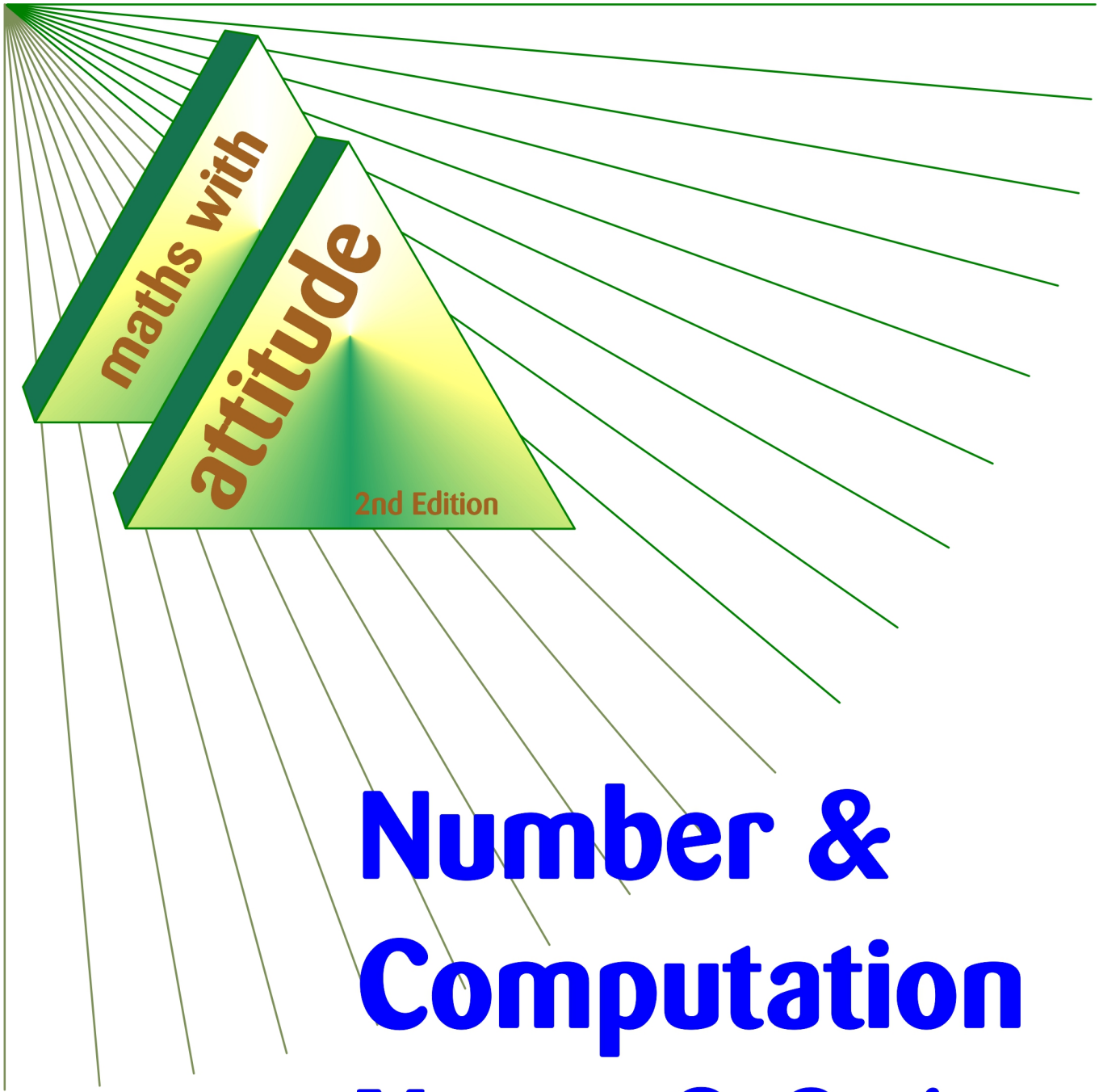

Number & Computation Years 3 & 4

Charles Lovitt
Doug Williams

Mathematics Task Centre & Maths300
helping to create happy healthy cheerful productive inspiring classrooms

Number & Computation

Years 3 & 4

In this kit:

- *Hands-on problem solving tasks*
- *Detailed curriculum planning*

Access from Maths300:

- *Extensive lesson plans*
- *Software*

Doug Williams
Charles Lovitt

The **Maths With Attitude** series has been developed by The Task Centre Collective and is published by Black Douglas Professional Education Services.

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Maths With Attitude includes reference to hands-on problem solving tasks and Maths300 material.

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Part 1: Preparing To Teach

Our Objective

- To support teachers, schools and systems wanting to create:

  happy, healthy, cheerful, productive, inspiring classrooms

Our Attitude

- to learning:

  learning is a personal journey stimulated by achievable challenge
- to learners:

  stimulated students are creative and love to learn
- to pedagogy:

  the art of choosing teaching strategies to involve and interest all students
- to mathematics:

  mathematics is concrete, visual and makes sense
- to learning mathematics:

  all students can learn to work like a mathematician
- to teachers:

  the teacher is the most important resource in education
- to professional development:

  teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

Our Objective in Detail

What do we mean by creating:

happy, healthy, cheerful, productive, inspiring classrooms

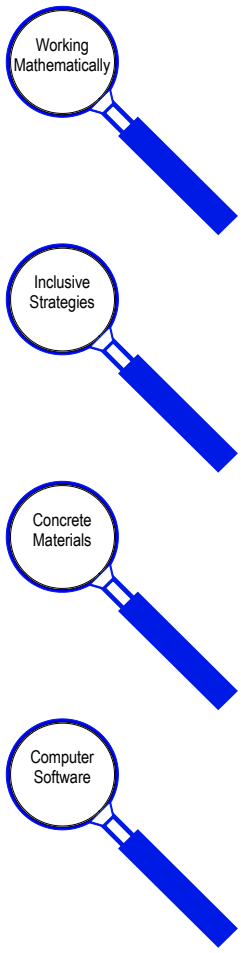
Happy…

means the elimination of the unnecessary fear of failure that hangs over so many students in their mathematics studies. Learning experiences *can* be structured so that all students see there is something in it for them and hence make a commitment to the learning. In so many 'threatening' situations, students see the impending failure and withhold their participation.

A phrase which describes the structure allowing all students to perceive something in it for them is *multiple entry points and multiple exit points*. That is, students can enter at a variety of levels, make progress and exit the problem having visibly achieved.

Healthy…

means *educationally healthy*. The learning environment should be a reflection of all that our community knows about how students learn. This translates into a rich array of teaching strategies that could and should be evident within the learning experience.



If we scrutinise the *exploration* through any lens, it should confirm to us that it is well structured or alert us to missed opportunities. For example, peering through a pedagogy lens we should see such features as:

- a story shell to embed the situation in a meaningful context
- significant active use of concrete materials
- a problem solving challenge which provides ownership for students
- small group work
- a strong visual component
- access to supportive software

Cheerful…

because we want 'happy' in the title twice!

Productive…

is the clear acknowledgment that students are working towards recognisable outcomes. They should know what these are and have guidelines to show they have either reached them or made progress. Teachers are accountable to these outcomes as well as to the quality of the learning environment.

Inspiring…

is about creating experiences that are uplifting or exalting; that actually *turn students on*. Experiences that make students feel great about themselves and empowered to act in meaningful ways.

Number & Computation Resources

To help you create

happy, healthy, cheerful, productive, inspiring classrooms

this kit contains

- 20 hands-on problem solving tasks from Mathematics Centre

and a Teachers' Manual which integrates the use of the tasks with

- 13 detailed lesson plans from Maths300

The kit offers **10 weeks** of Scope & Sequence planning in Number & Computation for *each* of Year 3 and Year 4. This is detailed in *Part 2: Planning Curriculum* which begins on Page 12. You are invited to map these weeks into your Year Planner.
Together, the four kits available for these levels provide 25 weeks of core curriculum in Working Mathematically (working like a mathematician).

Note: Membership of Maths300 is assumed.
The kit will be useful without it, but it will be much more useful with it.

Tasks

- Challenge
- Consecutive Sums
- Criss-Cross Numbers
- Crosses
- Domino Trails
- Eric The Sheep
- Guessing Colours Game
- Highest Number 1
- How Many Things?
- Make The Whole
- Nim
- Rectangle Fractions
- Rod Mats
- Row Points
- Soft Drink Crates
- Truth Tiles
- Truth Tiles 2
- What's It Worth?
- Which Floor
- Window Frames

Part 2 of this manual introduces each task. The latest information can be found at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm

Maths300 Lessons

- Counting Machines
- Domino Trails
- Eric The Sheep
- Highest Number
- Hunting For Stars
- Make A Million
- Nine & Over
- Rectangle Fractions
- Spiders & Ants
- Tackling Times Tables
- Take Away Of The Day
- Trial, Record & Improve
- Truth Tiles

Lessons with Software

- Highest Number
- Hunting For Stars
- Make A Million
- Nine & Over
- Rectangle Fractions
- Tackling Times Tables
- Truth Tiles

Part 2 of this manual introduces each lesson. Full details can be found at:

- http://www.maths300.com

Working Like A Mathematician

Our attitude is:

all students can learn to work like a mathematician

What does a mathematician's work actually involve? Mathematicians have provided their answer on Page 8. In particular we are indebted to Dr. Derek Holton for the clarity of his contribution to this description.

Perhaps the most important aspect of Working Mathematically is the recognition that *knowledge is created by a community and becomes part of the fabric of that community*. Recognising, and engaging in, the process by which that knowledge is generated can help students to see themselves as able to work like a mathematician. Hence Working Mathematically is the framework of **Maths With Attitude**.

Skills, Strategies & Working Mathematically

A Working Mathematically curriculum places learning mathematical skills and problem solving strategies in their true context. Skills and strategies are the tools mathematicians employ in their struggle to solve problems. Lessons on skills or lessons on strategies are not an end in themselves.

- **Our skill toolbox** can be added to in the same way as the mechanic or carpenter adds tools to their toolbox. Equally, the addition of the tools is not for the sake of collecting them, but rather for the purpose of getting on with a job. A mathematician's job is to attempt to solve problems, not to collect tools that might one day help solve a problem.
- **Our strategy toolbox** has been provided through the collective wisdom of mathematicians from the past. All mathematical problems (and indeed life problems) that have ever been solved have been solved by the application of this concise set of strategies.

About Tasks

Our attitude is:

mathematics is concrete, visual and makes sense

Tasks are from Mathematics Task Centre. They are an invitation to two students to work like a mathematician (see Page 8).

The Task Centre concept began in Australia in the late 1970s as a collection of rich tasks housed in a special room, which came to be called a Task Centre. Since that time hundreds of Australian teachers, and, more recently, teachers from other countries, have adapted and modified the concept to work in their schools. For example, the special purpose room is no longer seen as an essential component, although many schools continue to opt for this facility.

A brief history of Task Centre development, considerable support for using tasks, for example Task Cameos, and a catalogue of all currently available tasks can be found at:

- http://www.mathematicscentre.com/taskcentre

Key principles are:

- A good task is the tip of an iceberg
- Each task has three lives
- Tasks involve students in the Working Mathematically process

The Task Centre Room or the Classroom?

There are good reasons for using the tasks in a special room which the students visit regularly. There are also different good reasons for keeping the tasks in classrooms. Either system can work well if staff are committed to a core curriculum built around learning to work like a mathematician.

- A task centre room creates a focus and presence for mathematics in the school. Tasks are often housed in clear plastic 'cake storer' type boxes. Display space can be more easily managed. The visual impact can be vibrant and purposeful.
- However, tasks can be more readily integrated into the curriculum if teachers have them at their finger tips in the classrooms. In this case tasks are often housed in press-seal plastic bags which take up less space and are more readily moved from classroom to classroom.

Tip of an Iceberg

The initial problem on the card can usually be solved in 10 to 20 minutes. The investigation iceberg which lies beneath may take many lessons (even a lifetime!). Tasks are designed so that the original problem reveals just the 'tip of the iceberg'. Task Cameos and Maths300 lessons help to dig deeper into the iceberg.

We are constantly surprised by the creative steps teachers and students take that lead us further into a task. No task is ever 'finished'.

> *Most tasks have many levels of entry and exit and therefore offer an on-going invitation to revisit them, and, importantly, multiple levels of success for students.*

Three Lives of a Task

This phrase, coined by a teacher, captures the full potential and flexibility of the tasks. Teachers say they like using them in three distinct ways:

- As on the card, which is designed for two students.
- As a whole class lesson involving all students, as supported by outlines in the Task Cameos and in detail through the Maths300 site.
- Extended by an Investigation Guide (project), examples of which are included in both Task Cameos and Maths300.

The first life involves just the 'tip of the iceberg' of each task, but nonetheless provides a worthwhile problem solving challenge - one which 'demands' concrete materials in its solution. This is the invitation to work like a mathematician. Most students will experience some level of success and accomplishment in a short time.

The second life involves adapting the materials to involve the whole class in the investigation, in the first instance to model the work of a mathematician, but also to develop key outcomes or specific content knowledge. This involves choosing teaching craft to interest the students in the problem and then absorb them in it.

The third life challenges students to explore the 'rest of the iceberg' independently. Investigation Guides are used to probe aspects and extensions of the task and can be introduced into either the first or second life. Typically this involves providing suggestions for the direction the investigation might take. Students submit the 'story' of their work for 'portfolio assessment'. Typically a major criteria for assessment is application of the Working Mathematically process.

About Maths300

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

Maths300 is a subscription based web site. It is an attempt to collect and publish the 300 most 'interesting' maths lessons (K - 12).

- *Lessons have been successfully trialed in a range of classrooms.*
- *About one third of the lessons are supported by specially written software.*
- *Lessons are also supported by investigation sheets (with answers) and game boards where relevant.*
- *A 'living' Classroom Contributions section in each lesson includes the latest information from schools.*
- *The search engine allows teachers to find lessons by pedagogical feature, curriculum strand, content and year level.*
- *Lesson plans can be printed directly from the site.*
- *Each lesson supports teaches to model the Working Mathematically process.*

Modern internet facilities and computers allow teachers easy access to these lesson plans. Lesson plans need to be researched, reflected upon in the light of your own students and activated by collecting and organising materials as necessary.

Maths300 Software

Our attitude is:

> *stimulated students are creative and love to learn*

Pedagogically sound software is one feature likely to encourage enthusiastic learning and for that reason it has been included as an element in about one third of Maths300 lesson plans. The software is used to develop an investigation beyond its introduction and early exploration which is likely to include other pedagogical techniques such as concrete materials, physical involvement, estimation or mathematical conversation. The software is not the lesson plan. It is a feature of the lesson plan used at the teacher's discretion.

For school-wide use, the software needs to be downloaded from the site and installed in the school's network image. You will need to consult your IT Manager about these arrangements. It can also be downloaded to stand alone machines covered by the site licence, in particular a teacher's own laptop, from where it can be used with the whole class through a data projector.

Note:

- *Maths300 lessons and software may only be used by Maths300 members.*

Working Mathematically

First give me an interesting problem.

When mathematicians become interested in a problem they:

- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

Questions which help mathematicians learn more are:

- Can I check this another way?
- What happens if ...?
- How many solutions are there?
- How will I know when I have found them all?

When mathematicians have a problem they:

- Read & understand the problem.
- Plan a strategy to start the problem.
- Carry out their plan.
- Check the result.

A mathematician's strategy toolbox includes:

- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- Break a problem into smaller parts
- ...

If one way doesn't work, I just start again another way.

Professional Development Purpose

Our attitude is:

the teacher is the most important resource in education

> *We had our first study group on Monday. The session will be repeated again on Thursday. I had 15 teachers attend. We looked at the task Farmyard Friends (Task 129 from the Mathematics Task Centre). We extended it out like the questions from the companion Maths300 lesson suggested, and talked for quite a while about the concept of a factorial. This is exactly the type of dialog that I feel is essential for our elementary teachers to support the development of their math background. So anytime we can use the tasks to extend the teacher's math knowledge we are ahead of the game.*
> District Math Coordinator, Denver, Colorado

Research suggests that professional development most likely to succeed:

- is requested by the teachers
- takes place as close to the teacher's own working environment as possible
- takes place over an extended period of time
- provides opportunities for reflection and feedback
- enables participants to feel a substantial degree of ownership
- involves conscious commitment by the teacher
- involves groups of teachers rather than individuals from a school
- increases the participant's mathematical knowledge in some way
- uses the services of a consultant and/or critical friend

Maths With Attitude has been designed with these principles in mind. All the materials have been tried, tested and modified by teachers from a wide range of classrooms. We hope the resources will enable teacher groups to lead themselves further along the professional development road, and support systems to improve the learning outcomes for students K - 12.

With the support of Maths300 ETuTE, professional development can be a regular component of in-house professional development. See:

- http://www.mathematicscentre.com/taskcentre/resource.htm#etute

For external assistance with professional development, contact:

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Part 2: Planning Curriculum

Curriculum Planners

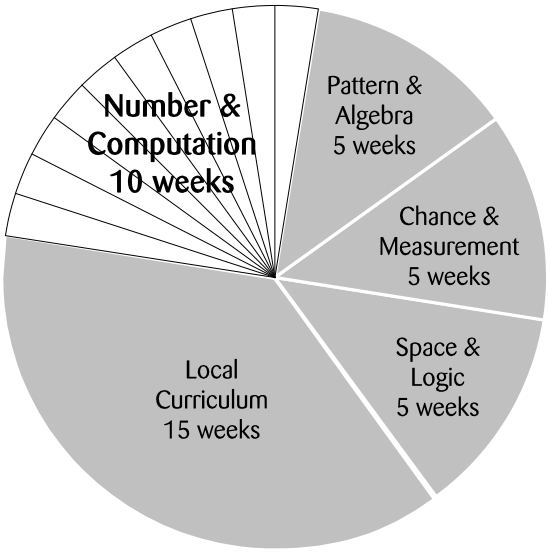
Our attitude is:

learning is a personal journey stimulated by achievable challenge

Curriculum Planners:

- show one way these resources can be integrated into your weekly planning
- provide a starting point for those new to these materials
- offer a flexible structure for those more experienced

You are invited to map Planner weeks into your school year planner as the core of the curriculum.



Planners:

- detail each week lesson by lesson
- offer structures for using tasks and lessons
- are sequenced from lesson to lesson, week to week and year to year to 'grow' learning

Teachers and schools will map the material in their own way, but all will be making use of extensively trialed materials and pedagogy.

Using Resources

- Your kit contains 20 hands-on problem solving tasks and reference to relevant Maths300 lessons.
- Tasks are introduced in this manual and supported by the Task Cameos at:
  http://www.mathematicscentre.com/taskcentre/iceberg.htm
- Maths300 lessons are introduced in this manual and supported by detailed lesson plans at:
  http://www.maths300.com

In your preparation, please note:

- Planners assume 4 lessons per week of about 1 hour each.
- Planners are *not* prescribing a continuous block of work.
- Weeks can be interspersed with other learning; perhaps a **Maths With Attitude** week from a different strand.
- Weeks can sometimes be interchanged within the planner.
- Lessons can sometimes be interchanged within weeks.
- The four **Maths With Attitude** kits available at each year level offer 25 weeks of a Working Mathematically core curriculum.

A Way to Begin

- Glance over the Planner for your class. Skim through the comments for each task and lesson as it is named. This will provide an overview of the kit.
- Task Comments begin after the Planners. Lesson Comments begin after Task Comments. The index will also lead you to any task or lesson comments.
- Select your preferred starting week - usually Week 1.
- Now plan in detail by researching the comments and web support. Enjoy!

Research, Reflect, Activate

Curriculum Planner

Number & Computation: Year 3

Note: Tasks in this kit are used in a special way. See Task Comments Pg 18.

	Session 1	Session 2	Session 3	Session 4
Week 1	*Highest Number*: The class plays a card game to begin a unit on place value concepts and skills. The game is the same as the task but the lesson adds much more. It may be sufficient for students to develop their 5 round scores then find two different ways to calculate the total '...using anything in the classroom to help'. Options within the software will be useful.			
Week 2	*Counting Machines*: Explores an abacus model for place value which focuses on the process of reaching the top of a column and moving one place in the next column to record you have done so. Provides background for the particular case of Nine & Over.			
Week 3	*Nine & Over and Take Away of the Day*: The focus is now on the abacus model in the Base 10 system. From *Take Away of the Day*, use the activity Predict A Count to support this work.			
Week 4	*Nine & Over*: Explore the lesson further, especially the many software options and the additional resource Win/Lose A Flat. Also consider using Classroom Shop and Change The Wood from *Take Away of the Day*.			
Week 5	*Domino Trails* and *Spiders & Ants*: Two lessons refreshing and extending number and operations on number.			
Weeks 6 & 7	*Take Away of the Day*: While continuing appropriate place value activities from above, introduce one subtraction each day in the manner described in the lesson. This approach to teaching subtraction builds on student-created methods. The lesson shows it is very powerful.			
Week 8	*Tackling Times Tables*: An array model approach to the conceptualisation of multiplication and development of times table facts.			
Weeks 9 & 10	*Rectangle Fractions*: Build and explore the concept of a fraction and equivalence using a hands-on approach supported by software and lots of mathematical conversation. Recording in natural language rather than symbols is a feature. Addition and subtraction of fractions using this model are included in Year 4.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Curriculum Planner

Number & Computation: Year 4

Note: Tasks in this kit are used in a special way. See Task Comments Pg 18.

	Session 1	Session 2	Session 3	Session 4
Week 1	*Make A Million*: Start your number work by exploring a very large number. Can we make a million dollars? Spend a million dollars? Collect a million screw caps? How long will it take to count to one million? The lesson is supported by software.			
Week 2	*Nine & Over*: Refresh the concept of place value with Camel Counting in the Additional Resources section of the lesson plan. Revise and extend further with activities and software already explored in Year 3.			
Week 3	*Trial, Record & Improve*: Designed as a Threaded Activity to use for a small amount of time each day. The lesson notes show that students of this age, and younger, can surprise with their ability to understand and solve equations. Calculators are essential.			
Week 4	*Take Away of the Day*: Select from the many activities in this lesson (which was used in Year 3) to extend number sense and continue to build on student-created algorithms. Work towards exploring adult algorithms for subtraction. The Epilogue Stories provide strong teacher support.			
Week 5	*Tackling Times Tables*: Review and extend Year 3 work, especially through the software. At this age we are moving closer to automatic response.			
Week 6	*Eric The Sheep*: A problem solving situation involving groups of 3, pattern and generalisation, other equation based work and graphing. The lesson provides a wonderful opportunity to follow the Working Mathematically process (Page 8) right through to publishing a report.			
Week 7	*Hunting For Stars*: Application of times tables, factors, multiples and primes in an attractive problem solving context. Supported by software.			
Week 8	*Truth Tiles*: Operations on number are continued in a problem solving situation that also raises the question of mathematical proof.			
Weeks 9 & 10	*Rectangle Fractions*: Review Year 3 and extend into addition and subtraction of fractions. Operations on fractions were not formally approached in Year 3. Option 3 of the software supports this work.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Planning Notes

Enhancing Maths With Attitude

Resources to support learning to work like a mathematician are extensive and growing. There are more tasks and lessons available than have been included in this Number & Computation kit. You could use the following to enhance this kit.

Additional Tasks

- Task 19, Cookie Count
  Place any number of cookies on a plate ready to offer equal shares to guests. The problem is: If I tell you any number of guests can you tell me the share they each receive?
- Task 139, Squound
  A square and a circle intersect to form a 'Squound'. The total number of counters is known as is the total in the square and circle. But how many in the Squound? Students must realise that the number in the intersection can't be counted twice and are led to this by a work sheet.
- Task 230, Pack Up Your Bears
  Simply and sweetly this task, which was designed for infants, has children exploring sums of three numbers that add to six (or seven, or eight, or...).
- Task 231, Flowers In The Field
  Also designed for infants, but with a twist that lifts it right up through the school. There are many problems on the card, including ones the students decide for themselves, but they all revolve around having a known number of different flowers and asking how many bunches of a different size can be made.
- Task 235, Tables For 25
  The familiar context of sitting in table groups in the classroom leads to problems with multiple solutions when the teacher adds conditions related to the size of the table group and the least number of boys (or girls) allowed to be in a group.
- Task 239, Money Charts
  Text book exercises on adding and subtracting money come to life in this hands-on logic challenge that is most effectively completed by patient application of if-then reasoning and working backwards. In effect there is a little equation solving going on too.

More information about these tasks may be available in the Task Cameo Library:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Additional Lessons

- Lesson 168, Truth Tiles 2
  Easy to state, easy to start and involves heaps of mathematics. What more could you ask for? Students only need to be able to add and subtract single digit numbers to be able to find initial success with this problem which is, given the digits 3 to 7 arrange any four of them to make this equation true ... _ + _ - _ = _ First we find one solution, then another. Then we ask how many there are altogether and how we will know when

we have found them all? Can we check our reasoning another way? And the investigation is extended even further with What happens if... ? *questions.*

- Lesson 169, Human Computer
  A group of students become the working parts of a computer - a human computer - which can be taught (programmed) to perform the four basic operations. The team develops their computing talents and then offers to answer challenges from an audience. The human computer shows how the simplicity of the binary system can be used as the basis of electronic computing. Physical involvement and group co-operation make it a most enjoyable and productive learning experience. A spin-off is the opportunity for students to revisit the underlying structure of the place value system.
- Lesson 172, Licorice Factory
  This fantasy story introduction to the concept of prime numbers highlights the power of a story shell to facilitate learning. Many students have experienced stretching a piece of licorice so the idea of a factory which has machines to stretch unit pieces to make them 2 times, 3 times, 4 times, ... 100 times longer seems quite natural. In some classes the visualisation of the factory becomes a reality for the students.
- Lesson 174, Cookie Count
  Children have a strong desire to make fair shares. This lesson capitalises on that desire by finding fair ways to make equal shares when presented with various plates of cookies. Challenges occur at a range of levels and the activity provides a reason for discussing fractions. The lesson is perfectly complemented by the children's story The Doorbell Rang, by Pat Hutchins, so the whole experience can become a rich integrated unit.

Keep in touch with new developments which enhance **Maths With Attitude** at:

- http://www.mathematicscentre.com/taskcentre/enhance.htm

Additional Materials

As stated, our attitude is that mathematics is concrete, visual and makes sense. We assume that all classrooms will have easy access to many materials beyond what we supply. For this unit you will need:

- Packs of playing cards, or numeral cards 1 - 9
- MAB10 blocks (also called Dienes blocks)
- Calculators
- Dice
- Balls of knitting wool
- Dominoes (but there is a master sheet in the lesson that will allow you to make sets by printing onto thin card and laminating)

You will also find it useful to begin a (large) collection of plastic screw caps from soft drink, fruit juice and other such containers.

Poly Plug can be used as a class set of equipment to support the Maths300 lessons *Eric The Sheep*, *Make A Million*, *Nine & Over*, *Rectangle Fractions* and *Tackling Times Tables*. If you don't yet have this multi-useful resource, you can find Poly Plug information at:

- http://www.mathematicscentre.com/taskcentre/polyplug.htm

Special Comments Year 3

- *Nine & Over*, Planner Week 2. Now is the time you could be asking a teacher aid to make you one or more Tin Can Odometers as shown in the lesson.
- *Take Away of the Day*, Planner Weeks 3 & 4. You may want to include Predict A Count and Classroom Shop as regular 'five minutes a day' activities from the beginning of the year. These Threaded Activities are sourced from Calculating Changes as are several other ideas in this kit. On the Calculating Changes site, Predict A Count is a Free Tour activity. Its link is in the lesson plan, but you can go to it directly using:

  http://www.mathematicscentre.com/calchange/cch_act/predict.htm

  Classroom Shop is from the Members section of Calculating Changes but it is also described in *Take Away of the Day*. However, if you want to find out more about Calculating Changes membership, visit:

  http://www.mathematicscentre.com/calchange/membserv.htm

- *Domino Trails*, Planner Week 5. To make best use of this lesson you will need dominoes and problem cards made from the masters provided in the lesson. It will take time to prepare these, especially if you are going to make your own dominoes from the masters provided, so, perhaps with the help of a parent, begin as early as possible.

Special Comments Years 3 & 4

- Tasks provided with the kit and *Domino Trails* task cards can be used in any week. See note at the beginning of Task Comments on the next page.

Special Comments Year 4

- If this is the first year that your students have experienced the **Maths With Attitude** curriculum support materials, then we suggest beginning with the Year 3 planner and extending each lesson appropriately.
- Some lessons were used in Year 3 and are deliberately revisited to continue to build concepts and skills in place value, times tables and fractions. If possible, it will be useful to chat with the students' Year 3 teacher to get a sense of the aspects of the lesson actually used at that level.
- *Hunting For Stars*, Planner Week 7. The lesson includes several printed sheets to guide the investigation. Decide in advance which ones, if any, you will use and make arrangements for printing. You might like to produce booklets for each student.

Task Comments

Task in this **Maths With Attitude** kit are not structured into the planner. They are provided in the expectation that teachers will find their own ways to weave them into their curriculum. Perhaps you will group your students and use the tasks at a work station. Perhaps you will have a task day each week. Perhaps you will use them as home lending experiences - maths around the kitchen table.

Learning to work like a mathematician involves times when the process is modelled and times when you are invited to try the process for yourself. Tasks are an invitation to two students to 'fall in love' with a problem in the way a mathematician does. For this kit, we have selected tasks which other children of this age have enjoyed. Task Comments below, which include links to Task Cameos where appropriate, will supply answers and ideas for extension.

Many schools print the Task Cameos they need and include them with this Maths With Attitude resource.

Modelling the work of a mathematician **is** mapped into the Planners and **is** extensively supported by the Lesson Comments and Maths300 lessons.

Tasks, lessons and unit plans prepare students for the more traditional skill practice lessons, which we invite you to weave into your curriculum. Teachers who have used practical, hands-on investigations as the focus of their curriculum, rather than focussing on the drill and practice diet of traditional mathematics, report success in referring to skill practice lessons as Toolbox Lessons. This links to the idea of a mathematician dipping into a toolbox to find and use skills to solve problems.

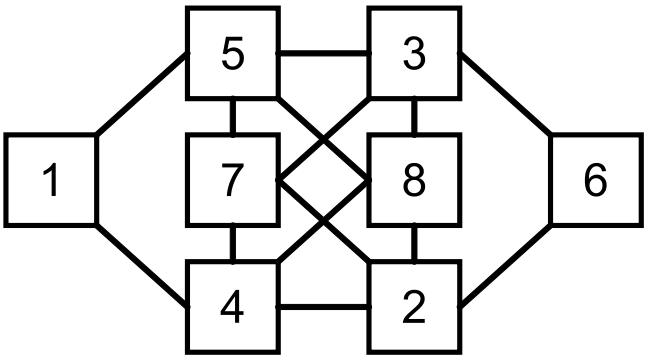
Challenge

Blocks are placed on the board so that the difference between the blocks on either end of any line is *not* 1.

- Can you find one solution?
- Can you find more than one solution?
- What happens if...?

Can the students create other similar puzzles of their own?

It is likely that children will find one solution using a trial and improve approach. However, if they are to find more than one, perhaps even *all* the solutions, a more efficient strategy will be needed. For example one might break the problem into smaller parts by choosing to place the smallest number (1) into the left hand box and then check all the consequent possibilities. One solution produced using this strategy is:



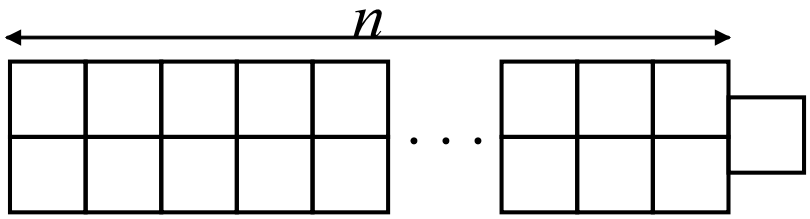
Continuing an organised search in this way, with each digit in turn taking its place in the left hand box, will produce several more solutions.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Consecutive Sums

While becoming more familiar with the idea of consecutive numbers, the students are encouraged by the challenge to organise their thinking to be sure that they have counted every possibility. In looking at the results of their organised search they should see patterns that lead to generalisations. For example Question 1 asks about two consecutive numbers and the sums of these within the discs provided is always an odd number. Students who notice this might go on to ask why and be led to represent the first number as n and the next as $n + 1$. The sum of these is therefore $2n + 1$. By the definition of even numbers, or pictorially:



students can reason that this must always be an odd number, as suggested by their own data.

Also, this type of generalisation is usually in the background of a type of word problem that involves *Think of a number, now add 6, multiply by 2 ... take off the number you first thought of ... and the answer is ...* Teachers will have their own favourites from among these and this may be a good opportunity to bridge into an investigation of them.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Criss-Cross Numbers

A hands-on example of a Cross Number (cf: Crossword) puzzle. Using the numbered counters makes it easier (and safer?) than a pencil and paper version. The problem is to fill the empty spaces knowing only the numbers in the completed puzzle and a few given digits.

The puzzle is equivalent to completing a crossword with no clues other than the words in the puzzle, a few correctly placed letters and sufficient scrabble tiles to complete all the words. One solution is shown. Are there more?

4	6	■	2	3	1
3	■	2	■	■	7
8	■	4	9	8	■
■	8	0	2	■	9
7	■	■	5	■	5
1	7	6	■	5	1

A great extension is for students make up similar puzzles of their own. What is the minimum number of clues required?

Find more information about this task in the Task Cameo Library at:

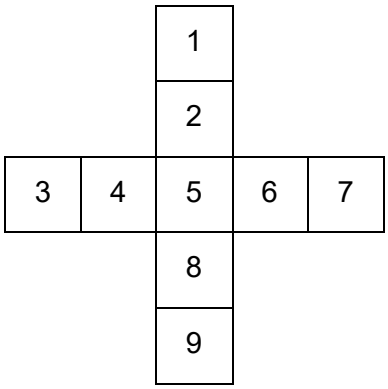
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Crosses

Digits 1 - 9 are placed on arms of a cross so that the partial sum of each arm is the same. Addition fact practice in a problem solving situation that has several solutions.

- How many solutions are there?
- How do you know when you have found them all?

Finding any solution is a success and most often children find this one first:



Challenge the students to convince you the arms are the same total and then ask *Can you check it another way?*. It is more powerful to be able to use several strategies to calculate, for example, 3 + 4 + 5 + 6 + 7 than it is to be able to find the answer and stop. The question, which of course is a mathematician's question, also encourages students to take charge of their own work.

As students continue to seek other solutions, they might realise that:

- The number in the middle has to be odd because there are 5 odd digits and 4 even digits.
- The grand total of the two arms must be the sum of the digits 1 - 9 (=45) plus a repeat of the middle number.
- The grand total of the two arms must be even because it has to be divided into two. Or, looking at it another way, it must be even because it is made up of two equal parts. If both parts are Even, the grand total will be even. If both parts are Odd, the grand total will also be even.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Domino Trails

Students are asked to make a limited trail of dominoes that lead to a given total. It is particularly open-ended, so there are many solutions. The driving questions are:

- What happens if we change the target number?
- What happens if we change the length of the trail?

The task involves lots of maths including one to one correspondence, recognition of dot patterns, addition and subtraction and mind stretching problem solving. *A sheet for recording trails is at the end of this manual.*

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Eric The Sheep

Eric is at the end of a long line of his fellow sheep, but he is too impatient to wait his turn to be shorn. Each time the shearer takes a sheep from the front of the line, Eric skips past two sheep. If there are fifty sheep in front of Eric, how many will be shorn before Eric gets to the front?

Most students set this problem up with materials and easily find the one answer required on the card. In doing so, they might begin to 'see' the words in the problem and realise that 3 sheep are removed from the line on each move. One taken by the shearer at the same time as two are passed by Eric.

Hence, the iceberg of the task involves multiples of 3, visual representation and algebraic generalisation which all develop from the *What happens if...?* questions:

- What happens if we change the number of sheep?
- What happens if we change the jumping rule?
- What happens if we change the number of shearers?

With each new question, the times table which solves it also changes. For example, a rule with the two shearers working and remove two each time, which Eric is simultaneously skipping past three, creates a multiples of 5 situation.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Guessing Colours Game

Negative numbers are likely to arise in the scoring system and students are likely to surprise with their ability to deal with these 'underground' numbers. Whether or not this does happen depends on how good the students are at guessing the colours.

As the Challenge suggests, the variation in the task comes from changing the ratio of discs in the bag but keeping the total at 15. The challenge can be extended using the problem solving question, *Which starting arrangement of discs makes for the best game?*. This requires making evidence based judgements.

One might also argue that the 15 points which start the game described on the card has been chosen because there are 15 discs in the bag, so:

- What happens if we change the collection of discs in the bag and start the game with the same number of points as the total of discs?

Guessing Colours Game also has a probability aspect which you may wish to follow up. The structure of the game involves drawing from a sample space without replacement, so there is plenty of opportunity to discuss chance. For example, the task suggests keeping a record using the columns Guess and Points. Teachers may wish students to add a column between these headed Chance of Correct Guess.

Highest Number 1

This task is included in the Year 3 planner for its place value and arithmetic skills components, although it also encourages probability discussion and can develop into data collection and statistics at other Year levels.

Students are encouraged to improve their five round game total by thoughtfully examining which of the hundreds, tens or ones columns they will place their card into on each roll. The task is structured to illustrate that working like a mathematician, rather than playing randomly, is likely to result in a better score.

The Challenge on the card further encourages working like a mathematician by encouraging students to check their addition of each set of five scores in another way. Each score is a hundreds number, so totalling five scores is likely to be more difficult than any textbook exercise. However, students usually tackle that challenge readily. Their methods can vary from using calculators to concrete materials to insightful regrouping of numbers based on place value.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

How Many Things?

Reminiscent of guessing the number of jelly beans in a jar at the church fete, the task offers practical experience with ways of estimating number. The cards suggests three methods - using volume, area and weight - and therefore could also be used in a measurement unit.

The task also represents the type of problem faced by researchers who can only sample a population and then must make inferences about the whole population. To strengthen the relationship of this task to the broader idea of sampling you could ask students to respond to challenges like:

> *You have been asked to find the average height of Australian females. Clearly you can't measure every person. Use your How Many Things? experience to help you design an approach to solving this problem.*

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Make The Whole

This task derives from **Rod Mats** (see later). The task is supplied with pieces from a rod mat which has Orange as the whole. The students engage in considerable equivalent fraction work and addition and subtraction of fractions as they play the game. The game also offers a model for making additional similar games.

A sheet of blanks is supplied at the end of this manual for teachers who want the students to create their own game. This can be copied onto thin card, laminated and cut up to make a blank set on which the students write with a marker. If a less permanent set is required, then paper copies will work for one game.

The steps in creating a **Make The Whole** variation are:

- choose a rod (or collection of rods) to be the whole
- make the rod mat for this whole
- record the colour of the whole
- record one fraction card (words and symbols) for each rod in the mat.

The game is now played with the same rules but a different whole and different cards.

Example: For the picture on the **Rod Mats** task card, students would prepare two one half cards, four one quarter cards and eight one eighth cards.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Nim

A well known and intriguing logic game which at this level will probably be enjoyed for the fun. However the game can be analysed and reveals a connection between winning strategies and odd and even numbers. Doing so revolves around realising that it is not the number of counters that are left at any stage that matters, but rather the number of possible moves that are left.

- For you to win your opponent has to remove the last counter, that is there must be one move remaining for them.
- Represented in numbers, this means your opponent must face 0 0 0 1, where the 1 could be in any row and means 1 counter remaining.
- If however, you leave them facing only two possible moves you will lose because they will do one of the moves and you must do the other.
- After only one or two games students usually pick up that 1 1 1 0 is a winning position to leave. This is an odd number of moves to the end and the 0 may be in any row.
- Another winning position is 0 0 2 2. Again, the order of the numbers doesn't matter and you can't lose if you make the best possible moves.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Rectangle Fractions

Whenever objects are arranged in equal rows, the complete array can be thought of as the whole. The rows, columns and individual pieces are then clearly parts of the whole and can be discussed in fraction language. This conceptualisation changes representation of fractions, equivalence and operations on fractions from symbolic symbol shoving to spatial perception. It encourages students to build a brain picture that is consistent with the manipulation of the numbers.

The advantage of the board is that many different rectangles can be revealed by changing the arrangement of plugs. The red board provides the framework of a rectangle and the yellow plugs are the loose objects.

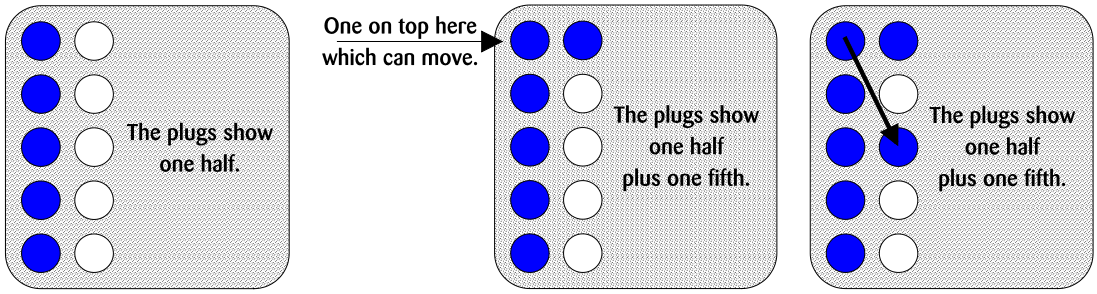
This same model is present in many other situations, for example, soft drink bottles in a crate, students lined up in twos, marching teams and vegetables planted in rows and columns. Students can think of more. When these models appear during the day, seizing teachable moments to pose a fraction challenge can encourage the students to 'see' and practise fractions for themselves.

To understand the first problem on the card it is necessary to notice that:

- each space is one tenth of the grid
- one half is one column - put in the yellow plugs
- one fifth is one row - put in the yellow plugs

This information is expressed in the way the student named on the card explains her inspiration; however, many students don't read far enough to discover this. Getting this far makes it clear that:

- the space needed by one plug in the one fifth row is already occupied by a plug from the one half column.
- you could place it on top and remember to count it.
- or, since one plug is worth one tenth wherever it is in this grid, you can see the total of one fifth + one half by shifting it to an empty space.



Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Rod Mats

The focus in this task is on mathematical conversation using fraction language. The key is that any rod can be the whole; and it will always be possible to make at least one row of rods of the same type which is the same length as the whole. The white rod can always be repeated to exactly fit the length of any chosen whole. The whole and its rows of 'same rods' make the rod mat. The task is closely linked to **Make The Whole**.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Row Points

Thirteen cells of a 5 x 5 array are occupied by counters. Points are given if the counters are in rows, columns or diagonals of 3, 4, or 5. Now many questions can be asked about the totals which can be formed. The task offers considerable mental arithmetic practice in a spatially based problem solving context.

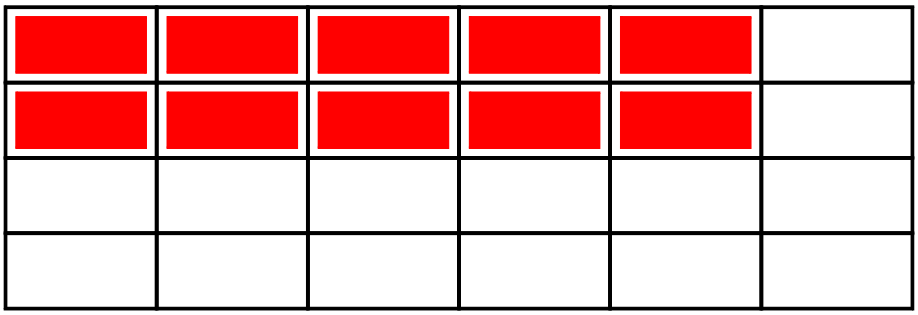
Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

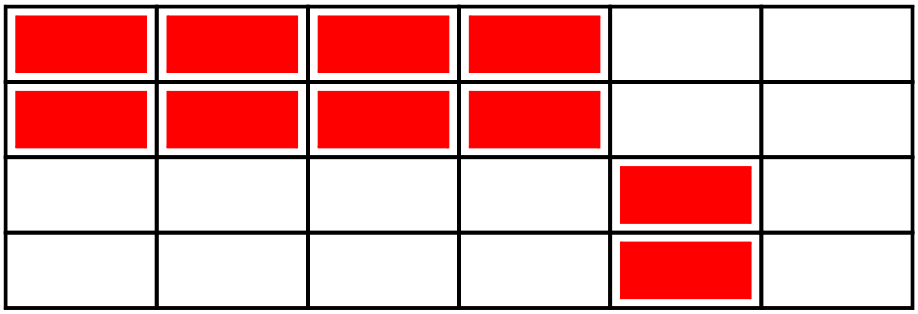
Soft Drink Crates

The number focus of the task is on odd and even numbers, but it also addresses space and logic outcomes. The task is simply stated, but the solution of the first problem is not easy. Think in terms of breaking the problem into smaller parts:

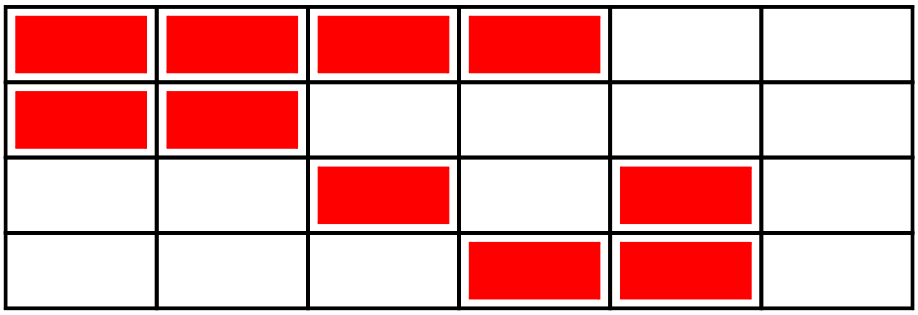
1. Make the columns correct...



2. Keep cans in the same column but alter their row...



3. Until the rows are also correct:



There are many variations on this solution, and the strategy is now applicable to other problems on the card. The task encourages high level thinking without a correspondingly high demand for mathematical skills.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Truth Tiles

Using only the digits 1 to 9, the students have to make three simple equations - an addition, a subtraction and a multiplication - true simultaneously. At one level the task engages students in simple arithmetic, but in successfully achieving the challenge, students will also demonstrate their understanding of basic mathematical properties.

This is also a maths problem with more than one answer and the 'big' challenge is to find all the solutions. It provides lots of opportunity to work like a mathematician and highlights several properties of numbers such as the commutative law and 1 as the multiplicative identity element.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Truth Tiles 2

Now we have three unknown numbers combined by addition and subtraction to make an unknown answer. However, all the numbers involved are from the set {3, 4, 5, 6, 7}. Lots of possible answers to explore and heaps of addition and subtraction fact practice.

Once one solution is found, we already have four solutions, since some numbers may be transposed. For example, 3 + 6 - 4 = 5 also gives:

- 3 + 6 - 5 = 4
- 4 + 5 - 3 = 6
- 4 + 5 - 6 = 3

If you also swap the order of the first two numbers in each case there are double the number of solutions. It is at this stage that you begin to realise that these are all versions of:

- 3 + 6 = 4 + 5

Equivalence of two operations and the commutative law does the rest.

It is the search for all solutions which unlocks the iceberg of this task. Students have to collect some initial solutions, see patterns in these, use those patterns to form hypotheses, then apply problem solving strategies to prove or disprove their theories (hypotheses) and finally communicate their findings.

The following comments from teachers indicate the interest the task has developed in some classes.

Concrete materials

> *I used to use this task as a pencil and paper item, but the simple addition of having tiles to move around (or ripped up pieces of paper) made the task more accessible to my students.*

Mixed ability

> *I also like the task because it is so open-ended. All my students can find some solutions and achieve a level of success, but the option of adding more tiles increases the depth of the task significantly and can challenge my very capable students.*

Student generated investigations

> *One student noticed that 3, 4, 5, 6, 7 was made up of 3 odd numbers and 2 evens. Would it make a difference if the sequence was 2, 3, 4, 5, 6 being 3 evens and 2 odds?*

Another student noticed that if you read any solution backwards it still worked. For example, 3 + 6 - 5 = 4. Writing this backwards gives 4 + 5 - 6 = 3. Why this observation always works became a small investigation.

Returning to the task card, there are either 12 solutions, or 24, depending on the definition of 'different'. If students feel that 3 + 6 - 5 = 4 and 6 + 3 - 5 = 4 are different, then this doubles the number of solutions. The 12 basic solutions are:

- 3 + 6 - 4 = 5
- 3 + 6 - 5 = 4
- 4 + 5 - 3 = 6
- 4 + 5 - 6 = 3
- 3 + 7 - 4 = 6
- 3 + 7 - 6 = 4
- 4 + 6 - 3 = 7
- 4 + 6 - 7 = 3
- 4 + 7 - 5 = 6
- 4 + 7 - 6 = 5
- 5 + 6 - 4 = 7
- 5 + 6 - 7 = 4

Several strategies can be used to find these and often students start with a simple idea such as testing every possible combination, and then adapt as they notice patterns.

Strategy 1

Put every digit in turn in the first spot and then try all the other numbers there in turn.

Strategy 2

Notice that the answers form groups. For example, 5 + 6 - 4 = 7 are based on a total of 11. The smallest group total is 9 (3 + 6 - 4 = 5), and the highest is 11 (4 + 7 - 5 = 6).

Strategy 3

Leave out the 7 to start with and just use the four smallest numbers 3, 4, 5, 6. This gives four solutions. Then include the 7 which means leaving out one of the others in turn.

Strategy 4

Transform the problem to:

☐ + ☐ = ☐ + ☐

and recognise the consecutive numbers in the problem can be represented as:

x, x + 1, x + 2, x + 3, x + 4

where x = 3.

Find more information about this task in the Task Cameo Library at:

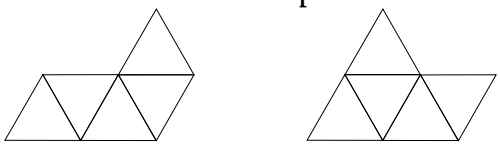
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

What's It Worth?

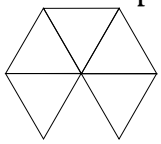
This task is another way of looking at fractions. The whole is worth 1, so the part is worth…?, a question which encourages fraction/decimal arithmetic.

As presented the task involves spatial problem solving as well as number work. For example, finding shapes worth $2.75 implies finding all the shapes that can be made with five triangles. Apart from the five in a row there are:

Four in a row with one placed as shown



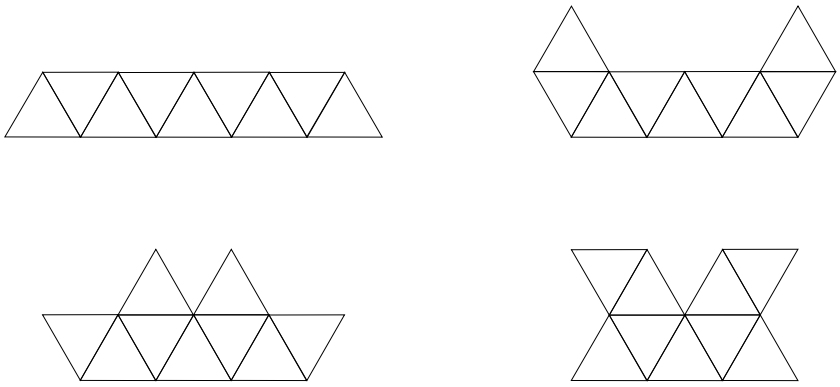
Three in a row with two placed as shown



All other shapes are rotations or flips of these. *A sheet for recording shapes is at the end of this manual*.

The value of $3.30 in Question 4 implies shapes made with 6 equilateral triangles at 55¢ each. These are the hexiamonds. The full set can be found by beginning with the four pentiamonds above and finding all the ways of adding a triangle.

The shape in Question 5 uses nine equilateral triangles. Since this is an odd number, one of the triangles must somehow be in the 'middle' to maintain symmetry. Some solutions are:



Rather than individuals searching for all the symmetric shapes as asked for on the card, teachers may prefer to begin a class display which grows as each pair of students finds one or two more. When there is plenty of data on the display, the class's attention could be drawn to looking for ways of classifying the shapes so that it can be decided whether they have all been found.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Which Floor
A variation on addition and subtraction on a number line that conceals lots of potential problem solving. The first question on the card is closed, but the Challenge is based on the question *How many solutions are there?*.

Find more information about this task in the Task Cameo Library at:
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Window Frames
Students explore the structure of a 0-99 number chart using frames which isolate particular cells of the chart. What emerges is that no matter where the frame is placed on the grid, special relationships exist between the numbers in the window. At one level the problem is about searching for and finding these relationships. At another it is about explaining their existence. The task is rich with operations on number and patterns within the frames.

Find more information about this task in the Task Cameo Library at:
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Lesson Comments

- These comments introduce you to each Maths300 lesson. The complete plan is easily accessed through the lesson library available to members at:
  http://www.maths300.com
  where they are listed alphabetically by lesson name.

Counting Machines
Understanding place value is more than just being able to manipulate hundreds, tens and ones. This lesson explores several counting machines which all operate on a place value system, but only one of them is 'exactly like our numbers'. The lesson begins with a Human Counting Machine which later becomes a desktop game between two students. Exploring several counting machines over a number of lessons allows the children to abstract key elements of place value such as:
- A limited set of digits starting at zero.
- Re-using the digit set to count the number of times the largest possible group has been formed.
- The right to left convention built into the machines to show increasing value.
- Continuing to reuse the digits to record groups of groups, groups of groups of groups ... and so on.
- If the limited set of digits is small many more columns are needed.
- If the limited set of digits is larger it takes more moves before the next column is used.

The lesson includes student work showing that children as young as six can develop an understanding of these sophistications.

Domino Trails

This lesson is almost an encyclopedia of ideas for using dominoes. Making domino trails which lead to a given total is only one of them. Some are described as whole class challenges and others are available as problem solving cards which you prepare in advance. (They will print one to a page.) If you do not have dominoes, they can be made in cardboard form from the master in the lesson. Also, the matching Task Cameo includes a set of printable floor dominoes to provide a large scale version to focus class discussion. (You will need a reasonable amount of floor space to use these, so consider using the multi-purpose room.)

Following a whole class introduction the focus can shift to independent exploration of the domino problem cards. If prepared in a sturdy form (on card and laminated?) these will also be available for any future task based work station.

Eric The Sheep

This investigation cries out for a physically involving start using the students as sheep and one specially chosen student as Eric. Gathering data in this way for a few different numbers of sheep in front of Eric leads into the question:

- If I tell you any number of sheep in front of Eric can you tell me the number that will be shorn when Eric reaches the front?

The lesson includes interesting Classroom Contributions from Year levels varying from Year 1/2 to Year 10.

Highest Number

The lesson is chosen for its place value component and because of the fun way it introduces a unit on place value.

- Teach the game to the whole class in the first lesson.
- Students play in pairs.
- After a few practice rounds, introduce the task of playing five rounds and recording a personal score (whether win or lose) for each round.
- Challenge the students to find their five round total in two different ways so that *… you won't have to ask me if you are right*. A calculator can be one method students choose.

One way to continue the week is to create three groups and three work stations and set up a rotation. The work stations could be:

- Software: Highest Number options 1 & 2
- Tasks: Truth Tiles 1 & 2, Highest Number, Criss-Cross Numbers, How Many Things?, Challenge, Guessing Colours Game, Which Floor, Consecutive Sums, Nim
- Place Value Activity: Win A Flat - a structured place value game using MAB10, dice and calculators which is described in *Nine & Over*.

This approach takes a little preparation, but one preparation session services several lessons. Team preparation reduces the load even further.

It is efficient to prepare Win A Flat materials in small tubs for each group of three. Tubs contain 2 dice and 2 calculators and sufficient MAB pieces for 2players. The third player is the Game Manager. The materials are stored with the boards between sessions. At the end of a time period students make a quick sketch of

where they were up to and whose turn it is and store it with their tub. Next session they just pull out their tub and board and continue.

Hunting For Stars

A group of students sit in a circle and a ball of wool is passed between them according to a certain rule. The wool trail either creates a star or it doesn't. Easy enough to start and quite intriguing when the stars appear. Can we predict which combinations of student number and passing rule produce stars?

The class first explores in groups with wool and then gravitates to individual or pair work using printed circles with different numbers of points marked on the circumference. These are provided in the lesson. Further experimentation can be carried out using the software supplied.

Make A Million

The title says it all. The lesson suggests many starting points for exploring a million, a number that is used often throughout students' experience. TV game shows suggest it is possible to win a million dollars, but if these coins were laid end to end from 'here', where would the line end? Could we collect a million screw caps or a million gum nuts or a million acorns? Could we count to one million? If so, how long would it take?

The lesson is supported by software that does count to one million and shows every number on the way as:

- an abacus position
- a numeral on a calculator
- a numeral on an odometer
- a word

and makes the rhythm of place value visual.

After the students estimate how long it will take to count to one million they can design an experiment to test their hypotheses on one machine set up in the corner of the room. Students can be given opportunities to change their estimate as the actual time unfolds.

Nine & Over

Nine & Over software is a development of Make A Million built around *What happens if...?*.

- What happens if we count in groups other than one?
- What happens if we can change the number of place value columns used?
- What happens if we can start and end at any number?

Perhaps start with the *Make A Million* software as a class display. Use the Demo option and encourage students to predict what will happen on the abacus, the calculator and in the text box each time the Space Bar is pressed.

- You may have to discuss with students that the use of 'and' in a number (eg: two hundred and fifty-four) is a British, Australian and Hispanic tradition. This convention appears in the text box, but a tick box allows the American tradition (without the 'and') to be shown.

Introduce the Tin Can Odometer. You might have organised in advance for sufficient materials from home so that each child can make their own. Introduce any of the first three software options from *Nine & Over* and organise for the software to become an on-going part of the classroom, perhaps as a work station. Encourage students to see connections between this software and *Make A Million*.

The main part of the lesson plan is a game with a similar structure to Win A Flat but played on an abacus model. If Win A Flat was not extracted from this lesson and used in conjunction with *Highest Number*, it can be introduced parallel with the *Nine & Over* game. The game also has variations in the companion software.

Note the assessment suggestions in the lesson and introduce those if you choose. How much time will you offer for the students to achieve these objectives? There is no suggestion in the notes that it needs to be at the end of this week. Remember how long it took mathematicians to understand place value. The resources of this lesson are designed to be used in small amounts often - perhaps in a rotation-based work station structure - well beyond the number of weeks shown for the lesson in the Planners.

Rectangle Fractions

It is important not to move too fast with this lesson. It will be threaded through both the Year 3 and the Year 4 curriculum. However, if you find the students 'running with it' the lesson does provide extension material.

Begin with the Setting The Context part of the lesson and explore, discuss and record situations like the Windows and Squads suggested in the lesson. Anything arranged in equal rows provides opportunity for such fraction talks. If the arrangement is the whole thing, then the rows and columns *are* parts and must be fractions of the whole.

Teachers who have used these materials strongly suggest:

- heaps of mathematical conversation based on concrete situations - students are more willing to take a risk in an oral situation rather than a written one,
- recording in words and allowing the students to 'slip into' shorthand, and later, full symbolic representations,
- allowing operations on fractions to occur naturally rather than pushing on to the addition and subtraction that is part of this lesson - there is always next year.

Options 1 & 2 of the software can be used and the tasks **Rectangle Fractions**, **What's It Worth?**, **Rod Mats** and **Make The Whole** can be added to the curriculum mix. These all directly support fraction talks using length, area and separate object representations as described in Setting The Context.

Spiders & Ants

This is an intriguing lesson for introducing ideas about multiplication and division without actually mentioning those skills. Many teachers are surprised by what the lesson reveals about their students' current understanding. They also find the range of problem solving approaches used to be the source of fruitful discussion. There are examples of this in the lesson. The value of the lesson is that it gives each student the opportunity to provide a different response to the same problem; instead of expecting the same (generally procedural) response from all students.

Tackling Times Tables

Tackling Times Tables provides several activities that can be threaded through the curriculum to develop a visual and concrete conceptual basis for multiplication and division which is built around an array model. At the same time, the desired objective of automatic response to times tables is building. Uncover Counting is one of these very powerful conceptual activities. However, its power comes from the repeated use of the activity (perhaps for 10 minutes a day, three days a week over several weeks) and the extended mathematical conversation that develops from it.

If you do not have the Poly Plug material pictured in the lesson, it is very easy to 'sticky tak' circular counters into place as a substitute. There is an example in the notes. However you can find more information about Poly Plug at:

- http://www.mathematicscentre.com/taskcentre/polyplug.htm

The lesson also includes a software program built on the array model and leading to finding all the whole number solutions to a problem like __ x __ = 48.

It also includes a Fun and Games link with Times Tables activities such as Catch Up and Racetrack which have proved successful in many classrooms. Sections such as Finger Multiplication, Times Tables Strips and an activity called Up To 100 could be reserved for Year 4. If possible, plan the use of this rich lesson as a Year 3 & 4 team.

Take Away Of The Day

The aim of this lesson is to provide students with time - lots of it - and challenge to help them develop and refine their own algorithms for subtraction. Carrying is included from the beginning. This may be an approach that seems alien to teachers, but it is based in Constructivist thinking and the Epilogue Stories provided for teacher research demonstrate that it can be successful. If you have ever felt that teaching subtraction comes down to explaining rules so that children can do 'take aways' even if they don't understand them, and that displeases you professionally, then this lesson is worth considering.

The focus is on the students asking the question:

> *Can I check this another way?*

That is, the focus is on collecting and organising personal conceptual data, not on 'the answer'. The answer is important, but the students learn to independently and reliably decide on its correctness, rather than slipping into the *Am I right Miss?* strategy.

Its success depends on extensive place value preparation as described in Highest Number, Counting Machines, Make A Million and Nine & Over. It also depends on exploring only one subtraction a day over a period of time. The lesson is built on the work of the Calculating Changes network in developing the concept of Threaded Activities.
Find more information about Calculating Changes at:

- http://www.mathematicscentre.com/calchange/cchbrief.htm

Trial, Record & Improve

Again developing from Calculating Changes, this lesson uses the Threaded Activity approach to encourage creation and solution of equations. It contains several games and activities which can be used to build a unit of work in this topic and the Classroom Contributions give examples of children's work from Years 2 and 3. Remarkably the children have included decimals in their work as a matter of course. Perhaps they have the confidence to do so because a key feature of the lesson is free use of simple four function calculators.

Truth Tiles

This lesson is an opportunity to pick up a task which students have previously seen in your arrangements for using tasks in this kit and to explore its life as a whole class lesson. The shift from a two person task to the whole class level is as easy as tearing a piece of paper into 9 roughly rectangular pieces.

> *I know you have all seen this but let's see what we can learn from it together.*

The focus is on modelling the process of Working Mathematically in order to answer the questions:

> *How many solutions are there?*
> *How will we know when we have found them all?*

The *Truth Tiles* lesson will take 1 or 2 sessions and can be partnered with *Truth Tiles 2* which is listed in the **Enhancing Maths With Attitude** section on Page 16.

Part 3: Value Adding

The Poster Problem Clinic

Maths With Attitude kits offer several models for building a Working Mathematically curriculum around tasks. Each kit uses a different model, so across the range of 16 kits, teachers' professional learning continues and students experience variety. The Poster Problem Clinic is an additional model. It can be used to lead students into working with tasks, or it can be used in a briefer form as a opening component of each task session.

> *I was apprehensive about using tasks when it seemed such a different way of working. I felt my children had little or no experience of problem solving and I wanted to prepare them to think more deeply. The Clinic proved a perfect way in.*

Careful thought needs to be given to management in such lessons. One approach to getting the class started on the tasks and giving it a sense of direction and purpose is to start with a whole class problem. Usually this is displayed on a poster that all can see, perhaps in a Maths Corner. Another approach is to print a copy for each person. A Poster Problem Clinic fosters class discussion and thought about problem solving strategies.

Starting the lesson this way also means that just prior to liberating the students into the task session, they are all together to allow the teacher to make any short, general observations about classroom organisation, or to celebrate any problem solving ideas that have arisen.

One teacher describes the session like this:

> *I like starting with a class problem - for just a few minutes - it focuses the class attention, and often allows me to introduce a particular strategy that is new or needs emphasis.*

It only takes a short time to introduce a poster and get some initial ideas going. The class discussion develops a way of thinking. It allows class members to hear, and learn from their peers, about problem solving strategies that work for them.

> *If we don't collectively solve the problem in 5 minutes, I will leave the problem 'hanging' and it gives a purpose to the class review session at the end.*
> *Sometimes I require everyone to work out and write down their solution to the whole class problem. The staggered finishing time for this allows me to get organised and help students get started on tasks without being besieged.*
> *I try to never interrupt the task session, but all pupils know we have a five minute review session at the end to allow them to comment on such things as an activity they particularly liked. We often close then with an agreed answer to our whole class problem.*

A Clinic in Action

The aims of the regular clinic are:

- to provide children with the opportunity to learn a variety of strategies
- to familiarise children with a process for solving problems.

The following example illustrates a structure which many teachers have found successful when running a clinic.

Preparation

For each session teachers need:

- a Strategy Board as below
- a How To Solve A Problem chart as below
- to choose a suitable problem and prepare it as a poster
- to organise children into groups of two or three.

The Strategy Board can be prepared in advance as a reference for the children, or may be developed *with* the children as they explore problem solving and suggest their own versions of the strategies.

The problem can be chosen from

- a book
- the task collection
- prepared collections such as Professor Morris Puzzles which can be viewed at: http://www.mathematicscentre.com/taskcentre/resource.htm#profmorr

The example which follows is from the task collection. The teacher copied it onto a large sheet of paper and asked some children to illustrate it. *The teacher also changed the number of sheep to sixty* to make the poster a little different from the one in the task collection.

The Strategy Board and the How To Solve A Problem chart can be used in any maths activity and are frequently referred to in Maths300 lessons.

The Clinic

The poster used for this example session is:

> Eric the Sheep is lining up to be shorn before the hot summer ahead. There are sixty [60] sheep in front of him. Eric can't be bothered waiting in the queue properly, so he decides to sneak towards the front.
>
> Every time one [1] sheep is taken to be shorn, Eric then sneaks past two [2] sheep. How many sheep will be shorn before Eric?

This Poster Problem Clinic approach is also extensively explored in Maths300 Lesson 14, *The Farmer's Puzzle*.

Strategy Board

DO I KNOW A SIMILAR PROBLEM?	ACT IT OUT
GUESS, CHECK AND IMPROVE	DRAW A PICTURE OR GRAPH
TRY A SIMPLER PROBLEM	MAKE A MODEL
WRITE AN EQUATION	LOOK FOR A PATTERN
MAKE A LIST OR TABLE	TRY ALL POSSIBILITIES
WORK BACKWARDS	SEEK AN EXCEPTION
BREAK INTO SMALLER PARTS	...

How To Solve A Problem

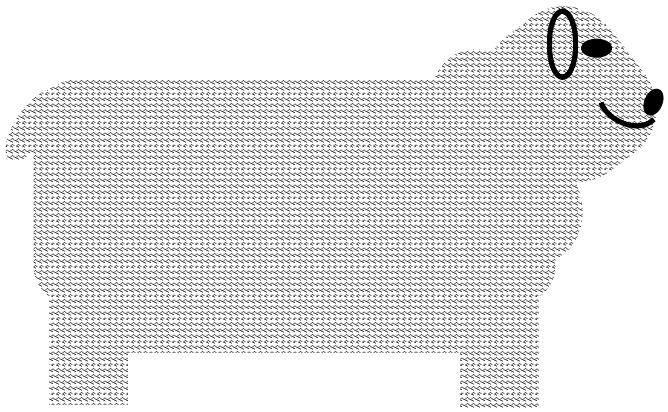
SEE & UNDERSTAND	Do I understand what the problem is asking? Discuss
PLANNING	Select a strategy from the board. Plan how you intend solving the problem.
DOING IT	Try out your idea.
CHECK IT	Did it work out? If so reflect on the activity. If not, go back to step one.

Step 1

- Tell the children that we are at Stage 1 of our four stage plan ... **See & Understand** ... Point to it! Read the problem with the class. Discuss the problem and clarify any misunderstandings.
- If children do not clearly understand what the problem is asking, they will not cope with the next stage. A good way of finding out if a child understands a problem is for her/him to retell it.
- Allow time for questions - approximately 3 to 5 minutes.

Step 2

- Tell the children that we are at Stage 2 of our four stage plan ... **Planning**. In their groups children select one or more strategies from the Strategy Board and discuss/organise how to go about solving the problem.
- Without guidance, children will often skip this step and go straight to Doing It. It is vital to emphasise that this stage is simply planning, not solving, the problem.
- After about 3 minutes, ask the children to share their plans.



Plan 1

Well we're drawing a picture and sort of making a model.

Can you give me more information please Brigid?

We're putting 60 crosses on our paper for sheep and the pen top will be Eric. Then Claire will circle one from that end, and I will pass two crosses with my pen top.

Plan 2

Our strategy is Guess and Check.

That's good Nick, but how are you going to check your guess?

Oh, we're making a model.

Go on ...

> *John's getting MAB smalls to be sheep and I'm getting a domino to be Eric and the chalk box to be the shed for shearing.*

Plan 3

> *We are doing it for 3 sheep then 4 sheep then 5 sheep and so on. Later we will look at 60.*

Great so you are going to try a simpler problem, make a table and look for a pattern.

This sharing of strategies is invaluable as it provides children who would normally feel lost in this type of activity with an opportunity to listen to their peers and make sense out of strategy selection. Note that such children are not given the answer. Rather they are assisted with understanding the power of selecting and applying strategies.

Step 3

- Tell the children that we are at Stage 3 of our four stage plan ... **Doing It**. Children collect what they need and carry out their plan.

Step 4

- Tell the children that we are at Stage 4 of our four stage plan ... **Check It**. Come together as a class for groups to share their findings. Again emphasis is on strategies.

> *We used the drawing strategy, but we changed while we were doing it because we saw a pattern.*

So Jake, you used the Look For A Pattern strategy. What was it?

> *We found that when Eric passed 10 sheep, 5 had been shorn, so 20 sheep meant 10 had been shorn ... and that means when Eric passes 40 sheep, 20 were shorn and that makes the 60 altogether.*

Great Jake. How would you work out the answer for 59 sheep or 62 sheep?

Sharing time is also a good opportunity to add in a strategy which no one may have used. For example:

> *Maybe we could've used the Number Sentence strategy, ie: 1 sheep goes to be shorn and Eric passes two sheep. That's 3 sheep, so perhaps, 60 divided into groups of 3, or 60 ÷ 3 gives the answer.*

Round off the lesson by referring to the Working Mathematically chart. There will be many opportunities to compliment the students on working like a mathematician.

Curriculum Planning Stories

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

In more than a decade of using tasks and many years of using the detailed whole class lessons of Maths300, teachers have developed several models for integrating tasks and whole class lessons. Some of those stories are retold here. Others can be found at:

- http://www.mathematicscentre.com/taskcentre/plans.htm

Story 1: Threading

> *Educational research caused me a dilemma. It tells us that students construct their own learning and that this process takes time. My understanding of the history of mathematics told me that certain concepts, such as place value and fractions, took thousands of years for mathematicians to understand. The dilemma was being faced with a textbook that expected students to 'get it' in a concentrated one, two or three week block of work and then usually not revisit the topic again until the next academic year.*

A Working Mathematically curriculum reflects the need to provide time to learn in a supportive, non-threatening environment and…

> *When I was involved in a Calculating Changes PD program I realised that:*
>
> - *choosing rich and revisitable activities, which are familiar in structure but fresh in challenge each time they are used, and*
> - *threading them through the curriculum over weeks for a small amount of time in each of several lessons per week*
>
> *resulted in deeper learning, especially when partnered with purposeful discussion and recording.*

Calculating Changes:

- http://www.mathematicscentre.com/calchange

Story 2: Your turn

Some teachers are making extensive use of a partnership between the whole class lessons of Maths300 and small group work with the tasks. Setting aside a lesson for using the tasks in the way they were originally designed now seems to have more meaning, as indicated by this teacher's story:

> *When I was thinking about helping students learn to work like a mathematician, my mind drifted to my daughter learning to drive. She*

needed me to model how to do it and then she needed lots of opportunity to try it for herself.

That's when the idea clicked of using the Maths300 lessons as a model and the tasks as a chance for the students to have their turn to be a mathematician.

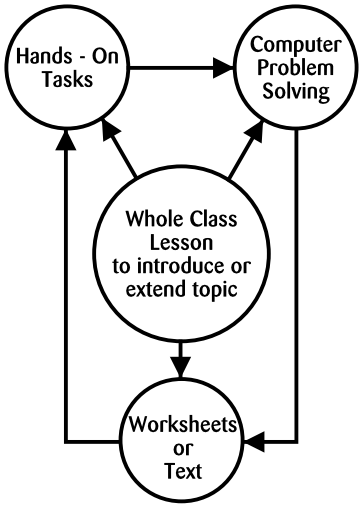
The Maths300 lessons illustrate how other teachers have modelled the process, so I felt I could do it too. Now the process is always on display on the wall or pasted inside the student's journal.

A session just using the tasks had seemed a bit like play time before this. Now I see it as an integral part of learning to work mathematically.

Story 3: Mixed Media

It was our staff discussion on Gardner's theory of Multiple Intelligences that led us into creating mixed media units. That and the access you have provided to tasks and Maths300 software.

We felt challenged to integrate these resources into our syllabus. There was really no excuse for a text book diet that favours the formal learners. We now often use four different modes of learning in the work station structure shown. It can be easily managed by one teacher, but it is better when we plan and execute it together.



Story 4: Replacement Unit

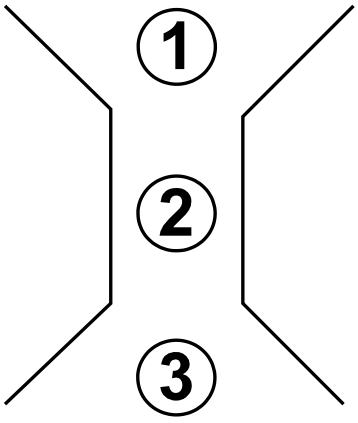
We started meeting with the secondary school maths teachers to try to make transition between systems easier for the students. After considerable discussion we contracted a consultant who suggested that school might look too much the same across the transition when the students were hoping for something new. On the other hand our experience suggested that there needed to be some consistency in the way teachers worked.

We decided to 'bite the bullet' and try a hands-on problem solving unit in one strand. We selected two menus of twenty hands-on tasks, one for the primary and one for the secondary, that became the core of the unit. We deliberately overlapped some tasks that we knew were very rich and added some new ones for the high school.

Class lessons and investigation sheets were used to extend the tasks, within a three week model.

It is important to note that although these teachers structured a 3 week unit for the students, they strongly advised an additional *Week Zero* for staff preparation. The units came to be called Replacement Units.

Week Zero - Planning



Staff familiarise themselves with the material and jointly plan the unit. This is not a model that can be 'planned on the way to class'.

> *Getting together turned out to be great professional development for our group.*

Week 1 - Introduction

Students explore the 20 tasks listed on a printed menu:

- students explore the tip of the task, as on the card
- students move from task to task following teacher questioning that suggests there is more to the task than the tip
- in discussion with students, teachers gather informal assessment information that guides lesson planning for the following week.

> *We gave the kids an 'encouragement talk' first about joining us in an experiment in ways of learning maths and then gave out the tasks. The response was intelligent and there was quite a buzz in the room.*

Week 2 - Formalisation

> *It was good for both us and the students that the lessons in this week were a bit more traditional. However, they weren't text book based. We used whole class lessons based on the tasks they had been exploring and taught the Working Mathematically process, content and report writing.*
>
> *Assessment was via standard teacher-designed tests, quizzes and homework.*

Week 3 - Investigations

> *We were most delighted with Week 3. Each student chose one task from the menu and carried out an in-depth investigation into the iceberg guided by an investigation sheet. They had to publish a report of their investigation and we were quite surprised at the outcomes. It was clear that the first two weeks had lifted the image of mathematics from 'boring repetition' to a higher level of intellectual activity.*

Story 5: Curriculum shift

> *I think our school was like many others. The syllabus pattern was 10 units of three weeks each through the year. We had drifted into that through a text book driven curriculum and we knew the students weren't responding.*

Our consultant suggested that there was sameness about the intellectual demands of this approach which gave the impression that maths was the pursuit of skills. We agreed to select two deeper investigations to add to each unit. It took some time and considerable commitment, but we know that we have now made a curriculum shift. We are more satisfied and so are the students.

The principles guiding this shift were:

- Agree

  The 20 particular investigations for the year are agreed to by all teachers. If, for example, Cube Nets is decided as one of these, then all the teachers are committed to present this within its unit.

- Publish

  The investigations are written into the published syllabus. Students and parents are made aware of their existence and expect them to occur.

- Commit

  Once agreed, teachers are required to present the chosen investigations. They are not a negotiable 'extra'.

- Value

  The investigations each illustrate an explicit form of the Working Mathematically process. This is promoted to students, constantly referenced and valued.

- Assess

  The process provides students with scaffolding for their written reports and is also known by them as the criteria for assessment. *(See next page.)*

- Report

  The assessment component features within the school reporting structure.

A Final Comment

Including investigations has become policy.

Why? Because to not do so is to offer a diminished learning experience.

The investigative process ranks equally with skill development and needs to be planned for, delivered, assessed and reported.

Perhaps most of all we are grateful to our consultant because he was prepared to begin where we were. We never felt as if we had to throw out the baby and the bath water.

Assessment

Our attitude is:

stimulated students are creative and love to learn

Regardless of the way you use your **Maths With Attitude** resource, a variety of procedures can be employed to assess this learning.

Where these assessment procedures are applied to task sessions and involve written responses from students, teachers will need to be careful that the writing does not become too onerous. Students who get bogged down in doing the writing may lose interest in doing the tasks.

In addition to the ideas below, useful references are:

- http://www.mathematicscentre.com/taskcentre/assess.htm
- http://www.mathematicscentre.com/taskcentre/report.htm

The first offers several methods of assessment with examples and the second is a detailed lesson plan to support students to prepare a Maths Report.

Journal Writing

Journal writing is a way of determining whether the task or lesson has been understood by the student. The pupil can comment on such things as:

- What I learned in this task.
- What strategies I/we tried (refer to the Strategy Board).
- What went wrong.
- How I/we fixed it.
- Jottings - ie: any special thoughts or observations

Some teachers may prefer to have the page folded vertically, so that children's reflective thoughts can be recorded adjacent to critical working.

Assessment Form

An assessment form uses questions to help students reflect upon specific issues related to a specific task.

Anecdotal Records

Some teachers keep ongoing records about how students are tackling the tasks. These include jottings on whether students were showing initiative, whether they were working co-operatively, whether they could explain ideas clearly, whether they showed perseverance.

Checklists

A simple approach is to create a checklist based on the Working Mathematically process. Teachers might fill it in following questioning of individuals, or the students may fill it in and add comments appropriately.

Pupil Self-Reflection

Many theorists value and promote metacognition, the notion that learning is more permanent if pupils deliberately and consciously analyse their own learning. The

deliberate teaching strategy of oral questioning and the way pupils record their work is an attempt to manifest this philosophy in action. The alternative is the tempting 'butterfly' approach which is to madly do as many activities as possible, mostly superficially, in the mistaken belief that quantity equates to quality.

> *I had to work quite hard to overcome previously entrenched habits of just getting the answer, any answer, and moving on to the next task.*

Thinking about *what* was learned *how* it was learned consolidates and adds to the learning.

When it follows an extensive whole class investigation, a reflection lesson such as this helps to shift entrenched approaches to mathematics learning. It is also an important component of the assessment process. On the one hand it gives you a lot of real data to assist your assessment. On the other it prepares the students for any formal assessment which you may choose to round off a unit.

Introduction

Ask students to recall what was done during the unit or lesson by asking a few individuals to say what *they* did, eg:

> *What did you do or learn that was new?*
> *What can you now do/understand that is new?*
> *What do you know now that you didn't know 1 (2, 3, ...) lesson ago?*

Continuing Discussion

Get a few ideas from the first students you ask, then:

- organise 5 -10 minute buzz groups of three or four students to chat together with one person to act as a recorder. These groups address the same questions as above.
- have a reporting session, with the recorder from each group telling the class about the group's ideas.

Student comments could be recorded on the board, perhaps in three groups.

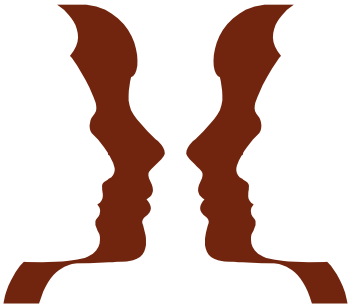
Ideas & Facts	Maths Skills	Process (learning) Skills

If you need more questions to probe deeper and encourage more thought about process, try the following:

> *What new things did you do that were part of* how *you learned?*
> *Who uses this kind of knowledge and skill in their work?*

Student Recording

Hand out the REFLECTION sheet (next page) and ask students to write their own reflection about what they did, based on the ideas shared by the class. Collect these for interest and, possibly, assessment information.



REFLECTION

me looking at me learning

NAME: CLASS:

Reproducible Page

Working With Parents

Balancing Problem Solving with Basic Skill Practice

Many schools find that parents respond well to an evening where they have an opportunity to work with the tasks and perhaps work a task together as a 'whole class'. Resourced by the materials in this kit, teachers often feel quite confident to run these practical sessions. Comments from parents like:

> *I wish I had learnt maths like this.*

are very supportive. Letting students 'host' the evening is an additional benefit to the home/school relationship.

The 4½ Minute Talk

Charles Lovitt has considerable experience working with parents and has developed a crisp, parent-friendly talk which he shares below. Many others have used it verbatim with great success.

Why the Four and a Half Minute Talk?

When talking with parents about Problem Solving or the meaning of the term Working Mathematically, I have often found myself in the position, after having promoted inquiry based or investigative learning, of the parents saying:

> *Well - that's all very well - BUT ...*

at which stage they often express their concern for basic (meaning arithmetic) skill development.

The weakness of my previous attempts has been that I have been unable to reassure parents that problem solving does not mean sacrificing our belief in the virtues of such basic skill development.

One of the unfortunate perceptions about problem solving is that if a student is engaged in it, then somehow they are not doing, or it may be at the expense of, important skill based work.

This Four and a Half Minute Talk to parents is an attempt to express my belief that basic skill practice and problem solving development can be closely intertwined and not seen as in some way mutually exclusive.

(I'm still somewhat uncomfortable using the expression 'basic skills' in the above way as I am certain that some thinking, reasoning, strategy and communication skills are also 'basic'.)

Another aspect of the following 'talk' is that, as teachers put more emphasis on including investigative problem solving into their courses, a question arises about the source of suitable tasks.

This talk argues that we can learn to create them for ourselves by 'tweaking' the closed tasks that heavily populate out existing text exercises, and hence not be dependent on external suppliers. (Even better if students begin to create such opportunities for themselves.)

The Talk

In preparation, write the following graphic on the board:

CLOSED	OPEN	EXTENDED INVESTIGATION
		How many solutions exist? How do you know you have found them all?

I would like to show you what teachers are beginning to do to achieve some of the thinking and reasoning and communication skills we hope students will develop. I would like to show you three examples.

Example One: *6 + 5 = ?*

I write this question under the 'closed' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 —		How many solutions exist? How do you know you have found them all?

And I ask:

What is the answer to this question?

I then explain that:

We often ask students many closed questions such as 6 + 5 = ?

The only response the students can tell us is "The answer is 11." … and as a reward for getting it correct we ask another twenty questions just like it.

What some teachers are doing is trying to *tweak* the question and ask it a different way, for example:

I have two counting numbers that add to 11. What might the numbers be?

[Counting numbers = positive whole numbers including zero]

I write this under the 'open' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 ___	? + ? 11	How many solutions exist? How do you know you have found them all?

What is the answer to the question now?

At this stage it becomes apparent there are several solutions:

The question is now a bit more open than it was before, allowing students to tell you things like 8 + 3, or 10 + 1, or 11 + 0 etc.

Let's see what happens if the teacher 'tweaks' it even further with the investigative challenge *or* extended investigation question:

How many solutions are there altogether?

and more importantly, and with greater emphasis on the second question:

How could you convince someone else that you have found them all?

Now the original question is definitely different - it still involves the skills of addition but now also involves thinking, reasoning and problem solving skills, strategy development and particularly communication skills.

Young students will soon tell you the answer is 'six different ones', but they must also confront the communication and reasoning challenge of convincing you that there are only six and no more.

Example Two: *Finding Averages*

Again, as I go through this example, I write it into the diagram on the board in the relevant sections.

The CLOSED question is: *11, 12, 13 - find the average*

Tweaking this makes it an OPEN question and it becomes:

I have three counting numbers whose average is 12. What might the numbers be?

Students will often say:

10, 12, 14 ... or 9, 12, 15 ... or even 12, 12, 12

After realising there are many answers, you can tweak it some more and turn it into an EXTENDED INVESTIGATION:

How many solutions exist? ... AND ...

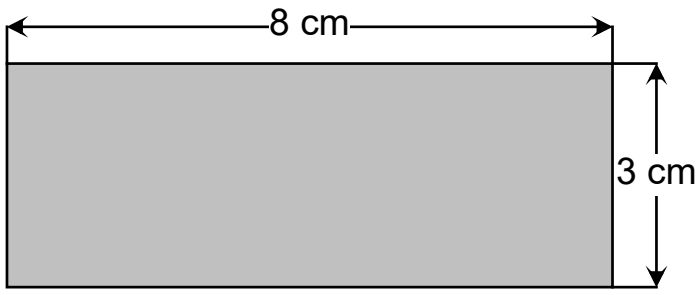
How do you know you have found them all?

Now the question is of a quite different nature. It still involves the arithmetic skill, but has something else as well - and that something else is the thinking, reasoning and communication skills necessary to find all of the combinations and convince someone else that you have done so.

By the time a student announces, with confidence, there are 127 different ways (which there are) that student will have engaged in all of these aspects, ie: the skill of calculating averages, (and some combination number theory) as well as significant strategy and reasoning experiences.

Example Three: *Finding the Area of a Rectangle*

A typical CLOSED question is:



Find the area. Find the perimeter.

The OPEN question is:

A rectangle has 24 squares inside:

What might its length and width be?

What might its perimeter be?

The EXTENDED INVESTIGATION version is:

Given they are whole number lengths, how many different rectangles are there? … AND …

How do you know you have found them all?

In summary, mathematics teachers are trying to convert *some* (not all) of the many closed questions that populate our courses and 'push' them towards the investigation direction. In doing so, we keep the skills we obviously value, but also activate the thinking, reasoning and justification skills we hope students will also develop.

This sequence of three examples hopefully shows two major features:

- That skills and problem solving can 'live alongside each other' and be developed concurrently.
- That the process of creating open-ended investigations can be done by anyone - just go to any source of closed questions and try 'tweaking' them as above. If it only worked for one question per page it would still provide a very large supply of investigations.

In terms of the effect of the talk on parents, I have usually found them to be reassured that we are not compromising important skill development (and nor do we want to). The only debate then becomes whether the additional skills of thinking, reasoning and communication are also desirable.

I've also been told that parents appreciate it because of the essential simplicity of the examples - no complicated theoretical jargon.

A Working Mathematically Curriculum

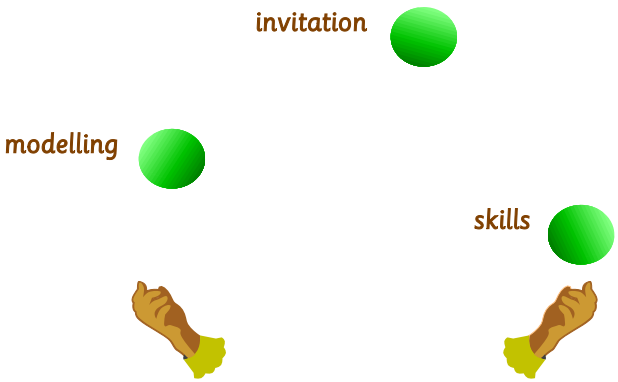
An Investigative Approach to Learning

The aim of a Working Mathematically curriculum is to help students learn to work like a mathematician. This process is detailed earlier (Page 8) in a one page document which becomes central to such a curriculum.

The change of emphasis brings a change of direction which *implies and requires* a balance between:

- the process of being a mathematician, and
- the development of skills needed to be a *successful* mathematician.

This journey is not two paths. It is one path made of two interwoven threads in the same way as DNA, the building block of life, is one compound made of two interwoven coils. To achieve a Working Mathematically curriculum teachers need to balance three components.



The task component of **Maths With Attitude** offers each pair of students an invitation to work like a mathematician.

The Maths300 component of **Maths With Attitude** assists teachers to model working like a mathematician.

Content skills are developed in context. They *are* important, but it is the application of skills within the process of Working Mathematically that has developed, and is developing, the human community's mathematical knowledge.

A focus for the Working Mathematically teacher is to help students develop mathematical skills in the context of problem posing and solving.

> *We are all 'born' with the same size mathematical toolbox, in the same way as I can own the same size toolbox as my motor mechanic. However, my motor mechanic has many more tools in her box than I and she has had more experience than I using them in context. Someone has helped her learn to use those tools while crawling under a car.*

Afzal Ahmed, Professor of Mathematics at Chichester, UK, once quipped:

> *If teachers of mathematics had to teach soccer, they would start off with a lesson on kicking the ball, follow it with lessons on trapping the ball and end with a lesson on heading the ball. At no time would they play a game of football.*

Such is not the case when teaching a Working Mathematically curriculum.

Elements of a Working Mathematically Curriculum

Working Mathematically is a K - 12 experience offering a balanced curriculum structured around the components below.

Hands-on Problem Solving Play

Mathematicians don't know the answer to a problem when they start it. If they did, it wouldn't be a problem. They have to play around with it. Each task invites students to play with mathematics 'like a mathematician'.

Skill Development

A mathematician needs skills to solve problems. Many teachers find it makes sense to students to place skill practice in the context of *Toolbox Lessons* which *help us better use the Working Mathematically Process* (Page 8).

Focus on Process

This is what mathematicians do; engage in the problem solving process.

Strategy Development

Mathematicians also make use of a strategy toolbox. These strategies are embedded in Maths300 lessons, but may also have a separate focus. Poster Problem Clinics are a useful way to approach this component.

Concept Development

A few major concepts in mathematics took centuries for the human race to develop and apply. Examples are place value, fractions and probability. In the past students have been expected to understand such concepts after having 'done' them for a two week slot. Typically they were not visited again until the next year. A Working Mathematically curriculum identifies these concepts and regularly 'threads' them through the curriculum.

Planning to Work Mathematically

The class, school or system that shifts towards a Working Mathematically curriculum will no longer use a curriculum document that looks like a list of content skills. The document would be clear in:

- choosing genuine problems to initiate investigation
- choosing a range of best practice teaching strategies to interest a wider range of students
- practising skills for the purpose of problem solving

Some teachers have found the planning template on the next page assists them to keep this framework at the forefront of their planning. It can be used to plan single lessons, or units built of several lessons. There are examples from schools in the Curriculum & Planning section of Maths300 and a Word document version of the template.

Unit Planning Page

Class

Topic

Pedagogy	Problem Solving In this topic how will I engage my students in the Working Mathematically process?	Skills
How do I create an environment where students know what they are doing and why they have accepted the challenge?		Does the challenge identify skills to practise? Are there other skills to practise in preparation for future problem solving?

Notes

As a general guide:

- Find a problem(s) to solve related to the topic.
- Choose the best teaching craft likely to engage the learners.
- Where possible link skill practice to the problem solving process.

More on Professional Development

For many teachers there will be new ideas within **Maths With Attitude**, such as unit structures, views of how students learn, teaching strategies, classroom organisation, assessment techniques and use of concrete materials. It is anticipated (and expected) that as teachers explore the material in their classrooms they will meet, experiment with and reflect upon these ideas with a view to long term implications for the school program and for their own personal teaching.

Being explored 'on-the-job' so to speak, in the teacher's own classroom, makes the professional development more meaningful and practical for the teacher. This is also a practical and economic alternative for a local authority.

Strategic Use by Systems

From Years 3 - 10, **Maths With Attitude** is designed as a professional development vehicle by schools or clusters or systems because it carries a variety of sound educational messages. They might choose **Maths With Attitude** because:

- It can be used to highlight how investigative approaches to mathematics can be built into balanced unit plans without compromising skill development and without being relegated to the margins of a syllabus as something to be done only after 'the real' content has been covered.
- It can be used to focus on how a balance of concept, skill and application work can all be achieved within the one manageable unit structure.
- It can be used to show how a variety of assessment practices can be used concurrently to build a picture of student progress.
- It can be used to focus on transition between primary and secondary school by moving towards harmony and consistency of approach.
- It can be used to raise and continue debate about the pedagogy (art of teaching) that supports deeper mathematical learning for a wider range of students.

Teachers in Years K - 2 are similarly encouraged in professional growth through **Working Mathematically with Infants**, which derives from Calculating Changes, a network of teachers enhancing children's number skills from Years K - 6.

In supporting its teachers by supplying these resources in conjunction with targeted professional development over time, a system can fuel and encourage classroom-based debate on improving outcomes. There is evidence that by exploring alternative teaching strategies and encouraging curriculum shift towards Working Mathematically, learners improve and teachers are more satisfied. For more detail visit Research & Stories at:

- http://www.mathematicscentre.com/taskcentre/do.htm

We would be happy to discuss professional development with system leaders.

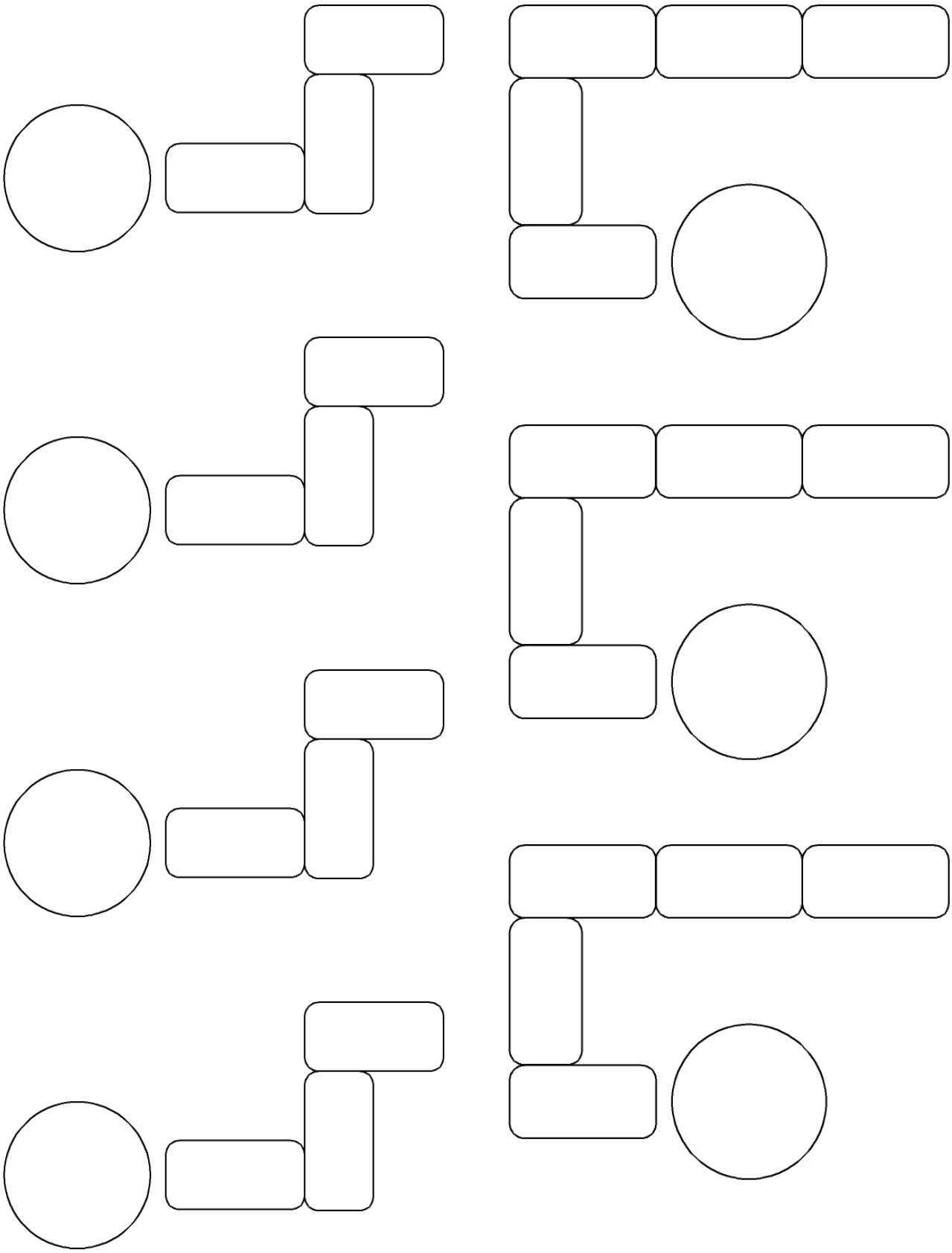
Web Reference

The starting point for all aspects of learning to work like a mathematician, including Calculating Changes, and the teaching craft which encourages it is:

- http://www.mathematicscentre.com/mathematicscentre

Appendix: Recording Sheets

Domino Trails Recording Sheet



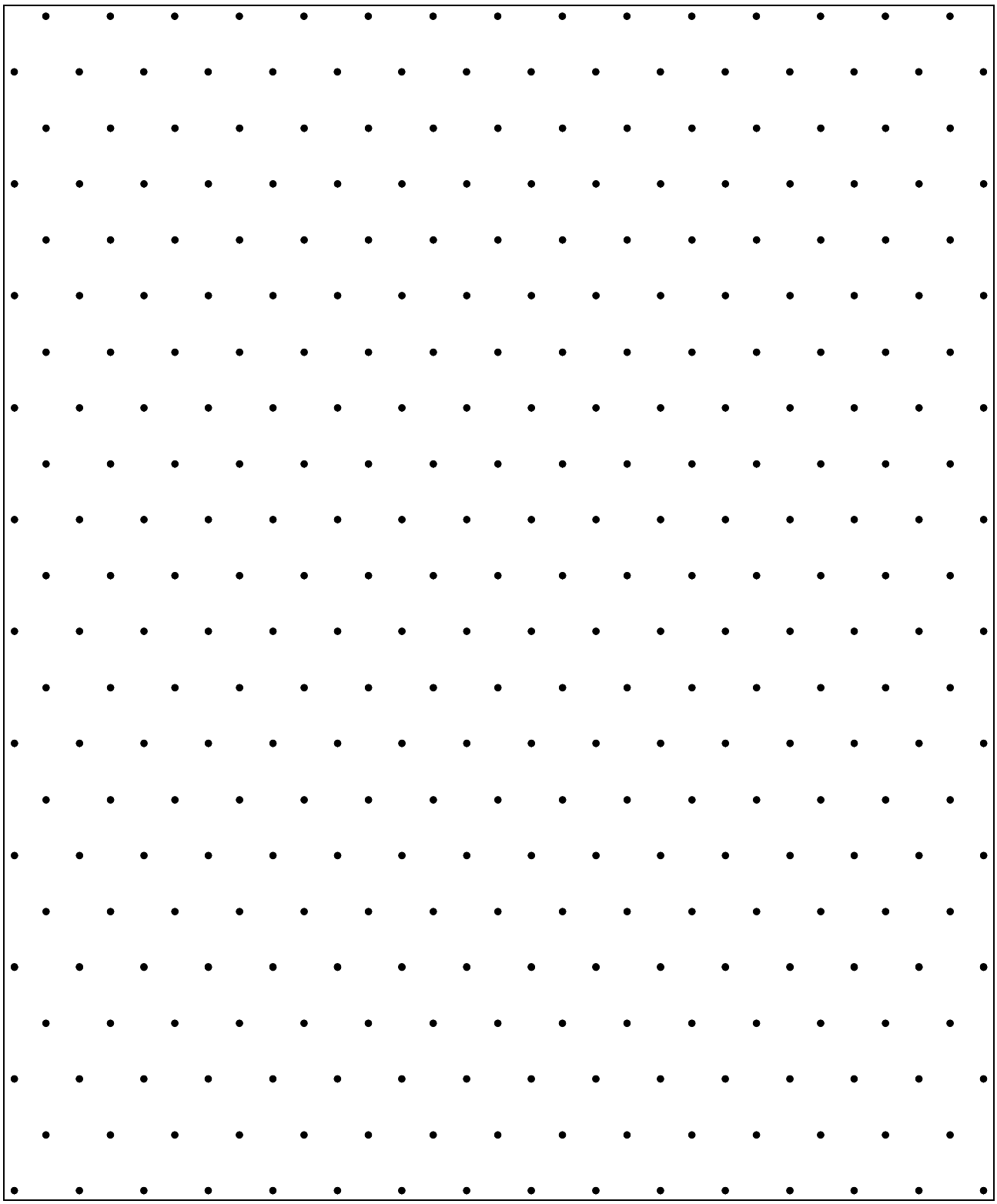
Make Your Own Make The Whole Game

Reproducible Page

MAKE THE WHOLE CARDS - SET	..... ROD IS WHOLE	

What's It Worth?

Reproducible Page



Names: ..... Class: .....