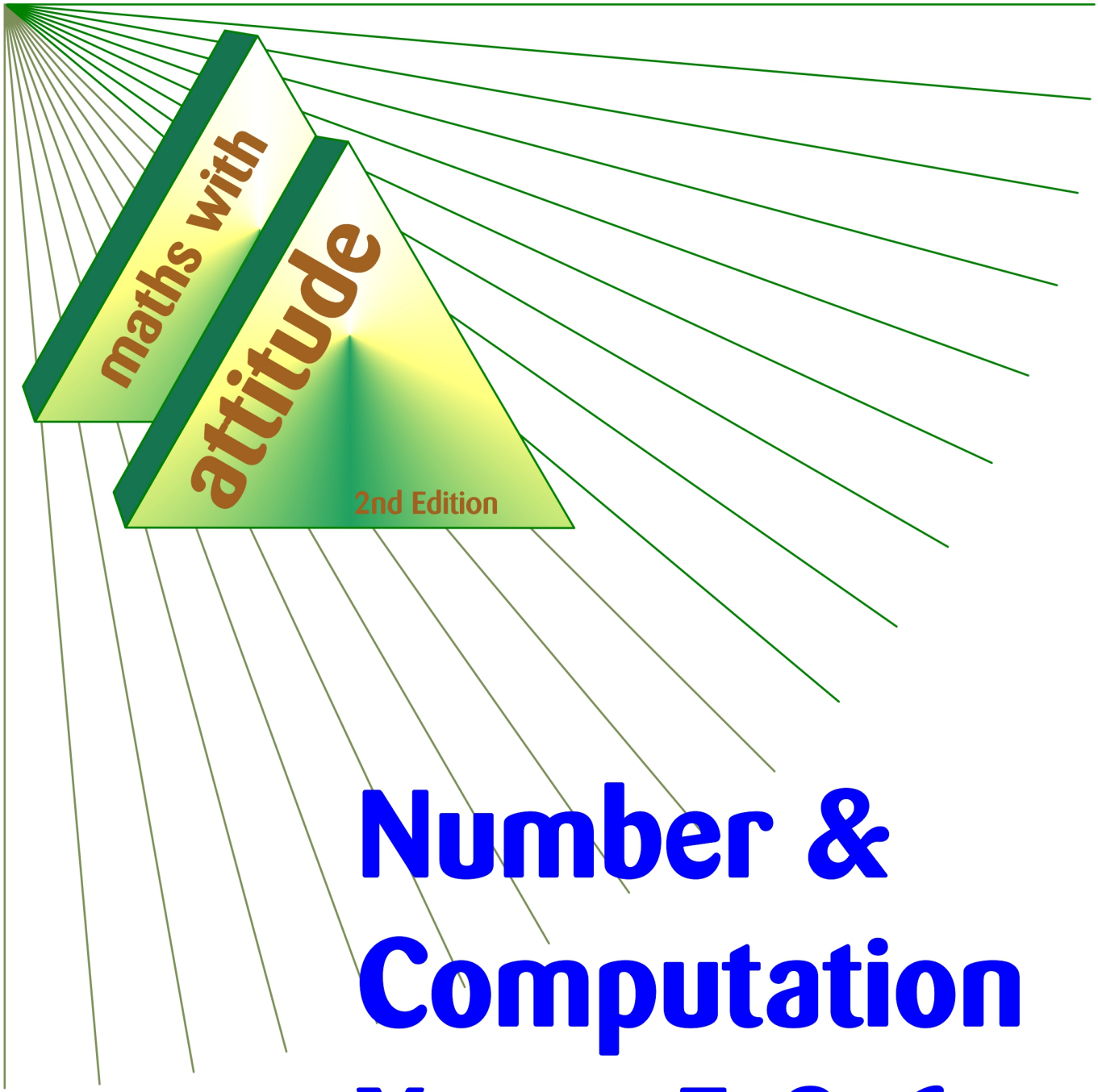

Number & Computation Years 5 & 6

Charles Lovitt
Doug Williams

Mathematics Task Centre & Maths300
helping to create happy healthy cheerful productive inspiring classrooms

Number & Computation

Years 5 & 6

In this kit:

- *Hands-on problem solving tasks*
- *Detailed curriculum planning*

Access from Maths300:

- *Extensive lesson plans*
- *Software*

Doug Williams
Charles Lovitt

The **Maths With Attitude** series has been developed by The Task Centre Collective and is published by Black Douglas Professional Education Services.

Prepared for free distribution February 2025.

Design and Desktop Publishing:
Black Douglas Professional Education Services

ISBN: 0-958-59797-9 ... 978-0-958-59797-5
First published 2002
2nd Edition 2012
2016: Designed for web delivery as a computer-based file

Maths With Attitude includes reference to hands-on problem solving tasks and Maths300 material.

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Part 1: Preparing To Teach

Our Objective

- To support teachers, schools and systems wanting to create:

  happy, healthy, cheerful, productive, inspiring classrooms

Our Attitude

- to learning:

  learning is a personal journey stimulated by achievable challenge
- to learners:

  stimulated students are creative and love to learn
- to pedagogy:

  the art of choosing teaching strategies to involve and interest all students
- to mathematics:

  mathematics is concrete, visual and makes sense
- to learning mathematics:

  all students can learn to work like a mathematician
- to teachers:

  the teacher is the most important resource in education
- to professional development:

  teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

Our Objective in Detail

What do we mean by creating:

happy, healthy, cheerful, productive, inspiring classrooms

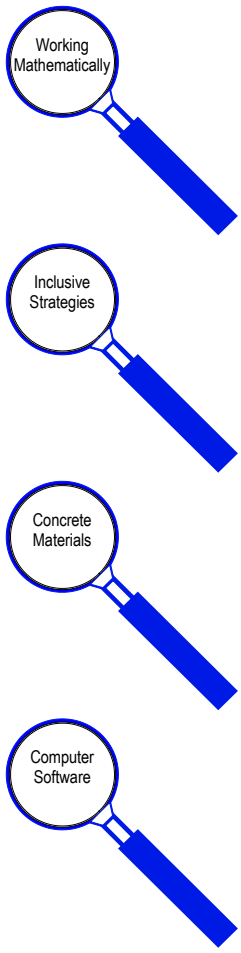
Happy…

means the elimination of the unnecessary fear of failure that hangs over so many students in their mathematics studies. Learning experiences *can* be structured so that all students see there is something in it for them and hence make a commitment to the learning. In so many 'threatening' situations, students see the impending failure and withhold their participation.

A phrase which describes the structure allowing all students to perceive something in it for them is *multiple entry points and multiple exit points*. That is, students can enter at a variety of levels, make progress and exit the problem having visibly achieved.

Healthy…

means *educationally healthy*. The learning environment should be a reflection of all that our community knows about how students learn. This translates into a rich array of teaching strategies that could and should be evident within the learning experience.



If we scrutinise the *exploration* through any lens, it should confirm to us that it is well structured or alert us to missed opportunities. For example, peering through a pedagogy lens we should see such features as:

- a story shell to embed the situation in a meaningful context
- significant active use of concrete materials
- a problem solving challenge which provides ownership for students
- small group work
- a strong visual component
- access to supportive software

Cheerful…

because we want 'happy' in the title twice!

Productive…

is the clear acknowledgment that students are working towards recognisable outcomes. They should know what these are and have guidelines to show they have either reached them or made progress. Teachers are accountable to these outcomes as well as to the quality of the learning environment.

Inspiring…

is about creating experiences that are uplifting or exalting; that actually *turn students on*. Experiences that make students feel great about themselves and empowered to act in meaningful ways.

Number & Computation Resources

To help you create

happy, healthy, cheerful, productive, inspiring classrooms

this kit contains

- 20 hands-on problem solving tasks from Mathematics Centre

and a Teachers' Manual which integrates the use of the tasks with

- 14 detailed lesson plans from Maths300

The kit offers **8 weeks** of Scope & Sequence planning in Number & Computation for *each* of Year 5 and Year 6. This is detailed in *Part 2: Planning Curriculum* which begins on Page 12. You are invited to map these weeks into your Year Planner.
Together, the four kits available for these levels provide 25 weeks of core curriculum in Working Mathematically (working like a mathematician).

Note: Membership of Maths300 is assumed.
The kit will be useful without it, but it will be much more useful with it.

Tasks

- 4 & 20 Blackbirds
- Add The Pack
- A Dollar To Spend
- Bob's Buttons
- Change
- Cover Up
- Doctor Dart
- Dominoes
- Doug's Tablecloth
- Eureka
- Magic Square
- Making Fractions 2
- Martian Maths
- Monkeys & Bananas
- Number Game
- Number Tiles
- Pascal's Triangle In Asia
- Peg & Tape Fractions
- Pick A Box
- Steps

Part 2 of this manual introduces each task. The latest information can be found at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm

Maths300 Lessons

- 4 & 20 Blackbirds
- Bob's Buttons
- Doctor Dart
- Estimating Fractions
- Eureka
- First Principles Percent
- Gauss Beats The Teacher
- Multo
- Number Charts
- Number Tiles
- Palindromes
- Rod Mats
- Steps
- What's It Worth?

Lessons with Software

- Bob's Buttons
- Doctor Dart
- Estimating Fractions
- Multo
- Number Charts
- Number Tiles
- Palindromes
- Steps
- What's It Worth?

Part 2 of this manual introduces each lesson. Full details can be found at:

- http://www.maths300.com

Working Like A Mathematician

Our attitude is:

all students can learn to work like a mathematician

What does a mathematician's work actually involve? Mathematicians have provided their answer on Page 8. In particular we are indebted to Dr. Derek Holton for the clarity of his contribution to this description.

Perhaps the most important aspect of Working Mathematically is the recognition that *knowledge is created by a community and becomes part of the fabric of that community*. Recognising, and engaging in, the process by which that knowledge is generated can help students to see themselves as able to work like a mathematician. Hence Working Mathematically is the framework of **Maths With Attitude**.

Skills, Strategies & Working Mathematically

A Working Mathematically curriculum places learning mathematical skills and problem solving strategies in their true context. Skills and strategies are the tools mathematicians employ in their struggle to solve problems. Lessons on skills or lessons on strategies are not an end in themselves.

- **Our skill toolbox** can be added to in the same way as the mechanic or carpenter adds tools to their toolbox. Equally, the addition of the tools is not for the sake of collecting them, but rather for the purpose of getting on with a job. A mathematician's job is to attempt to solve problems, not to collect tools that might one day help solve a problem.
- **Our strategy toolbox** has been provided through the collective wisdom of mathematicians from the past. All mathematical problems (and indeed life problems) that have ever been solved have been solved by the application of this concise set of strategies.

About Tasks

Our attitude is:

mathematics is concrete, visual and makes sense

Tasks are from Mathematics Task Centre. They are an invitation to two students to work like a mathematician (see Page 8).

The Task Centre concept began in Australia in the late 1970s as a collection of rich tasks housed in a special room, which came to be called a Task Centre. Since that time hundreds of Australian teachers, and, more recently, teachers from other countries, have adapted and modified the concept to work in their schools. For example, the special purpose room is no longer seen as an essential component, although many schools continue to opt for this facility.

A brief history of Task Centre development, considerable support for using tasks, for example Task Cameos, and a catalogue of all currently available tasks can be found at:

- http://www.mathematicscentre.com/taskcentre

Key principles are:

- A good task is the tip of an iceberg
- Each task has three lives
- Tasks involve students in the Working Mathematically process

The Task Centre Room or the Classroom?

There are good reasons for using the tasks in a special room which the students visit regularly. There are also different good reasons for keeping the tasks in classrooms. Either system can work well if staff are committed to a core curriculum built around learning to work like a mathematician.

- A task centre room creates a focus and presence for mathematics in the school. Tasks are often housed in clear plastic 'cake storer' type boxes. Display space can be more easily managed. The visual impact can be vibrant and purposeful.
- However, tasks can be more readily integrated into the curriculum if teachers have them at their finger tips in the classrooms. In this case tasks are often housed in press-seal plastic bags which take up less space and are more readily moved from classroom to classroom.

Tip of an Iceberg

The initial problem on the card can usually be solved in 10 to 20 minutes. The investigation iceberg which lies beneath may take many lessons (even a lifetime!). Tasks are designed so that the original problem reveals just the 'tip of the iceberg'. Task Cameos and Maths300 lessons help to dig deeper into the iceberg.

We are constantly surprised by the creative steps teachers and students take that lead us further into a task. No task is ever 'finished'.

> *Most tasks have many levels of entry and exit and therefore offer an on-going invitation to revisit them, and, importantly, multiple levels of success for students.*

Three Lives of a Task

This phrase, coined by a teacher, captures the full potential and flexibility of the tasks. Teachers say they like using them in three distinct ways:

1. As on the card, which is designed for two students.
2. As a whole class lesson involving all students, as supported by outlines in the Task Cameos and in detail through the Maths300 site.
3. Extended by an Investigation Guide (project), examples of which are included in both Task Cameos and Maths300.

The first life involves just the 'tip of the iceberg' of each task, but nonetheless provides a worthwhile problem solving challenge - one which 'demands' concrete materials in its solution. This is the invitation to work like a mathematician. Most students will experience some level of success and accomplishment in a short time.

The second life involves adapting the materials to involve the whole class in the investigation, in the first instance to model the work of a mathematician, but also to develop key outcomes or specific content knowledge. This involves choosing teaching craft to interest the students in the problem and then absorb them in it.

The third life challenges students to explore the 'rest of the iceberg' independently. Investigation Guides are used to probe aspects and extensions of the task and can be introduced into either the first or second life. Typically this involves providing suggestions for the direction the investigation might take. Students submit the 'story' of their work for 'portfolio assessment'. Typically a major criteria for assessment is application of the Working Mathematically process.

About Maths300

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

Maths300 is a subscription based web site. It is an attempt to collect and publish the 300 most 'interesting' maths lessons (K - 12).

- *Lessons have been successfully trialed in a range of classrooms.*
- *About one third of the lessons are supported by specially written software.*
- *Lessons are also supported by investigation sheets (with answers) and game boards where relevant.*
- *A 'living' Classroom Contributions section in each lesson includes the latest information from schools.*
- *The search engine allows teachers to find lessons by pedagogical feature, curriculum strand, content and year level.*
- *Lesson plans can be printed directly from the site.*
- *Each lesson supports teaches to model the Working Mathematically process.*

Modern internet facilities and computers allow teachers easy access to these lesson plans. Lesson plans need to be researched, reflected upon in the light of your own students and activated by collecting and organising materials as necessary.

Maths300 Software

Our attitude is:

> *stimulated students are creative and love to learn*

Pedagogically sound software is one feature likely to encourage enthusiastic learning and for that reason it has been included as an element in about one third of Maths300 lesson plans. The software is used to develop an investigation beyond its introduction and early exploration which is likely to include other pedagogical techniques such as concrete materials, physical involvement, estimation or mathematical conversation. The software is not the lesson plan. It is a feature of the lesson plan used at the teacher's discretion.

For school-wide use, the software needs to be downloaded from the site and installed in the school's network image. You will need to consult your IT Manager about these arrangements. It can also be downloaded to stand alone machines covered by the site licence, in particular a teacher's own laptop, from where it can be used with the whole class through a data projector.

Note:

- *Maths300 lessons and software may only be used by Maths300 members.*

Working Mathematically

First give me an interesting problem.

When mathematicians become interested in a problem they:

- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

Questions which help mathematicians learn more are:

- Can I check this another way?
- What happens if ...?
- How many solutions are there?
- How will I know when I have found them all?

When mathematicians have a problem they:

- Read & understand the problem.
- Plan a strategy to start the problem.
- Carry out their plan.
- Check the result.

A mathematician's strategy toolbox includes:

- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- Break a problem into smaller parts
- ...

If one way doesn't work, I just start again another way.

Professional Development Purpose

Our attitude is:

the teacher is the most important resource in education

> *We had our first study group on Monday. The session will be repeated again on Thursday. I had 15 teachers attend. We looked at the task Farmyard Friends (Task 129 from the Mathematics Task Centre). We extended it out like the questions from the companion Maths300 lesson suggested, and talked for quite a while about the concept of a factorial. This is exactly the type of dialog that I feel is essential for our elementary teachers to support the development of their math background. So anytime we can use the tasks to extend the teacher's math knowledge we are ahead of the game.*
> District Math Coordinator, Denver, Colorado

Research suggests that professional development most likely to succeed:

- is requested by the teachers
- takes place as close to the teacher's own working environment as possible
- takes place over an extended period of time
- provides opportunities for reflection and feedback
- enables participants to feel a substantial degree of ownership
- involves conscious commitment by the teacher
- involves groups of teachers rather than individuals from a school
- increases the participant's mathematical knowledge in some way
- uses the services of a consultant and/or critical friend

Maths With Attitude has been designed with these principles in mind. All the materials have been tried, tested and modified by teachers from a wide range of classrooms. We hope the resources will enable teacher groups to lead themselves further along the professional development road, and support systems to improve the learning outcomes for students K - 12.

With the support of Maths300 ETuTE, professional development can be a regular component of in-house professional development. See:

- http://www.mathematicscentre.com/taskcentre/resource.htm#etute

For external assistance with professional development, contact:

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E: doug@blackdouglas.com.au

Part 2: Planning Curriculum

Curriculum Planners

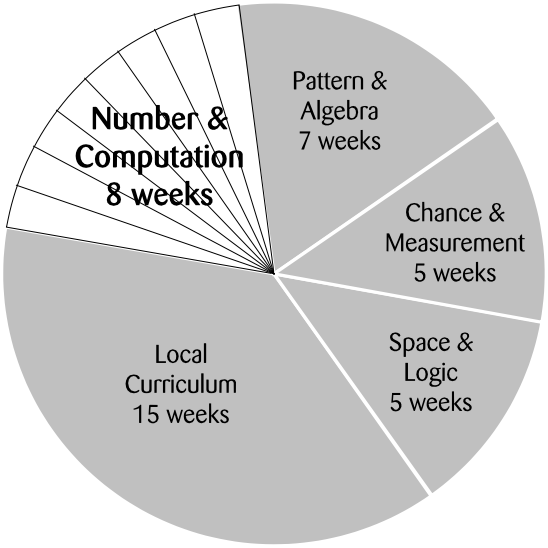
Our attitude is:

learning is a personal journey stimulated by achievable challenge

Curriculum Planners:

- show one way these resources can be integrated into your weekly planning
- provide a starting point for those new to these materials
- offer a flexible structure for those more experienced

You are invited to map Planner weeks into your school year planner as the core of the curriculum.



Planners:

- detail each week lesson by lesson
- offer structures for using tasks and lessons
- are sequenced from lesson to lesson, week to week and year to year to 'grow' learning

Teachers and schools will map the material in their own way, but all will be making use of extensively trialed materials and pedagogy.

Using Resources

- Your kit contains 20 hands-on problem solving tasks and reference to relevant Maths300 lessons.
- Tasks are introduced in this manual and supported by the Task Cameos at:
  http://www.mathematicscentre.com/taskcentre/iceberg.htm
- Maths300 lessons are introduced in this manual and supported by detailed lesson plans at:
  http://www.maths300.com

In your preparation, please note:

- Planners assume 4 lessons per week of about 1 hour each.
- Planners are *not* prescribing a continuous block of work.
- Weeks can be interspersed with other learning; perhaps a **Maths With Attitude** week from a different strand.
- Weeks can sometimes be interchanged within the planner.
- Lessons can sometimes be interchanged within weeks.
- The four **Maths With Attitude** kits available at each year level offer 25 weeks of a Working Mathematically core curriculum.

A Way to Begin

- Glance over the Planner for your class. Skim through the comments for each task and lesson as it is named. This will provide an overview of the kit.
- Task Comments begin after the Planners. Lesson Comments begin after Task Comments. The index will also lead you to any task or lesson comments.
- Select your preferred starting week - usually Week 1.
- Now plan in detail by researching the comments and web support. Enjoy!

Research, Reflect, Activate

Curriculum Planner

Number & Computation: Year 5

	Session 1	Session 2	Session 3	Session 4
Week 1	**Whole Class Investigations:** *Gauss, Steps & Number Charts* emphasise number facts, number bonds, number sense in a series of problem solving situations. The aim is to establish the Working Mathematically focus and include plenty of skill practice in new and interesting situations. Can include additional material from local text.			
Weeks 2 & 3	**Self-directed Maths Journey I:** For two weeks, students journey through a mathematical landscape in pairs and keep a diary of their adventures. See Page 18 for details. The journey includes: tasks, software, skill practice, opportunity to follow up ideas introduced in Week 1 and Rod Mat Chats with the teacher. See notes about these chats in the *Rod Mats* lesson. Tasks suggested for Year 5 are: **4 & 20 Blackbirds**, **Add The Pack**, **Bob's Buttons**, **Change**, **Doctor Dart**, **Making Fractions 2**, **Number Game**, **Peg & Tape Fractions**, **Pick A Box**, **Steps**.			
Week 4	**Whole Class Investigation:** The class works together to explore *Doctor Dart* which is a problem with many extensions. This lesson can take from 1 to 4 sessions. Balance its use with other local maths curriculum needs.			
Week 5	**Whole Class Investigations:** *Bob's Buttons, Multo & What's It Worth?* offer problem solving challenges based around multiplicative thinking. *What's It Worth?* can also involve fractions and decimals. These investigations establish a basis for SMJ II.			
Weeks 6 & 7	**Self-directed Maths Journey II:** Extending the problem solving opportunities of Week 5, exploring the software further, revisiting the tasks above and continuing the Rod Mat Chats. Can include skill practice exercises from local text.			
Week 8	**Skill Development:** *First Principles Percent* focuses on developing the concept of x% percent as x out of every 100. Plenty of skill practice guided by the worksheets provided. Students are encouraged to verbalise their thinking and to develop their own short cuts rather than being taught rules.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Curriculum Planner

Number & Computation: Year 6

	Session 1	Session 2	Session 3	Session 4
Week 1	**Whole Class Investigations:** In many ways *Number Tiles, Palindromes & Multo* review and extend the work of the Year 5 Planner. *Palindromes* is based on an appealing curiosity that involves about 10% problem solving and 90% excuse for addition practice. *Number Tiles* also involves addition and place value, but reverses the balance in favour of problem solving. The process of Working Mathematically is highlighted. *Multo* is reviewed and the search for the 'best' placement strategy continues.			
Weeks 2 & 3	**Self-directed Maths Journey III:** For two weeks, students journey through a mathematical landscape in pairs and keep a diary of their adventures. See Page 18 for details. The journey includes: tasks, software, skill practice, opportunity to follow up ideas introduced in Week 1 and Rod Mat Chats with the teacher. See notes about these chats in the *Rod Mats* lesson. Tasks suggested for Year 6 are: **A Dollar To Spend**, **Cover Up**, **Dominoes**, **Doug's Tablecloth**, **Eureka**, **Magic Square**, **Martian Maths**, **Monkeys & Bananas**, **Number Tiles**, **Pascal's Triangle In Asia**.			
Week 4	**Whole Class Investigation:** The class works together to explore *4 & 20 Blackbirds*, a problem with many extensions. This lesson can take from 1 to 4 sessions. Balance its use with other local maths curriculum needs.			
Week 5	**Whole Class Investigations:** *Estimating Fractions, Number Charts & What's It Worth?* offer problem solving challenges based around fractions, decimals and percent. *Estimating Fractions* is a whole class extension of the Year 5 task **Peg & Tape Fractions**. *Number Charts* and *What's It Worth?* review and extend Year 5.			
Weeks 6 & 7	**Self-directed Maths Journey IV:** Extending the problem solving opportunities of Week 5, exploring the software further, revisiting the tasks above and continuing the Rod Mat Chats. Can include skill practice exercises from local text.			
Week 8	**Whole Class Investigation:** Working together as a class to explore the problem *Eureka* which has many extensions. This lesson can take from 1 to 4 sessions. Balance its use with other local maths curriculum needs.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Planning Notes

Enhancing Maths With Attitude

Resources to support learning to work like a mathematician are extensive and growing. There are more tasks and lessons available than have been included in this Number & Computation kit. You could use the following to enhance this kit.

Additional Tasks

- Task 213, Chains

  The board is six squares in a line and the equipment is six cubes. The arrangement in which cubes are placed scores points. What is the lowest possible score? The highest? Try boards of different lengths.
- Task 236, Star Numbers

  Star Numbers are constructed by adding Triangle Numbers to each side of a Square Number. In this sense, they have to be made as shapes to be understood. As the size of the Star Number increases, so does the number of plugs needed to make it. Students can explore particular cases, thereby applying number skills, or can generalise for any size Star Number.
- Task 240, Less Then Fractions

  Number tiles (1 - 9) allow students to experiment with fractions less than 1 in a non-threatening, open-ended way. Early success is guaranteed because there are 36 possible answers and the obvious one is 1/2. But the greater challenge is to add two fractions (each tile can be used only once) and still get an answer less than one. How many solutions are there? How do you know when you have found them all?

More information about these tasks may be available in the Task Cameo Library:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Additional Lessons

- Task 70, Pick A Box

  The boxes have been packed, wrapped and numbered. They have been sorted into groups for delivery to different areas. Then - oh no! - a gust of wind and they are scattered and mixed up. Not to worry. Santa's #1 Elf remembers the sorting rules. The groups can be made again just by following the rules. Unfortunately, although old #1 doesn't know it, the rules lead to more than one regrouping. How many ways of grouping the boxes can the class find?
- Task 120, 1089

  With due flamboyance the teacher announces that this lesson will be special because the answer to all the questions has already been written out. It soon becomes clear that starting with (almost) any 3-digit number, followed by a subtraction and an addition, the answer is always 1089. The young mathematicians in the class are challenged to recognise that this can't be magic, but rather there must be a mathematical explanation for why this happens. The challenge of the lesson is to explain why the answer is always the same and to communicate this to others.

- Task 130, The Mushroom Hunt

  Six people go looking for mushrooms. Each one has a basket. When they return they compare the numbers of mushrooms in their baskets. They discover two things: (a) Adding up the numbers gives a total of 63. (b) Grouping the baskets in different ways gives sub-totals equal to every number from 1 to 63. The problem is, of course, how many mushrooms are in each basket. The problem can be tackled at every level from an exploration of doubling, to an introduction to powers and indices, to the concept of binary numbers, to an investigation of multiplicative (exponential-like) growth.

- Task 145, Estimating Averages

  This concept lesson is cleverly constructed to focus on understanding the idea of an average rather than on the algorithmic skill. There are two central features: (a) The use of estimation followed by a discussion of strategy leading to a second round of estimates. Students almost invariably score many more points in the second round. (b) The use of concrete materials in the middle stages of the lesson, involving students with differing numbers of blocks sharing (in silence) until all players have the same number.

- Task 146, Division Boxes

  Place digits 1 to 9 into three boxes (no repeats) to make a three digit number so that: (a) the number in the first box is divisible by 1, and (b) the number in the first two boxes is divisible by 2, and (c) the whole three digit number is divisible by 3. There are many solutions, eg: 621, however 612 is not a solution since the 61 is not divisible by 2. In this form the puzzle invokes knowledge of divisibility tests. However by adding more boxes and finding all possible solutions, it turns into an extended open-ended investigation which can suit many levels. There are solutions for each string of boxes up to ten. However, for the 10-box problem there is just one unique solution and part of the lesson is the search for this special number.

- Task 150, Fermi Problems

  Fermi Problems have the characteristic that most people who encounter them respond that it is a problem that they couldn't solve without recourse to outside information. Students see that through a series of simple steps using only common sense and their experience they can quite often come up with reasonable estimates for the answers. The lesson is structured around a common problem to illustrate the process, followed by group attempts to solve and report on a problem of their choice.

- Task 162, Multiplication in a Table Format

  This very interesting technique for multiplication has a strong theoretical connection to the distributive property and progresses naturally into factorising and expanding algebraic expressions. The technique arises from a concrete array and has strong visual imagery behind the process. It is also very practical, easy to learn and easy to generate understanding of each step.

- Task 178, Tables For 25

  Working in groups, a context familiar to all students, is the focus of the lesson. Mr. Edwards begins his class of 25 by asking them to organise into groups of 5 at each table, with at least two boys in each group. How the table arrangements work out will depend on how many boys there are in the class altogether, which is what opens the door to a broad investigation and a personal project for each student.
- Task 181, Natalie's LCM Task

  Take a closed question such as:

  What is the lowest common multiple of 45 and 60?

  turn it into an open question:

  The lowest common multiple is 180, what are the two numbers?

  and you have an open investigation. The investigation is applicable at many levels and is supported by software.
- Task 182, Fractions to Decimals (on a rope!)

  This lesson builds a strong conceptual understanding of equivalence when converting fractions to decimals. It offers students concrete, hands-on and visually-based experiences before moving to a formal algorithm. Students use a rope with pegs as markers to first establish ten parts to represent decimals on a number line extending from 0 to 1. Then they fold the rope and explore by estimation how common fractions convert to decimals using the markers on the rope. The lesson features group work, estimation, problem solving and concrete and visual learning.

Keep in touch with new developments which enhance **Maths With Attitude** at:

- http://www.mathematicscentre.com/taskcentre/enhance.htm

Additional Materials

As stated, our attitude is that mathematics is concrete, visual and makes sense. We assume that all classrooms will have easy access to many materials beyond what we supply. For this unit you will need:

- Linking cubes like Unifix or Multilink
- Square tiles
- Calculators
- Cuisenaire Rods (at least one set for the teacher to use in Rod Mat Chats). Cuisenaire Rods can be obtained from Mathematics Centre by special order. Alternatively, other materials in fraction relation to each other may be used, provided the materials are not stamped with specific fractions.
- Pegs and lengths of tape as in **Peg & Tape Fractions**.
- Poly Plug is useful for converting **4 & 20 Blackbirds** and **Bob's Buttons** to whole class lessons.

Find Poly Plug information at:

- http://www.mathematicscentre.com/taskcentre/polyplug.htm

Find Mathematics Centre resources at:

- http://www.mathematicscentre.com/taskcentre/resource.htm

Self-directed Maths Journey

Students are asked to choose their own activities within a limited, but challenging, mathematical landscape. It is just like being put into a broad, fenced environmental landscape and being given free reign to explore.

Each Self-directed Maths Journey (SMJ) is a loosely structured unit plan designed to encourage independent, self-directed mathematics learners. A feature is the development of mathematics in a language context - a model which helps teachers 'work smarter' by simultaneously achieving numeracy and literacy objectives.

Students keep a diary of their adventures and teachers encourage detailed entry in the diary by allowing sufficient time to write it. The diary has a dual purpose:

- In writing 'for an 'audience' students reprocess the work they have been doing and this helps to enhance learning.
- It provides a record that contributes to evaluation evidence.

The two week SMJ is prepared in the previous week, just as a journey into an unknown physical landscape is prepared beforehand. This occurs in two ways - by class exploration of problems which leave room for further exploration during the journey and by preparation of the diary.

Preparation of the Diary

In a mathematics session, introduce a problem that will need more than one session to explore its iceberg. For example, for SMJ I on the Year 5 Planner, this could be *Gauss Beats The Teacher* leading into *Steps*. Leave the investigation unfinished but with clear indication of the directions it can go. Explain that this investigation will be one of the activities to choose during next week's Self-directed Maths Journey.

Follow up in a language session with the concept of keeping a diary of the Self-directed Maths Journey. Collect examples of diaries from the library - literary diaries like 'Diary of Anne Frank', or professional diaries like that kept by Joseph Banks, the botanist on Captain Cook's explorations of Australia. Diaries/notebooks also feature in 'popular culture'. For example the father's diary is central to the plot in the film 'Indiana Jones and the Last Crusade', a movie well known to most students, and the film has several shots of the book showing text, sketches and maps. Note items like these as features of a diary and add others suggested by students such as the importance of dates and the possible inclusion of photographs. Encourage the use of the school's cameras to record significant mathematical moments. Consider also the possibility of an electronic diary if you are interested in achieving Information Technology outcomes in conjunction with mathematics and language.

Introduce the idea of a diary published for others, such as that of Joseph Banks, being prepared at a later time from notes taken at important moments. Perhaps use the image of a hiker travelling with a diary in their backpack and a notepad and pencil in their shirt pocket. The pad is used to record significant moments in the day and the diary is filled in later around the camp fire.

Provide 'provisions' for the SMJ:

- small spiral notepad (or equivalent)
- a more substantial book to use as the diary
- an outline of the mathematical landscape, such as the examples below
- the Working Mathematically process page

Provide time to decorate and personalise the diary. The Working Mathematically process and the landscape page are placed in the front pages of the diary. Students are told that the next page is to be a Map Page and thereafter they record each day in text, drawing and photo as they wish.

The Map Page records each date and its activities in name only as a summary of the adventure. These items are arranged on the page as the student wishes and are linked with pathways and other landscape features in an imaginative way.

On The Journey

- Students work in pairs - it is usual to go on a journey with a companion - but keep separate diaries.
- It doesn't matter which order students tackle the activities, or which ones they tackle, or how many they tackle, or even if some activities are left started, but unfinished. What matters is that they demonstrate through their diary (and interview if you wish) how they have worked like a mathematician in each session.
- Some students may need help getting started because the range of choice may be daunting. In such cases, choose a hands-on task for them and sit down together to ask questions that will help them begin.
- Plan a compulsory teacher-led activity that students have to attend for a given number of sessions in the journey. Rod Mat Chats is the example below. These chats only take about 15 minutes so use the remainder of your time to join groups and share their journeys.
- Allow time every maths session to begin diary writing and follow up in language time. Encourage students to work further on the diaries in their own time.
- Collect diaries regularly and annotate with encouraging comment and suggestions. Celebrate examples of student diary work that you wish to encourage in the group.
- You may also wish to continue using diaries as the source of language activities.

Following The SMJ

- Allow time to finalise diaries. Collect, read, comment and display.
- Use the ideas in 'Pupil Self Reflection' in the Assessment section of this manual.

Examples of SMJ Landscapes

These sample landscapes are provided as a guide. They are not prescriptive. You are invited to construct your own from the resources provided, or include other resources of your own. To be true to the model, any material you include should be as open as possible.

Year 5

Self-directed Maths Journey I

Mathematical Landscape

- Complete the *Gauss Beats The Teacher* investigation sheet.
- From any starting number create a sequence of consecutive numbers. Add the sequence in 3 different ways.
- *Steps*: How many solutions are there? How do you know when you have found them all?
- *Number Charts* software: Screen capture and print the results for your diary.
- **Tasks**: 4 & 20 Blackbirds, Add The Pack, Bob's Buttons, Change, Doctor Dart, Making Fractions 2, Number Game, Peg & Tape, Fractions, Pick A Box, Steps.
- From Text: Exercises...
- Compulsory: 2 Rod Mat Chats with your teacher.

Self-directed Maths Journey II

Mathematical Landscape

- *Bob's Buttons*: Search for a way to predict the pattern for any given pair of groups and left overs. The software will help your search.
- *Multo*: Search for the best way to arrange the numbers in the grid. The software will help your search.
- *What's It Worth?*: Make a puzzle card.
- *What's It Worth?*: Try someone else's puzzle card.
- *What's It Worth?*: Try some puzzles from the software. Screen capture and print the results for your diary.
- **Tasks**: 4 & 20 Blackbirds, Add The Pack, Bob's Buttons, Change, Doctor Dart, Making Fractions 2, Number Game, Peg & Tape, Fractions, Pick A Box, Steps.
- From Text: Exercises...
- Compulsory: 2 Rod Mat Chats with your teacher.

Year 6

Self-directed Maths Journey III

Mathematical Landscape

- *Number Tiles*: Search for more solutions. How many solutions are there? How do you know when you have found them all? The software may help your search.
- *Palindromes*: Search for a starting number that *does not* produce a palindrome. The software will help your search.
- *Multo*: Continue searching for the best way to arrange the numbers in the grid. The software will help your search.
- **Tasks**: A Dollar To Spend, Cover Up, Dominoes, Doug's Tablecloth, Eureka, Magic Square, Martian Maths, Monkeys & Bananas, Number Tiles, Pascal's Triangle In Asia.
- Investigate other number systems. How do they relate to our decimal system? What are their advantages and disadvantages?
- From Text: Exercises...
- Compulsory: 2 Rod Mat Chats with your teacher.

Self-directed Maths Journey IV

Mathematical Landscape

- *Estimating Fractions*: Use the software a lot and demonstrate that your estimates are improving in fractions, decimals and percent.
- *Number Charts*: Use the Fraction Addition option. Screen capture and print the results for your diary.
- *What's It Worth?*: Make a puzzle card that includes fractions and decimals.
- *What's It Worth?*: Try someone else's puzzle card that includes fractions and decimals.
- *What's It Worth?*: Try some fraction and decimal puzzles from the software. Screen capture and print the results for your diary.
- **Tasks**: A Dollar To Spend, Cover Up, Dominoes, Doug's Tablecloth, Eureka, Magic Square, Martian Maths, Monkeys & Bananas, Number Tiles, Pascal's Triangle In Asia.
- Investigate other number systems. How do they relate to our decimal system? What are their advantages and disadvantages?
- From Text: Exercises...
- Compulsory: 2 Rod Mat Chats with your teacher.

Task Comments

- Tasks, lessons and unit plans prepare students for the more traditional skill practice lessons, which we invite you to weave into your curriculum. Teachers who have used practical, hands-on investigations as the focus of their curriculum, rather than focussing on the drill and practice diet of traditional mathematics, report success in referring to skill practice lessons as Toolbox Lessons. This links to the idea of a mathematician dipping into a toolbox to find and use skills to solve problems.

4 & 20 Blackbirds

Many students need to work on this problem by physically stacking and shifting the bird counters. Others will drift to recording number patterns. Each is valid. The problem solving in the task involves some of the key elements of the Working Mathematically process. There are many solutions to the original problem, so there is a level of success for anyone using a guess and check strategy. However, those who tackle the mathematician's questions:

> *How many solutions are there?*
> *How do I know when I have found them all?*

will become more deeply involved in the process. Even when this is explored, we are reminded that a mathematical problem may never be finished. Question three asks

> *What happens if...?*

and opens the door to further investigation.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Add The Pack

Many number sense opportunities in this task. Perhaps the very slowest way to find the total is for one student to shuffle the pack then place the cards on the table one at a time while the other student keeps a progressive total, either orally, or with a calculator. From there, guided by the card, the students will think of several ways of applying strategies that reach the total more efficiently. In effect the task encourages students to ask the mathematician's question:

> *Can I check this another way?*

Question 4 deliberately encourages a mind experiment because only cards 7 - K are supplied, but if necessary a full card pack may be used. Whichever approach the students take, the teacher is encouraged to refrain from answering yes or no to the question *Is that right?*. The response *Can you check it another way?* is more

appropriate. It encourages students to employ more than one strategy and it encourages independent learning. The task has a direct link to *Gauss Beats The Teacher*, which is a Maths300 lesson assigned to this kit.

A Dollar To Spend

In the first instance this task is an opportunity to practice number skills. However the problem in the last question requires some thought to obtain an answer such as 1 pencil, 90 paper clips and 9 hairpins (10c + 45c + 45c). The focus can then shift to a variety of strategies that could be used to obtain this answer, or indeed, whether there are other answers. Asking *What happens if...?* might lead to wondering whether there was a solution for 90 items for 90 cents, or even whether a similar problem could be constructed using alternative prices for the same objects.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Bob's Buttons

At first sight the task has only one answer, ie: 26 buttons. The task could in fact be a text book question and some students will solve it by application of 'automatic' number knowledge. Others will need to arrange and rearrange the buttons. Both methods are valid. However, there are two hints on the card that there may be more than one answer. One is the restriction 'more than ten buttons'. The other is that the question asks for the smallest number of buttons Bob could have. Seeking other solutions leads to a much deeper investigation.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Change

Number in the context of money. Working out all the combinations of change for $2 requires an organised approach. For some the key will be starting with forty 5c coins and planning a sequence of substitutions.

The tokens are representative of Australian money and Australia does not use a coin equivalent to 25c. Hence a similar task using all the denominations available in the USA, for example, would have different answers. The task can be tackled as is, then the idea of introducing the quarter dollar coin can be introduced as an extension activity.

Cover Up

The task encourages students to think about pairs of numbers which sum to make the digits 1 - 9. The winner is the person who accumulates a score of 45 and students eventually realise that this is the sum of the digits 1 - 9. The twist is that if a dice roll results in a 9, eg: 6 and 3, all the tiles are covered so their score is zero and therefore the 45 score can never be achieved in one roll.

It adds a twist to the game if the rule is first to 45 loses. That means students would be better to score zero each round, which can only happen with a roll of 9. Whichever way the game is played, it leads to the chances of rolling a total of 9 and hence an examination of the probabilities associated with rolling two dice.

Exploring number pairs and the sum of 1 - 9 is also basic to the tasks **Steps** and **Number Tiles** and to the Maths300 lesson *Gauss Beats The Teacher*.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Doctor Dart

You may find you have to interpret this task with the students. Experience shows that the language of the card may be a bit daunting for some. For example the story shell usually makes more sense to the students once they are shown the blocks stacked upwards in three columns on the desk, rather than lying flat as might be interpreted from the picture. Also, the image of the anti-gravity door being steel or titanium and having a key pad as in the picture also makes links for many students. Once under way, most students are able to work independently. Much of the iceberg of the task is on the card.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Dominoes

Easy to start and the questions take the students deeper and deeper into the investigation. The task is a concrete situation requiring the application of number facts and place value. Not asked on the card, but always lurking are the questions

> *How many solutions are there?*
> *How do I know when I have found them all?*

For example, how many combinations are there that involve carrying?

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Doug's Tablecloth

Number in a measurement context, although the numbers need not be used to solve the problem. Most students use a trial and improve strategy and eventually try making the first fold of the cloth into thirds rather than halves. This is crucial to the solution. However there are many clues in the numbers and once solved it is important to look back and ask *What might I have learned from the numbers in the problem?*. The numbers can also be used to offer hints to students who are struggling, for example, *The material has to exactly fit the depth of the draw, so how does the depth relate to the original measurements?*. The challenge on the card invites students to work backwards, unfolding the cloth as they keep a tally of the growth of the length from 16cm.

Fractions arise naturally when discussing the way to fold the whole cloth the first time - halves? thirds? Once the problem is solved, fractions can be explored further by examining the equal sections that have been created in the whole cloth by folding. For students who have experienced *Rectangle Fractions* in **Maths With**

Attitude, Number & Computation, Years 3 & 4, there is a direct visual link with earlier activities.

Note: You will sometimes have to iron the cloth.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Eureka

Enjoyable arithmetic practice supported by the concrete presence of the 'nuggets'. With a little mathematical reasoning, for example realising that Paul's total must be even because it is twice something, an answer can be found. One solution is: Peter: (5, 8, 9) = 22 Paul: (6, 10, 11, 17) = 44 ... Mary: (7, 14, 15, 16, 40) = 92 The puzzle becomes more interesting as soon as it is realised that there is more than one solution. Finding more than one is the next level of success, and finding all the solutions is a challenge that requires significant thought to develop a strategy, and a good deal of disciplined examination of all possibilities; an approach that is perhaps best achieved by the joint efforts of all the students.

Magic Square

This task often appears as a pencil and paper exercise, but the magic sum is usually given. We believe the concrete approach using tiles and a board encourages more students to persevere for longer with the deeper problem. The first sub-problem is to decide on the magic total. This is another place where the sum of the digits 1 - 9 becomes important (see **Cover Up** above). If, for example the columns were added every digit would be counted once. Therefore the sum of these three column totals must be the same as the sum of all nine digits. To find the magic total this grand sum has to be shared equally among the three columns. Parallel reasoning obtains the magic sum in the 4 x 4 case. The next problem is to arrange the tiles to achieve the magic total. An approach that some find successful is to first fix one direction, say the columns, and then swap within each column to also get the rows correct. Finally a little more jiggling obtains the diagonals.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Making Fractions 2

This task builds on the same principle as **What's It Worth?** which was included in Number & Computation, Years 3 & 4. *What's It Worth?* is also a Maths300 lesson planned into this kit. The principle is that of taking a whole that is divided into equal parts and choosing any of those parts to have a value of 1. In effect, the task is based on the concept that any piece, not necessarily the largest one, can be the whole. Making Fractions 2 is based on halves, quarters and eighths to make the relationships as simple as possible. The completed table opens discussion about reciprocal fractions.

Note that the pieces are not intended to fit in the cells at the top of each column. However, some students do stand them on their edge outside those cells. Drawings, or a code such as LS for larger square go in the top cells. An additional spatial challenge is provided by trying to fit the pieces back into the frame at pack up time. This should be encouraged as a check that all the pieces are there.

Martian Maths

There are two versions of this card, either 98a or 98b. The problem is essentially the same on either card. The two forms exist in response to accessibility of materials. The answer for Question 2 depends, therefore, on the form of the card you have:
98a: Square = 25, Triangle = 5, Circle = 1
98b: Circle = 25, Triangle = 5, Square = 1
The solution can be found by either trial and improve, or in the case of advanced students, solving simultaneous equations.

The problem is designed to inspire discussion about alternative number systems in order to better understand the efficiency of our own. It seems the Martian system is different from ours because it has a base of 5. Is that better than 10? They wouldn't have so many digits to use. Do the Martians have a symbol for zero? How would their system, or ours, be affected if there was no symbol for zero? What is the history of the development of our zero?

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Monkeys & Bananas

There are many ways to tackle this problem, but the door is open to more students through the presence of the 'bananas', than would be the case if the problem were presented in a book. The following approach highlights thinking that uses fractions in a conceptual way. However, this is not the only approach and some students avoid thinking about the fractions altogether by thinking in terms of multiplication; an approach that is equally valid.

Working backwards and trying possibilities: In the morning, the number of bananas must be a multiple of 3, so try 3 as the first possibility.

- If 3, then the third monkey, after eating 1, was looking at a pile of bananas that could be divided into three equal parts. One of those parts she hid. So two of the parts became the 3 that might have been left in the morning.
- But if two parts is 3, then one part is 1.5 and the pile she was looking at must be 4.5 in total.
- This isn't allowed because the monkeys were dealing with whole bananas.
- The next possible 'morning number' is 6.
- If 6, then the third monkey, after eating 1, was looking at a pile of bananas two equal parts of which was 6.
- But if two parts is 6, then one part is 3 and the pile must be 9 in total.
- This is allowed so, counting the third monkey's night snack, the pile she saw when waking up must have been 10.
- But this third monkey number (10) came from the second monkey's night time adventure.
- If 10, then the second monkey, after eating 1, was looking at a pile of bananas two equal parts of which was 10.
- But if two parts is 10, then one part is 5 and the pile must be 15 in total.
- This is allowed so, counting the second monkey's night snack, the pile he saw when waking up must have been 16.

- But this second monkey number (16) came from the first monkey's night time adventure.
- If 16, then the first monkey, after eating 1, was looking at a pile of bananas two equal parts of which was 16.
- But if two parts is 16, then one part is 8 and the pile must be 24 in total.
- This is allowed so, counting the first monkey's night snack, the pile she saw when waking up must have been 25.
- Therefore the monkeys must have collected 25 bananas during the day.

This may be problem enough for some, but there are many other possibilities. It is important when using tasks to leave the students with the conviction that there is more that can be done with a problem, even if we choose not to do it at this moment.

In this case, we might ask:

- Suppose the 'morning number' was 9. Could we find a solution?
- Are there other 'morning numbers' that provide solutions?
- Can we investigate the number of hidden bananas?
- Suppose there were four, five, six... monkeys who behaved in the same way. Do the key numbers (fourth monkey number, third monkey number...) form a pattern?
- Suppose the three monkeys hid a fourth of the pile each time after eating one and were still able to share equally in the morning. Does this problem have a solution?
- Suppose there were four, five, six... monkeys who behaved in the same way. Do the key numbers (fourth monkey number, third monkey number...) form a pattern?

Number Game

The task allows a wide variety of students to be involved and creative. There is enough probability in the game to give the less able a chance of success against the more able. The game encourages application and development of number sense and some teachers have used moments from particular games to generate number talks with the class. The scoring rules are designed to encourage use of larger numbers and division. Larger scores are obtained by using times tables! Calculators could be used and it may be necessary to place a limit on the time allowed to construct equations in each deal. The task links well with popular activities at this level such as using the digits of the current year to make all the digits from 1 to 100.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Number Tiles

There is no such suggestion on the card, but students often try to do this task without carrying. In fact the only way to find a solution is with carrying, so you may have to suggest that including carrying could help. The card hints at the iceberg by asking for six solutions altogether. Finding these provides the opportunity to ask:

How many solutions are there?
How do I know when I have found them all?

There are in fact 168 solutions.

Find more information about this task in the Task Cameo Library at:
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Pascal's Triangle in Asia

Another task (see also **Martian Maths**) that highlights alternative number systems so that we can better understand our own. It also contains other subtle learning related to:
- the contribution to mathematical knowledge of cultures other than the Western influence.
- the flow of knowledge between cultures in past centuries - did Pascal know of Murai, or was it the other way around, or did they both learn from another?
- mathematics as a body of knowledge that continually grows.

Aside from all this, the pattern itself is fascinating and offers many investigations.

Find more information about this task in the Task Cameo Library at:
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Peg & Tape Fractions

A conceptual approach to fractions using a linear whole. In fact, the task offers two wholes so students can discover that their folding strategy is independent of the size of the whole. There is more than one way to do the folding involved in checking the estimate. It could be by jiggling to obtain equal segments, which is essentially using a guess and check strategy, or it could be by using complementary fractions. For example, *if* the peg placed at one third from end A is correct, the other section must be two thirds. Folding that section in half would produce a one third measurer, which would correctly match the estimated third if, and only if, the estimate was correct. This approach combines previous knowledge of fractions and wholes with if-then reasoning.

Pick A Box

Students tend to find this task quite engaging, although they will often reach what they think is a solution, only to find that on a second or third check there is a condition not satisfied. It may be necessary to read the 'frustration level' for some students and decide to intervene. One such opportunity is the third clue. Adding numbers in the three groups counts each of the fifteen numbers once. Therefore the total of these totals must be the same as the total of the numbers 1 through 15, which is 120 and therefore the three totals must be 39, 40 and 41.

Adding consecutive numbers like this is involved in several other tasks and lessons in the kit, so students should have several strategies for finding this total. Apart from this sort of thinking, and the number practice involved, another nice thing about this task is the multiple solutions. Again the students are able to ask:

How many solutions are there?
How do I know when I have found them all?

even if they choose not to follow it through at this moment.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Steps

This is another task for which a solution can be found by guess and check. However, application of problem strategies leads to a solution more efficiently and suggests a way of finding more than one. Once again the total of the numbers 1 through 9 is involved. The four partial sums on the steps board must total the sum of the numbers 1 - 9 BUT include the three corner numbers twice. In other words the grand total must be greater than the sum of 1 to 9 which is 45. Further, this grand total has to be equally shared between the four pieces of the step board, so the smallest possible grand total is 48, which implies each line sum is 12.

Since 4 x 12 = 48, the extra 3 above 45 must be caused by counting the corner numbers twice. Is it possible to put digits in these corners so they sum to 3? Clearly not, hence the line sum cannot be 12. Could it be 13, or 14, or...? Examining these other possibilities leads to all the solutions.

Lesson Comments

- These comments introduce you to each Maths300 lesson. The complete plan is easily accessed through the lesson library available to members at:
  http://www.maths300.com
  where they are listed alphabetically by lesson name.

4 & 20 Blackbirds

The story shell surrounding the lesson is that after the pie was open, the Queen decided that she liked the blackbirds' song so much that she built feeding platforms for them in the Royal Garden. The platforms were built at the four corners and the four mid-points of the rectangular Royal Garden. Each morning when the Queen came to listen to the singing, she counted 24 blackbirds. She also noticed that they always arranged themselves in a certain way. The lesson involves working out all the possible ways the blackbirds could be arranged.

The story seems to capture student interest, and with simple concrete materials it turns into a large scale whole class investigation. It also encourages mathematical conversation and idea sharing and sets up the problem so that everyone understands what is required when the time comes to work in groups.

This is not a SMJ week activity. It is an extensive problem with many solutions and is included so teachers can work with the class to illustrate the process of working like a mathematician. It is worth at least two sessions. Alternatively, you might proceed, as suggested by one trial teacher, by using the whole class approach to gather and record data for the first 10-15 minutes of each lesson, then in the last session of the week follow through with the rest of the lesson plan.

The Classroom Contributions offer software support for the lesson designed and contributed by a teacher.

Bob's Buttons

This lesson is used to open a line of investigation which is followed independently in Self-directed Maths Journey II. It involves a considerable amount of work on multiplication, and division with remainders, and is an excellent model of the Working Mathematically process. The Investigation Sheets provided with the lesson could be used to guide the self-directed work on this problem in SMJ II.

The lesson begins by physically involving students in a game of making groups based on the total number in the class today. On a signal from the teacher the children form into groups of a stated size and the number of groups and the 'remainder' number of ungrouped children is recorded. This is played several times, then the teacher poses a problem that involves working backwards from groups/remainder data to find the original number in the class. The problem is easily understandable because the students are physically involved in setting it up (kinaesthetic learning). Once the initial problem is solved, there is an extensive set of investigations which develop.

For students less confident with their number facts, the investigation can be continued with concrete materials and Option 1 of the software provides a companion 'electronic table top' that reflects the same way students work with

materials. This means that the same problem can be tackled at different levels in a mixed ability class and each student can feel successful.

Doctor Dart

Doctor Dart is on an adventure. She must get through the anti-gravity door to save the universe, but the Evil Professor has locked the door with an electronic key pad. He explains the rules for operating the pad, then challenges her, with appropriate melodramatic cackling, to find the score that opens the door.

The initial problem is usually solved by the class in 10 - 15 minutes. Then, the door which Doctor Dart metaphorically opens leads to a universe of extensions which are supported by the companion software. These include student's designing their own 'Doctor Dart' problems, which can be done with, or without, computer extensions. The software also introduces Tree Diagrams as a mathematician's strategy tool used in situations where there is a choice.

There is much more than one session possible with this Investigation, but no matter how many are used, it is important to review the lesson from the perspective of the Working Mathematically process. This is suggested in the lesson plan. It is important that students are continually reminded by their personal and joint successes that they can work like a mathematician.

Estimating Fractions

This investigation is very rich because it involves the concept of a fraction, estimation and measurement as well as data collection and number work. It is a whole class version of the task **Peg & Tape Fractions**. The lesson needs to be revisited to explore its depth, so it is included in Weeks 5, 6 and 7 of Year 6.

Students are challenged to estimate parts of a real 'touchable' whole such as a length of cord, the perimeter of a building, the amount of turn about a point (angle) or a distance between two points. The spirit of the challenge is to demonstrate increasing closeness (better estimation) rather than insisting on an exact answer as found by measurement and calculation. In fact, the class is asked to think of ways to test the estimates before using the 'exact' approach. This promotes mathematical conversation which helps to develop a conceptual understanding of fractions. The lesson software provides a wide range of challenges which capture this same spirit. Mathematical conversation is further encouraged if two students work on one computer.

You will need to think ahead to collect the necessary class set of clothes pegs and strips of non-stretchable material such as bias binding, or lengths of cord.

Eureka

This is another investigation with many answers which is included so the class can work together as mathematicians. The story shell that introduces the problem is that Peter, Paul and Mary have struck gold. Between them they have found 12 nuggets, each of which has a different mass. Peter has found 3 nuggets, Paul has found 4 nuggets and Mary has found 5 nuggets. Knowing the various masses and special connections between them, the students are asked to discover which nuggets Mary found.

Asking the mathematician's questions:

- How many solutions are there?
- How will we know when we have found them all?

leads to an extensive investigation which can be tackled as a team by the whole class. There are over 100 solutions so there is plenty of scope for success for all students.

One student comments:

> *Earlier in the year I did an assignment on a maths game entitled Eureka. It consisted of getting twelve different numbers (5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17 and 40) and dividing them into three groups making sure one group got 5 numbers, the second group got 4 numbers and the third group got 3 numbers. The hard part was you had to make the second group's numbers double the third group's numbers. It was pretty easy to find one combination, but it was difficult to find all the combinations. This was a very challenging assignment that I really enjoyed.*

One teacher comments:

> *We have just found a wonderful new investigation - Eureka. I included it because we were doing a theme on the Gold Rush, but it has developed a life of its own. The hunt for all the solutions has kept us all engaged. It is almost as if each solution we found was like another gold nugget!*

First Principles Percent

The challenge in this lesson is to become better at calculating percentages. Following an introduction that highlights the frequency of their use in the media, the emphasis is on the meaning of x% as x in every 100. It is represented in a visual/concrete form and verbalised in every calculation. This verbalisation (strong link with language) is a central theme in keeping the focus on the underlying meaning. Students learn to look for the 100s in the calculation and thereby build the final answer from a set of answers to sub-problems. Mathematicians often use this approach of dealing with sub-problems to build towards a final answer.

The lesson is not about learning a recipe for calculating percentages. Rather it is about students working each calculation from first principles to reinforce the conceptual understanding that underpins these recipes. The lesson pivots on students discussing and justifying their calculations and, in their own time, developing short cuts. After establishing the process, teachers are encouraged to take a back seat. From the beginning, calculators are used in partnership with the discussion to allow students to test their hypotheses in a non-threatening way. The lesson is supported by four worksheets that introduce a sequence of more and more challenging calculations. Once the class has published their 'How To Do Percentages' instructions as suggested in the lesson plan, they can be invited to apply their procedure to text book problems.

Gauss Beats The Teacher

The investigation explores many strategies for adding lists of consecutive numbers that start at 1, or indeed any list with a built in pattern. Visual/concrete models are explored for each of the methods and the Investigation Sheet is included in the sample mathematical landscape of SMJ I. One particular outcome is discovering that the sum of the numbers 1 through 10 is 55. This leads to easily calculating the sum of numbers 1 - 9, which is an important fact used in the complete solution of **Steps** and **Number Tiles**.

Another outcome is the link with the history of mathematics through retelling the legend of the mathematician Carl Gauss as a young student.

From a Working Mathematically viewpoint, the emphasis is on:

- Can I check this another way?
- Simplifying a problem.
- Making and testing hypotheses.

The first of these has an important management component, namely, that independent learners who can check their own work are less likely to put a strain on your time with frequent queries of *Is this right Miss?*.

Multo

Multo is much better than Bingo. The teacher uses 100 cards marked from 0x0 to 9x9. The students draw up a 4row/4column grid and enter 16 numbers which would be answers to these cards. No repeats. The teacher draws cards one at a time and at a regular pace announces each random times table in turn. If they have the answer on their grid, students mark it off. Multo is either:

- 4 in a row horizontally
- 4 in a row vertically
- 4 in a row diagonally
- all four corners

The game involves plenty of times tables practice, but this is soon subservient to the greater challenge of finding the best Multo grid. Software aids the search and can be visited and revisited because there are so many subtleties in the 'best' grid that there is a sense in which the problem is never solved.

The linked assessment sheet provides teachers with considerable assessment information. You might like to include it as a feature in a Self-directed Maths Journey landscape, or as a component of a post-SMJ assessment package.

Number Charts

The lesson plan introduces the skill practice exercise of filling a 4x4 addition or multiplication grid from the limited number of clues given. However it places this practice in the context of exploring the if-then reasoning required to solve the puzzle and the deeper question of how this type of puzzle is constructed. The software opens many possibilities for self-directed work, and students seem to respond well to this twist, which is why it has been included in the activities for a Self-directed Maths Journey for both Years 5 & 6.

Number Tiles

A simply stated problem unfolds a world of mathematics adaptable to a wide range of students. The problem uses:

- nine tiles numbered 1 through 9 and a 3 x 3 grid addition frame

Place the tiles into the frame so that they form a correct addition. That is, the last three digit number is the sum of the other two.

Tearing a piece of paper into nine squares numbered 1 to 9 makes the task very concrete and seems to open the door to participation by all students.

There are many solutions. As a puzzle, finding one or even several solutions provides plenty of practice in basic arithmetic skills. However, the challenge of finding and justifying all solutions (168 in all) lifts the puzzle into an extended investigation and a shift from arithmetic to concepts of strategy and proof.

There are several ways to go about the investigation. One suggestion is to introduce the puzzle and begin the search for solutions as a whole class, then include the continued search for solutions as part of Self-directed Maths Journey III.

However rather than leaving the puzzle at this personal level, the requirement could be to add new solutions to a class display and, following the SMJ week, the class could return to this expanded data set and examine the problem more deeply.

One upper primary class realised that the digits in the answer line of the solutions always added to 18. From there they:

- looked at all the possible 3 digit numbers with a digit sum of 18 (981, 918, 891, 872,...),
- assigned one of these number to each group to investigate
- worked backwards to find the addition with this answer line, if it existed.

Each group's result was recorded on poster paper and the total display was apparently impressive enough for the local paper to run an article on how the class had worked like mathematicians.

Palindromes

- Take any number, say 39.
- Reverse it to give 93.
- Now add these, which gives 132.
- Now reverse 132 to get 231.
- Add these two, which gives 363.

363 is palindromic. A palindromic numeral reads the same forwards or backwards.

The challenge behind this investigation is:

- Does every sequence of reversing and adding always produce a palindromic numeral?

Once students have tested for all the numbers say from 1 to 200, the supporting computer program allows them to test large numbers to look for either exceptions or confirmation of the conjecture.

To date no one has been able to prove the conjecture - one of your students could be the first - but neither has anyone found a number that doesn't work. There are likely candidates, 196 for instance, but computers have been used to calculate the reversals for several thousand digits and still have not reached a conclusion. The

software has an option which emulates this real world approach. It is also impressive for the size of the numbers it can display. This may be the first time students have seen what really, really large numbers look like.

Palindromes also exist in language as words or sentences that read the same forwards and backwards. This gives the lesson a useful link to the language curriculum.

Rod Mats

Consistent with both the concept of 'threading' (see Story 1 in the section Curriculum Planning Stories) and the historical development of human understanding of fractions over a very long time, this lesson is included as teacher-led small group work (Rod Mat Chat) in each of the Self-directed Maths Journey examples. The other reason for this approach is that the investigation is materials-dependent and Cuisenaire Rods, which are essential to the lesson, are not as available in schools as they once were. Other material can be substituted *provided* the wholes and parts are *not marked* with specific fraction relationships.

The key to the Rod Mat Chat is that any rod (or collection of rods) can be chosen as the whole, so pre-marked pieces are of no use. There are several reasons why Cuisenaire Rods are the best material for this lesson, so teachers are encouraged to hunt out, or purchase, at least one set. If the school has **Maths With Attitude** Number and Computation Years 3 & 4, the tasks **Rod Mats** and **Make The Whole** are included. They contain sufficient equipment to carry out the Rod Mat Chats with a small group.

Steps

See the notes on the task **Steps** on Page 29 for an outline. The Maths300 lesson plan expands this description into a whole class investigation and includes software that can aid the search for solutions. Using concrete materials, and the fact that there are 96 solutions to the challenge usually provides sufficient scope for all students to participate productively. The software also, as with *Number Tiles*, demonstrates that the problem solving strategy of examining all possible cases can be used, albeit with the help of a computer programmer.

What's It Worth?

The principle behind this threaded investigation is simple. Create a shape of some sort that is made up of equal size parts.

- If each Part is worth a given amount, what is the total shape worth?

This is a straightforward whole number question IF the Part is worth a whole number. But the part could be worth a decimal, or a fraction. Further, suppose we have to work backwards, ie: we know the value of the Whole and have to calculate the value of the Part.

This open-ended, rich and revisitable lesson begins in a spatial/concrete setting using an intriguing shape called a Sphinx which is provided as a reproducible master. There are possible side trips into spatial investigations as a result and the lesson offers endless value relations opportunities.

Students are encouraged to create their own value relations puzzles for others to solve, and are offered a large range of these puzzles in the companion software.

The investigation is included in two of the four SMJs - one in Year 5 and one in Year 6. Again this is consistent with the pedagogy of threading 'small' challenges more frequently through the curriculum to give students time and reason to construct and reconstruct their concept of ratio and fractions, and the ways to operate on them.

Find more sphinx shapes in the Mathematics Centre Resources link:

- http://www.mathematicscentre.com/taskcentre/resource.htm

Part 3: Value Adding

The Poster Problem Clinic

Maths With Attitude kits offer several models for building a Working Mathematically curriculum around tasks. Each kit uses a different model, so across the range of 16 kits, teachers' professional learning continues and students experience variety. The Poster Problem Clinic is an additional model. It can be used to lead students into working with tasks, or it can be used in a briefer form as a opening component of each task session.

> *I was apprehensive about using tasks when it seemed such a different way of working. I felt my children had little or no experience of problem solving and I wanted to prepare them to think more deeply. The Clinic proved a perfect way in.*

Careful thought needs to be given to management in such lessons. One approach to getting the class started on the tasks and giving it a sense of direction and purpose is to start with a whole class problem. Usually this is displayed on a poster that all can see, perhaps in a Maths Corner. Another approach is to print a copy for each person. A Poster Problem Clinic fosters class discussion and thought about problem solving strategies.

Starting the lesson this way also means that just prior to liberating the students into the task session, they are all together to allow the teacher to make any short, general observations about classroom organisation, or to celebrate any problem solving ideas that have arisen.

One teacher describes the session like this:

> *I like starting with a class problem - for just a few minutes - it focuses the class attention, and often allows me to introduce a particular strategy that is new or needs emphasis.*

It only takes a short time to introduce a poster and get some initial ideas going. The class discussion develops a way of thinking. It allows class members to hear, and learn from their peers, about problem solving strategies that work for them.

> *If we don't collectively solve the problem in 5 minutes, I will leave the problem 'hanging' and it gives a purpose to the class review session at the end.*
> *Sometimes I require everyone to work out and write down their solution to the whole class problem. The staggered finishing time for this allows me to get organised and help students get started on tasks without being besieged.*
> *I try to never interrupt the task session, but all pupils know we have a five minute review session at the end to allow them to comment on such things as an activity they particularly liked. We often close then with an agreed answer to our whole class problem.*

A Clinic in Action

The aims of the regular clinic are:

- to provide children with the opportunity to learn a variety of strategies
- to familiarise children with a process for solving problems.

The following example illustrates a structure which many teachers have found successful when running a clinic.

Preparation

For each session teachers need:

- a Strategy Board as below
- a How To Solve A Problem chart as below
- to choose a suitable problem and prepare it as a poster
- to organise children into groups of two or three.

The Strategy Board can be prepared in advance as a reference for the children, or may be developed *with* the children as they explore problem solving and suggest their own versions of the strategies.

The problem can be chosen from

- a book
- the task collection
- prepared collections such as Professor Morris Puzzles which can be viewed at: http://www.mathematicscentre.com/taskcentre/resource.htm#profmorr

The example which follows is from the task collection. The teacher copied it onto a large sheet of paper and asked some children to illustrate it. *The teacher also changed the number of sheep to sixty* to make the poster a little different from the one in the task collection.

The Strategy Board and the How To Solve A Problem chart can be used in any maths activity and are frequently referred to in Maths300 lessons.

The Clinic

The poster used for this example session is:

> Eric the Sheep is lining up to be shorn before the hot summer ahead. There are sixty [60] sheep in front of him. Eric can't be bothered waiting in the queue properly, so he decides to sneak towards the front.
>
> Every time one [1] sheep is taken to be shorn, Eric then sneaks past two [2] sheep. How many sheep will be shorn before Eric?

This Poster Problem Clinic approach is also extensively explored in Maths300 Lesson 14, *The Farmer's Puzzle*.

Strategy Board

DO I KNOW A SIMILAR PROBLEM?	ACT IT OUT
GUESS, CHECK AND IMPROVE	DRAW A PICTURE OR GRAPH
TRY A SIMPLER PROBLEM	MAKE A MODEL
WRITE AN EQUATION	LOOK FOR A PATTERN
MAKE A LIST OR TABLE	TRY ALL POSSIBILITIES
WORK BACKWARDS	SEEK AN EXCEPTION
BREAK INTO SMALLER PARTS	...

How To Solve A Problem

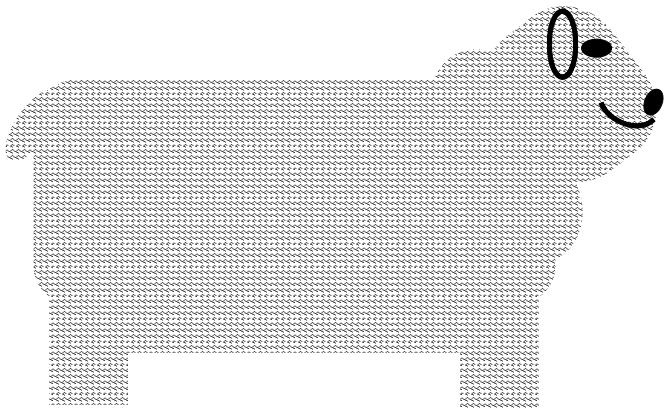
SEE & UNDERSTAND	Do I understand what the problem is asking? Discuss
PLANNING	Select a strategy from the board. Plan how you intend solving the problem.
DOING IT	Try out your idea.
CHECK IT	Did it work out? If so reflect on the activity. If not, go back to step one.

Step 1

- Tell the children that we are at Stage 1 of our four stage plan ... **See & Understand** ... Point to it! Read the problem with the class. Discuss the problem and clarify any misunderstandings.
- If children do not clearly understand what the problem is asking, they will not cope with the next stage. A good way of finding out if a child understands a problem is for her/him to retell it.
- Allow time for questions - approximately 3 to 5 minutes.

Step 2

- Tell the children that we are at Stage 2 of our four stage plan ... **Planning**. In their groups children select one or more strategies from the Strategy Board and discuss/organise how to go about solving the problem.
- Without guidance, children will often skip this step and go straight to Doing It. It is vital to emphasise that this stage is simply planning, not solving, the problem.
- After about 3 minutes, ask the children to share their plans.



Plan 1

Well we're drawing a picture and sort of making a model.

Can you give me more information please Brigid?

We're putting 60 crosses on our paper for sheep and the pen top will be Eric. Then Claire will circle one from that end, and I will pass two crosses with my pen top.

Plan 2

Our strategy is Guess and Check.

That's good Nick, but how are you going to check your guess?

Oh, we're making a model.

Go on ...

> *John's getting MAB smalls to be sheep and I'm getting a domino to be Eric and the chalk box to be the shed for shearing.*

Plan 3

> *We are doing it for 3 sheep then 4 sheep then 5 sheep and so on. Later we will look at 60.*

Great so you are going to try a simpler problem, make a table and look for a pattern.

This sharing of strategies is invaluable as it provides children who would normally feel lost in this type of activity with an opportunity to listen to their peers and make sense out of strategy selection. Note that such children are not given the answer. Rather they are assisted with understanding the power of selecting and applying strategies.

Step 3

- Tell the children that we are at Stage 3 of our four stage plan ... **Doing It**. Children collect what they need and carry out their plan.

Step 4

- Tell the children that we are at Stage 4 of our four stage plan ... **Check It**. Come together as a class for groups to share their findings. Again emphasis is on strategies.

> *We used the drawing strategy, but we changed while we were doing it because we saw a pattern.*

So Jake, you used the Look For A Pattern strategy. What was it?

> *We found that when Eric passed 10 sheep, 5 had been shorn, so 20 sheep meant 10 had been shorn ... and that means when Eric passes 40 sheep, 20 were shorn and that makes the 60 altogether.*

Great Jake. How would you work out the answer for 59 sheep or 62 sheep?

Sharing time is also a good opportunity to add in a strategy which no one may have used. For example:

> *Maybe we could've used the Number Sentence strategy, ie: 1 sheep goes to be shorn and Eric passes two sheep. That's 3 sheep, so perhaps, 60 divided into groups of 3, or 60 ÷ 3 gives the answer.*

Round off the lesson by referring to the Working Mathematically chart. There will be many opportunities to compliment the students on working like a mathematician.

Curriculum Planning Stories

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

In more than a decade of using tasks and many years of using the detailed whole class lessons of Maths300, teachers have developed several models for integrating tasks and whole class lessons. Some of those stories are retold here. Others can be found at:

- http://www.mathematicscentre.com/taskcentre/plans.htm

Story 1: Threading

> *Educational research caused me a dilemma. It tells us that students construct their own learning and that this process takes time. My understanding of the history of mathematics told me that certain concepts, such as place value and fractions, took thousands of years for mathematicians to understand. The dilemma was being faced with a textbook that expected students to 'get it' in a concentrated one, two or three week block of work and then usually not revisit the topic again until the next academic year.*

A Working Mathematically curriculum reflects the need to provide time to learn in a supportive, non-threatening environment and…

> *When I was involved in a Calculating Changes PD program I realised that:*
>
> - *choosing rich and revisitable activities, which are familiar in structure but fresh in challenge each time they are used, and*
> - *threading them through the curriculum over weeks for a small amount of time in each of several lessons per week*
>
> *resulted in deeper learning, especially when partnered with purposeful discussion and recording.*

Calculating Changes:

- http://www.mathematicscentre.com/calchange

Story 2: Your turn

Some teachers are making extensive use of a partnership between the whole class lessons of Maths300 and small group work with the tasks. Setting aside a lesson for using the tasks in the way they were originally designed now seems to have more meaning, as indicated by this teacher's story:

> *When I was thinking about helping students learn to work like a mathematician, my mind drifted to my daughter learning to drive. She*

needed me to model how to do it and then she needed lots of opportunity to try it for herself.

That's when the idea clicked of using the Maths300 lessons as a model and the tasks as a chance for the students to have their turn to be a mathematician.

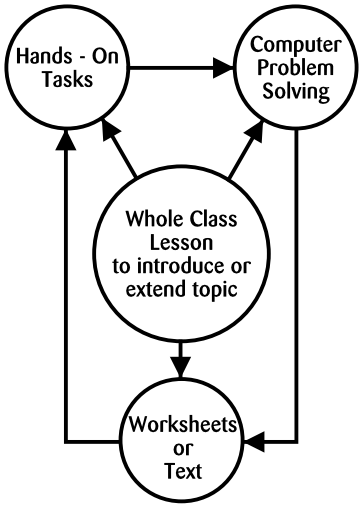
The Maths300 lessons illustrate how other teachers have modelled the process, so I felt I could do it too. Now the process is always on display on the wall or pasted inside the student's journal.

A session just using the tasks had seemed a bit like play time before this. Now I see it as an integral part of learning to work mathematically.

Story 3: Mixed Media

It was our staff discussion on Gardner's theory of Multiple Intelligences that led us into creating mixed media units. That and the access you have provided to tasks and Maths300 software.

We felt challenged to integrate these resources into our syllabus. There was really no excuse for a text book diet that favours the formal learners. We now often use four different modes of learning in the work station structure shown. It can be easily managed by one teacher, but it is better when we plan and execute it together.



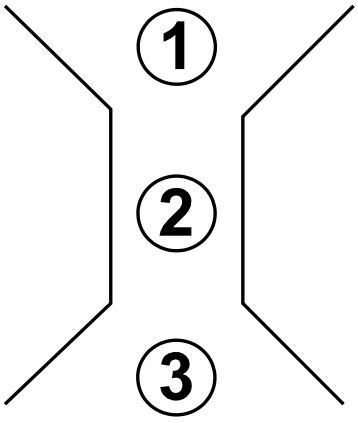
Story 4: Replacement Unit

We started meeting with the secondary school maths teachers to try to make transition between systems easier for the students. After considerable discussion we contracted a consultant who suggested that school might look too much the same across the transition when the students were hoping for something new. On the other hand our experience suggested that there needed to be some consistency in the way teachers worked.

We decided to 'bite the bullet' and try a hands-on problem solving unit in one strand. We selected two menus of twenty hands-on tasks, one for the primary and one for the secondary, that became the core of the unit. We deliberately overlapped some tasks that we knew were very rich and added some new ones for the high school.

Class lessons and investigation sheets were used to extend the tasks, within a three week model.

It is important to note that although these teachers structured a 3 week unit for the students, they strongly advised an additional *Week Zero* for staff preparation. The units came to be called Replacement Units.



Week Zero - Planning

Staff familiarise themselves with the material and jointly plan the unit. This is not a model that can be 'planned on the way to class'.

> *Getting together turned out to be great professional development for our group.*

Week 1 - Introduction

Students explore the 20 tasks listed on a printed menu:

- students explore the tip of the task, as on the card
- students move from task to task following teacher questioning that suggests there is more to the task than the tip
- in discussion with students, teachers gather informal assessment information that guides lesson planning for the following week.

> *We gave the kids an 'encouragement talk' first about joining us in an experiment in ways of learning maths and then gave out the tasks. The response was intelligent and there was quite a buzz in the room.*

Week 2 - Formalisation

> *It was good for both us and the students that the lessons in this week were a bit more traditional. However, they weren't text book based. We used whole class lessons based on the tasks they had been exploring and taught the Working Mathematically process, content and report writing.*
>
> *Assessment was via standard teacher-designed tests, quizzes and homework.*

Week 3 - Investigations

> *We were most delighted with Week 3. Each student chose one task from the menu and carried out an in-depth investigation into the iceberg guided by an investigation sheet. They had to publish a report of their investigation and we were quite surprised at the outcomes. It was clear that the first two weeks had lifted the image of mathematics from 'boring repetition' to a higher level of intellectual activity.*

Story 5: Curriculum shift

> *I think our school was like many others. The syllabus pattern was 10 units of three weeks each through the year. We had drifted into that through a text book driven curriculum and we knew the students weren't responding.*

Our consultant suggested that there was sameness about the intellectual demands of this approach which gave the impression that maths was the pursuit of skills. We agreed to select two deeper investigations to add to each unit. It took some time and considerable commitment, but we know that we have now made a curriculum shift. We are more satisfied and so are the students.

The principles guiding this shift were:

- Agree

  The 20 particular investigations for the year are agreed to by all teachers. If, for example, Cube Nets is decided as one of these, then all the teachers are committed to present this within its unit.

- Publish

  The investigations are written into the published syllabus. Students and parents are made aware of their existence and expect them to occur.

- Commit

  Once agreed, teachers are required to present the chosen investigations. They are not a negotiable 'extra'.

- Value

  The investigations each illustrate an explicit form of the Working Mathematically process. This is promoted to students, constantly referenced and valued.

- Assess

  The process provides students with scaffolding for their written reports and is also known by them as the criteria for assessment. (See next page.)

- Report

  The assessment component features within the school reporting structure.

A Final Comment

Including investigations has become policy.

Why? Because to not do so is to offer a diminished learning experience.

The investigative process ranks equally with skill development and needs to be planned for, delivered, assessed and reported.

Perhaps most of all we are grateful to our consultant because he was prepared to begin where we were. We never felt as if we had to throw out the baby and the bath water.

Assessment

Our attitude is:

stimulated students are creative and love to learn

Regardless of the way you use your **Maths With Attitude** resource, a variety of procedures can be employed to assess this learning.

Where these assessment procedures are applied to task sessions and involve written responses from students, teachers will need to be careful that the writing does not become too onerous. Students who get bogged down in doing the writing may lose interest in doing the tasks.

In addition to the ideas below, useful references are:

- http://www.mathematicscentre.com/taskcentre/assess.htm
- http://www.mathematicscentre.com/taskcentre/report.htm

The first offers several methods of assessment with examples and the second is a detailed lesson plan to support students to prepare a Maths Report.

Journal Writing

Journal writing is a way of determining whether the task or lesson has been understood by the student. The pupil can comment on such things as:

- What I learned in this task.
- What strategies I/we tried (refer to the Strategy Board).
- What went wrong.
- How I/we fixed it.
- Jottings - ie: any special thoughts or observations

Some teachers may prefer to have the page folded vertically, so that children's reflective thoughts can be recorded adjacent to critical working.

Assessment Form

An assessment form uses questions to help students reflect upon specific issues related to a specific task.

Anecdotal Records

Some teachers keep ongoing records about how students are tackling the tasks. These include jottings on whether students were showing initiative, whether they were working co-operatively, whether they could explain ideas clearly, whether they showed perseverance.

Checklists

A simple approach is to create a checklist based on the Working Mathematically process. Teachers might fill it in following questioning of individuals, or the students may fill it in and add comments appropriately.

Pupil Self-Reflection

Many theorists value and promote metacognition, the notion that learning is more permanent if pupils deliberately and consciously analyse their own learning. The

deliberate teaching strategy of oral questioning and the way pupils record their work is an attempt to manifest this philosophy in action. The alternative is the tempting 'butterfly' approach which is to madly do as many activities as possible, mostly superficially, in the mistaken belief that quantity equates to quality.

> *I had to work quite hard to overcome previously entrenched habits of just getting the answer, any answer, and moving on to the next task.*

Thinking about *what* was learned *how* it was learned consolidates and adds to the learning.

When it follows an extensive whole class investigation, a reflection lesson such as this helps to shift entrenched approaches to mathematics learning. It is also an important component of the assessment process. On the one hand it gives you a lot of real data to assist your assessment. On the other it prepares the students for any formal assessment which you may choose to round off a unit.

Introduction

Ask students to recall what was done during the unit or lesson by asking a few individuals to say what *they* did, eg:

> *What did you do or learn that was new?*
> *What can you now do/understand that is new?*
> *What do you know now that you didn't know 1 (2, 3, ...) lesson ago?*

Continuing Discussion

Get a few ideas from the first students you ask, then:

- organise 5 -10 minute buzz groups of three or four students to chat together with one person to act as a recorder. These groups address the same questions as above.
- have a reporting session, with the recorder from each group telling the class about the group's ideas.

Student comments could be recorded on the board, perhaps in three groups.

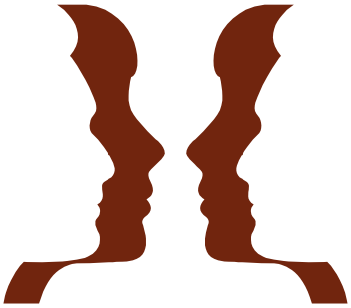
Ideas & Facts	Maths Skills	Process (learning) Skills

If you need more questions to probe deeper and encourage more thought about process, try the following:

> *What new things did you do that were part of* how *you learned?*
> *Who uses this kind of knowledge and skill in their work?*

Student Recording

Hand out the REFLECTION sheet (next page) and ask students to write their own reflection about what they did, based on the ideas shared by the class. Collect these for interest and, possibly, assessment information.



REFLECTION

me looking at me learning

NAME: CLASS:

Reproducible Page

Working With Parents

Balancing Problem Solving with Basic Skill Practice

Many schools find that parents respond well to an evening where they have an opportunity to work with the tasks and perhaps work a task together as a 'whole class'. Resourced by the materials in this kit, teachers often feel quite confident to run these practical sessions. Comments from parents like:

> *I wish I had learnt maths like this.*

are very supportive. Letting students 'host' the evening is an additional benefit to the home/school relationship.

The 4½ Minute Talk

Charles Lovitt has considerable experience working with parents and has developed a crisp, parent-friendly talk which he shares below. Many others have used it verbatim with great success.

Why the Four and a Half Minute Talk?

When talking with parents about Problem Solving or the meaning of the term Working Mathematically, I have often found myself in the position, after having promoted inquiry based or investigative learning, of the parents saying:

> *Well - that's all very well - BUT ...*

at which stage they often express their concern for basic (meaning arithmetic) skill development.

The weakness of my previous attempts has been that I have been unable to reassure parents that problem solving does not mean sacrificing our belief in the virtues of such basic skill development.

One of the unfortunate perceptions about problem solving is that if a student is engaged in it, then somehow they are not doing, or it may be at the expense of, important skill based work.

This Four and a Half Minute Talk to parents is an attempt to express my belief that basic skill practice and problem solving development can be closely intertwined and not seen as in some way mutually exclusive.

(I'm still somewhat uncomfortable using the expression 'basic skills' in the above way as I am certain that some thinking, reasoning, strategy and communication skills are also 'basic'.)

Another aspect of the following 'talk' is that, as teachers put more emphasis on including investigative problem solving into their courses, a question arises about the source of suitable tasks.

This talk argues that we can learn to create them for ourselves by 'tweaking' the closed tasks that heavily populate out existing text exercises, and hence not be dependent on external suppliers. (Even better if students begin to create such opportunities for themselves.)

The Talk

In preparation, write the following graphic on the board:

CLOSED	OPEN	EXTENDED INVESTIGATION
		How many solutions exist? How do you know you have found them all?

I would like to show you what teachers are beginning to do to achieve some of the thinking and reasoning and communication skills we hope students will develop. I would like to show you three examples.

Example One: *6 + 5 = ?*

I write this question under the 'closed' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 —		How many solutions exist? How do you know you have found them all?

And I ask:

What is the answer to this question?

I then explain that:

We often ask students many closed questions such as 6 + 5 = ?

The only response the students can tell us is "The answer is 11." … and as a reward for getting it correct we ask another twenty questions just like it.

What some teachers are doing is trying to *tweak* the question and ask it a different way, for example:

I have two counting numbers that add to 11. What might the numbers be?

[Counting numbers = positive whole numbers including zero]

I write this under the 'open' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 ___	? + ? 11	How many solutions exist? How do you know you have found them all?

What is the answer to the question now?

At this stage it becomes apparent there are several solutions:

The question is now a bit more open than it was before, allowing students to tell you things like 8 + 3, or 10 + 1, or 11 + 0 etc.

Let's see what happens if the teacher 'tweaks' it even further with the investigative challenge *or* extended investigation question:

How many solutions are there altogether?

and more importantly, and with greater emphasis on the second question:

How could you convince someone else that you have found them all?

Now the original question is definitely different - it still involves the skills of addition but now also involves thinking, reasoning and problem solving skills, strategy development and particularly communication skills.

Young students will soon tell you the answer is 'six different ones', but they must also confront the communication and reasoning challenge of convincing you that there are only six and no more.

Example Two: *Finding Averages*

Again, as I go through this example, I write it into the diagram on the board in the relevant sections.

The CLOSED question is: *11, 12, 13 - find the average*

Tweaking this makes it an OPEN question and it becomes:

I have three counting numbers whose average is 12. What might the numbers be?

Students will often say:

10, 12, 14 … or 9, 12, 15 … or even 12, 12, 12

After realising there are many answers, you can tweak it some more and turn it into an EXTENDED INVESTIGATION:

How many solutions exist? … AND …

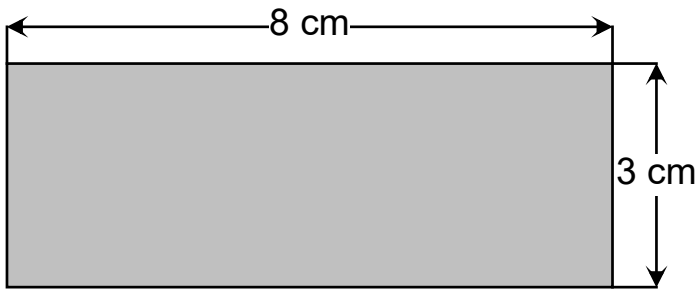
How do you know you have found them all?

Now the question is of a quite different nature. It still involves the arithmetic skill, but has something else as well - and that something else is the thinking, reasoning and communication skills necessary to find all of the combinations and convince someone else that you have done so.

By the time a student announces, with confidence, there are 127 different ways (which there are) that student will have engaged in all of these aspects, ie: the skill of calculating averages, (and some combination number theory) as well as significant strategy and reasoning experiences.

Example Three: *Finding the Area of a Rectangle*

A typical CLOSED question is:



Find the area. Find the perimeter.

The OPEN question is:

A rectangle has 24 squares inside:

What might its length and width be?

What might its perimeter be?

The EXTENDED INVESTIGATION version is:

Given they are whole number lengths, how many different rectangles are there? … AND …

How do you know you have found them all?

In summary, mathematics teachers are trying to convert *some* (not all) of the many closed questions that populate our courses and 'push' them towards the investigation direction. In doing so, we keep the skills we obviously value, but also activate the thinking, reasoning and justification skills we hope students will also develop.

This sequence of three examples hopefully shows two major features:

- That skills and problem solving can 'live alongside each other' and be developed concurrently.
- That the process of creating open-ended investigations can be done by anyone - just go to any source of closed questions and try 'tweaking' them as above. If it only worked for one question per page it would still provide a very large supply of investigations.

In terms of the effect of the talk on parents, I have usually found them to be reassured that we are not compromising important skill development (and nor do we want to). The only debate then becomes whether the additional skills of thinking, reasoning and communication are also desirable.

I've also been told that parents appreciate it because of the essential simplicity of the examples - no complicated theoretical jargon.

A Working Mathematically Curriculum

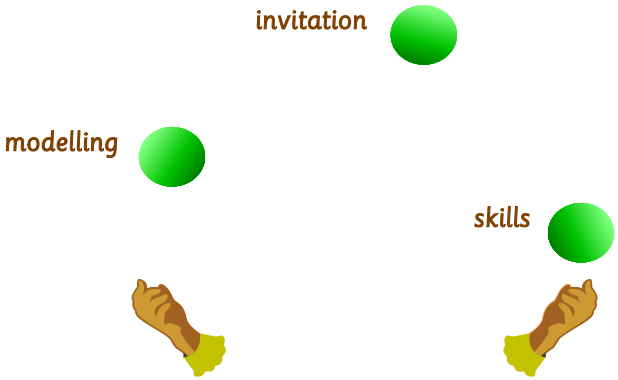
An Investigative Approach to Learning

The aim of a Working Mathematically curriculum is to help students learn to work like a mathematician. This process is detailed earlier (Page 8) in a one page document which becomes central to such a curriculum.

The change of emphasis brings a change of direction which *implies and requires* a balance between:

- the process of being a mathematician, and
- the development of skills needed to be a *successful* mathematician.

This journey is not two paths. It is one path made of two interwoven threads in the same way as DNA, the building block of life, is one compound made of two interwoven coils. To achieve a Working Mathematically curriculum teachers need to balance three components.



The task component of **Maths With Attitude** offers each pair of students an invitation to work like a mathematician.

The Maths300 component of **Maths With Attitude** assists teachers to model working like a mathematician.

Content skills are developed in context. They *are* important, but it is the application of skills within the process of Working Mathematically that has developed, and is developing, the human community's mathematical knowledge.

A focus for the Working Mathematically teacher is to help students develop mathematical skills in the context of problem posing and solving.

> *We are all 'born' with the same size mathematical toolbox, in the same way as I can own the same size toolbox as my motor mechanic. However, my motor mechanic has many more tools in her box than I and she has had more experience than I using them in context. Someone has helped her learn to use those tools while crawling under a car.*

Afzal Ahmed, Professor of Mathematics at Chichester, UK, once quipped:

> *If teachers of mathematics had to teach soccer, they would start off with a lesson on kicking the ball, follow it with lessons on trapping the ball and end with a lesson on heading the ball. At no time would they play a game of football.*

Such is not the case when teaching a Working Mathematically curriculum.

Elements of a Working Mathematically Curriculum

Working Mathematically is a K - 12 experience offering a balanced curriculum structured around the components below.

Hands-on Problem Solving Play

Mathematicians don't know the answer to a problem when they start it. If they did, it wouldn't be a problem. They have to play around with it. Each task invites students to play with mathematics 'like a mathematician'.

Skill Development

A mathematician needs skills to solve problems. Many teachers find it makes sense to students to place skill practice in the context of *Toolbox Lessons* which *help us better use the Working Mathematically Process* (Page 8).

Focus on Process

This is what mathematicians do; engage in the problem solving process.

Strategy Development

Mathematicians also make use of a strategy toolbox. These strategies are embedded in Maths300 lessons, but may also have a separate focus. Poster Problem Clinics are a useful way to approach this component.

Concept Development

A few major concepts in mathematics took centuries for the human race to develop and apply. Examples are place value, fractions and probability. In the past students have been expected to understand such concepts after having 'done' them for a two week slot. Typically they were not visited again until the next year. A Working Mathematically curriculum identifies these concepts and regularly 'threads' them through the curriculum.

Planning to Work Mathematically

The class, school or system that shifts towards a Working Mathematically curriculum will no longer use a curriculum document that looks like a list of content skills. The document would be clear in:

- choosing genuine problems to initiate investigation
- choosing a range of best practice teaching strategies to interest a wider range of students
- practising skills for the purpose of problem solving

Some teachers have found the planning template on the next page assists them to keep this framework at the forefront of their planning. It can be used to plan single lessons, or units built of several lessons. There are examples from schools in the Curriculum & Planning section of Maths300 and a Word document version of the template.

Unit Planning Page

Class

Topic

Pedagogy	Problem Solving In this topic how will I engage my students in the Working Mathematically process?	Skills
How do I create an environment where students know what they are doing and why they have accepted the challenge?		Does the challenge identify skills to practise? Are there other skills to practise in preparation for future problem solving?

Notes

As a general guide:

- Find a problem(s) to solve related to the topic.
- Choose the best teaching craft likely to engage the learners.
- Where possible link skill practice to the problem solving process.

More on Professional Development

For many teachers there will be new ideas within **Maths With Attitude**, such as unit structures, views of how students learn, teaching strategies, classroom organisation, assessment techniques and use of concrete materials. It is anticipated (and expected) that as teachers explore the material in their classrooms they will meet, experiment with and reflect upon these ideas with a view to long term implications for the school program and for their own personal teaching.

Being explored 'on-the-job' so to speak, in the teacher's own classroom, makes the professional development more meaningful and practical for the teacher. This is also a practical and economic alternative for a local authority.

Strategic Use by Systems

From Years 3 - 10, **Maths With Attitude** is designed as a professional development vehicle by schools or clusters or systems because it carries a variety of sound educational messages. They might choose **Maths With Attitude** because:

- It can be used to highlight how investigative approaches to mathematics can be built into balanced unit plans without compromising skill development and without being relegated to the margins of a syllabus as something to be done only after 'the real' content has been covered.
- It can be used to focus on how a balance of concept, skill and application work can all be achieved within the one manageable unit structure.
- It can be used to show how a variety of assessment practices can be used concurrently to build a picture of student progress.
- It can be used to focus on transition between primary and secondary school by moving towards harmony and consistency of approach.
- It can be used to raise and continue debate about the pedagogy (art of teaching) that supports deeper mathematical learning for a wider range of students.

Teachers in Years K - 2 are similarly encouraged in professional growth through **Working Mathematically with Infants**, which derives from Calculating Changes, a network of teachers enhancing children's number skills from Years K - 6.

In supporting its teachers by supplying these resources in conjunction with targeted professional development over time, a system can fuel and encourage classroom-based debate on improving outcomes. There is evidence that by exploring alternative teaching strategies and encouraging curriculum shift towards Working Mathematically, learners improve and teachers are more satisfied. For more detail visit Research & Stories at:

- http://www.mathematicscentre.com/taskcentre/do.htm

We would be happy to discuss professional development with system leaders.

Web Reference

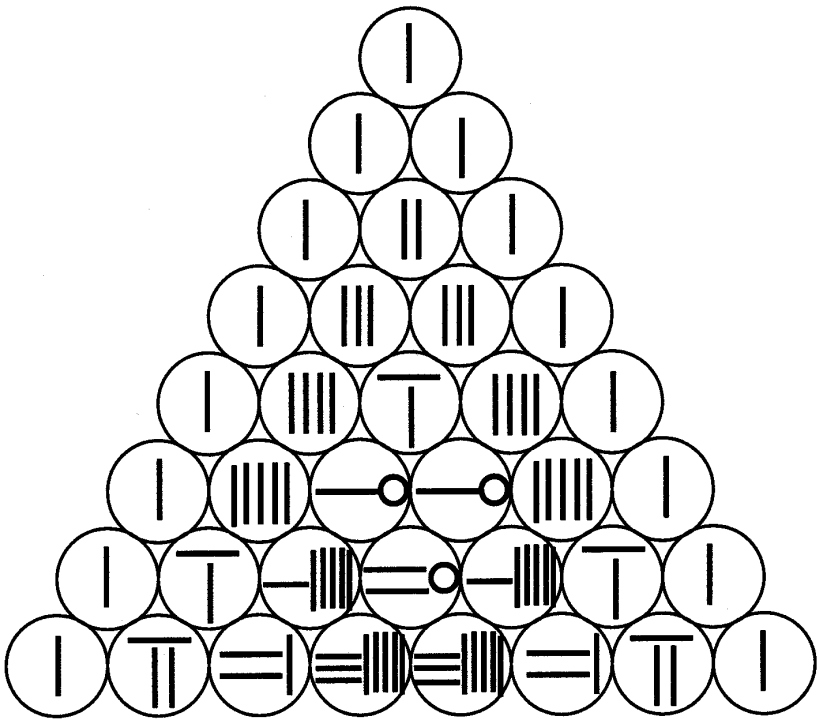
The starting point for all aspects of learning to work like a mathematician, including Calculating Changes, and the teaching craft which encourages it is:

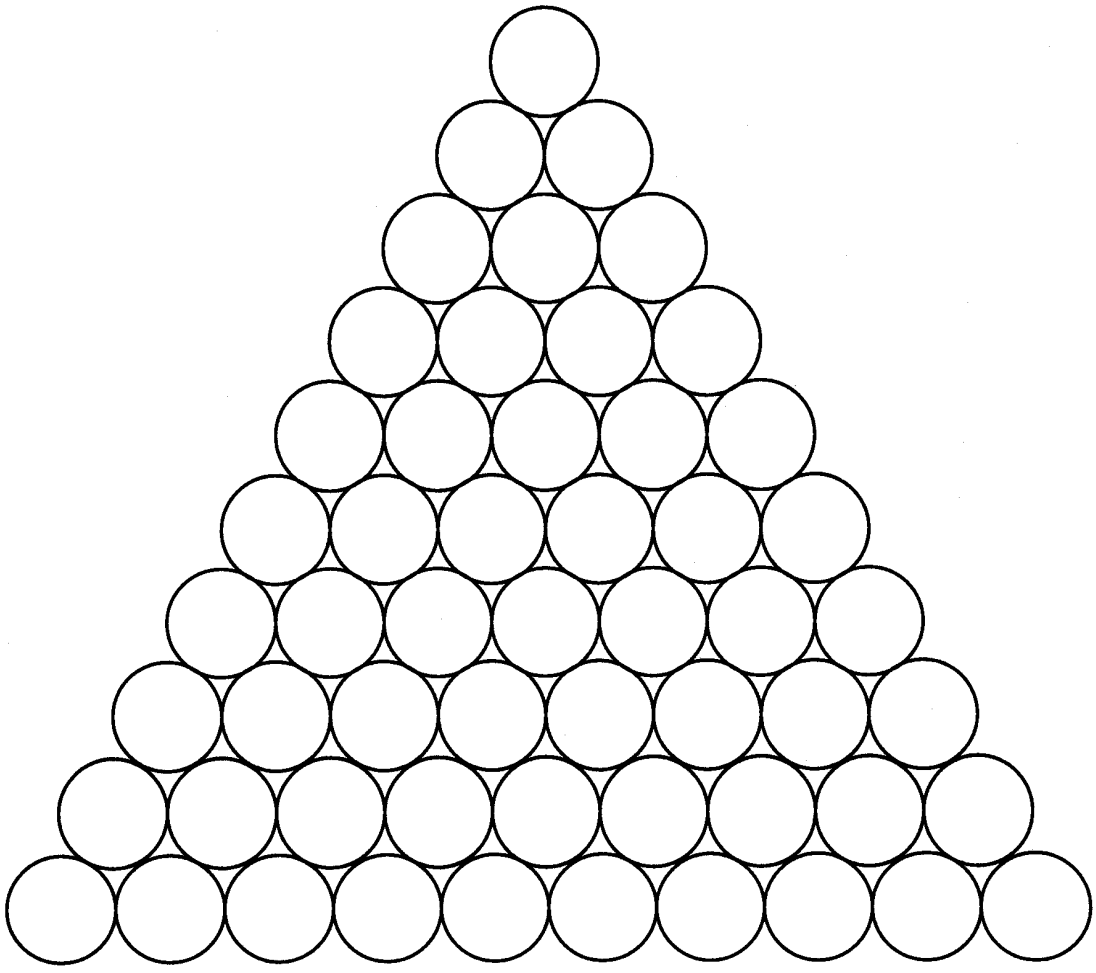
- http://www.mathematicscentre.com/mathematicscentre

Appendix: Recording Sheets

Pascal's Triangle in Asia

Names: ..... Class: .....





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