and flexibility of the tasks. Teachers say they like using them in three distinct ways:

1. As on the card, which is designed for two students.
2. As a whole class lesson involving all students, as supported by outlines in the Task Cameos and in detail through the Maths300 site.
3. Extended by an Investigation Guide (project), examples of which are included in both Task Cameos and Maths300.

**The first life** involves just the 'tip of the iceberg' of each task, but nonetheless provides a worthwhile problem solving challenge - one which 'demands' concrete materials in its solution. This is the invitation to work like a mathematician. Most students will experience some level of success and accomplishment in a short time.

**The second life** involves adapting the materials to involve the whole class in the investigation, in the first instance to model the work of a mathematician, but also to develop key outcomes or specific content knowledge. This involves choosing teaching craft to interest the students in the problem and then absorb them in it.

**The third life** challenges students to explore the 'rest of the iceberg' independently. Investigation Guides are used to probe aspects and extensions of the task and can be introduced into either the first or second life. Typically this involves providing suggestions for the direction the investigation might take. Students submit the 'story' of their work for 'portfolio assessment'. Typically a major criteria for assessment is application of the Working Mathematically process.

# About Maths300

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

Maths300 is a subscription based web site. It is an attempt to collect and publish the 300 most 'interesting' maths lessons (K - 12).

- *Lessons have been successfully trialed in a range of classrooms.*
- *About one third of the lessons are supported by specially written software.*
- *Lessons are also supported by investigation sheets (with answers) and game boards where relevant.*
- *A 'living' Classroom Contributions section in each lesson includes the latest information from schools.*
- *The search engine allows teachers to find lessons by pedagogical feature, curriculum strand, content and year level.*
- *Lesson plans can be printed directly from the site.*
- *Each lesson supports teaches to model the Working Mathematically process.*

Modern internet facilities and computers allow teachers easy access to these lesson plans. Lesson plans need to be researched, reflected upon in the light of your own students and activated by collecting and organising materials as necessary.

## Maths300 Software

Our attitude is:

> *stimulated students are creative and love to learn*

Pedagogically sound software is one feature likely to encourage enthusiastic learning and for that reason it has been included as an element in about one third of Maths300 lesson plans. The software is used to develop an investigation beyond its introduction and early exploration which is likely to include other pedagogical techniques such as concrete materials, physical involvement, estimation or mathematical conversation. The software is not the lesson plan. It is a feature of the lesson plan used at the teacher's discretion.

For school-wide use, the software needs to be downloaded from the site and installed in the school's network image. You will need to consult your IT Manager about these arrangements. It can also be downloaded to stand alone machines covered by the site licence, in particular a teacher's own laptop, from where it can be used with the whole class through a data projector.

**Note:**

- *Maths300 lessons and software may only be used by Maths300 members.*

# Working Mathematically

First give me an interesting problem.

When mathematicians become interested in a problem they:

- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

Questions which help mathematicians learn more are:

- Can I check this another way?
- What happens if ...?
- How many solutions are there?
- How will I know when I have found them all?

When mathematicians have a problem they:

- Read & understand the problem.
- Plan a strategy to start the problem.
- Carry out their plan.
- Check the result.

A mathematician's strategy toolbox includes:

- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- Break a problem into smaller parts
- ...

If one way doesn't work, I just start again another way.

# Professional Development Purpose

Our attitude is:

the teacher is the most important resource in education

> *We had our first study group on Monday. The session will be repeated again on Thursday. I had 15 teachers attend. We looked at the task Farmyard Friends (Task 129 from the Mathematics Task Centre). We extended it out like the questions from the companion Maths300 lesson suggested, and talked for quite a while about the concept of a factorial. This is exactly the type of dialog that I feel is essential for our elementary teachers to support the development of their math background. So anytime we can use the tasks to extend the teacher's math knowledge we are ahead of the game.*
> District Math Coordinator, Denver, Colorado

Research suggests that professional development most likely to succeed:

- is requested by the teachers
- takes place as close to the teacher's own working environment as possible
- takes place over an extended period of time
- provides opportunities for reflection and feedback
- enables participants to feel a substantial degree of ownership
- involves conscious commitment by the teacher
- involves groups of teachers rather than individuals from a school
- increases the participant's mathematical knowledge in some way
- uses the services of a consultant and/or critical friend

**Maths With Attitude** has been designed with these principles in mind. All the materials have been tried, tested and modified by teachers from a wide range of classrooms. We hope the resources will enable teacher groups to lead themselves further along the professional development road, and support systems to improve the learning outcomes for students K - 12.

With the support of Maths300 ETuTE, professional development can be a regular component of in-house professional development. See:

- http://www.mathematicscentre.com/taskcentre/resource.htm#etute

For external assistance with professional development, contact:

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# Part 2: Planning Curriculum

# Curriculum Planners

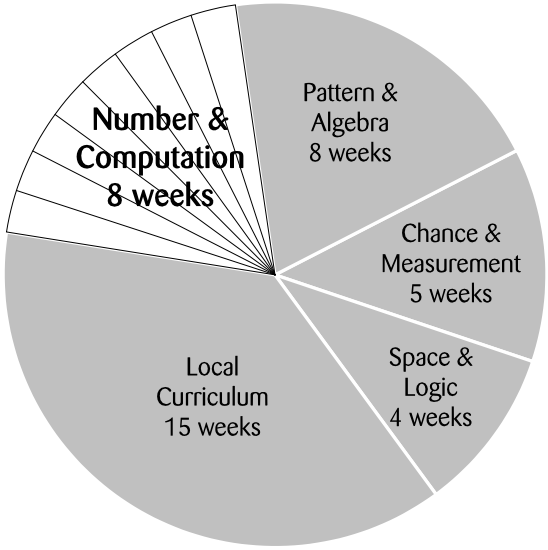
Our attitude is:

*learning is a personal journey stimulated by achievable challenge*

Curriculum Planners:

- show one way these resources can be integrated into your weekly planning
- provide a starting point for those new to these materials
- offer a flexible structure for those more experienced

You are invited to map Planner weeks into your school year planner as the core of the curriculum.




Planners:

- detail each week lesson by lesson
- offer structures for using tasks and lessons
- are sequenced from lesson to lesson, week to week and year to year to 'grow' learning

Teachers and schools will map the material in their own way, but all will be making use of extensively trialed materials and pedagogy.

## Using Resources

- Your kit contains 20 hands-on problem solving tasks and reference to relevant Maths300 lessons.
- Tasks are introduced in this manual and supported by the Task Cameos at:
  http://www.mathematicscentre.com/taskcentre/iceberg.htm
- Maths300 lessons are introduced in this manual and supported by detailed lesson plans at:
  http://www.maths300.com

In your preparation, please note:

- Planners assume 4 lessons per week of about 1 hour each.
- Planners are *not* prescribing a continuous block of work.
- Weeks can be interspersed with other learning; perhaps a **Maths With Attitude** week from a different strand.
- Weeks can sometimes be interchanged within the planner.
- Lessons can sometimes be interchanged within weeks.
- The four **Maths With Attitude** kits available at each year level offer 25 weeks of a Working Mathematically core curriculum.

## A Way to Begin

- Glance over the Planner for your class. Skim through the comments for each task and lesson as it is named. This will provide an overview of the kit.
- Task Comments begin after the Planners. Lesson Comments begin after Task Comments. The index will also lead you to any task or lesson comments.
- Select your preferred starting week - usually Week 1.
- Now plan in detail by researching the comments and web support. Enjoy!

Research, Reflect, Activate

# Curriculum Planner

## Number & Computation: Year 7

| | Session 1 | Session 2 | Session 3 | Session 4 |
|---|---|---|---|---|
| **Week 1** | **Whole Class Investigations:** *Gauss Beats The Teacher, Row Points* and *Truth Tiles* are easy to prepare investigations which help to extend students' automatic response of number facts through problem solving situations. Number properties such as the commutative and distributive laws become apparent in context, giving teachers an opportunity to evaluate and compliment students on their current grasp of the number system. *Truth Tiles* provides software to extend the problem solving aspect of the lesson. | | | |
| **Weeks 2 - 4** | **Mixed Media Unit A:** The structure is described on page 19 and the ten tasks for this unit are listed. The focus of the unit is on refreshing the students' skills with the four operations and then extending their ability to use them with integers rather than just whole numbers. The suggested whole class lessons for this unit are *Multo* and *Protons & Anti-Protons*. Both provide software for the software work station. One text-based work station each week backs up the task and software stations. | | | |
| **Week 5** | **Whole Class Investigations:** *Palindromes* and *Odds & Evens* are curious number puzzles which build on the whole number work in the unit above. Both have software support and both provide plenty of opportunity for the students to work like a mathematician. *First Principles Percent* builds a first principles approach to evaluating percentages. Investigation sheets are provided and the lesson reflects a mindful, rather than mindless, approach to this skill work. | | | |
| **Weeks 6 - 8** | **Mixed Media Unit B:** The structure is described on page 19 and the ten tasks for this unit are listed. The focus of this unit is on understanding and operating with fractions, decimals and percentages. The suggested whole class lessons for this unit are *Rod Mats* and *Fraction Estimation*. There are 8 sub-programs in the *Fraction Estimation* software so it provides plenty of challenge for the software work station. One text-based work station each week backs up the task and software stations. | | | |

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

# Curriculum Planner

## Number & Computation: Year 8

| | Session 1 | Session 2 | Session 3 | Session 4 |
|---|---|---|---|---|
| **Week 1** | **Whole Class Investigations:** *Simple, Elegant, Elusive* is a software supported lesson which encourages students to dip into their skill toolbox and use their knowledge of multiples, factors and primes to solve a spatial problem. *Arithmagons* and *Dominoes* draw on addition, subtraction and place value to investigate their particular problems. | | | |
| **Weeks 2 - 4** | **Mixed Media Unit C:** The structure is described on page 19 and the ten tasks for this unit are listed. This unit will refresh and extend the work in Mixed Media A presented in Year 7. The suggested whole class lessons for this unit are *Number Charts* and *Walk The Plank*. Both provide software for the software work station. One text-based work station each week backs up the task and software stations. See the Classroom Contributions of *Protons & Anti-Protons* for the value one teacher found in following this lesson with *Walk The Plank*. | | | |
| **Week 5** | **Whole Class Investigations:** *Goldbach's Conjecture* offers another curious number puzzle to build whole number work. It is supported by software and there is plenty of opportunity for the students to work like a mathematician. That opportunity is continued in *Fraction Magic Square* which is focused on the problem solving process. Due to the way the lesson is crafted, students who may feel less confident with fractions are still able to enter the problem. *Chocolate Cake* is a fun way to review students' understanding of fractions prior to the next Mixed Media unit. The Classroom Contributions section of this lesson also has a tale to tell. | | | |
| **Weeks 6 - 8** | **Mixed Media Unit D:** The structure is described on page 19 and the ten tasks for this unit are listed. The focus of this unit is on reviewing and developing the students' understanding of, and ability to operate with, fractions and decimals. The suggested whole class lessons for this unit are *Rectangle Fractions* and *What's It Worth?*. Both offer integrated software and challenges at a broad range of levels. One text-based work station each week backs up the task and software stations. | | | |

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

# Planning Notes

## Enhancing Maths With Attitude

Resources to support learning to work like a mathematician are extensive and growing. There are more tasks and lessons available than have been included in this Number & Computation kit. You could use the following to enhance this kit.

In fact, there is almost enough additional material here to create a second Number & Computation kit at this level. **Maths With Attitude** is deliberately limited to 25 weeks of core curriculum at each level, to allow teachers choice and flexibility - choice in which activities to use and flexibility in how to use them. This additional material broadens that choice and flexibility significantly further.

### Additional Tasks

- Task 195, Stop At 4
  *The task is presented as a set of instructions which appear to be a card trick. Indeed, although the procedure could be memorised and used to surprise an audience, further analysis shows it is actually an ingenious device to arrange that the number of cards left in a pile reveal the number on a particular card. Full analysis requires symbolic representation of the instructions.*
- Task 213, Chains
  *The board is six squares in a line and the equipment is six cubes. The arrangement in which cubes are placed scores points. What is the lowest possible score? The highest? Try boards of different lengths.*
- Task 231, Flowers In The Field
  *Also designed for infants, but with a twist that lifts it right up through the school. There are many problems on the card, including ones the students decide for themselves, but they all revolve around having a known number of different flowers and asking how many bunches of a different size can be made.*
- Task 235, Tables For 25
  *The familiar context of sitting in table groups in the classroom leads to problems with multiple solutions when the teacher adds conditions related to the size of the table group and the least number of boys (or girls) allowed to be in a group.*
- Task 237, Trisquares
  *Students explore the creation of new shapes from this simple shape made of three squares joined as an L. This leads to the problem of finding all the new shapes that can be made with just 2 Trisquares and to an investigation of area and perimeter. Each of these aspects of the investigation has a number/computation component.*
- Task 239, Money Charts
  *Text book exercises on adding and subtracting money come to life in this hands-on logic challenge that is most effectively completed by patient application of if-then reasoning and working backwards. In effect there is a little equation solving going on too.*

More information about these tasks may be available in the Task Cameo Library:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## Additional Lessons

- Lesson 70, Pick A Box

  *The boxes have been packed, wrapped and numbered. They have been sorted into groups for delivery to different areas. Then - oh no! - a gust of wind and they are scattered and mixed up. Not to worry. Santa's #1 Elf remembers the sorting rules. The groups can be made again just by following the rules. Unfortunately, although Old #1 doesn't know it, the rules lead to more than one regrouping. How many ways of grouping the boxes can the class find?*

- Lesson 104, Factorgrams

  *Factorgrams are an interesting visual way to illustrate the factor relationships that exist within numbers. As well as the practise of division and multiplication facts, there is considerable opportunity for exploring prime factors. Several problem solving situations are presented.*

- Lesson 145, Estimating Averages

  *This concept lesson is cleverly constructed to focus on understanding the idea of an average rather than on the algorithmic skill. There are two central features: (a) The use of estimation followed by a discussion of strategy leading to a second round of estimates. Students almost invariable score many more points in the second round. (b) The use of concrete materials in the middle stages of the lesson, involving students with differing numbers of blocks sharing (in silence) until all players have the same number.*

- Lesson 146, Division Boxes

  *Place digits 1 to 9 into three boxes (no repeats) to make a three digit number so that: (a) the number in the first box is divisible by 1, and (b) the number in the first two boxes is divisible by 2, and (c) the whole three digit number is divisible by 3. There are many solutions, eg: 621, however 612 is not a solution since the 61 is not divisible by 2. In this form the puzzle invokes knowledge of divisibility tests. However by adding more boxes and finding all possible solutions, it turns into an extended open-ended investigation which can suit many levels. There are solutions for each string of boxes up to ten. However, for the 10-box problem there is just one unique solution and part of the lesson is the search for this special number.*

- Lesson 162, Multiplication in a Table Format

  *This very interesting technique for multiplication has a strong theoretical connection to the distributive property and progresses naturally into factorising and expanding algebraic expressions. The technique arises from a concrete array and has strong visual imagery behind the process. It is also very practical, easy to learn and easy to generate understanding of each step.*

- Lesson 168, Truth Tiles 2

  *Easy to state, easy to start and involves heaps of mathematics. What more could you ask for? Students only need to be able to add and subtract single digit numbers to find initial success with this problem. Given the digits 3 to 7 arrange any four of them to make this equation true ... _ + _ - _ = _. First we find one solution, then another. Then we ask how many there are altogether and how we will know when we have found them all.*

*Can we check our reasoning another way? Then come the What happens if... ? questions.*

- Lesson 169, Human Computer

  *A group of students become the working parts of a computer - a human computer - which can be taught (programmed) to perform the four basic operations. The team develops their computing talents and then offers to answer challenges from an audience. The human computer shows how the simplicity of the binary system can be used as the basis of electronic computing. Physical involvement and group co-operation makes it a most enjoyable and productive learning experience. A spin-off is the opportunity for students to revisit the underlying structure of the place value system.*

- Lesson 172, Licorice Factory

  *This fantasy story introduction to the concept of prime numbers highlights the power of a story shell to facilitate learning. Many students have experienced stretching a piece of licorice so the idea of a factory which has machines to stretch unit pieces to make them 2 times, 3 times, 4 times, ... 100 times longer seems quite natural. In some classes the visualisation of the factory becomes a reality for the students.*

- Lesson 177, Birth Year Puzzle

  *This popular puzzle offers many advantages. It shows the mathematical investigative process, has huge opportunity to practise basic skills, can develop creativity, offers 'point-of-need' opportunity for teachers and caters for mixed ability classes. The challenge is to use the four digits of your Birth Year in that order, and with any legitimate mathematical operations, to generate all the numbers from 0 to 100.*

- Lesson 178, Tables For 25

  *Working in groups, a context familiar to all students, is the focus of the lesson. Mr. Edwards begins his class of 25 by asking the students to organise into groups of 5 at each table, with at least two boys in each group. How the table arrangements work out will depend on how many boys there are in the class altogether, which is what opens the door to a broad investigation and a personal project for each student.*

- Lesson 182, Fractions to Decimals on a Rope

  *This lesson builds a strong conceptual understanding of equivalence when converting fractions to decimals. It offers students concrete, hands-on and visually-based experiences before moving to a formal algorithm. Students use a rope with pegs as markers to first establish ten parts to represent decimals on a number line extending from 0 to 1. Then they fold the rope and explore by estimation how common fractions convert to decimals using the markers on the rope. The lesson features group work, estimation, problem solving and concrete and visual learning.*

Keep in touch with new developments which enhance **Maths With Attitude** at:

- http://www.mathematicscentre.com/taskcentre/enhance.htm

## Additional Materials

As stated, our attitude is that mathematics is concrete, visual and makes sense. We assume that all classrooms will have easy access to many materials beyond what we supply. For this unit you will need:

- Counters and a printed 5 x 5 grid, or Poly Plug
- Unifix or Multi-Link or the like
- Counters with a different colour either side, or Poly Plug
- Cuisenaire Rods for *Rod Mats*
- Tape & pegs for *Fraction Estimation*
- Dominoes
- Sphinx pieces, an optional resource for *What's It Worth?*

## Special Comments Year 7

- *Row Points*, Week 1, will require a 5 x 5 grid for each student. There is one provided in the lesson. Perhaps you will want to plan far enough ahead to make a laminated set that can be used in future years. Alternatively, use Poly Plug if you have it.
- Look ahead to Planner Weeks 2 - 4. The *Multo* cards the teacher will need are provided in the lesson, but to be useful, they will have to be printed on card, laminated and cut.
- Look ahead to Planner Weeks 6 - 8. *Rod Mats* cannot really be tackled as written without Cuisenaire Rods. If the sets provided in the tasks of this kit are not enough, plan ahead to order more. The pegs and tapes needed for *Fraction Estimation* are easily available from supermarkets and haberdashery shops.

Poly Plug, Sphinx, Dominoes and Cuisenaire Rods can be obtained from Mathematics Centre at:

- http://www.mathematicscentre.com/taskcentre/resource.htm

## Special Comments Year 7 & 8

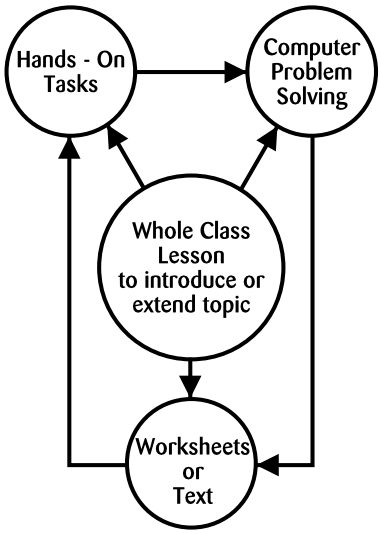
- The Mixed Media Units may be a new structure for you and the students so look ahead and plan carefully. Trial teachers suggest the planning is more effective and efficient when carried out as a team.

## Special Comments Year 8

- The domino sets needed for Week 1 may be available from school families either on loan or as a gift.

# Mixed Media Unit

Mixed Media Mathematics has been created as *one* structure which allows teachers to integrate problem solving tasks into the curriculum.




The design incorporates four different modes of learning into a structure which can be readily managed by one teacher, but which is enhanced when prepared and executed by a team.

A three week Mixed Media Unit includes:

- whole class lessons
- hands-on problem solving
- problem solving software
- skill practice worksheets (or text material)
- time to reflect on learning
- assessment opportunities

If this is the first time such a structure has been used in your classroom, it is a good idea to prepare the students in a manner which 'brings them into the experiment'.

A vital element of the process is to reflect on *what* is learned and *how* it is learned *before* the final assessment of the learning. Guidance with respect to assessment is also provided in this manual. In particular, the Pupil Self-Reflection information in the Assessment section of Part 3 was designed by teachers who trialed the original Mixed Media units. In general the structure above is run for two weeks. The third week begins with a discussion lesson around the question: *What do you know now that you didn't know two weeks ago?* Responses to this discussion guide teachers in rounding off the three week unit during the remaining sessions.

## Mixed Media Units A & C

The tasks suggested for these units are:

- Add The Pack
- Arithmagons 2
- Consecutive Sums
- Dominoes
- Find My Pattern
- Guessing Colours Game
- Number Game
- Protons & Anti-Protons
- Red & Black Card Game
- Row Points

## Mixed Media Units B & D

The tasks suggested for these units are:

- Doug's Tablecloth
- Four Bead Mix
- Fraction Magic Square
- Make The Whole
- Making Fractions 2
- Making Fractions 3
- Peg & Tape Fractions
- Rectangle Fractions
- Rod Mats
- What's It Worth?

# Task Comments

- Tasks, lessons and unit plans prepare students for the more traditional skill practice lessons, which we invite you to weave into your curriculum. Teachers who have used practical, hands-on investigations as the focus of their curriculum, rather than focussing on the drill and practice diet of traditional mathematics, report success in referring to skill practice lessons as Toolbox Lessons. This links to the idea of a mathematician dipping into a toolbox to find and use skills to solve problems.

## Add The Pack

There are many number sense opportunities in this task. Perhaps the very slowest way to find the total is for one student to shuffle the pack then place the cards on the table one at a time while the other student keeps a progressive total, either orally, or with a calculator. From there, guided by the card, the students will think of several ways of applying strategies that reach the total more efficiently. In effect the task encourages students to ask the mathematician's question:

> *Can I check this another way?*

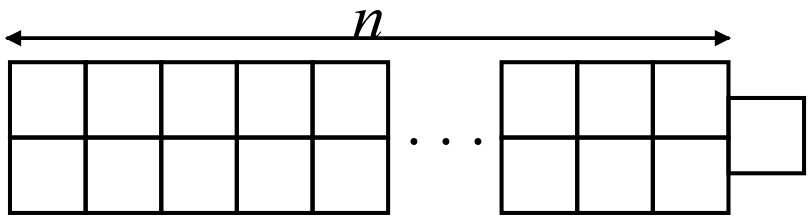
Question 4 deliberately encourages a mind experiment because only cards 7 - K are supplied, but if necessary a full card pack may be used. Whichever approach the students take, the teacher is encouraged to refrain from answering yes or no to the question *Is that right?*. The response *Can you check it another way?* is more appropriate. It encourages students to employ more than one strategy and it encourages independent learning. The task has a direct link to *Gauss Beats The Teacher*, which is a Maths300 lesson assigned to this kit.

## Arithmagons 2

What appears at first as a task to practise addition and subtraction facts opens into a pattern and algebra problem. As students record solutions to the parts of Question 1, they should start to see connections that will help them solve the working backwards examples of Question 2. This is an opportunity to reinforce that a mathematician needs skills to be able to solve problems. Skill development is not simply an end in itself and tasks like this demonstrate that your curriculum is offering genuine problems in which skills practised elsewhere can be applied. There are extensive notes about the task in the companion Maths300 lesson.

## Consecutive Sums

While becoming more familiar with the idea of consecutive numbers, the students are encouraged by the challenge to organise their thinking to be sure that they have counted every possibility. In looking at the results of their organised search they should see patterns that lead to generalisations. For example Question 1 asks about two consecutive numbers and the sums of these within the discs provided is always an odd number. Students who notice this might go on to ask why and be led to represent the first number as $n$ and the next as $n + 1$. The sum of these is therefore $2n + 1$. By the definition of even numbers, or pictorially:




students can reason that this must always be an odd number, as suggested by their own data.

Also, this type of generalisation is usually in the background of a type of word problem that involves *Think of a number, now add 6, multiply by 2 ... take off the number you first thought of ... and the answer is ...* Teachers will have their own favourites from among these and this may be a good opportunity to bridge into an investigation of them.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## Dominoes

Easy to start and the questions take the students deeper and deeper into the investigation. The task is a concrete situation requiring the application of number facts and place value. Not asked on the card, but always lurking, are the questions

> *How many solutions are there?*
> *How do I know when I have found them all?*

For example, how many combinations are there that involve carrying? There are extensive notes in the companion Maths300 lesson. The Classroom Contributions link also contains two 'entries' in the Great Domino Sum Challenge.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## Doug's Tablecloth

Number in a measurement context, although the numbers need not be used to solve the problem. Most students use a trial and improve strategy and eventually try making the first fold of the cloth into thirds rather than halves. This is crucial to the solution. However there are many clues in the numbers and once solved it is important to look back and ask *What might I have learned from the numbers in the problem?*. The numbers can also be used to offer hints to students who are struggling. For example, *The material has to exactly fit the depth of the drawer, so how does the depth relate to the original measurements?*. The Challenge on the card invites students to work backwards, unfolding the cloth as they keep a tally of the growth of the length from 16cm.

One way of obtaining the solution is:

- Lay the cloth out on the table beneath the card so that its long edge is running the same direction as the long edge of the picture of the drawer.

Fold only from here. Do not turn it at all for the remainder of the instructions.
- Fold the cloth to halve this long edge.
- Fold again, once from the left and once from the right to make thirds.
- Fold upwards in half once and then in half again and the bundle of cloth will fit the depth of the drawer.

Retracing the folding steps above will reveal the actions which affect the long edge of the cloth. They are the first halving and then the thirds. We know from the picture that the width of the final bundle of cloth must be 16cm, so we have to find a starting length such that when it is first folded in half and then in thirds its measurement will be 16cm. This is a classic working backwards question which leads to the answer of 96cm. So the cloth has to be 36 x 96 if it is to fit exactly into the drawer.

### Extension

This task is about experiencing fractions as an action rather than operating with them as symbols. However for those who want to explore the task further, ask the students to orient a piece of plain A4 paper the same way that the material is to start with.

> *Now do the same sequence of folds as you did with the table cloth. Press each fold tightly because when you unfold you have to be able to see the fold lines clearly.*

When the students unfold the paper they will have one whole rectangle marked into equal parts by the fold lines. Discuss the main parts (rows, columns and cells) and the fraction of the whole that each represents. Ask the students to write all the fraction stories they can find in this whole.

A working backwards question is to ask how many folds have to be made in a piece of paper to make a whole which helps you work out: $^1/_2 + ^1/_3$

> *This activity proved useful in considering combinations which make up to 24 by doubling, tripling etc. Students found it difficult to start with, some needing a gentle prod in the right direction, but most were able to work it out for themselves. Comments such as: "It was hard until you worked it out. It was fun." were fairly typical.*
> Warragul Regional College

For students who have experienced **Rectangle Fractions** either as a task or through the companion Maths300 lesson, there is a direct visual link with earlier activities.

Note: You will sometimes have to iron the cloth.

Find more information about this task in the Task Cameo Library at:
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## Find My Pattern

This task was invented by a group of Year 4 students. You will soon find their patterns are more complex than those teachers would usually offer. The task offers plenty of opportunity for mental and written arithmetic. There are many questions you could ask which will help students to see that there is more to the iceberg. At some stage, you might also like to use this lesson for its algebra and co-ordinate geometry connections.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## Four Bead Mix

At one level this ratio task can be accessed simply through counting. There are enough beads provided to allow the students to count out sets of beads in the given ratio. Finding that there are 18 in a set offers a clue to a quick way of finding how many beads of each type there are in a can of 180 beads. However, if students need to, they can continue to count the beads in sets.

Question 1 investigates 'nice ratios' and, in part (c), encourages the students to explore and apply this principle by making up their own. They will discover that 'nice ratios' are ones such that the sum of the parts is a factor of 180. It is quite a challenge to ask:

> *How many 'nice ratios' are there?*
> *How will we know when we have found them all?*

Question 2 is an example of a backwards question. If you know the ratio, what total amounts of beads are possible.

Once the students begin to talk about the sum of the parts in reference to a complete, or whole, set of beads, the way is open to talk in fraction language. As with all fraction problems, defining the whole is important. For example if we consider just two types of beads (A and B) in the ratio 2 : 3, it is correct to say $^2/_5$ of the collection is As; and it is also correct to say that there are $^2/_3$ as many As as Bs. In the first statement the total collection is the whole; in the second the Bs are being considered as the whole. In the second case, what would be the fraction statement if the As were considered as the whole?

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## Fraction Magic Square

Basic number skills, equivalent fractions and addition of fractions are all involved in the solution of this task. Students are usually familiar with Magic Squares and this knowledge can be the starting point of the solution. A mathematician would ask:

> *Have I seen a similar problem before?*

and then perhaps work backwards to relate the current task to previous knowledge.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## Guessing Colours Game

The task is included at this level because the scoring system involves additions and subtractions that can lead to negative numbers. Whether or not this does happen depends on how good the students are at guessing the colours. This aspect of the game relates to Maths300 lesson *Walk The Plank*, which is assigned to this kit. It also involves the same arithmetic practice as **Red & Black Card Game**. Further, the problem solving question in this task, *How long is a good game?*, can be applied to **Guessing Colours Game**. The variation in the task would come from changing the number of discs in the bag and consequently, the starting number of points.

**Guessing Colours Game** also has a probability aspect which you may wish to follow up. The structure of the game involves drawing from a sample space without replacement, so there is plenty of opportunity to discuss chance. For example, the task suggests keeping a record using the columns Guess and Points. Teachers may wish students to add a column between these headed Chance of Correct Guess.

Find more information about Red & Black Card Game in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## Make The Whole

This task derives from **Rod Mats** (see later). The task is supplied with pieces from a rod mat which has Orange as the whole. The students engage in considerable equivalent fraction work and addition and subtraction of fractions as they play the game. The game also offers a model for making additional similar games.

A worksheet of blanks is supplied at the end of this manual for teachers who want the students to create their own game. This can be copied onto thin card, laminated and cut up to make a blank set on which the students write with a marker. If a less permanent set is required, then paper copies will work for one game.

The steps in creating a **Make The Whole** variation are:

- choose a rod (or collection of rods) to be whole
- make the rod mat for this whole
- record the colour of the whole
- record one fraction card (words and symbols) for each rod in the mat.

The game is now played with the same rules but a different whole and different cards.

Example: For the picture on the **Rod Mats** task card, students would prepare two one half cards, four one quarter cards and eight one eighth cards.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## Making Fractions 2

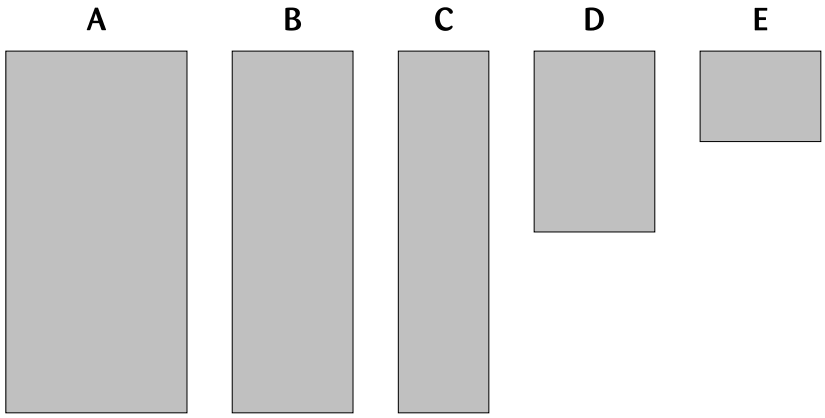
This task builds on the same principle as **What's It Worth?** which is included in this kit both as a task and as a lesson. The principle is to take a whole that is divided into parts and choose any of those parts to have a value of 1. In effect, the task is based on the concept that any piece, not necessarily the largest one, can be the whole. **Making Fractions 2** is based on halves, quarters and eighths to make the relationships as simple as possible. The completed table opens discussion about reciprocal fractions.

Note that the intention is drawings, or a code such as LS for larger square, go in the top cells of each column of the board included with the task. The pieces are not intended to fit in these cells. However, some students do stand them on their edge outside the cells. An additional spatial challenge is provided by trying to fit the pieces back into the frame at pack up time. This should be encouraged because it is a check that all the pieces are there.

## Making Fractions 3

This task is a more difficult partner of the one above. Its pieces are halves, thirds, fourths, sixths and twelfths of a whole that is not included in the pieces. Some comparisons of these pieces will be easier than others. For example, the half pieces compared to the quarters, sixths or twelfths will be straightforward, but comparing them to the thirds will require working through another piece, which, in effect, will be their common denominator.

From largest to smallest the blocks are:




Some of the relationships between the blocks are straightforward, for example:

- *Two* of Block E fit on top of one of Block D, so D is double E or E is half of D (the reciprocal relationship)
- *Three* of Block B fit on top of two of Block A, so A is one and a half times B or B is two thirds of A (the reciprocal relationship)

The first row of the table is:

| | | | | |
|---|---|---|---|---|
| 1 | $^2/_3$ | $^1/_2$ | $^1/_3$ | $^1/_6$ |

The full table is:

| | | | | |
|---|---|---|---|---|
| 1 | $^2/_3$ | $^1/_2$ | $^1/_3$ | $^1/_6$ |
| $1^1/_2$ | 1 | $^3/_4$ | $^1/_2$ | $^1/_4$ |
| 2 | $1^1/_3$ | 1 | $^2/_3$ | $^1/_3$ |
| 3 | 2 | $1^1/_2$ | 1 | $^1/_2$ |
| 6 | 4 | 3 | 2 | 1 |

The patterns in this table include:

- the ones down the leading diagonal
- the doubling and halving between the last two columns (and last two rows)
- the reciprocals in a mirror pattern across the leading diagonal

The final question on the card relates the fraction question to decimals. It also involves making a decision about rounding off. Per unit, prices like $16^2/_3$ cents occur. Should these be rounded or would the manufacturer of the pieces leave the figure in the fraction form on the grounds that pieces would be sold in large quantities?

If rounding off is used, the table becomes:

| | | | | |
|---|---|---|---|---|
| \$1 | 67¢ | 50¢ | 33¢ | 17¢ |
| \$1.50 | \$1 | 75¢ | 50¢ | 25¢ |
| \$2 | \$1.33 | \$1 | 67¢ | 33¢ |
| \$3 | \$2 | \$1.50 | \$1 | 50¢ |
| \$6 | \$4 | \$3 | \$2 | \$1 |

### Extension

Although the card doesn't suggest it, there is considerable opportunity here for the students to create fraction number stories. They must first choose one piece as the whole, then find the parts of it that are represented by the other blocks. This is equivalent to focusing on just one row of the table. Encourage students to record as many equations as they can for that one row and ask them to justify some. Their responses should provide some assessment information.

> *It was a bit confusing at the start but after reading it again it made sense. It was good having the blocks to help because you could use different blocks to work out the others. It took a while to understand but I enjoyed it.*
> Declan, Year 6

## Number Game

The task allows a wide variety of students to be involved and creative. The game encourages application and development of number sense and some teachers have used moments from particular games to generate number talks with the class. The scoring rules are designed to encourage use of larger numbers and division. Larger scores are obtained by using times tables! Calculators could be used and it may be necessary to place a limit on the time allowed to construct equations in each deal. The task links well with popular Maths Display Board activities at this level, such as making all the digits from 1 to 100 using the digits of the current year.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## Peg & Tape Fractions

A conceptual approach to fractions using a linear whole. In fact, the task offers two wholes so students can discover that their folding strategy is independent of the size of the whole. Extensive notes and software are provided in the companion Maths300 lesson *Fraction Estimation*.

## Protons & Anti-Protons

When equal collections of protons and anti-protons meet their combined value is zero. The converse, which is the strength of the task, is that zero can be created from any equal number of protons and anti-protons. Hence if you don't have enough of one type of atomic particle to subtract what you are asked to subtract, you can introduce the extra you need by prudently choosing a collection worth zero. Take away what you need and a collection of the opposite particle will remain.

There is no need at all to mention positive and negative numbers when using this task. However it does lay a firm mathematical foundation for conceptualising integer arithmetic.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## Rectangle Fractions

Whenever objects are arranged in equal rows, the complete array can be thought of as the whole. The rows, columns and individual pieces are then clearly parts of the whole and can be discussed in fraction language. This conceptualisation changes operations on fractions from symbolic symbol shoving to spatial perception. It encourages students to build a brain picture that is consistent with the manipulation of the numbers.

The advantage of the board is that many different rectangles can be revealed by changing the arrangement of plugs. The red board provides the framework of a rectangle and the yellow plugs are the loose objects.

This same model is present in many other situations, for example, soft drink bottles in a crate, students lined up in twos, marching teams and vegetables planted in rows and columns. Students can think of more. When these models

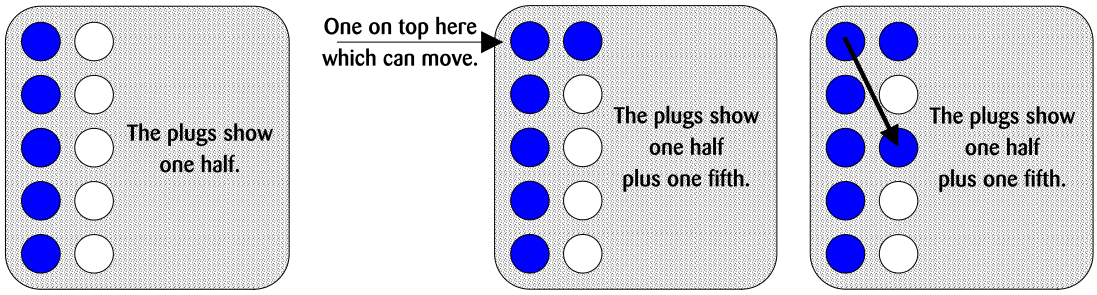
appear during the day, seizing teachable moments to pose a fraction challenge can encourage the students to 'see' and practise fractions for themselves.

To understand the first problem on the card it is necessary to notice that:

- each space is one tenth of the grid
- one half is one column - put in the yellow plugs
- one fifth is one row - put in the yellow plugs

This information is expressed in the way the student recorded on the card explains her inspiration; however, many students don't read far enough to discover this. Getting this far makes it clear that:

- the space needed by one plug in the one fifth row is already occupied by a plug from the one half column.
- you could place it on top and remember to count it.
- or, since one plug is worth one tenth wherever it is in this grid, you can see the total of one fifth + one half by shifting it to an empty space.




Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## Red & Black Card Game

This task has a double value. At the same time as it provides opportunity to practise the arithmetic of positive and negative numbers, it also provides data for a statistical investigation. If the challenge on the card is to be answered on the basis of evidence, then the students will need to experiment, collect and organise data and make and test hypotheses based on the evidence. They will be working like a mathematician as they do this.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## Rod Mats

The focus in this task is on mathematical conversation using fraction language. The key is that any rod can be the whole; and it will always be possible to make at least one row of rods of the same type which is the same length as the whole. The white rod can always be repeated to exactly fit the length of any chosen whole. The whole and its rows of 'same rods' make the rod mat. The task is closely linked to **Make The Whole**.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## Row Points

Thirteen cells of a 5 x 5 array are occupied by counters. Points are given if the counters are in rows, columns or diagonals of 3, 4, or 5. Now many questions can be asked about the totals which can be formed.

The task provides considerable mental arithmetic practice in a spatially based problem solving context. The challenges suggested on the card and the questions that grow from them are non-trivial, and yet accessible to almost all students.

Find more information about this task in the Task Cameo Library at:

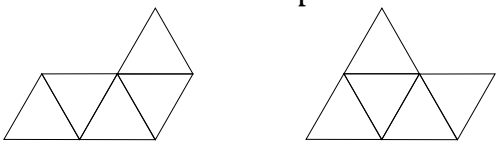
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## What's It Worth?

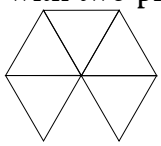
This task is a partner to Making Fractions 2 & 3 because it is based on the same value relations concept. The part is worth 1, so the whole is worth…?. Or, the whole is worth 1, so the part is worth...? The first of these encourages whole number/integer arithmetic. The second encourages fraction/decimal arithmetic.

As presented the task involves spatial problem solving as well as number work. For example, finding shapes worth $2.75 implies finding all the shapes that can be made with five triangles. Apart from the five in a row there are:

Four in a row with one placed as shown



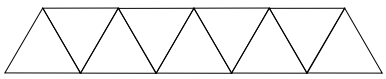
Three in a row with two placed as shown

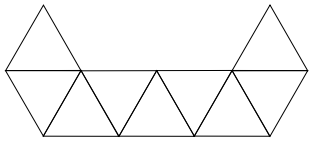


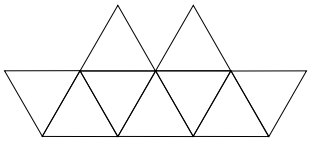
All other shapes are rotations or flips of these.

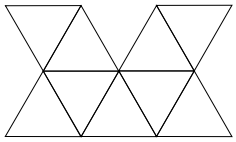
The value of $3.30 in Question 4 implies shapes made with 6 equilateral triangles at 55¢ each. These are the hexiamonds. The full set can be found by beginning with the four pentiamonds above and finding all the ways of adding one more triangle.

The shape in Question 5 uses nine equilateral triangles. Since this is an odd number, one of the triangles must somehow be in the 'middle' to maintain symmetry. Some solutions are:









Rather than individuals searching for all the symmetric shapes as asked for on the card, teachers may prefer to begin a class display which grows as each pair of students finds one or two more. When there is plenty of data on the display, the class's attention could be drawn to looking for ways of classifying the shapes so that it can be decided whether they have all been found.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

### Extensions

Any of the shapes students make in carrying out this task could be seen as a whole thing (the shape) which is divided into equal parts (the triangles). Students can be invited to:

Make any shape with the tiles:

- This is your whole.
- Cut it out from the triangle paper and paste it into your journal.
- Write all the fraction stories you can from this shape.
- Make sure you can demonstrate each one with the tiles.

Make any shape with *ten* tiles:

- This is your whole.
- Cut it out from the triangle paper and paste it into your journal.
- Write all the decimal stories you can from this shape.
- Make sure you can demonstrate each one with the tiles.

Make a whole shape which would allow you to work out:

- $^1/_2 + {}^1/_3$
- What other fraction problems can you work out with this whole?
- How about: $^1/_2 + {}^2/_3$ or $^2/_3 + {}^1/_2$?

# Lesson Comments

- These comments introduce you to each Maths300 lesson. The complete plan is easily accessed through the lesson library available to members at:
  http://www.maths300.com
  where they are listed alphabetically by lesson name.

## Arithmagons

Although these delightful puzzles require only the ability to add whole numbers up to 20, they are non-routine because the student needs to hold two conditions true simultaneously. The simple content level invites a wide range of students to participate and therefore become engaged in the process of Working Mathematically. The problems also succumb to algebraic representation and analysis. The only equipment needed is scrap paper and the recording sheets. The recording sheets are provided in the lesson plan.

*Just about finished Arithmagons lesson with my Year 7 class. Has gone very well. Much impressed with the lesson notes. Kids enjoyed it a lot. The LOTE teacher complained that the kids were drawing those so and so triangle things in Japanese. Kids currently finishing off by producing an explanatory poster, which must include some Arithmagons to be completed by another person such as their Mum or Dad.*

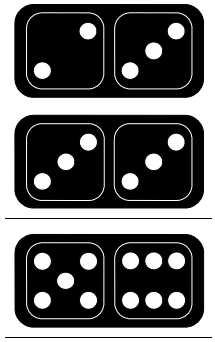
## Chocolate Cake

In the context of a make believe party at which six rectangular chocolate cakes are shared, the students investigate making fractions. The cakes are represented by sheets of A4 paper and different groups of students are challenged to divide different numbers of cakes into various fractions. Measurement, folding, discussion and consensus are encouraged. The lesson is a wonderful introduction to a fraction unit because it reveals a considerable amount about the students' current understanding of the topic.

*It was obvious from the moment that I first trialed this lesson with grade 5/6 students that it was exciting.*

## Dominoes

These three dominoes form a correct addition:



The lesson develops this observation into an extended investigation about other domino combinations that also form correct additions.

Dominoes are familiar and popular. They offer a wealth of number puzzles to reinforce place value, basic arithmetic skills and problem solving strategies. The lesson is written on the assumption that the teacher has access to a standard 28-piece double six set for each pair of students. However, if material is limited, the lesson can be modified to use as a task within Mixed Media Unit A or C.

Alternatively, there is a master sheet for printing domino pictures at the end of this manual and a slightly larger set, which is easier to cut, can be printed from Lesson 95, *Domino Trails*. In combination with the Floor Dominoes (each domino is one A4 page) available from the **Domino Trails** task cameo it is possible to run the whole class investigation without actual dominoes.

Find **Domino Trails** cameo in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

## First Principles Percent

The challenge in this lesson is to become better at calculating percentages. Following an introduction that highlights the frequency of their use in the media, the emphasis is on the meaning of x% as x in every 100. It is represented in a visual/concrete form and verbalised in every calculation. This verbalisation (strong link with language) is a central theme in keeping the focus on the underlying meaning. Students learn to look for the 100s in the calculation and thereby build the final answer from a set of answers to sub-problems. Mathematicians often use this approach of dealing with sub-problems to build towards a final answer.

The lesson is not about learning a recipe for calculating percentages. Rather it is about students working each calculation from first principles to reinforce the conceptual understanding that underpins these recipes. The lesson pivots on students discussing and justifying their calculations and, in their own time, developing short cuts. After establishing the process, teachers are encouraged to take a back seat. From the beginning, calculators are used in partnership with the discussion to allow students to test their hypotheses in a non-threatening way. The lesson is supported by four worksheets that introduce a sequence of more and more challenging calculations.

Once the class has published their 'How To Do Percentages' instructions as suggested in the lesson plan, they can be invited to apply their procedure to text book problems.

## Fraction Estimation

This investigation is very rich because it involves the concept of a fraction, estimation and measurement as well as data collection and number work. The lesson needs to be revisited to explore its depth, so it is included in Mixed Media Unit B. Teachers are invited to use it at other times as well.

Students are challenged to estimate parts of a real 'touchable' whole such as a length of cord, the perimeter of a building, the amount of turn about a point (angle) or a distance between two points. The spirit of the challenge is to

demonstrate increasing closeness (better estimation) rather than insisting on an exact answer as found by measurement and calculation. In fact, the class is asked to think of ways to test the estimates before using the 'exact' approach. This promotes mathematical conversation which helps to develop a conceptual understanding of fractions. The lesson software provides a wide range of challenges which capture this same spirit. Mathematical conversation is further encouraged if two students work on one computer.

You will need to think ahead to collect the necessary class set of clothes pegs and strips of non-stretchable material such as bias binding, or lengths of cord.

## Fraction Magic Square

Magic Squares are often familiar to students. This lesson uses the familiarity to highlight one aspect of problem solving, that is, *Have I seen a similar problem?*. The unfamiliar fraction magic square is linked to the familiar 3x3 whole number example. Students refresh their fraction equivalence and addition and move on to use these skills to create their own magic square.

## Gauss Beats The Teacher

The investigation explores many strategies for adding lists of consecutive numbers that start at 1, or indeed any list with a built in pattern. Visual/concrete models are explored for each of the methods and an Investigation Sheet is included. One outcome is discovering that the sum of the numbers 1 through 9 is 45. This is an important fact often used in the solution of number problems. Another outcome is the link with the history of mathematics through retelling the legend of the mathematician Carl Gauss as a young student.

From a Working Mathematically viewpoint, the emphasis is on:

- Can I check this another way?
- Simplifying a problem.
- Making and testing hypotheses.

The first of these has an important pedagogical component, namely, that independent learners who can check their own work are less likely to put a strain on your time with frequent queries of *Is this right Miss?*.

## Goldbach's Conjecture

8 = 5 + 3 and 10 = 7 + 3 and 12 = 7 + 5. What is common to all these additions? ... Each is an even number that is the sum of two prime numbers. This is the conjecture attributed to Goldbach (a Prussian mathematician, 1690 - 1764) and which bears his name. In this lesson, it is presented as the investigative challenge:

> *Can every even number greater than 4 be written as the sum of two primes?*

The lesson is essentially about recognising prime numbers and it includes much skill practice in addition. However, the investigation and genuine history behind the challenge give it a richness beyond just skill practice. In fact Goldbach's Conjecture remains unsolved and this, combined with the tantalising information that a rich prize was once available for its solution in conjunction with the

publication of *Uncle Petros and Goldbach's Conjecture* by Apostolos Doxiadis, often captures student interest.

Once students have searched for all the even numbers from say 2 to 200, the companion software allows them to test large numbers looking for either exceptions or confirmation of the conjecture.

*Number Charts*, *Odds and Evens* and *Palindromes* are Maths300 lessons of a similar genre.

## Multo

Multo is much better than Bingo. The teacher uses 100 cards marked from 0x0 to 9x9. A master for these is included with the lesson. The students draw up a 4row/4column grid and enter 16 numbers which would be answers to these cards. No repeats. The teacher draws cards one at a time and at a regular pace announces each random times table in turn. If they have the answer on their grid, students mark it off. Multo is either:

- 4 in a row horizontally
- 4 in a row vertically
- 4 in a row diagonally
- all four corners

The game involves plenty of times tables practice, but this is soon subservient to the greater challenge of finding the best Multo grid. Software aids the search and can be visited and revisited because there are so many subtleties in the 'best' grid that there is a sense in which the problem is never solved.

The linked assessment sheet provides teachers with considerable assessment information.

## Number Charts

The lesson plan introduces the skill practice exercise of filling a 4x4 addition or multiplication grid from the limited number of clues given. However it places this practice in the context of exploring the if-then reasoning required to solve the puzzle and the deeper question of how this type of puzzle is constructed. The software opens many possibilities for self-directed work, and students seem to respond well to this twist. The software can be used for whole number addition or multiplication or fraction addition. Therefore, even though it is written into Mixed Media Unit C which largely focuses on integer arithmetic, it is likely the students will use it to also challenge themselves with fraction arithmetic.

## Odds & Evens

This number puzzle requires simple arithmetic skills, yet offers considerable opportunity to highlight the features of the process of Working Mathematically. The students dip into their skill toolbox to recognise odd and even numbers and then either divide by 2 (if even) or multiply by 3 and add 1 (if odd). For example starting with the number 13 produces this chain:

*13 --> 40 --> 20 --> 10 --> 5 --> 16 --> 8 --> 4 --> 2 --> 1*

The problem solving challenge is to make a reasoned judgement about whether all chains will end in one. An investigation sheet, group work and software are used to support the investigation.

## Palindromes

- Take any number, say 39.
- Reverse it to give 93.
- Now add these, which gives 132.
- Now reverse 132 to get 231.
- Add these two, which gives 363.

363 is palindromic. A palindromic numeral reads the same forwards or backwards.

The challenge behind this investigation is:

- Does every sequence of reversing and adding always produce a palindromic numeral?

Once students have tested for all the numbers say from 1 to 200, the supporting computer program allows them to test large numbers to look for either exceptions or confirmation of the conjecture.

To date no one has been able to prove the conjecture - one of your students could be the first - but neither has anyone found a number that doesn't work. There are likely candidates, 196 for instance. In 2002 Benjamin Després reported taking the 196 Palindrome Quest to 45 million digits without success, which may be strong evidence, but is certainly not proof, that 196 will never become a palindromic numeral. A number which doesn't become a palindrome - if such a number is ever discovered - will already have a name. It will be called a Lychrel Number. 196 is the smallest candidate for that title.

The Maths300 software emulates this real world approach. It is also impressive for the size of the numbers it can display. This may be the first time students have seen what really, really large numbers look like.

Palindromes also exist in language as words or sentences that read the same forwards and backwards. This gives the lesson a useful link to the language curriculum.

## Protons & Anti-Protons

The lesson begins with a newspaper article. Science suggests that a Proton/Anti-Proton pair transforms into a packet of energy which has zero effect on the total of Protons and Anti-Protons in a collection. This scientific idea is tweaked a little to allow 'energy to return to matter', ie: zero reveals that it is composed of a Proton/Anti-Proton pair. (The Classroom Contribution for this lesson gives a reference to a scientific experiment that actually confirms this 'tweak'.)

With the assistance of such teaching craft techniques as kinaesthetic learning, concrete materials and companion software, the lesson builds a valid, useable and mathematically sound model of positive and negative numbers. The emphasis of the lesson is on the personal construction of the concept. Time spent on this makes the development of the integer arithmetic skills easier.

*Walk The Plank* and *Number Charts* intersect with this lesson.

## Rectangle Fractions

It is important not to move too fast with this lesson. Students may have already developed an impression that fractions are difficult and concentrating on the spatial features of a rectangle (rows, columns and cells) may be just the visualisation they need to regain confidence. However, it may take time to first overcome the expectation that fraction arithmetic seems like mindless manipulation of symbols, before it is possible to build confidence in a new visualisation.

Begin with the **Setting The Context** part of the lesson and explore, discuss and record situations like the **Windows** and **Squads** suggested in the lesson. Anything arranged in equal rows provides opportunity for such fraction talks. If the arrangement is the whole thing, then the rows and columns *are* parts and must be fractions of the whole.

Teachers who have used these materials strongly suggest:

- heaps of mathematical conversation based on concrete situations - students are more willing to take a risk in an oral situation than a written one
- recording in words and allowing the students to 'slip into' shorthand, and later, full symbolic representations
- bringing in operations on fractions only when the students seem firm in the geometric parts of any rectangle

Options 1 & 2 of the software help the visual image of whole and parts to develop and Option 3 applies that to addition and subtraction of fractions. Eventually students can be asked symbolic addition and subtraction questions and encouraged to either use graph paper to support a visualisation of the solution, or even just picture it in their head.

## Rod Mats

This is a lesson designed to promote conversation in and around the meaning of fractions, equivalent fractions and operations on fractions. Although it possibly works best in a small group situation, by using silhouettes on an overhead projector, or pseudo-rods on an electronic white board, the whole class can be engaged in discussion and recording as in the lesson. If you have sufficient sets of rods, students can be invited to continue the investigation of fraction relationships in their small groups. One way of working in such a situation is to provide butcher's paper and marking pens and ask each group to start with the same rod mat.

The challenge becomes writing more fraction equations than the other groups. Of course, groups must be able to demonstrate any recording with the rods.

The lesson has the potential to show just how much the students do know about fractions. In this way it can help to break the *I can't do fractions* cycle.

The key to further investigation is that any rod (or collection of rods) can be chosen as the whole, so pre-marked pieces are of no use. There are several reasons why Cuisenaire Rods are the best material for this lesson, so teachers are encouraged to hunt out, or purchase, more rods to add to those in the tasks **Rod Mats** and **Make The Whole**. Sets of transparent overhead projector rods are also available commercially in some places.

## Row Points

Such an easy puzzle to begin. It is exactly the same as the task of the same name and simple to convert to a whole class investigation. To start, place 13 counters on a 5x5 grid. Then add a scoring system for 5 in a row, 4 in a row and 3 in a row. Now the challenge becomes to find the highest score, or the lowest score, or all the scores in between or….

This is great simple arithmetic practice and yet the problem is non-trivial and is perfect for modelling how a mathematician works.

## Simple, Elegant, Elusive

Students have a circle with 10 points numbered from 1 to 10. (The lesson includes a master document with circles already prepared). A pattern is generated by using a multiplication rule such as 'Times 3'. Starting at 1, 1 times 3 = 3, so draw a line from 1 to 3. 2 times 3 = 6 so draw a line from 2 to 6. Continuing this for all other numbers from 1 to 10 produces an elegant and interesting pattern. The pattern is symmetric and has two separate sets of interlocking triangles.

- But what if the rule is changed to 'Times 4', or 'Times 7'. What happens to the pattern?
- And what if we change the number of points in the circle to 6, or 11, or any other number?

Exploring these questions converts the activity into an extended investigation involving many number patterns. The openness of the investigation invites many levels of challenge and offers opportunity for students to plan their own investigation.

The main challenge is the generalisation: *Predict the pattern for any number of points on the circle (P) and any rule (times T)*.

The companion software allows theories about the shapes and the relationship between P and T to be developed and tested. This lesson elegantly links geometry patterns, number theory (factors and ratios) and algebra (the generalisation of the patterns). It is also an excellent example of creating a learning environment to generate an 'aha' experience for students. The teacher cannot create the 'aha' - this occurs at different times for different students, but the environment can be orchestrated to increase the chances of it.

> *Using the times table on the circle points I have had success with a disenfranchised girl who has done nothing for years … she loved it! Take care - I love the lessons - even if I am so hassled that I can't remember their names! The kids do…*

## Truth Tiles

This lesson is an opportunity to model how a mathematician works without the students' number skills being a barrier. The arithmetic involved is within the grasp of almost every student. The whole class lesson can begin with the little effort that it takes to tear a piece of paper into 9 roughly rectangular pieces.

The focus is on modelling the process of Working Mathematically in order to answer the questions:

*How many solutions are there?*
*How will we know when we have found them all?*

As this investigation unfolds the students will call upon their knowledge of the properties of numbers. The companion software offers a representation of the problem that may engage students not captured by the paper pieces presentation. Option 3 is a powerful application of the strategy of try every possibility. In using it, the teacher can highlight that the class is considerably smarter than the machine because *We know that there are some cases not worth trying*.

## Walk The Plank

Concepts such as subtracting a negative can be difficult for students to understand. This outdoor game employs physical involvement to help students construct an image of what such operations on integers could mean. A game usually takes five to ten minutes using six marks in each direction. The kinaesthetic nature of the activity helps to build a visual image. The visual image can eventually be used to evaluate symbolic equations.

The Expert level of Number Charts provides plenty of opportunity to apply this new found skill of visualising answers to integer arithmetic questions.

This activity is mostly about integer arithmetic at this level, however, especially because of the software, the lesson describes a second fruitful path which can lead to statistics and probability.

## What's It Worth?

The principle behind this investigation is simple. Create a shape of some sort that is made up of equal size parts.

- If each Part is worth a given amount, what is the total shape worth?

This is a straightforward whole number question IF the Part is worth a whole number. But the Part could be worth a decimal, or a fraction. Further, suppose we have to work backwards, that is, we know the value of the Whole and have to calculate the value of the Part.

This open-ended, rich and revisitable lesson begins in a spatial/concrete setting using an intriguing shape called a Sphinx which is provided as a reproducible master. There are possible side trips into spatial investigations as a result and the lesson offers endless value relations opportunities.

Students are encouraged to create their own value relations puzzles for others to solve, and are offered a large range of these puzzles in the companion software.

# Part 3: Value Adding

# The Poster Problem Clinic

**Maths With Attitude** kits offer several models for building a Working Mathematically curriculum around tasks. Each kit uses a different model, so across the range of 16 kits, teachers' professional learning continues and students experience variety. The Poster Problem Clinic is an additional model. It can be used to lead students into working with tasks, or it can be used in a briefer form as a opening component of each task session.

> *I was apprehensive about using tasks when it seemed such a different way of working. I felt my children had little or no experience of problem solving and I wanted to prepare them to think more deeply. The Clinic proved a perfect way in.*

Careful thought needs to be given to management in such lessons. One approach to getting the class started on the tasks and giving it a sense of direction and purpose is to start with a whole class problem. Usually this is displayed on a poster that all can see, perhaps in a Maths Corner. Another approach is to print a copy for each person. A Poster Problem Clinic fosters class discussion and thought about problem solving strategies.

Starting the lesson this way also means that just prior to liberating the students into the task session, they are all together to allow the teacher to make any short, general observations about classroom organisation, or to celebrate any problem solving ideas that have arisen.

One teacher describes the session like this:

> *I like starting with a class problem - for just a few minutes - it focuses the class attention, and often allows me to introduce a particular strategy that is new or needs emphasis.*

It only takes a short time to introduce a poster and get some initial ideas going. The class discussion develops a way of thinking. It allows class members to hear, and learn from their peers, about problem solving strategies that work for them.

> *If we don't collectively solve the problem in 5 minutes, I will leave the problem 'hanging' and it gives a purpose to the class review session at the end.*
> *Sometimes I require everyone to work out and write down their solution to the whole class problem. The staggered finishing time for this allows me to get organised and help students get started on tasks without being besieged.*
> *I try to never interrupt the task session, but all pupils know we have a five minute review session at the end to allow them to comment on such things as an activity they particularly liked. We often close then with an agreed answer to our whole class problem.*

# A Clinic in Action

The aims of the regular clinic are:

- to provide children with the opportunity to learn a variety of strategies
- to familiarise children with a process for solving problems.

The following example illustrates a structure which many teachers have found successful when running a clinic.

## Preparation

For each session teachers need:

- a Strategy Board as below
- a How To Solve A Problem chart as below
- to choose a suitable problem and prepare it as a poster
- to organise children into groups of two or three.

The Strategy Board can be prepared in advance as a reference for the children, or may be developed *with* the children as they explore problem solving and suggest their own versions of the strategies.

The problem can be chosen from

- a book
- the task collection
- prepared collections such as Professor Morris Puzzles which can be viewed at: http://www.mathematicscentre.com/taskcentre/resource.htm#profmorr

The example which follows is from the task collection. The teacher copied it onto a large sheet of paper and asked some children to illustrate it. *The teacher also changed the number of sheep to sixty* to make the poster a little different from the one in the task collection.

The Strategy Board and the How To Solve A Problem chart can be used in any maths activity and are frequently referred to in Maths300 lessons.

## The Clinic

The poster used for this example session is:

> Eric the Sheep is lining up to be shorn before the hot summer ahead. There are sixty [60] sheep in front of him. Eric can't be bothered waiting in the queue properly, so he decides to sneak towards the front.
>
> Every time one [1] sheep is taken to be shorn, Eric then sneaks past two [2] sheep. How many sheep will be shorn before Eric?

This Poster Problem Clinic approach is also extensively explored in Maths300 Lesson 14, *The Farmer's Puzzle*.

# Strategy Board

| | |
|---|---|
| DO I KNOW A SIMILAR PROBLEM? | ACT IT OUT |
| GUESS, CHECK AND IMPROVE | DRAW A PICTURE OR GRAPH |
| TRY A SIMPLER PROBLEM | MAKE A MODEL |
| WRITE AN EQUATION | LOOK FOR A PATTERN |
| MAKE A LIST OR TABLE | TRY ALL POSSIBILITIES |
| WORK BACKWARDS | SEEK AN EXCEPTION |
| BREAK INTO SMALLER PARTS | ... |

# How To Solve A Problem

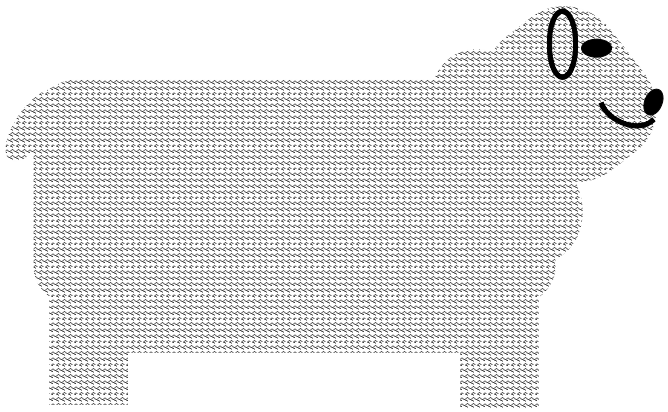
| | |
|---|---|
| SEE & UNDERSTAND | Do I understand what the problem is asking? Discuss |
| PLANNING | Select a strategy from the board. Plan how you intend solving the problem. |
| DOING IT | Try out your idea. |
| CHECK IT | Did it work out? If so reflect on the activity. If not, go back to step one. |

### Step 1

- Tell the children that we are at Stage 1 of our four stage plan ... **See & Understand** ... Point to it! Read the problem with the class. Discuss the problem and clarify any misunderstandings.
- If children do not clearly understand what the problem is asking, they will not cope with the next stage. A good way of finding out if a child understands a problem is for her/him to retell it.
- Allow time for questions - approximately 3 to 5 minutes.

### Step 2

- Tell the children that we are at Stage 2 of our four stage plan ... **Planning**. In their groups children select one or more strategies from the Strategy Board and discuss/organise how to go about solving the problem.
- Without guidance, children will often skip this step and go straight to Doing It. It is vital to emphasise that this stage is simply planning, not solving, the problem.
- After about 3 minutes, ask the children to share their plans.



**Plan 1**

| |
|---|
| *Well we're drawing a picture and sort of making a model.* |

Can you give me more information please Brigid?

| |
|---|
| *We're putting 60 crosses on our paper for sheep and the pen top will be Eric. Then Claire will circle one from that end, and I will pass two crosses with my pen top.* |

**Plan 2**

| |
|---|
| *Our strategy is Guess and Check.* |

That's good Nick, but how are you going to check your guess?

| |
|---|
| *Oh, we're making a model.* |

Go on ...

> *John's getting MAB smalls to be sheep and I'm getting a domino to be Eric and the chalk box to be the shed for shearing.*

**Plan 3**

> *We are doing it for 3 sheep then 4 sheep then 5 sheep and so on. Later we will look at 60.*

Great so you are going to try a simpler problem, make a table and look for a pattern.

This sharing of strategies is invaluable as it provides children who would normally feel lost in this type of activity with an opportunity to listen to their peers and make sense out of strategy selection. Note that such children are not given the answer. Rather they are assisted with understanding the power of selecting and applying strategies.

### Step 3

- Tell the children that we are at Stage 3 of our four stage plan ... **Doing It**. Children collect what they need and carry out their plan.

### Step 4

- Tell the children that we are at Stage 4 of our four stage plan ... **Check It**. Come together as a class for groups to share their findings. Again emphasis is on strategies.

> *We used the drawing strategy, but we changed while we were doing it because we saw a pattern.*

So Jake, you used the Look For A Pattern strategy. What was it?

> *We found that when Eric passed 10 sheep, 5 had been shorn, so 20 sheep meant 10 had been shorn ... and that means when Eric passes 40 sheep, 20 were shorn and that makes the 60 altogether.*

Great Jake. How would you work out the answer for 59 sheep or 62 sheep?

Sharing time is also a good opportunity to add in a strategy which no one may have used. For example:

> *Maybe we could've used the Number Sentence strategy, ie: 1 sheep goes to be shorn and Eric passes two sheep. That's 3 sheep, so perhaps, 60 divided into groups of 3, or 60 ÷ 3 gives the answer.*

Round off the lesson by referring to the Working Mathematically chart. There will be many opportunities to compliment the students on working like a mathematician.

# Curriculum Planning Stories

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

In more than a decade of using tasks and many years of using the detailed whole class lessons of Maths300, teachers have developed several models for integrating tasks and whole class lessons. Some of those stories are retold here. Others can be found at:

- http://www.mathematicscentre.com/taskcentre/plans.htm

## Story 1: Threading

> *Educational research caused me a dilemma. It tells us that students construct their own learning and that this process takes time. My understanding of the history of mathematics told me that certain concepts, such as place value and fractions, took thousands of years for mathematicians to understand. The dilemma was being faced with a textbook that expected students to 'get it' in a concentrated one, two or three week block of work and then usually not revisit the topic again until the next academic year.*

A Working Mathematically curriculum reflects the need to provide time to learn in a supportive, non-threatening environment and…

> *When I was involved in a Calculating Changes PD program I realised that:*
>
> - *choosing rich and revisitable activities, which are familiar in structure but fresh in challenge each time they are used, and*
> - *threading them through the curriculum over weeks for a small amount of time in each of several lessons per week*
>
> *resulted in deeper learning, especially when partnered with purposeful discussion and recording.*

Calculating Changes:

- http://www.mathematicscentre.com/calchange

## Story 2: Your turn

Some teachers are making extensive use of a partnership between the whole class lessons of Maths300 and small group work with the tasks. Setting aside a lesson for using the tasks in the way they were originally designed now seems to have more meaning, as indicated by this teacher's story:

> *When I was thinking about helping students learn to work like a mathematician, my mind drifted to my daughter learning to drive. She*

needed me to model how to do it and then she needed lots of opportunity to try it for herself.

That's when the idea clicked of using the Maths300 lessons as a model and the tasks as a chance for the students to have their turn to be a mathematician.

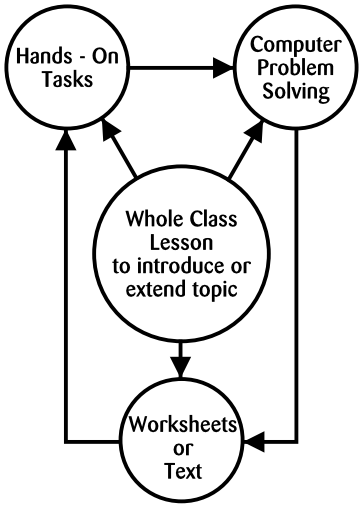
The Maths300 lessons illustrate how other teachers have modelled the process, so I felt I could do it too. Now the process is always on display on the wall or pasted inside the student's journal.

A session just using the tasks had seemed a bit like play time before this. Now I see it as an integral part of learning to work mathematically.

## Story 3: Mixed Media

It was our staff discussion on Gardner's theory of Multiple Intelligences that led us into creating mixed media units. That and the access you have provided to tasks and Maths300 software.

We felt challenged to integrate these resources into our syllabus. There was really no excuse for a text book diet that favours the formal learners. We now often use four different modes of learning in the work station structure shown. It can be easily managed by one teacher, but it is better when we plan and execute it together.




## Story 4: Replacement Unit

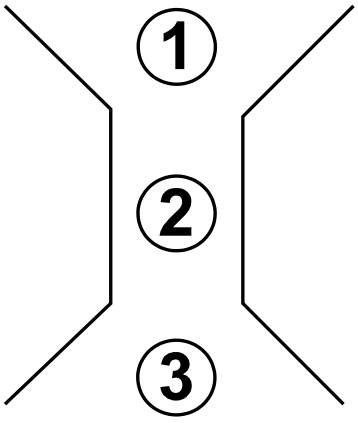
We started meeting with the secondary school maths teachers to try to make transition between systems easier for the students. After considerable discussion we contracted a consultant who suggested that school might look too much the same across the transition when the students were hoping for something new. On the other hand our experience suggested that there needed to be some consistency in the way teachers worked.

We decided to 'bite the bullet' and try a hands-on problem solving unit in one strand. We selected two menus of twenty hands-on tasks, one for the primary and one for the secondary, that became the core of the unit. We deliberately overlapped some tasks that we knew were very rich and added some new ones for the high school.

Class lessons and investigation sheets were used to extend the tasks, within a three week model.

It is important to note that although these teachers structured a 3 week unit for the students, they strongly advised an additional *Week Zero* for staff preparation. The units came to be called Replacement Units.

### Week Zero - Planning




Staff familiarise themselves with the material and jointly plan the unit. This is not a model that can be 'planned on the way to class'.

> *Getting together turned out to be great professional development for our group.*

### Week 1 - Introduction

Students explore the 20 tasks listed on a printed menu:

- students explore the tip of the task, as on the card
- students move from task to task following teacher questioning that suggests there is more to the task than the tip
- in discussion with students, teachers gather informal assessment information that guides lesson planning for the following week.

> *We gave the kids an 'encouragement talk' first about joining us in an experiment in ways of learning maths and then gave out the tasks. The response was intelligent and there was quite a buzz in the room.*

### Week 2 - Formalisation

> *It was good for both us and the students that the lessons in this week were a bit more traditional. However, they weren't text book based. We used whole class lessons based on the tasks they had been exploring and taught the Working Mathematically process, content and report writing.*
>
> *Assessment was via standard teacher-designed tests, quizzes and homework.*

### Week 3 - Investigations

> *We were most delighted with Week 3. Each student chose one task from the menu and carried out an in-depth investigation into the iceberg guided by an investigation sheet. They had to publish a report of their investigation and we were quite surprised at the outcomes. It was clear that the first two weeks had lifted the image of mathematics from 'boring repetition' to a higher level of intellectual activity.*

## Story 5: Curriculum shift

> *I think our school was like many others. The syllabus pattern was 10 units of three weeks each through the year. We had drifted into that through a text book driven curriculum and we knew the students weren't responding.*

*Our consultant suggested that there was sameness about the intellectual demands of this approach which gave the impression that maths was the pursuit of skills. We agreed to select two deeper investigations to add to each unit. It took some time and considerable commitment, but we know that we have now made a curriculum shift. We are more satisfied and so are the students.*

The principles guiding this shift were:

- Agree

  *The 20 particular investigations for the year are agreed to by all teachers. If, for example, Cube Nets is decided as one of these, then all the teachers are committed to present this within its unit.*

- Publish

  *The investigations are written into the published syllabus. Students and parents are made aware of their existence and expect them to occur.*

- Commit

  *Once agreed, teachers are required to present the chosen investigations. They are not a negotiable 'extra'.*

- Value

  *The investigations each illustrate an explicit form of the Working Mathematically process. This is promoted to students, constantly referenced and valued.*

- Assess

  *The process provides students with scaffolding for their written reports and is also known by them as the criteria for assessment. (See next page.)*

- Report

  *The assessment component features within the school reporting structure.*

**A Final Comment**

*Including investigations has become policy.*

*Why? Because to not do so is to offer a diminished learning experience.*

*The investigative process ranks equally with skill development and needs to be planned for, delivered, assessed and reported.*

*Perhaps most of all we are grateful to our consultant because he was prepared to begin where we were. We never felt as if we had to throw out the baby and the bath water.*

# Assessment

Our attitude is:

*stimulated students are creative and love to learn*

Regardless of the way you use your **Maths With Attitude** resource, a variety of procedures can be employed to assess this learning.

Where these assessment procedures are applied to task sessions and involve written responses from students, teachers will need to be careful that the writing does not become too onerous. Students who get bogged down in doing the writing may lose interest in doing the tasks.

In addition to the ideas below, useful references are:

- http://www.mathematicscentre.com/taskcentre/assess.htm
- http://www.mathematicscentre.com/taskcentre/report.htm

The first offers several methods of assessment with examples and the second is a detailed lesson plan to support students to prepare a Maths Report.

## Journal Writing

Journal writing is a way of determining whether the task or lesson has been understood by the student. The pupil can comment on such things as:

- What I learned in this task.
- What strategies I/we tried (refer to the Strategy Board).
- What went wrong.
- How I/we fixed it.
- Jottings - ie: any special thoughts or observations

Some teachers may prefer to have the page folded vertically, so that children's reflective thoughts can be recorded adjacent to critical working.

## Assessment Form

An assessment form uses questions to help students reflect upon specific issues related to a specific task.

## Anecdotal Records

Some teachers keep ongoing records about how students are tackling the tasks. These include jottings on whether students were showing initiative, whether they were working co-operatively, whether they could explain ideas clearly, whether they showed perseverance.

## Checklists

A simple approach is to create a checklist based on the Working Mathematically process. Teachers might fill it in following questioning of individuals, or the students may fill it in and add comments appropriately.

## Pupil Self-Reflection

Many theorists value and promote metacognition, the notion that learning is more permanent if pupils deliberately and consciously analyse their own learning. The

deliberate teaching strategy of oral questioning and the way pupils record their work is an attempt to manifest this philosophy in action. The alternative is the tempting 'butterfly' approach which is to madly do as many activities as possible, mostly superficially, in the mistaken belief that quantity equates to quality.

> *I had to work quite hard to overcome previously entrenched habits of just getting the answer, any answer, and moving on to the next task.*

Thinking about *what* was learned *how* it was learned consolidates and adds to the learning.

When it follows an extensive whole class investigation, a reflection lesson such as this helps to shift entrenched approaches to mathematics learning. It is also an important component of the assessment process. On the one hand it gives you a lot of real data to assist your assessment. On the other it prepares the students for any formal assessment which you may choose to round off a unit.

### Introduction

Ask students to recall what was done during the unit or lesson by asking a few individuals to say what *they* did, eg:

> *What did you do or learn that was new?*
> *What can you now do/understand that is new?*
> *What do you know now that you didn't know 1 (2, 3, ...) lesson ago?*

### Continuing Discussion

Get a few ideas from the first students you ask, then:

- organise 5 -10 minute buzz groups of three or four students to chat together with one person to act as a recorder. These groups address the same questions as above.
- have a reporting session, with the recorder from each group telling the class about the group's ideas.

Student comments could be recorded on the board, perhaps in three groups.

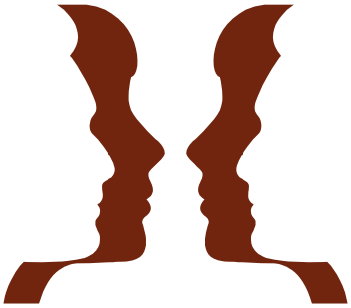
| Ideas & Facts | Maths Skills | Process (learning) Skills |
|---|---|---|

If you need more questions to probe deeper and encourage more thought about process, try the following:

> *What new things did you do that were part of* how *you learned?*
> *Who uses this kind of knowledge and skill in their work?*

### Student Recording

Hand out the REFLECTION sheet (next page) and ask students to write their own reflection about what they did, based on the ideas shared by the class. Collect these for interest and, possibly, assessment information.



# REFLECTION

## me looking at me learning

NAME: CLASS:

Reproducible Page

# Working With Parents

## Balancing Problem Solving with Basic Skill Practice

Many schools find that parents respond well to an evening where they have an opportunity to work with the tasks and perhaps work a task together as a 'whole class'. Resourced by the materials in this kit, teachers often feel quite confident to run these practical sessions. Comments from parents like:

> *I wish I had learnt maths like this.*

are very supportive. Letting students 'host' the evening is an additional benefit to the home/school relationship.

## The 4½ Minute Talk

Charles Lovitt has considerable experience working with parents and has developed a crisp, parent-friendly talk which he shares below. Many others have used it verbatim with great success.

### Why the Four and a Half Minute Talk?

*When talking with parents about Problem Solving or the meaning of the term Working Mathematically, I have often found myself in the position, after having promoted inquiry based or investigative learning, of the parents saying:*

> *Well - that's all very well - BUT ...*

*at which stage they often express their concern for basic (meaning arithmetic) skill development.*

*The weakness of my previous attempts has been that I have been unable to reassure parents that problem solving does not mean sacrificing our belief in the virtues of such basic skill development.*

*One of the unfortunate perceptions about problem solving is that if a student is engaged in it, then somehow they are not doing, or it may be at the expense of, important skill based work.*

*This Four and a Half Minute Talk to parents is an attempt to express my belief that basic skill practice and problem solving development can be closely intertwined and not seen as in some way mutually exclusive.*

*(I'm still somewhat uncomfortable using the expression 'basic skills' in the above way as I am certain that some thinking, reasoning, strategy and communication skills are also 'basic'.)*

*Another aspect of the following 'talk' is that, as teachers put more emphasis on including investigative problem solving into their courses, a question arises about the source of suitable tasks.*

*This talk argues that we can learn to create them for ourselves by 'tweaking' the closed tasks that heavily populate out existing text exercises, and hence not be dependent on external suppliers. (Even better if students begin to create such opportunities for themselves.)*

**The Talk**

In preparation, write the following graphic on the board:

| CLOSED | OPEN | EXTENDED INVESTIGATION |
|---|---|---|
| | | How many solutions exist?<br><br>How do you know you have found them all? |

*I would like to show you what teachers are beginning to do to achieve some of the thinking and reasoning and communication skills we hope students will develop. I would like to show you three examples.*

**Example One:** *6 + 5 = ?*

*I write this question under the 'closed' label on the diagram:*

| CLOSED | OPEN | EXTENDED INVESTIGATION |
|---|---|---|
| 6<br>+ 5<br>— | | How many solutions exist?<br><br>How do you know you have found them all? |

And I ask:

*What is the answer to this question?*

I then explain that:

*We often ask students many closed questions such as 6 + 5 = ?*

The only response the students can tell us is "The answer is 11." … and as a reward for getting it correct we ask another twenty questions just like it.

What some teachers are doing is trying to *tweak* the question and ask it a different way, for example:

*I have two counting numbers that add to 11. What might the numbers be?*

[Counting numbers = positive whole numbers including zero]

I write this under the 'open' label on the diagram:

| CLOSED | OPEN | EXTENDED INVESTIGATION |
|---|---|---|
| 6<br>+ 5<br>___ | ?<br>+ ?<br>11 | How many solutions exist?<br>How do you know you have found them all? |

*What is the answer to the question now?*

At this stage it becomes apparent there are several solutions:

*The question is now a bit more open than it was before, allowing students to tell you things like 8 + 3, or 10 + 1, or 11 + 0 etc.*

Let's see what happens if the teacher 'tweaks' it even further with the investigative challenge *or* extended investigation question:

*How many solutions are there altogether?*

and more importantly, and with greater emphasis on the second question:

*How could you convince someone else that you have found them all?*

Now the original question is definitely different - it still involves the skills of addition but now also involves thinking, reasoning and problem solving skills, strategy development and particularly communication skills.

Young students will soon tell you the answer is 'six different ones', but they must also confront the communication and reasoning challenge of convincing you that there are only six and no more.

**Example Two:** *Finding Averages*

Again, as I go through this example, I write it into the diagram on the board in the relevant sections.

The CLOSED question is: *11, 12, 13 - find the average*

Tweaking this makes it an OPEN question and it becomes:

*I have three counting numbers whose average is 12. What might the numbers be?*

Students will often say:

10, 12, 14 ... or 9, 12, 15 ... or even 12, 12, 12

After realising there are many answers, you can tweak it some more and turn it into an EXTENDED INVESTIGATION:

*How many solutions exist?* ... AND ...

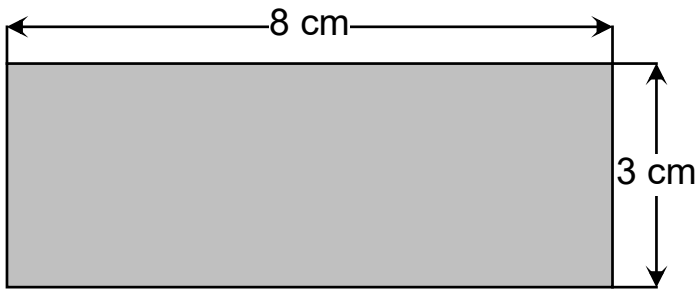
*How do you know you have found them all?*

Now the question is of a quite different nature. It still involves the arithmetic skill, but has something else as well - and that something else is the thinking, reasoning and communication skills necessary to find all of the combinations and convince someone else that you have done so.

By the time a student announces, with confidence, there are 127 different ways (which there are) that student will have engaged in all of these aspects, ie: the skill of calculating averages, (and some combination number theory) as well as significant strategy and reasoning experiences.

**Example Three:** *Finding the Area of a Rectangle*

A typical CLOSED question is:




*Find the area. Find the perimeter.*

The OPEN question is:

*A rectangle has 24 squares inside:*

*What might its length and width be?*

*What might its perimeter be?*

The EXTENDED INVESTIGATION version is:

*Given they are whole number lengths, how many different rectangles are there? …* AND …

*How do you know you have found them all?*

In summary, mathematics teachers are trying to convert *some* (not all) of the many closed questions that populate our courses and 'push' them towards the investigation direction. In doing so, we keep the skills we obviously value, but also activate the thinking, reasoning and justification skills we hope students will also develop.

This sequence of three examples hopefully shows two major features:

- That skills and problem solving can 'live alongside each other' and be developed concurrently.
- That the process of creating open-ended investigations can be done by anyone - just go to any source of closed questions and try 'tweaking' them as above. If it only worked for one question per page it would still provide a very large supply of investigations.

In terms of the effect of the talk on parents, I have usually found them to be reassured that we are not compromising important skill development (and nor do we want to). The only debate then becomes whether the additional skills of thinking, reasoning and communication are also desirable.

I've also been told that parents appreciate it because of the essential simplicity of the examples - no complicated theoretical jargon.

# A Working Mathematically Curriculum

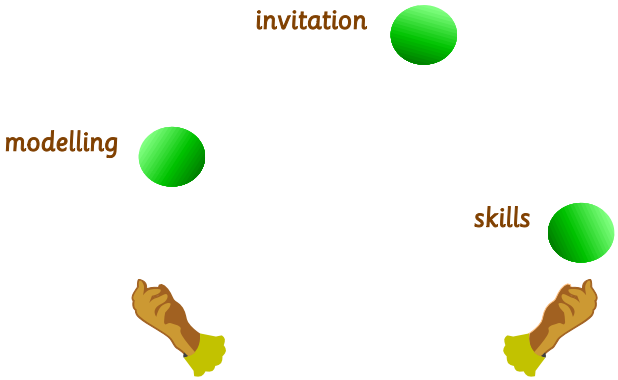
## An Investigative Approach to Learning

The aim of a Working Mathematically curriculum is to help students learn to work like a mathematician. This process is detailed earlier (Page 8) in a one page document which becomes central to such a curriculum.

The change of emphasis brings a change of direction which *implies and requires* a balance between:

- the process of being a mathematician, and
- the development of skills needed to be a *successful* mathematician.

This journey is not two paths. It is one path made of two interwoven threads in the same way as DNA, the building block of life, is one compound made of two interwoven coils. To achieve a Working Mathematically curriculum teachers need to balance three components.




The task component of **Maths With Attitude** offers each pair of students an invitation to work like a mathematician.

The Maths300 component of **Maths With Attitude** assists teachers to model working like a mathematician.

Content skills are developed in context. They *are* important, but it is the application of skills within the process of Working Mathematically that has developed, and is developing, the human community's mathematical knowledge.

A focus for the Working Mathematically teacher is to help students develop mathematical skills in the context of problem posing and solving.

> *We are all 'born' with the same size mathematical toolbox, in the same way as I can own the same size toolbox as my motor mechanic. However, my motor mechanic has many more tools in her box than I and she has had more experience than I using them in context. Someone has helped her learn to use those tools while crawling under a car.*

Afzal Ahmed, Professor of Mathematics at Chichester, UK, once quipped:

> *If teachers of mathematics had to teach soccer, they would start off with a lesson on kicking the ball, follow it with lessons on trapping the ball and end with a lesson on heading the ball. At no time would they play a game of football.*

Such is not the case when teaching a Working Mathematically curriculum.

## Elements of a Working Mathematically Curriculum

Working Mathematically is a K - 12 experience offering a balanced curriculum structured around the components below.

*Hands-on Problem Solving Play*

Mathematicians don't know the answer to a problem when they start it. If they did, it wouldn't be a problem. They have to play around with it. Each task invites students to play with mathematics 'like a mathematician'.

*Skill Development*

A mathematician needs skills to solve problems. Many teachers find it makes sense to students to place skill practice in the context of *Toolbox Lessons* which *help us better use the Working Mathematically Process* (Page 8).

*Focus on Process*

This is what mathematicians do; engage in the problem solving process.

*Strategy Development*

Mathematicians also make use of a strategy toolbox. These strategies are embedded in Maths300 lessons, but may also have a separate focus. Poster Problem Clinics are a useful way to approach this component.

*Concept Development*

A few major concepts in mathematics took centuries for the human race to develop and apply. Examples are place value, fractions and probability. In the past students have been expected to understand such concepts after having 'done' them for a two week slot. Typically they were not visited again until the next year. A Working Mathematically curriculum identifies these concepts and regularly 'threads' them through the curriculum.

## Planning to Work Mathematically

The class, school or system that shifts towards a Working Mathematically curriculum will no longer use a curriculum document that looks like a list of content skills. The document would be clear in:

- choosing genuine problems to initiate investigation
- choosing a range of best practice teaching strategies to interest a wider range of students
- practising skills for the purpose of problem solving

Some teachers have found the planning template on the next page assists them to keep this framework at the forefront of their planning. It can be used to plan single lessons, or units built of several lessons. There are examples from schools in the Curriculum & Planning section of Maths300 and a Word document version of the template.

# Unit Planning Page



**Class**

**Topic**

| Pedagogy | Problem Solving<br>In this topic how will I engage my students in the Working Mathematically process? | Skills |
|---|---|---|
| How do I create an environment where students know what they are doing and why they have accepted the challenge? | | Does the challenge identify skills to practise? Are there other skills to practise in preparation for future problem solving? |

## Notes

As a general guide:

- Find a problem(s) to solve related to the topic.
- Choose the best teaching craft likely to engage the learners.
- Where possible link skill practice to the problem solving process.

# More on Professional Development

For many teachers there will be new ideas within **Maths With Attitude**, such as unit structures, views of how students learn, teaching strategies, classroom organisation, assessment techniques and use of concrete materials. It is anticipated (and expected) that as teachers explore the material in their classrooms they will meet, experiment with and reflect upon these ideas with a view to long term implications for the school program and for their own personal teaching.

Being explored 'on-the-job' so to speak, in the teacher's own classroom, makes the professional development more meaningful and practical for the teacher. This is also a practical and economic alternative for a local authority.

## Strategic Use by Systems

From Years 3 - 10, **Maths With Attitude** is designed as a professional development vehicle by schools or clusters or systems because it carries a variety of sound educational messages. They might choose **Maths With Attitude** because:

- It can be used to highlight how investigative approaches to mathematics can be built into balanced unit plans without compromising skill development and without being relegated to the margins of a syllabus as something to be done only after 'the real' content has been covered.
- It can be used to focus on how a balance of concept, skill and application work can all be achieved within the one manageable unit structure.
- It can be used to show how a variety of assessment practices can be used concurrently to build a picture of student progress.
- It can be used to focus on transition between primary and secondary school by moving towards harmony and consistency of approach.
- It can be used to raise and continue debate about the pedagogy (art of teaching) that supports deeper mathematical learning for a wider range of students.

Teachers in Years K - 2 are similarly encouraged in professional growth through **Working Mathematically with Infants**, which derives from Calculating Changes, a network of teachers enhancing children's number skills from Years K - 6.

In supporting its teachers by supplying these resources in conjunction with targeted professional development over time, a system can fuel and encourage classroom-based debate on improving outcomes. There is evidence that by exploring alternative teaching strategies and encouraging curriculum shift towards Working Mathematically, learners improve and teachers are more satisfied. For more detail visit Research & Stories at:

- http://www.mathematicscentre.com/taskcentre/do.htm

We would be happy to discuss professional development with system leaders.

### Web Reference

The starting point for all aspects of learning to work like a mathematician, including Calculating Changes, and the teaching craft which encourages it is:

- http://www.mathematicscentre.com/mathematicscentre

# Appendix: Recording Sheets

# Dominoes

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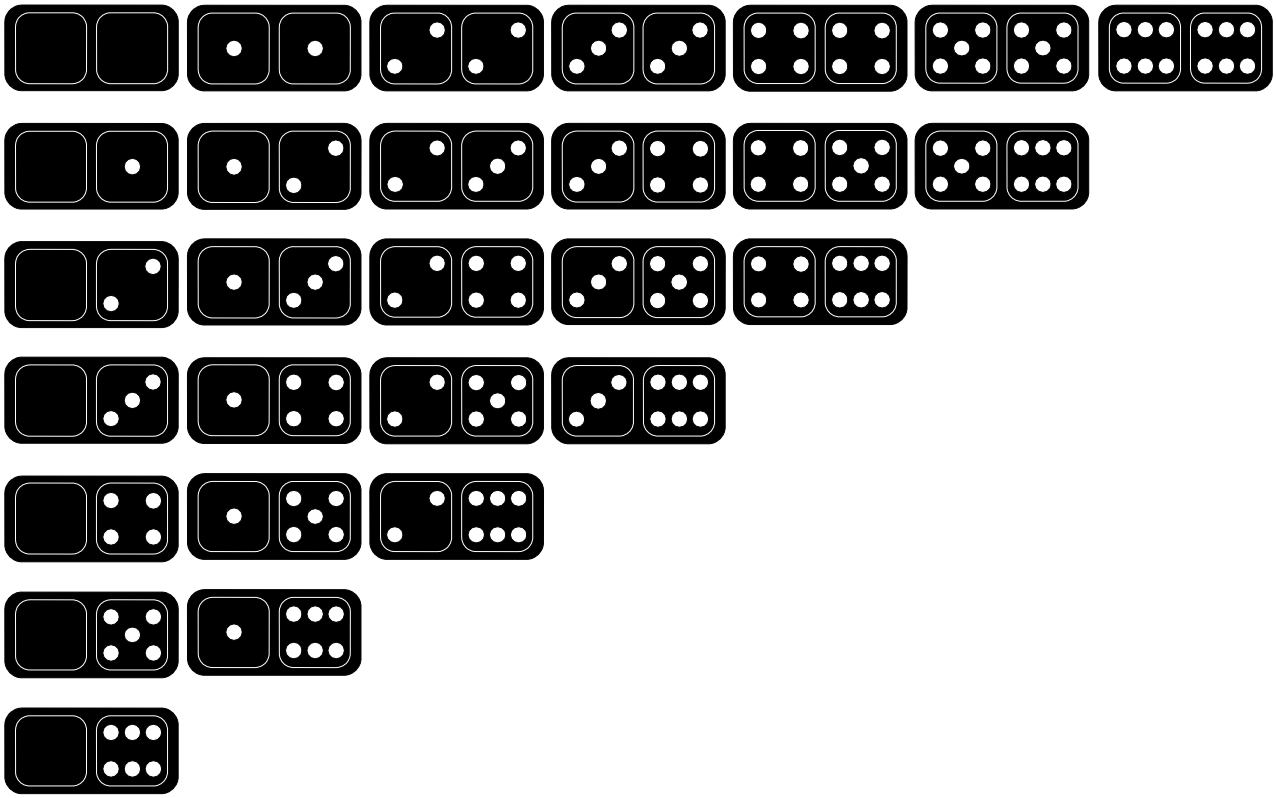

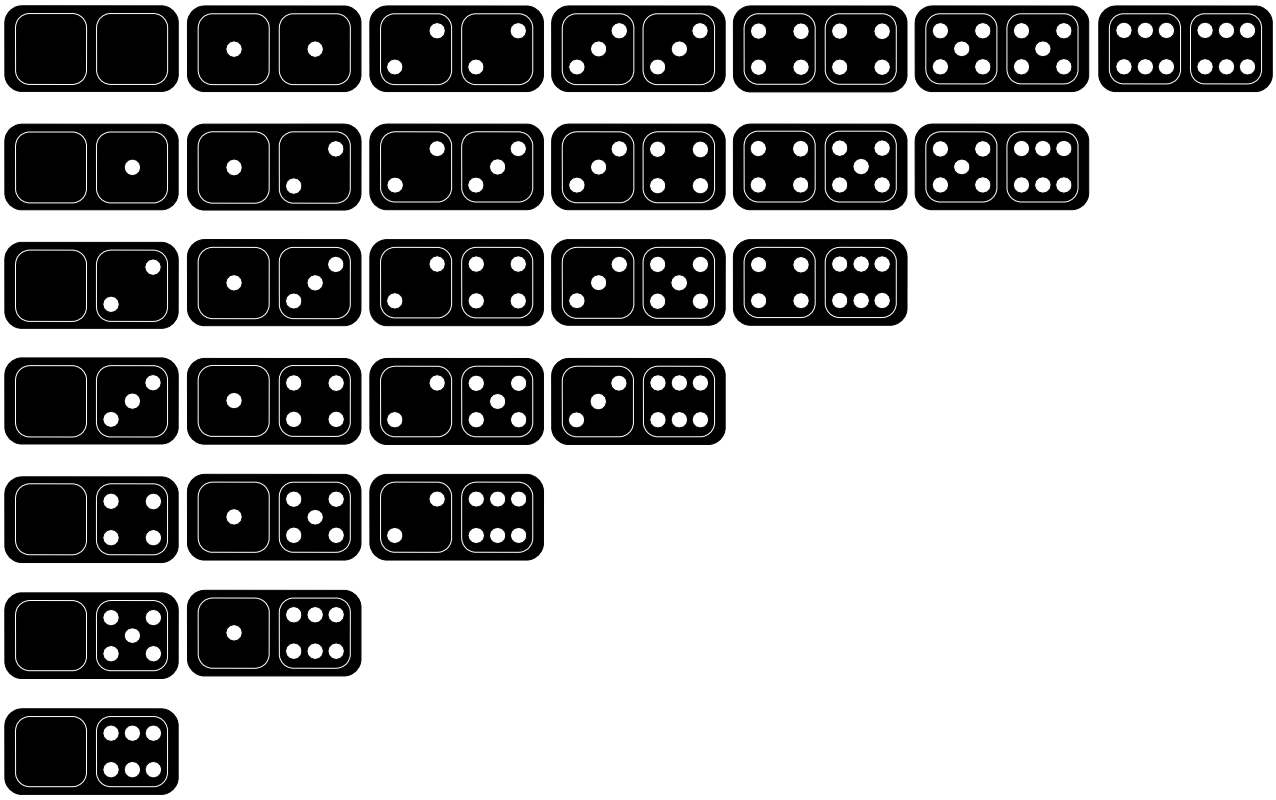

# Make Your Own Make The Whole Game

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| MAKE THE WHOLE CARDS - SET ..... | ..... ROD IS WHOLE | |
|---|---|---|
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# What's It Worth?

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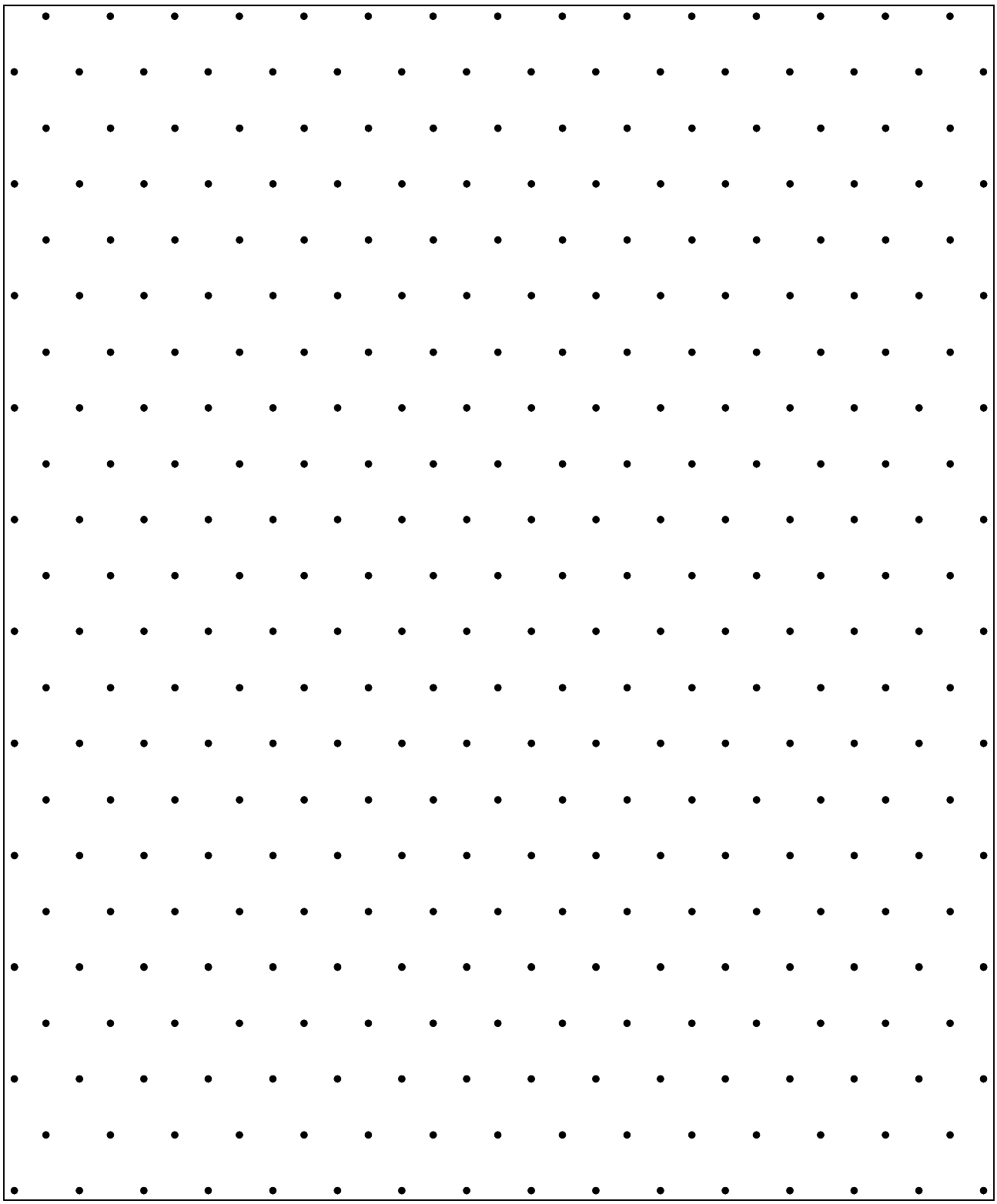

Names: ..... Class: .....