maths with attitude

2nd Edition

Chance & Measurement Years 9 & 10

Charles Lovitt
Doug Williams

Mathematics Task Centre & Maths300
helping to create happy healthy cheerful productive inspiring classrooms

maths with attitude

2nd Edition

Chance & Measurement

Years 9 & 10

In this kit:

- *Hands-on problem solving tasks*
- *Detailed curriculum planning*

Access from Maths300:

- *Extensive lesson plans*
- *Software*

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Maths With Attitude includes reference to hands-on problem solving tasks and Maths300 material.

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- To support teachers, schools and systems wanting to create:

  happy, healthy, cheerful, productive, inspiring classrooms

Our Attitude

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  learning is a personal journey stimulated by achievable challenge
- to learners:

  stimulated students are creative and love to learn
- to pedagogy:

  the art of choosing teaching strategies to involve and interest all students
- to mathematics:

  mathematics is concrete, visual and makes sense
- to learning mathematics:

  all students can learn to work like a mathematician
- to teachers:

  the teacher is the most important resource in education
- to professional development:

  teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

Our Objective in Detail

What do we mean by creating:

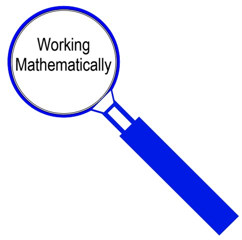
happy, healthy, cheerful, productive, inspiring classrooms

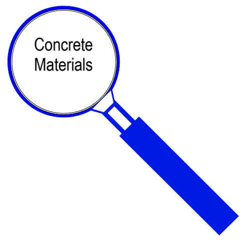
Happy…

means the elimination of the unnecessary fear of failure that hangs over so many students in their mathematics studies. Learning experiences *can* be structured so that all students see there is something in it for them and hence make a commitment to the learning. In so many 'threatening' situations, students see the impending failure and withhold their participation.

A phrase which describes the structure allowing all students to perceive something in it for them is *multiple entry points and multiple exit points*. That is, students can enter at a variety of levels, make progress and exit the problem having visibly achieved.

Healthy…





means *educationally healthy*. The learning environment should be a reflection of all that our community knows about how students learn. This translates into a rich array of teaching strategies that could and should be evident within the learning experience.

If we scrutinise the *exploration* through any lens, it should confirm to us that it is well structured or alert us to missed opportunities. For example, peering through a pedagogy lens we should see such features as:

- a story shell to embed the situation in a meaningful context
- significant active use of concrete materials
- a problem solving challenge which provides ownership for students
- small group work
- a strong visual component
- access to supportive software

Cheerful…

because we want 'happy' in the title twice!

Productive…

is the clear acknowledgment that students are working towards recognisable outcomes. They should know what these are and have guidelines to show they have either reached them or made progress. Teachers are accountable to these outcomes as well as to the quality of the learning environment.

Inspiring…

is about creating experiences that are uplifting or exalting; that actually *turn students on*. Experiences that make students feel great about themselves and empowered to act in meaningful ways.

Chance & Measurement Resources

To help you create

happy, healthy, cheerful, productive, inspiring classrooms

this kit contains

- 20 hands-on problem solving tasks from Mathematics Centre

and a Teachers' Manual which integrates the use of the tasks with

- 18 detailed lesson plans from Maths300

The kit offers **7 weeks** of Scope & Sequence planning in Chance & Measurement for *each* of Year 9 and Year 10. This is detailed in *Part 2: Planning Curriculum* which begins on Page 12. You are invited to map these weeks into your Year Planner.

Together, the four kits available for these levels provide 25 weeks of core curriculum in Working Mathematically (working like a mathematician).

Note: Membership of Maths300 is assumed.
The kit will be useful without it, but it will be much more useful with it.

Tasks

- 64 = 65
- Cross & Square
- Dice Differences
- Dice Footy
- Final Eight
- First Down The Mountain
- Game Show
- Growing Tricubes
- Have A Hexagon
- Photo Angles
- Planets
- Pythagoras 1
- Pythagoras 2
- Pythagoras Rods
- Rectangle Nightmare
- Same Or Different
- Take A Chance
- The Hole In The Triangle
- Time Swing
- What's In The Bag?

Part 2 of this manual introduces each task. The latest information can be found at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm

Maths300 Lessons

- Biggest Volume
- Cylinder Volumes
- Dice Differences
- Dice Footy
- Famous Math'ticians
- First Down... Mountain
- Game Show
- Greedy Pig
- Have A Hexagon
- Newspaper Pathways
- Planets
- Pythagoras...Polygons
- Radioactivity
- Same Or Different
- Sporting Finals
- The Grubby Pgs Effect
- Trigonometry Walk
- What's In The Bag?

Lessons with Software

- Biggest Volume
- Cylinder Volumes
- Dice Differences
- Dice Footy
- First Down...Mountain
- Game Show
- Greedy Pig
- Have A Hexagon
- Radioactivity
- Same Or Different
- Sporting Finals
- Trigonometry Walk

Part 2 of this manual introduces each lesson. Full details can be found at:

- http://www.maths300.com

Working Like A Mathematician

Our attitude is:

all students can learn to work like a mathematician

What does a mathematician's work actually involve? Mathematicians have provided their answer on Page 8. In particular we are indebted to Dr. Derek Holton for the clarity of his contribution to this description.

Perhaps the most important aspect of Working Mathematically is the recognition that *knowledge is created by a community and becomes part of the fabric of that community*. Recognising, and engaging in, the process by which that knowledge is generated can help students to see themselves as able to work like a mathematician. Hence Working Mathematically is the framework of **Maths With Attitude.**

Skills, Strategies & Working Mathematically

A Working Mathematically curriculum places learning mathematical skills and problem solving strategies in their true context. Skills and strategies are the tools mathematicians employ in their struggle to solve problems. Lessons on skills or lessons on strategies are not an end in themselves.

- **Our skill toolbox** can be added to in the same way as the mechanic or carpenter adds tools to their toolbox. Equally, the addition of the tools is not for the sake of collecting them, but rather for the purpose of getting on with a job. A mathematician's job is to attempt to solve problems, not to collect tools that might one day help solve a problem.
- **Our strategy toolbox** has been provided through the collective wisdom of mathematicians from the past. All mathematical problems (and indeed life problems) that have ever been solved have been solved by the application of this concise set of strategies.

About Tasks

Our attitude is:

mathematics is concrete, visual and makes sense

Tasks are from Mathematics Task Centre. They are an invitation to two students to work like a mathematician (see Page 8).

The Task Centre concept began in Australia in the late 1970s as a collection of rich tasks housed in a special room, which came to be called a Task Centre. Since that time hundreds of Australian teachers, and, more recently, teachers from other countries, have adapted and modified the concept to work in their schools. For example, the special purpose room is no longer seen as an essential component, although many schools continue to opt for this facility.

A brief history of Task Centre development, considerable support for using tasks, for example Task Cameos, and a catalogue of all currently available tasks can be found at:

- http://www.mathematicscentre.com/taskcentre

Key principles are:

- A good task is the tip of an iceberg
- Each task has three lives
- Tasks involve students in the Working Mathematically process

The Task Centre Room or the Classroom?

There are good reasons for using the tasks in a special room which the students visit regularly. There are also different good reasons for keeping the tasks in classrooms. Either system can work well if staff are committed to a core curriculum built around learning to work like a mathematician.

- A task centre room creates a focus and presence for mathematics in the school. Tasks are often housed in clear plastic 'cake storer' type boxes. Display space can be more easily managed. The visual impact can be vibrant and purposeful.
- However, tasks can be more readily integrated into the curriculum if teachers have them at their finger tips in the classrooms. In this case tasks are often housed in press-seal plastic bags which take up less space and are more readily moved from classroom to classroom.

Tip of an Iceberg

The initial problem on the card can usually be solved in 10 to 20 minutes. The investigation iceberg which lies beneath may take many lessons (even a lifetime!). Tasks are designed so that the original problem reveals just the 'tip of the iceberg'. Task Cameos and Maths300 lessons help to dig deeper into the iceberg.

We are constantly surprised by the creative steps teachers and students take that lead us further into a task. No task is ever 'finished'.

> *Most tasks have many levels of entry and exit and therefore offer an on-going invitation to revisit them, and, importantly, multiple levels of success for students.*

Three Lives of a Task

This phrase, coined by a teacher, captures the full potential and flexibility of the tasks. Teachers say they like using them in three distinct ways:

1. As on the card, which is designed for two students.
2. As a whole class lesson involving all students, as supported by outlines in the Task Cameos and in detail through the Maths300 site.
3. Extended by an Investigation Guide (project), examples of which are included in both Task Cameos and Maths300.

The first life involves just the 'tip of the iceberg' of each task, but nonetheless provides a worthwhile problem solving challenge - one which 'demands' concrete materials in its solution. This is the invitation to work like a mathematician. Most students will experience some level of success and accomplishment in a short time.

The second life involves adapting the materials to involve the whole class in the investigation, in the first instance to model the work of a mathematician, but also to develop key outcomes or specific content knowledge. This involves choosing teaching craft to interest the students in the problem and then absorb them in it.

The third life challenges students to explore the 'rest of the iceberg' independently. Investigation Guides are used to probe aspects and extensions of the task and can be introduced into either the first or second life. Typically this involves providing suggestions for the direction the investigation might take. Students submit the 'story' of their work for 'portfolio assessment'. Typically a major criteria for assessment is application of the Working Mathematically process.

About Maths300

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

Maths300 is a subscription based web site. It is an attempt to collect and publish the 300 most 'interesting' maths lessons (K - 12).

- *Lessons have been successfully trialed in a range of classrooms.*
- *About one third of the lessons are supported by specially written software.*
- *Lessons are also supported by investigation sheets (with answers) and game boards where relevant.*
- *A 'living' Classroom Contributions section in each lesson includes the latest information from schools.*
- *The search engine allows teachers to find lessons by pedagogical feature, curriculum strand, content and year level.*
- *Lesson plans can be printed directly from the site.*
- *Each lesson supports teaches to model the Working Mathematically process.*

Modern internet facilities and computers allow teachers easy access to these lesson plans. Lesson plans need to be researched, reflected upon in the light of your own students and activated by collecting and organising materials as necessary.

Maths300 Software

Our attitude is:

> *stimulated students are creative and love to learn*

Pedagogically sound software is one feature likely to encourage enthusiastic learning and for that reason it has been included as an element in about one third of Maths300 lesson plans. The software is used to develop an investigation beyond its introduction and early exploration which is likely to include other pedagogical techniques such as concrete materials, physical involvement, estimation or mathematical conversation. The software is not the lesson plan. It is a feature of the lesson plan used at the teacher's discretion.

For school-wide use, the software needs to be downloaded from the site and installed in the school's network image. You will need to consult your IT Manager about these arrangements. It can also be downloaded to stand alone machines covered by the site licence, in particular a teacher's own laptop, from where it can be used with the whole class through a data projector.

Note:

- *Maths300 lessons and software may only be used by Maths300 members.*

Working Mathematically

First give me an interesting problem.

When mathematicians become interested in a problem they:

- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

Questions which help mathematicians learn more are:

- Can I check this another way?
- What happens if ...?
- How many solutions are there?
- How will I know when I have found them all?

When mathematicians have a problem they:

- Read & understand the problem.
- Plan a strategy to start the problem.
- Carry out their plan.
- Check the result.

A mathematician's strategy toolbox includes:

- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- Break a problem into smaller parts
- ...

If one way doesn't work, I just start again another way.

Professional Development Purpose

Our attitude is:

the teacher is the most important resource in education

> *We had our first study group on Monday. The session will be repeated again on Thursday. I had 15 teachers attend. We looked at the task Farmyard Friends (Task 129 from the Mathematics Task Centre). We extended it out like the questions from the companion Maths300 lesson suggested, and talked for quite a while about the concept of a factorial. This is exactly the type of dialog that I feel is essential for our elementary teachers to support the development of their math background. So anytime we can use the tasks to extend the teacher's math knowledge we are ahead of the game.*
> District Math Coordinator, Denver, Colorado

Research suggests that professional development most likely to succeed:

- is requested by the teachers
- takes place as close to the teacher's own working environment as possible
- takes place over an extended period of time
- provides opportunities for reflection and feedback
- enables participants to feel a substantial degree of ownership
- involves conscious commitment by the teacher
- involves groups of teachers rather than individuals from a school
- increases the participant's mathematical knowledge in some way
- uses the services of a consultant and/or critical friend

Maths With Attitude has been designed with these principles in mind. All the materials have been tried, tested and modified by teachers from a wide range of classrooms. We hope the resources will enable teacher groups to lead themselves further along the professional development road, and support systems to improve the learning outcomes for students K - 12.

With the support of Maths300 ETuTE, professional development can be a regular component of in-house professional development. See:

- http://www.mathematicscentre.com/taskcentre/resource.htm#etute

For external assistance with professional development, contact:

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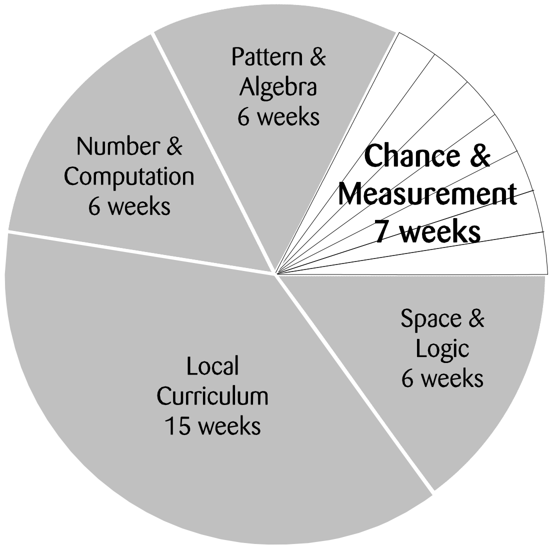
Part 2:
Planning
Curriculum

Curriculum Planners

Our attitude is:

learning is a personal journey stimulated by achievable challenge



Curriculum Planners:

- show one way these resources can be integrated into your weekly planning
- provide a starting point for those new to these materials
- offer a flexible structure for those more experienced

You are invited to map Planner weeks into your school year planner as the core of the curriculum.

Planners:

- detail each week lesson by lesson
- offer structures for using tasks and lessons
- are sequenced from lesson to lesson, week to week and year to year to 'grow' learning

Teachers and schools will map the material in their own way, but all will be making use of extensively trialed materials and pedagogy.

Using Resources

- Your kit contains 20 hands-on problem solving tasks and reference to relevant Maths300 lessons.
- Tasks are introduced in this manual and supported by the Task Cameos at:
  http://www.mathematicscentre.com/taskcentre/iceberg.htm
- Maths300 lessons are introduced in this manual and supported by detailed lesson plans at:
  http://www.maths300.com

In your preparation, please note:

- Planners assume 4 lessons per week of about 1 hour each.
- Planners are *not* prescribing a continuous block of work.
- Weeks can be interspersed with other learning; perhaps a **Maths With Attitude** week from a different strand.
- Weeks can sometimes be interchanged within the planner.
- Lessons can sometimes be interchanged within weeks.
- The four **Maths With Attitude** kits available at each year level offer 25 weeks of a Working Mathematically core curriculum.

A Way to Begin

- Glance over the Planner for your class. Skim through the comments for each task and lesson as it is named. This will provide an overview of the kit.
- Task Comments begin after the Planners. Lesson Comments begin after Task Comments. The index will also lead you to any task or lesson comments.
- Select your preferred starting week - usually Week 1.
- Now plan in detail by researching the comments and web support. Enjoy!

Research, Reflect, Activate

Curriculum Planner

Chance & Measurement: Year 9

	Session 1	Session 2	Session 3	Session 4
Weeks 1 - 3	**Data On Display - Mixed Media Unit A:** The Mixed Media teaching model is explained on Page 18. It assumes ready access to computers for one third of the class. If these are not a fixture in the room, schools have adopted alternatives such as (a) making arrangements for students to visit computer sites within the school, or, (b) collecting an appropriate number of notebook computers (5 or 6 for a class of 30). The focus is recording and displaying data - various forms of distribution and the statistics which describe them. Tasks used are **Dice Differences**, **Dice Footy**, **Final Eight**, **First Down The Mountain**, **Game Show**, **Have A Hexagon**, **Same Or Different**, **Take A Chance**, **Time Swing**, **What's In The Bag?** A Mixed Media Unit includes one whole class lesson each week and in this unit teachers can choose from *Dice Differences*, *Greedy Pig* or *Have A Hexagon*.			
Week 4	**What Can I Expect?:** Two whole class investigations. *Famous Mathematicians* explores a mathematical model to explain the chances of collecting a complete set of swap cards. It has strong cross-curriculum links and highlights real mathematicians and their lives. *What's in the Bag?* explores a mathematical model related to the sampling techniques used in 'real world' surveys.			
Weeks 5 & 6	**Pythagoras & Trigonometry - Self-Directed Maths Journey I:** For two weeks, students journey through a mathematical landscape in pairs and keep a diary of their adventures. See Page 20 for details. The journey includes tasks, software, skill practice, and text exercises. The unit uses the lessons *Pythagoras & Other Polygons* and *Trigonometry Walk* and the tasks **Cross & Square**, **Photo Angles**, **Pythagoras 1**, **Pythagoras** 2 and **Pythagoras Rods**.			
Week 7	**Focus On Football:** Two whole class investigations. *Dice Footy* introduces a mathematical model of Aussie Rules Football and uses this to explore many chance and data concepts. *Sporting Finals* investigates a Final Eight model that could be used to decide the champion team in many sports, including Aussie Rules.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Curriculum Planner
Chance & Measurement: Year 10

	Session 1	Session 2	Session 3	Session 4
Weeks 1 & 2	**Puzzles & Paradoxes - Self-Directed Maths Journey II:** For two weeks, students journey through a mathematical landscape in pairs and keep a diary of their adventures. See Page 20 for details. The journey includes tasks, software, skill practice, and text exercises. The unit uses the lessons *Game Show* and *The Grubby Pages Effect* and the tasks **64 = 65**, **Planets**, **Rectangle Nightmare**, **The Hole In The Triangle** and **Growing Tricubes**.			
Weeks 3 - 5	**Data On Display - Mixed Media Unit B:** The Mixed Media teaching model is explained on Page 18. It assumes ready access to computers for one third of the class. If these are not a fixture in the room, schools have adopted alternatives such as (a) making arrangements for students to visit computer sites within the school, or, (b) collecting an appropriate number of notebook computers (5 or 6 for a class of 30). The content focus is recording and displaying data - various forms of distribution and the statistics which describe them. The unit also refreshes many important aspects of chance and probability. Tasks used are **Dice Differences**, **Dice Footy**, **Final Eight**, **First Down The Mountain**, **Game Show**, **Have A Hexagon**, **Same Or Different**, **Take A Chance**, **Time Swing**, **What's In The Bag?** A Mixed Media Unit includes one whole class lesson each week and in this unit teachers can choose from *First Down The Mountain*, *Radioactivity* or *Same Or Different*.			
Weeks 6 & 7	**Make Mine Measurement:** Four whole class investigations. *Newspaper Pathways*, *Planets*, *Biggest Volume*, and *Cylinder Volumes* apply measurement to very large numbers and involve conversion of units. A side benefit is that the lessons introduce a need to find ways of representing large numbers, which opens the door to content such as Standard Form. *Biggest Volume* and *Cylinder Volumes* establish ground for later calculus by exploring the effect of incremental changes in measurement.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Planning Notes

Enhancing Maths With Attitude

Resources to support learning to work like a mathematician are extensive and growing. There are more tasks and lessons available than have been included in this Chance & Measurement kit. You could use the following to enhance this kit.

Additional Tasks

- Task 12, Matching Cards
  The initial challenge is to correctly match given pieces, then to order the correct matches. But how many not-matches are possible? Originally designed as an example of what a task card for Infants (Years K - 2) might look like, this one simple question opens the task to much older students - even students in senior high school.
- Task 241, Sicherman Dice
  It's easy enough to work out the possible results when two usual cube dice are rolled and the numbers added. It's also not too much of a challenge to work out the distribution of those sums and the associated probability of each outcome. But what happens if I asked you to find other cube dice that could be rolled and summed to get the same probabilities? And what happens if I restrict the dice faces to non-zero whole numbers? These investigations are the focus of Sicherman Dice.

More information about these tasks may be available in the Task Cameo Library:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Additional Lessons

- Lesson 111, Baby In The Car
  Babies, toddlers and animals must never be locked in cars on a hot day. Why? Because they will quickly dehydrate. While there are physiological reasons for this, one major factor is mathematical; the smaller the volume, the greater the proportional surface area. Mathematical modelling is used in this lesson to inform discussion of a real-life social issue. Concrete materials help students see, understand and explain why the surface area to volume ratio of babies is much greater than that of adults.
- Lesson 126, Make A Moke
  Played in a similar way to the parlour game Beetle (Lesson 121), Make A Moke is used to challenge student misconceptions about probability related to a cube dice. In particular the lesson focuses on the misconception that 'the six never comes up for me' and explores the reality behind this idea both empirically and theoretically.
- Lesson 132, 64 = 65
  We all know 64 does not equal 65! But this jigsaw puzzle visually suggests that it is true. This paradox puzzle is one of a genre of 'missing square puzzles'. Cut the 4 pieces from an 8x8 frame and the area is clearly 64. Place the same pieces into a 13x5 frame and the area now appears to be 65. Where did the extra square come from? The search for, and explanation of, the extra square involves many different mathematical tools and suits many year levels. However, noticing that

the key numbers involved (3, 5, 8 and 13) are successive terms of the Fibonacci sequence is the starting point for an extended investigation.

- Lesson 159, Chance With Crosses

  At random, (face down) place the digits 1 to 9 in the shape of a 'plus' sign. What is the chance that when turned face up, the sum of the digits on the horizontal arm will equal the sum of the digits on the vertical arm? The results are quite counter-intuitive when compared to students' initial expectations. The companion software allows the chances to be explored at significantly greater depth and allows the empirical results to be compared with theoretical calculations.
- Lesson 170, Take A Chance

  This lesson is based around a card game which involves risking counters at each play. The game situation initiates interest and the requirement to risk counters is a measure of student understanding of the chances involved. As they experience the game and make judgements, each student develops a notion of 'good chance'. Challenging students to state their clues for this 'good chance' opens a range of possible explorations, including whole class investigation, small group work guided by an Investigation Sheet or a major project.
- Lesson 175, Dice Cricket

  This game simulates real limited-over cricket formats such as one-day cricket or Twenty20. Played as a game between two students, the context provides a rich array of mathematical ideas from a simple starting point. The lesson involves collecting and analysing class data which allows analysis of several aspects of the mathematics. Using the computer simulation of the game, long-term patterns can be explored, and empirical results compared with theoretical expectations.
- Lesson 180, Maths of Lotto

  Gambling, in all forms, is a significant social issue. This lesson focuses on the Lotto-style game and takes the approach that if players better understand their chances they are likely to make better choices about how much of their income to commit to the chance of winning. The game is presented as a whole class investigation involving probability, combination theory, statistics and working mathematically. Companion software extends the problem solving possibilities.
- Lesson 183, Snakes & Ladders

  This famous childhood game originated in ancient India around 2 CE and is riddled with opportunities for mathematical investigation. After playing, exploring and analysing the game, students can take on the role of 'mathematical game board designers' and produce their own game boards to meet desired specifications. Using a computer, students can add snakes or ladders wherever they choose and analyse the effects of doing so. Hypotheses can be created and tested, both theoretically and empirically.

Keep in touch with new developments which enhance **Maths With Attitude** at:

- http://www.mathematicscentre.com/taskcentre/enhance.htm

Additional Materials

As stated, our attitude is that mathematics is concrete, visual and makes sense. We assume that all classrooms will have easy access to many materials beyond what we supply. For this unit you will need:

- Calculators
- Counters
- Dice
- Length of rope (Trigonometry Walk)
- Long measuring tape, say 30m or more (Planets)
- Lots of 2cm cubes in a range of colours. For *Cylinder Volumes* it is best if these are wooden cubes, but Unifix cubes could work. For *Same Or Different* and *What's In The Bag?* either type is fine.
- Newspapers
- Opaque plastic cups, or equivalent (for Game Show)

Special Comments Year 9

- Mixed Media Unit: Planner Weeks 1 - 3. A Mixed Media unit takes a little extra preparation in the beginning, but this is repaid during the unit because it runs for several days. Schools which have used the model find team preparation eases any burden, ie:
  - one teacher ensures the software station is prepared and runs effectively during the unit
  - one gets the tasks in order and keeps them that way during the unit
  - one selects appropriate material from the text and prepares the 'contract' sheet
  - one makes sure all teachers are resourced with whatever is necessary for the whole class lessons.
- If you are going to use *Have A Hexagon* as a whole class lesson in the Mixed Media Unit, you will need to print playing boards. Perhaps it would be useful to do this on thin card and laminate them so they are available for future years.
- *Famous Mathematicians*, Planner Week 4. The posters and 'swap' cards supplied in the lesson will take time to prepare well.
- Self-directed Maths Journey: Planner Weeks 5 & 6. A Self-directed Maths Journey may be a new teaching strategy for you, so you will need to take time to think it through.
- *Sporting Finals*, Planner Week 7. This will work well if timed to match the Final Series time of the year of the appropriate local sport. There are also playing boards for these lessons which might be best prepared on thin card and laminated so they are available for future years. The *Sporting Finals* board is more amiable to use if it is enlarged to A3.

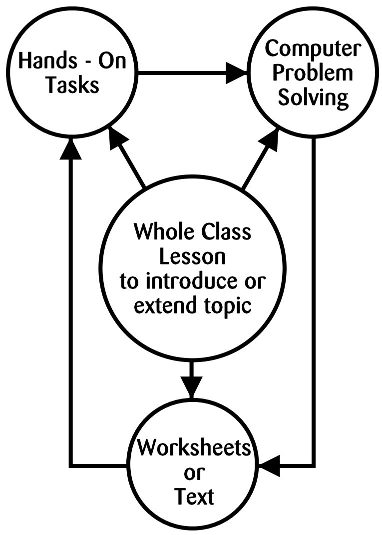
Special Comments Year 10

- Self-directed Maths Journey Planner Weeks 1 & 2. A Self-directed Maths Journey may be a new teaching strategy for you, so you will need to take time to think it through. You will also need the plastic cups or alternative for *Game Show*.

- Mixed Media Unit: Planner Weeks 3 - 5. See comments above for the Year 9 Mixed Media Units.
- If you choose *Same Or Different* as one of the whole class investigations, you will need to prepare sets of cubes (or the like) in advance. Perhaps a class set suitable for the lesson could be prepared once, then stored until the next time the lesson is used.
- *Newspaper Pathways*, Planner Weeks 6 & 7. You will need newspapers and cubes in significant numbers.
- *Planets*, Planner Weeks 6 & 7. To present this lesson you may want to print in colour and make some overhead transparencies or slides for the electronic white board. Again, consider making a permanent class set which can be used again in future years. Some teachers like to increase the 'scale' of this lesson by using a 60 metre rope instead of a 6 metre string. Working outside in this way may also take some forward planning.

Mixed Media Unit

Mixed Media Mathematics has been created as *one* structure which allows teachers to integrate problem solving tasks into the curriculum.



The design incorporates four different modes of learning into a structure which can be readily managed by one teacher, but which is enhanced when prepared and executed by a team.

A three week Mixed Media Unit includes:

- whole class lessons
- hands-on problem solving
- problem solving software
- skill practice worksheets (or text material)
- time to reflect on learning
- assessment opportunities

If this is the first time such a structure has been used in your classroom, it is a good idea to prepare the students in a manner which 'brings them into the experiment'.

A vital element of the process is to reflect on *what* is learned and *how* it is learned *before* the final assessment of the learning. Guidance with respect to assessment is also provided in this manual. In particular, the Pupil Self-Reflection information in the Assessment section of Part 3 was designed by teachers who trialed the original Mixed Media units.

Mixed Media Unit A

Dice Differences, *Greedy Pig* and *Have A Hexagon* are the whole class lessons suggested for this unit. You will certainly need to use two of them over the first two weeks of the unit, but whether you use the third will depend on whether you choose an assessment focus for the third week. If so, the whole class lesson in this week could be built around the Pupil Self-Reflection information (Page 57).

Each of the lessons is well documented in Maths300 and none requires anything beyond simple arithmetic to become involved. In that sense they are non-threatening and suit a broad range of abilities. All three include software support and an Investigation Sheet. The Investigation Sheet can be used to guide students into a deeper investigation of the lesson while at the software station. It can therefore form part of the assessment requirements of the unit.

You might choose:

- *Dice Differences* because the software uses averages and range and includes an unusual distribution based on the difference between two dice.
- *Greedy Pig* because the lesson and the software involve stem & leaf graphs and box & whisker plots.
- *Have A Hexagon* because the software involves percentages and the use of a bar graph to make a decision about how to alter the game to make it fairer.

Each of these lessons has developed from a task listed below and it doesn't really matter if the students have used that task before. Teachers have found that 'tucking one of these under the arm' and going into class to run a whole class investigation around it helps students to see the curriculum as more integrated. In whole class lessons teachers model how a mathematician works; when students open a task they are accepting the invitation to put the model into practice.

The tasks suggested for this unit are:

- Dice Differences
- Dice Footy
- Final Eight
- First Down The Mountain
- Game Show
- Have A Hexagon
- Same Or Different
- Take A Chance
- Time Swing
- What's In The Bag?

The third group in the rotation will be working for one period on probability and statistics work from your text. Students work in pairs on the tasks and the computers. Some teachers are happy for this collegiate approach to continue at the text station as well.

The same tasks are used in all weeks of the unit because, in the main, it will take one session to complete one task fully, so no students will be able to use them all in this unit. *Consequently the same tasks are used again in Year 10 for Mixed Media Unit B*. It has already been mentioned that you will need to choose a new class lesson for the second week, and you may also need new text work. Alternatively it may be appropriate for students to continue some aspects of their software-based deeper investigation during the period at the text station.

Teachers often find the third week needs to be flexible. You could continue the exploratory nature of the unit and introduce one more whole class lesson and follow up investigation from the set above. Or, if the unit is going to have a strong assessment component based on the deeper investigation, students may need time scheduled for report writing. In fact, it may be necessary to use a lesson time this week to model report writing.

Student Publishing

It is inappropriate to simply expect students to publish a report of their investigation. We have to devote lesson time to teaching how to keep a journal while investigating and how to plan and present a report. The Recording & Publishing section of Mathematics Task Centre includes two different approaches to scaffolding this process with the class. Both include sample student work and suggest that a report can be presented in forms other than pencil and paper, for example PowerPoint. The links are titled 'Learning to Write a Maths Report' and 'Learning to Write a Maths Report 2' and can be found at:

- http://www.mathematicscentre.com/taskcentre/record.htm

In the preparation of this unit some teachers create a 'contract' sheet for students which sets out the expectations of the unit, for example:

- Participate in two whole class lessons.
- Keep a diary of your work on at least three tasks.
- Complete a written report of one software-based investigation.
- Complete Exercises ... from the text.

Mixed Media Unit B

Tasks are as above. As the whole class lessons you might choose:

- *First Down The Mountain* because probability experiments and a histogram are used to explore a result that some find counter intuitive.
- *Radioactivity* because the software displays an exponential decay graph.
- *Same Or Different* because exploring the fairness of random events leads to a result unexpectedly controlled by the discriminant of a quadratic.

Self-directed Maths Journey

Students are asked to choose their own activities within a limited, but challenging, mathematical landscape. It is just like being put into a broad, fenced environmental landscape and being given free reign to explore.

Each Self-directed Maths Journey (SMJ) is a loosely structured unit plan designed to encourage independent, self-directed mathematics learners. A feature is the development of mathematics in a language context - a model which helps teachers 'work smarter' by simultaneously achieving numeracy and literacy objectives.

Students keep a diary of their adventures and teachers encourage detailed entry in the diary by allowing sufficient time to write it. The diary has a dual purpose:

- In writing 'for an audience' students reprocess the work they have been doing and this helps to enhance learning.
- It provides a record that contributes to evaluation evidence.

The two week SMJ is prepared in the previous week, just as a journey into an unknown physical landscape is prepared beforehand. This occurs in two ways - by class exploration of problems which leave room for further exploration during the journey and by preparation of the diary.

Preparation of the Diary

In a mathematics session, introduce a problem that will need more than one session to explore its iceberg. For example, for SMJ I on the Year 9 Planner, this could be *Trigonometry Walk*, which requires an outdoor (or gymnasium) lesson with physical activity to establish the concepts. Explain that exploring the software related to this investigation will be one of the activities to choose during next week's Self-directed Maths Journey.

Follow up during this preparation week with a language-focussed session on the concept of keeping a diary of the Self-directed Maths Journey. Collect examples of diaries from the library - literary diaries like 'Diary of Anne Frank', or professional diaries like that kept by Joseph Banks, the botanist on Captain Cook's explorations of Australia. Diaries/notebooks also feature in 'popular culture'. For example the father's diary is central to the plot in the film 'Indiana Jones and the Last Crusade', a movie well known to most students, and the film has several shots of the book showing text, sketches and maps. Note items like these as features of a diary and add others suggested by students such as the importance of dates and the possible inclusion of photographs. Encourage the use of the school's cameras to record significant mathematical moments. Consider also the possibility of an electronic diary if you are interested in achieving Information Technology outcomes in conjunction with mathematics and language. There is a great deal of potential here for cross-faculty planning that can make all courses more relevant to students.

Introduce the idea of a diary published for others, such as that of Joseph Banks, being prepared at a later time from notes taken at important moments. Perhaps use the image of a hiker travelling with a diary in their backpack and a notepad and pencil in their shirt pocket. The pad is used to record significant moments in the day and the diary is filled in later around the camp fire.

Provide 'provisions' for the SMJ:

- small spiral notepad (or equivalent)
- a more substantial book to use as the diary
- an outline of the mathematical landscape, such as the examples below
- the Working Mathematically process page (See Page 8)

Invite students to decorate and personalise the diary. This could be a homework exercise. The Working Mathematically process and the landscape page are placed in the front pages of the diary. Students are told that the next page is to be a Map Page and thereafter they record each day in text, drawing and photo as they wish.

The Map Page records each date and its activities in name only as a summary of the adventure. These items are arranged on the page as the student wishes and are linked with pathways and other landscape features in an imaginative way.

It may be unusual to expect this level of presentation in mathematics, but perhaps the combination of encouraging pride in displaying work for others, and some freedom in choosing their investigative work, will lead to more enjoyment of the subject and more willingness to participate and learn. If the current practices don't seem to be engaging students, perhaps planning an SMJ, especially with a faculty team is worth a try.

On The Journey

- Students work in pairs - it is usual to go on a journey with a companion - but keep separate diaries.
- It doesn't matter in which order students tackle the activities, or which ones they tackle, or how many they tackle, or even if some activities are left started, but unfinished. What matters is that they demonstrate through their diary (and interview if you wish) how they have worked like a mathematician in each session.
- Some students may need help getting started because the range of choices may be daunting. In such cases, choose a hands-on task for them and sit down together. Ask questions that will help them begin.
- Plan a compulsory teacher-led activity that students have to attend for a given number of sessions in the journey. Pythagoras & Other Polygons is an example below. Build on the Pythagoras experiences in the tasks and open an investigation (guided by the Investigation Sheets provided) which is added to the 'landscape'. These chats only take about 15 minutes with a tutorial-size group and go a long way towards supporting an adult to adult atmosphere in the classroom. This short time in tutorial leaves plenty of time to join groups and share their journeys.
- Allow time every maths session to begin diary writing. Encourage students to work further on the diaries in their own time.
- Collect diaries regularly and annotate with encouraging comments and suggestions. You don't need to collect every diary every session. Celebrate examples of student diary work that you wish to encourage in the group.
- Provide reference material from the library or staff resources so students can more easily carry out the research components of each SMJ below. You may also wish to supply a list of suitable web sites.

Following The SMJ

- Allow time to finalise diaries. Collect, read, comment and display.
- Use the ideas in 'Pupil Self Reflection' in the Assessment section of this manual.

Examples of SMJ Landscapes

The following sample landscapes are provided as a guide. They are not prescriptive. You are invited to construct your own from the resources provided, or include other resources of your own. To be true to the model, any material you include should be as open as possible.

Year 9

Self-directed Maths Journey I - Pythagoras & Trigonometry

Mathematical Landscape

- Explore the *Trigonometry Walk* software. Provide evidence of how accurate you are at estimating sines and cosines. (You could screen capture and print to include results in your diary.)
- **Tasks**: At least two of Cross & Square, Photo Angles, Pythagoras 1, Pythagoras 2 and Pythagoras Rods.
- Compulsory: *Pythagoras & Other Polygons* tutorial with your teacher. Continue the investigation begun in the tutorial.
- Find a relationship that links your Pythagoras knowledge with your Trigonometry knowledge.
- Research a real world application of Pythagoras Theorem and report.
- Research a real world application of Trigonometry and report.
- From Text: Exercises...

Year 10

Self-directed Maths Journey II - Puzzles & Paradoxes

Mathematical Landscape

- The investigation *Game Show* was introduced to you last week. Use Option 1 of the software to develop an hypothesis about whether you should change your mind. Use Option 2 to test your hypothesis. Report.
- **Tasks**: At least two of 64 = 65, Planets, Rectangle Nightmare, The Hole In The Triangle, and Growing Tricubes.
- Compulsory: (a) Do the homework exercise of collecting data for your *Grubby Pages Effect* tutorial. (b) Attend the tutorial. (c) Follow up using the web sites given and report on applications of this mathematics.
- Research and make an example of a mathematical puzzle or paradox.
- From Text: Exercises...

Task Comments

- Tasks, lessons and unit plans prepare students for the more traditional skill practice lessons, which we invite you to weave into your curriculum. Teachers who have used practical, hands-on investigations as the focus of their curriculum, rather than focussing on the drill and practice diet of traditional mathematics, report success in referring to skill practice lessons as Toolbox Lessons. This links to the idea of a mathematician dipping into a toolbox to find and use skills to solve problems.

The twenty tasks in this **Maths With Attitude** unit are grouped as follows:

Measurement

- 64 = 65
- Cross & Square
- Growing Tricubes
- Photo Angles
- Planets
- Pythagoras 1
- Pythagoras 2
- Pythagoras Rods
- Rectangle Nightmare
- The Hole In The Triangle

Chance & Data

- Dice Differences
- Dice Footy
- Final Eight
- First Down The Mountain
- Game Show
- Have A Hexagon
- Same Or Different
- Take A Chance
- Time Swing
- What's In The Bag?

64 = 65

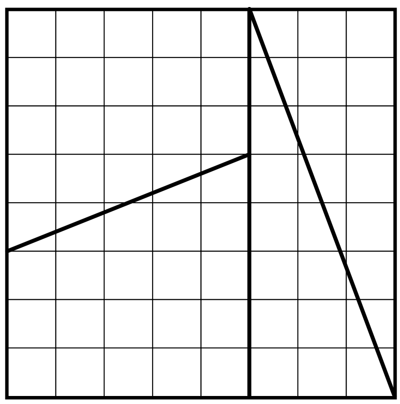
One of several paradoxical puzzles the resolution of which partly depends on the application of Pythagoras' Theorem. There are many of this type of puzzle in mathematical literature. Pythagoras is introduced in Year 9 through a Self-directed Maths Journey. This task then appears in a similar journey in Year 10 as an opportunity for students to put their knowledge into action.

An extra unit square appears to materialise in the task, but, it simply isn't possible for a square of 64 square units to turn into a rectangle of 65. How has the extra square appeared?

There are two levels of answer to this; 'by eye' and by calculation. By eye, if the 'diagonal' of the rectangle is closely examined, there is a long thin parallelogram in the middle ... and you can guess the area of this parallelogram.

The unit square in the problem is actually 2 centimetres each side. A 13 x 5 rectangle of these unit squares has been provided on a worksheet at the end of this manual to help students line up the rectangle in this way.

However, a preferred level of answer at Year 10 to the explanation of the extra square relies on Pythagoras' Theorem and Trigonometry. The dissection in the task is based on an 8 x 8 square of unit squares as shown.

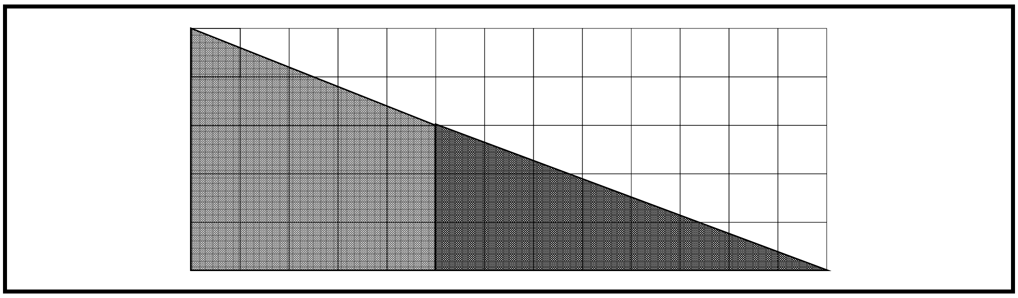


The diagonals in the dissection are the key. They become four diagonal lengths when cut and one of each makes up the diagonal of the supposed 13 x 5 rectangle. So, if the rectangle actually exists, the sum of the two diagonal lengths above should equal the diagonal of a 13 x 5 rectangle.

Sum of diagonals above	=	Square Root $(2^2 + 5^2)$ + Square Root $(3^2 + 8^2)$
	=	SQR(29) + SQR(73)
	=	5.385 + 8.544
	=	13.929
Diagonal of 13 x 5 rectangle	=	Square Root$(13^2 + 5^2)$
	=	SQR(194)
	=	13.928

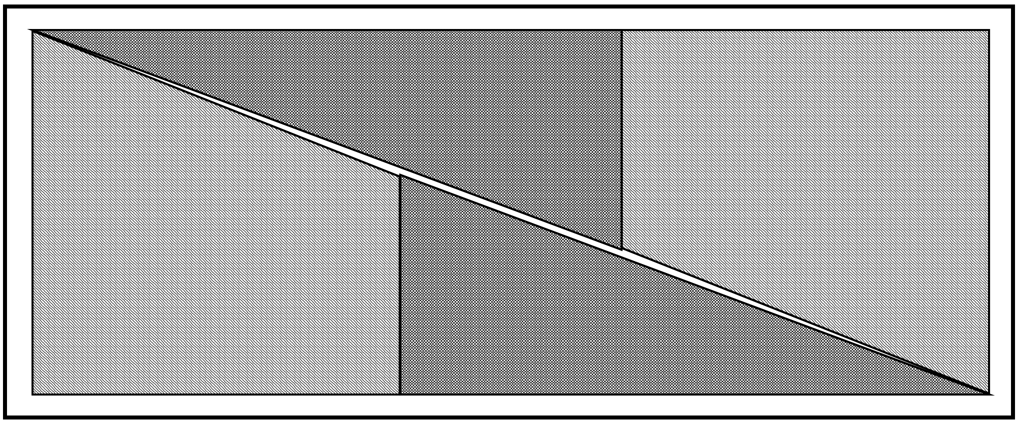
Close! Do we conclude therefore that the rectangle can be made because if only two decimal places are used, the lengths would appear to be the same?

We have to turn to trigonometry, or ratio, for more information. There are four pieces in the above dissection, but they come in pairs - 2 trapeziums and 2 triangles. One from each pair makes half of the supposed 13 x 5 rectangle like this:



If the hypotenuse of the shaded area is one straight line, then the slope of the section contributed by the triangle and the slope of the section contributed by the trapezium must be the same.

Triangle: $^{Rise}/_{Run} = {}^{3}/_{8} = 0.375$
Triangle section of trapezium : $^{Rise}/_{Run} = {}^{2}/_{5} = 0.4$
Almost the same, but just different enough to prove that the 13 x 5 rectangle can only exist if there is a long thin parallelogram 'up the diagonal'.



Teachers might expect students to go further and prove that the area of this parallelogram is indeed one square unit.

A major extension of the problem, which can turn it into an extended investigation, comes from noticing a link in the key numbers of the problem. The square is 8 x 8 and the rectangle is 13 x 5. The key numbers are 5, 8 and 13. A student might notice that the first two sum to the third. It is not much data, but it might lead to investigating whether 3 other numbers might exist that could generate this 'extra square phenomenon'.

- One might consider asking whether ?, 5 and 8 could be the three numbers. The ? would then have to be 3 if the sum of the first two is to equal the third.
- In the original problem it was the middle number that was the square. The rectangle was formed from the other two.
- If 3, 5 and 8 are to work, the square would be 25 square units and the rectangle 24. Hmmm, one square different. It might be possible to do a construction to create a new 'extra square phenomenon'.
- If this worked, the next smaller set of numbers would be ?, 3, 5, implying the set 2, 3 and 5.
- In this case the square would be 9 square units and the rectangle 10. But this time the rectangle is bigger than the square. In the previous set of numbers it was smaller, and in the original problem it was bigger. Hmmm.
- Continuing the exploration leads to the sets 1, 2 and 3 and 1, 1 and 2. Of course we could also investigate larger sets than the original 5, 8 and 13; such as 8, 13 and 21.

Looking back over the numbers generated in this way, ie: 1, 1, 2, 3, 5, 8, 13, 21..., identifies Fibonacci's sequence ... and therein lies an additional wealth of possibilities.

It is a property of the Fibonacci sequence that for any three terms of the series, the product of the first and third differs from the square of the middle term by 1. For example:

♦ 2, 3, 5	5 x 2 = 10	3 x 3 = 9
♦ 3, 5, 8	3 x 8 = 24	5 x 5 = 25
♦ 5, 8, 13	5 x 13 = 65	8 x 8 = 64
♦ 8, 13, 21	8 x 21 = 168	13 x 13 = 169
♦ 13, 21, 34	13 x 34= 442	21 x 21 = 441

Students could continue this pattern for several more terms or write a spreadsheet to continue for say the next 100 terms.

An interesting pattern within this is that the square of the middle term alternates between being bigger by 1 and smaller by 1; a connection perhaps with the 'disappearing' unit of area. Could it be therefore that the dissection of the 8 x 8 square in the puzzle can be reproduced for any three numbers in the sequence? A hypothesis worthy of testing.

If so, how would students dissect a 13 x 13 square to turn it into a 21 x 8 rectangle; thus ‘proving' that 169 = 168 or reducing the area (apparently) from 169 to 168. Is the missing unit square once again a long thin parallelogram? Does the alternating sequence above somehow show up in the presence of this parallelogram?

Similarly students can be challenged to build the dissection for a 21 x 21 square, and hence see how the dissection can be generalised across the Fibonacci sequence.

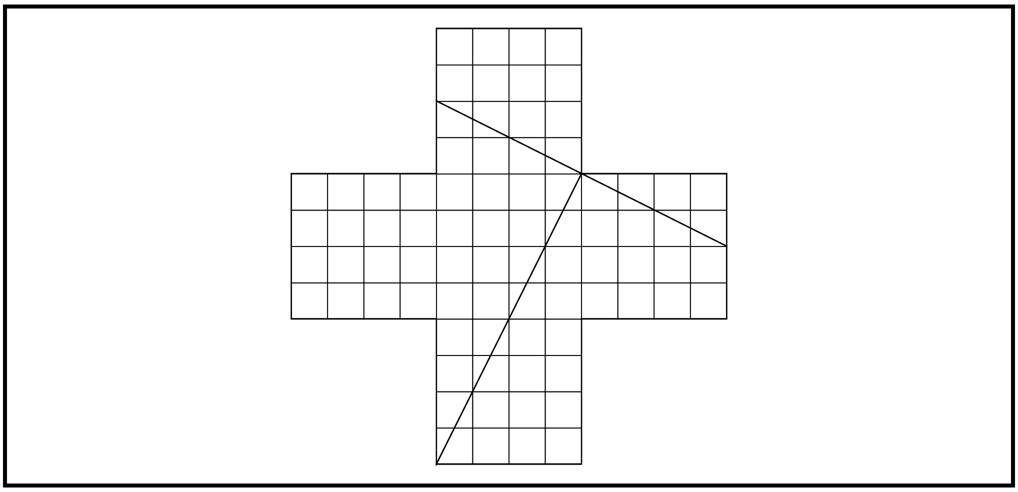
To visually highlight the parallelogram aspect of the puzzle, try the dissection on a 5 x 5 square and 'prove' that 25 = 24. The parallelogram (overlapping in this case) is clearly visible.

Cross & Square

(The investigation sheet you may need for this task is at the end of this manual.)

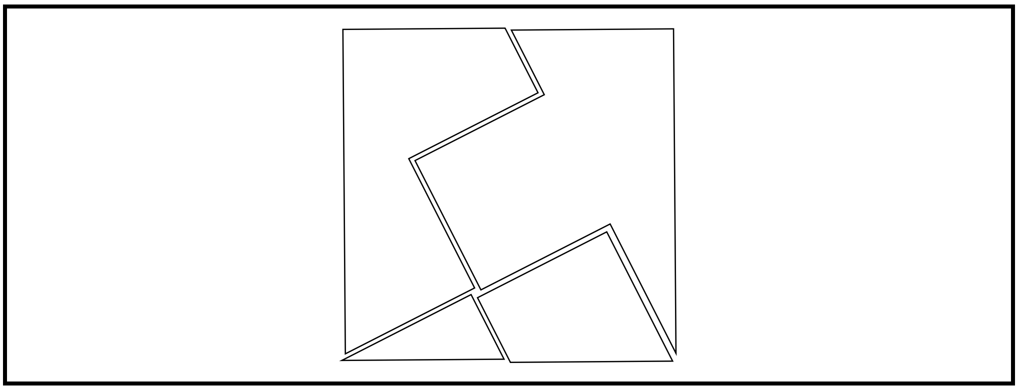
It is the comparison of the apparent area measurements of the cross and the square which lifts the puzzle from a four piece spatial puzzle into an application of Pythagoras' Theorem.

The dissection begins from the cross like this:



Clearly the area is 80 square units. However, when the four pieces are rearranged into a square, the square appears to be 9 x9 = 81 square units. Where did the additional square come from?

The pieces fit together to make the apparent square like this:



The right and left sides of this apparent square come from the diagonal which begins at the bottom left corner of the cross above. The length of the diagonal is calculated by Pythagoras to be:

Diagonal from left corner	=	Square Root$(8^2 + 4^2)$
	=	SQR(80)
	=	8.99427

Which is not quite the 9 that it needs to be to form the 9 x9 apparent square.

In a similar way, the top and bottom of the apparent square, which come from the other diagonal cut in the dissection, are also SQR(80). So, if the students can show that the appropriate angles in the new shape are 90º, then the new shape is a square of area 80 as expected. (Reference to the standard {1, 2, SQR(5)} right triangle in trigonometry should be sufficient to conclude that the necessary angles are 90º.)

So as the square apparently sits on a 9 x 9 square frame, the missing unit is the sum of the small gap around the outer edge of the square.

An extension challenge (or area measurement exercise) might be to situate the new square in the centre of the 9 x 9 frame and then calculate the areas of the border sections to prove they add to 1.

Dice Differences

What is the best strategy for placing six counters in six cells numbered 0 - 5, given that:

- the counters are to be removed in the least number of rolls of two dice
- the difference between the dice determines the cell from which one counter will be removed on each roll

The outcomes of the experiment are not equally likely. There are, for example, far more chances of scoring a difference of 1 than there are of scoring a difference of 5. The task invites the students to design an experiment, keep data and decide on statistics which help them make decisions about which placement strategies work better.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Dice Footy

A game situation which can be easily learned even by students who don't know about Australian Rules Football. The task is a mathematical model of the range of scores likely in the real game, rather than a model of the way the game is played. The task is a true tip of the iceberg situation because it is easy to play but has content including:

- Whole number skills
- Simple probability comparisons and calculations
- Collection and organisation of data
- Probability distributions
- Concept of expected results
- Calculation of averages

Student pairs playing the game in the task form sets the scene for the whole class investigation which is included as a Maths300 lesson.

Final Eight

Every team sport has to find a way to fairly decide its overall annual champion each year. Mathematicians have spent many years designing finals systems to achieve this end. This task is an example of one way to run the final series in a league that includes eight teams in the final play off.

Mathematicians have to model, or simulate, these real life situations. They need to gather and analyse data about the possible outcomes of alternative structures before they can recommend a particular one. It is an empirical approach that solves such problems, rather than a theoretical one.

Such approaches are underpinned with assumptions and, in the fun of collecting the data in this task, it should not be forgotten that the assumption is each team in any match has an equal chance of winning.

The structure in this task has been used by the Australian Football League (AFL) to decide its premier team. Was it fair, or did the finishing position at the end of the season load the chances of winning the flag? That investigation is the main thrust of the task. Working through the experimentation puts the students in the same position as professional mathematicians. The children are reminded several times to record the position of each team at the end of the season. That can then be compared to the team that becomes the Premiers. However, you may have to remind them again of the sense of this recording, because they can easily get lost in the fun of 'running' the finals.

Extensions

- A Final Four

  The league from which the AFL is derived once used a final system that only included the top four teams on the ladder. Investigate the chance of becoming premiers from each finishing position in this system:

  Round 1

  1st vs 2nd and 3rd vs 4th. The winner of 1st and 2nd had a bye in Round 2. The loser of 3rd and 4th was eliminated.

  Round 2

  Loser (1 & 2) vs Winner (3 & 4). Loser is eliminated.

  Round 3

  Winner (1 & 2 - Round 1) vs Winner Round 2. Winner of this match is the Premiership team and the loser is Runner-up.

- Suppose that one team in the final series 'had the edge', ie: there was not an equal chance of winning when playing this team. How would that affect the data gathered in the task? One way to model this is to complete Question 1 as indicated, record this Final Eight, then put the team names in a 'hat' and randomly choose one to be the team with the edge. Now use only one dice to play each match. If a match includes this team, then 1, 2, 3, 4 wins for the team with the edge and only 5 or 6 wins for the other team. If a match does not include this team, then 1, 2, 3 wins for Team A and 4, 5, 6 wins for Team B.
- There are two Maths300 lessons called Sporting Finals. One is for the AFL and one for the game of rugby as played in the NRL.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

First Down The Mountain

This task can keep players engaged in serious discussion at all levels from Infant to University. Encourage students to guess before each trial which player will win. Exploring their reasoning will provide insight into their current understanding. Younger students are more likely to reason that 2 and 12 are closest to the Win square and therefore are more likely to win. That reasoning is counter-intuitive for many older students - let's include teachers there too, since your authors reasoned incorrectly when first exploring the task. Our initial reasoning was that the steps are arranged so all players have an equal chance of reaching the square one before the Win, so 7 is most likely to win because it is most likely to be rolled. The fascinating thing is that the infants are correct in their hypothesis, even if their reasoning is incomplete.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Game Show

This task is likely to cause lots of discussion. Everyone thinks they understand what the chances are in this game, which came from a real life US TV show, but many people are inaccurate in their reasoning. When the TV show was first broadcast in the United States any number of people wrote to the public media to advise the contestant whether or not to change their mind. Even the mathematicians among them failed to agree on the correct theory to analyse the problem.

The task invites students to design an experiment and gather empirical data. Sufficient data begins to suggest that one of the options - change your mind or don't change your mind - is more likely to occur. If this empirical approach is carried out by many pairs and the results collated, a picture grows of the better action to take. At this level it is also appropriate to investigate the theory that supports this experimental result.

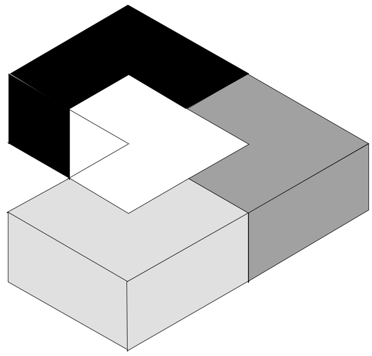
Maths300 supplies a companion lesson which includes software. The extensive notes and software support deeper investigation and supply the theoretical support for the problem.

Growing Tricubes

Volume, surface area and base area are three important measures in three dimensional space. This task explores all three and the relationships between them. They have many applications in biology, engineering and packaging. Often too, in these applications, the emphasis is on optimum values of the measures, an aspect reflected in the task.

The early parts of the task explore simple problems to introduce discussion involving these measures. There may be more than one answer to each question and recording paper is supplied at the end of this manual. This set of questions finishes with a spatial challenge that leads into the second section of the card.

The solution to Question 6 is shown on the next page...



...but this is not a Tricube!

A Tricube is made from three cubes and the new shape is made from three *cuboid blocks which are 1 x 2 x 2*. Students will need to make this twice to form a true Size 2 Tricube.

There are enough Tricubes in the task to make two of these Size 2s. Then the visual challenge is to use these to visualise the next size and decide its measurements. Graphing each measure for the three sizes encourages prediction of the base area, surface area and volume for larger Tricubes.

If you have more Tricubes there is still more to explore. For example, if the Growing Tricubes are named by the number of units of length along one side of the base, then they would be Size 1 (the single piece), Size 2 and Size 4 (these two being answers to the Challenge questions on the card).

- What happened to Size 3?
- If it could be made, how many Size 1 Tricubes would you need?

If you have additional Tricubes, students can try building Size 3.

Have A Hexagon

This is another task where the result is counter-intuitive for most students. The best hexagon to choose is actually the centre one which has the sections 1 - 6. The arithmetic behind the task is the products which can be made by multiplying the numbers on two standard dice. Gathering sufficient data by playing the game several times reveals that, although the other hexagons initially look more attractive, several of the numbers on them can only be made in a limited number of ways. The chance of filling either of these hexagons first is actually less than the chance of filling the centre one. Having gathered evidence to support that hypothesis, the what if question becomes: *What if we change the arrangement of the numbers on the hexagon? Could we make a fair playing board?*.

Exploring this investigative question leads to considering that the chances of winning a hexagon may be influenced not only by the number of ways a product can be formed, but also by the number of products on a particular hexagon which can only be formed in one way from the dice.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Photo Angles

This task generates considerable discussion and can be approached from at least three directions. Reasoning, in the beginning, tends to be along the lines of: *It must be about here because...* Estimation and approximation are important skills for a mathematician.

Encouraging students to justify such statements can lead to an experimental approach which sets up the objects on the grid and reproduces the lighting effects with a lamp, and the camera angle with the school's digital camera, or a student phone camera. When the approximately correct shadows and pictures are discovered, the position of the light and the camera can be measured.

At this level, it is also possible to encourage application of similar triangles, trigonometry and Pythagoras in three dimensional space to *calculate* the position of the light and camera. In this context such calculations are likely to be more relevant than the traditional spider crawling from one corner to a diagonally opposite corner of a room. Further, in attempting Photo Angles by calculation, the 'spider problems' may develop more meaning.

Planets

The measurements on the card are taken at a moment in time and represent an average. The movement of the planets is more complex than suggested by the table; for example not all the planets orbit the earth in the same plane, but occasionally they are in an alignment that allows the image of the string to be valid. Some students with an interest in astronomy may wish to discuss such details.

One of the exciting things about this task is that although it deals with very large numbers, judicious application of estimation (rounding off) and multiplication go a long way towards uncovering the wonder of the creation that resides within our solar system. The string is about 6 metres long. The solar system is about 6,000 million kilometres from the Sun to Pluto. This sets a scale:

- 1 metre = 1,000 million kilometres (roughly)

Half of the length to Pluto is about 3000 million kilometres and remarkably Uranus is at about that position. Half of the distance between Uranus and Pluto is 4500 million kilometres, and again a planet, Neptune, is there. Continuing this proportional reasoning helps to find a position for each of the other planets, with the only break from the pattern occurring between Mars and Jupiter. It was, in fact, this break from the pattern which led astronomers to look more carefully in this region and eventually discover the asteroids, which by some judgements are the fragments of a destroyed planet.

There is an opportunity here to link with work students may be doing in science related to the formulas which govern the gravitational forces between planets.

An important question to a mathematician is *Can I check it another way?*. In this case the placement of the planet cards along the string can be checked by application of the scale in a more precise manner, namely:

- 1 metre = 1,000 million kilometres (roughly)
- 1 centimetre = 10 million kilometres
- 1 millimetre = 1 million kilometres

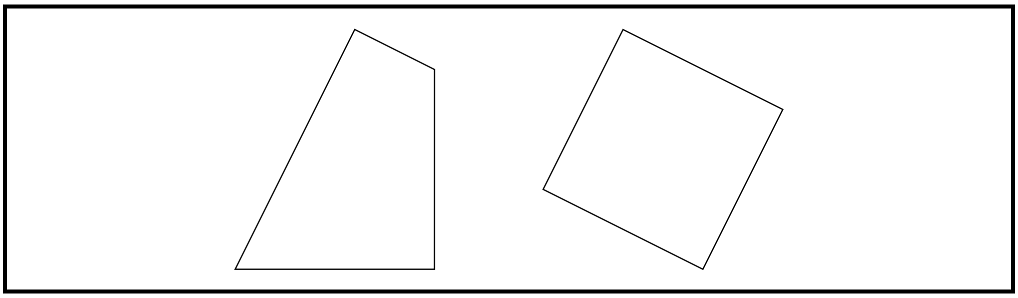
So on this scale, Mercury would be 5.8 centimetres from the Sun, Venus would be 10.9cm and Earth would be 15cm. That is a very small proportion of the total 6m distance, and yet Earth is the only place in the Universe where we are sure there is life.

An extension of the task develops from the realisation that the cards are all the same size and therefore don't truly represent the diameters of the planets. Research in an encyclopedia will reveal the diameters of the planets and the students can work out what these would become if the solar system was only as big as the string.

Find more information about this task in the Task Cameo Library at:

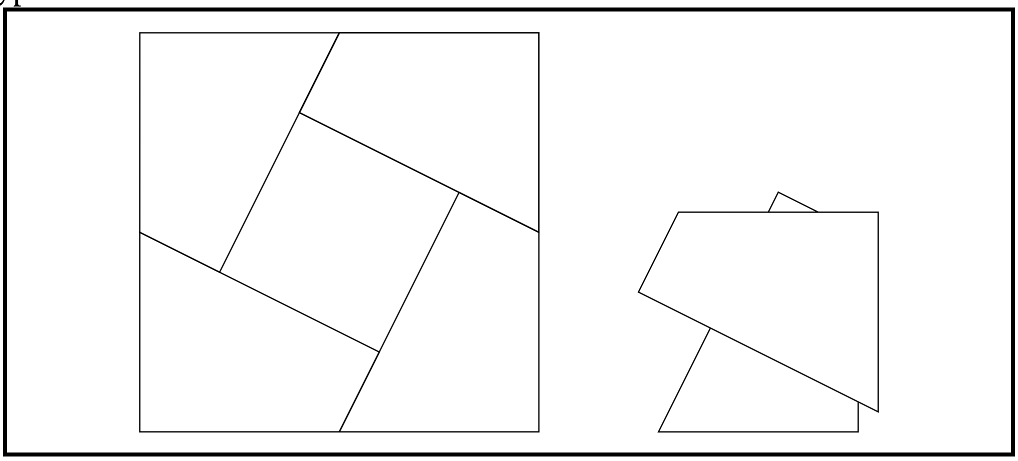
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Pythagoras 1



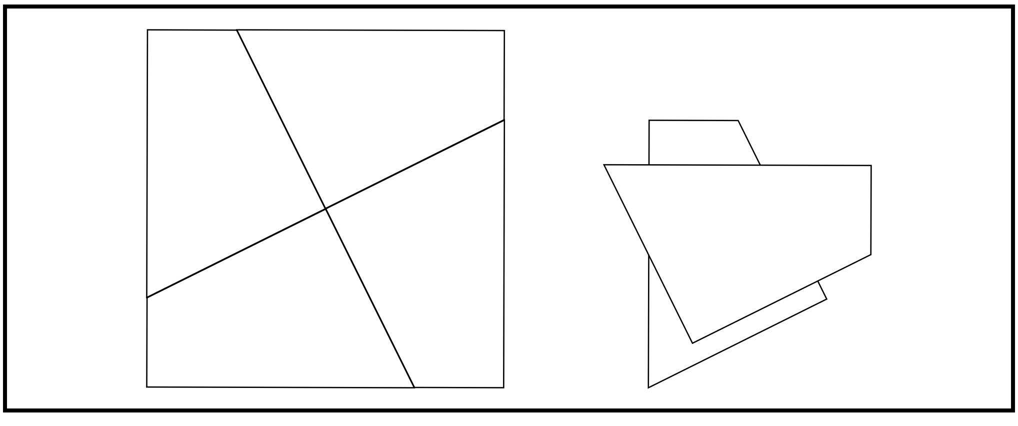
The four quadrilaterals and the square have to be made into a bigger square, then the four quadrilaterals on their own have to be made into a smaller square. If students are having trouble with this part, help them identify that the quadrilaterals are congruent and that each contains two right angles. Right angles are what is needed to make squares, so perhaps in one of the squares the right angles formed by the *longer* sides are 'hidden' inside, while in the other square, the right angles formed by the *smaller* sides are hidden inside.

The square on the hypotenuse is made like this:



Help students to see that the quadrilateral has been rotated 90° anticlockwise about the centre of its bounding rectangle and then translated to a new position. (Students familiar with creating shapes in drawing software will be used to the idea of a bounding rectangle.) Students can learn to stack the quadrilaterals, rotate all except the bottom one, translate and repeat the process to make the square.

The other square is made like this:



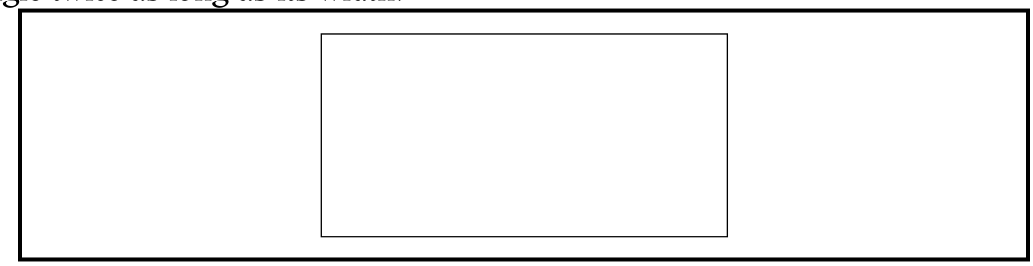
In this case the quadrilateral has been rotated 90° clockwise about the centre of its bounding rectangle and then translated to a new position. Again a stacking rotating and translating process can be used to create the square.

Of course, the measurement in the task is related to Pythagoras' Theorem. The visual/kinaesthetic demonstration illustrates that the area of the square built on the hypotenuse can be separated into the areas of the squares built on the other two sides. At this level, students need to think about whether this is a proof in the strict sense, especially since the eye has been fooled in other tasks such as **64 = 65** and **Cross & Square**. It will be important to have that brief discussion with each pair that tries the task.

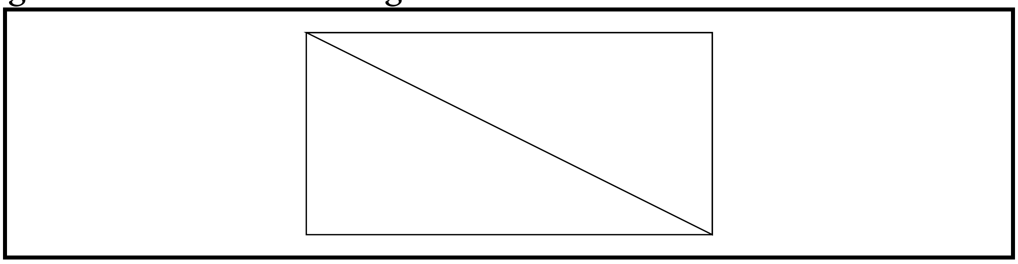
However, although the task is not a proof, it helps students to develop a visualisation on which to hang their understanding so that the geometric origins of Pythagoras are not lost in algebraic manipulations.

The last question on the card asks students to discover how the five pieces were designed. One way to think about this is to:

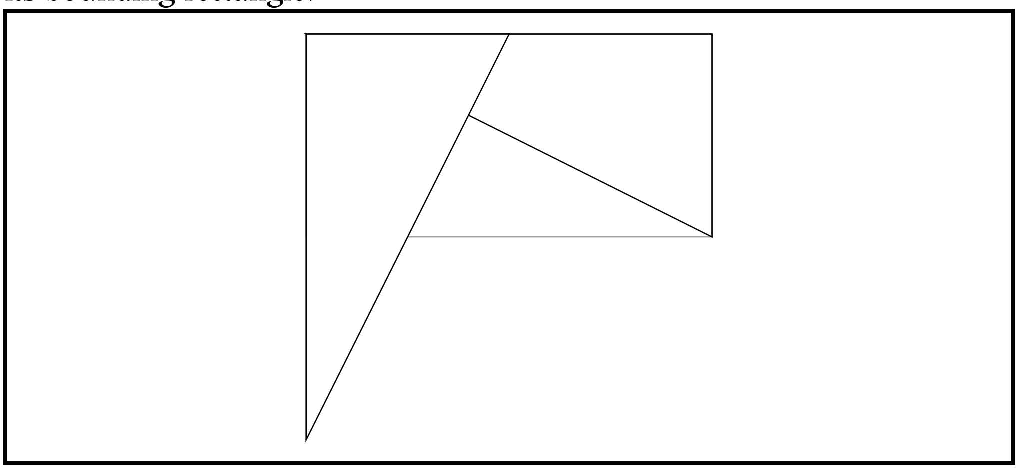
1. Draw a rectangle twice as long as its width.



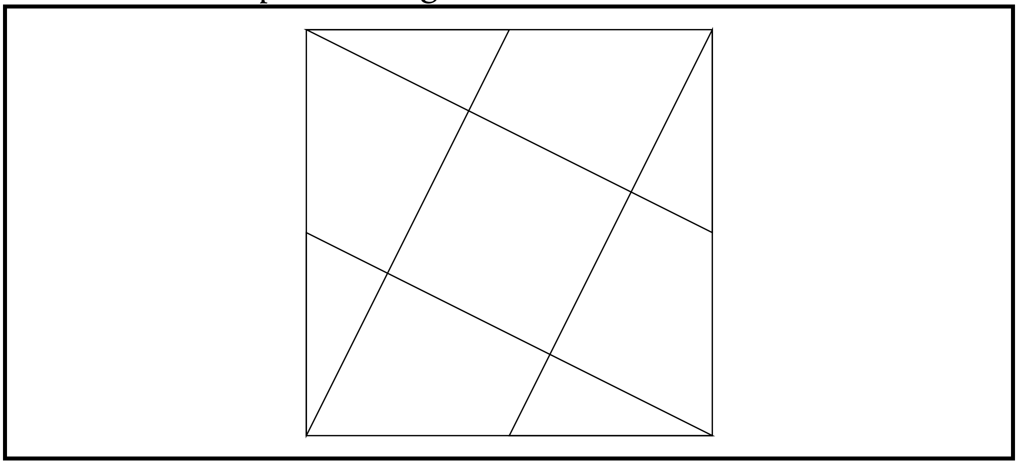
2. Draw in the diagonal to make the two triangles.



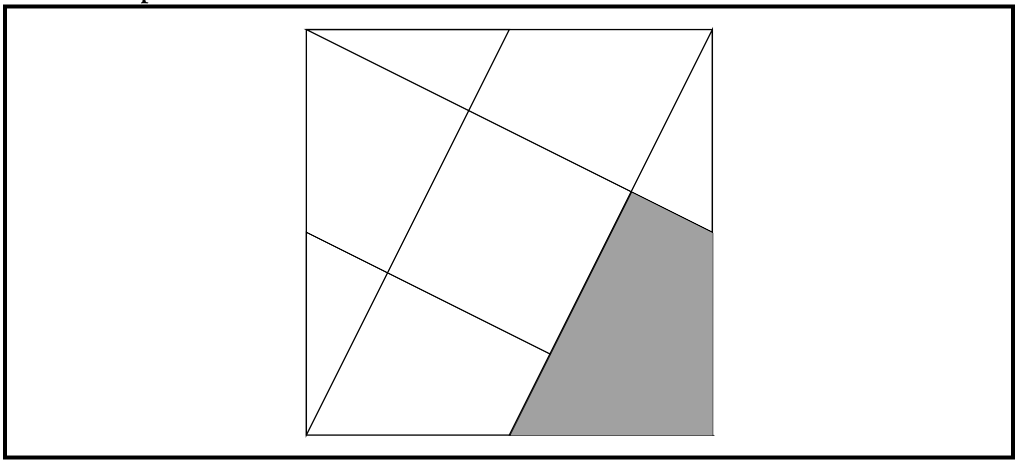
3. Use one (say the top one) and rotate it 90° anticlockwise around the bottom centre point of its bounding rectangle.



4. Repeat the rotation twice more to provide the guidelines…



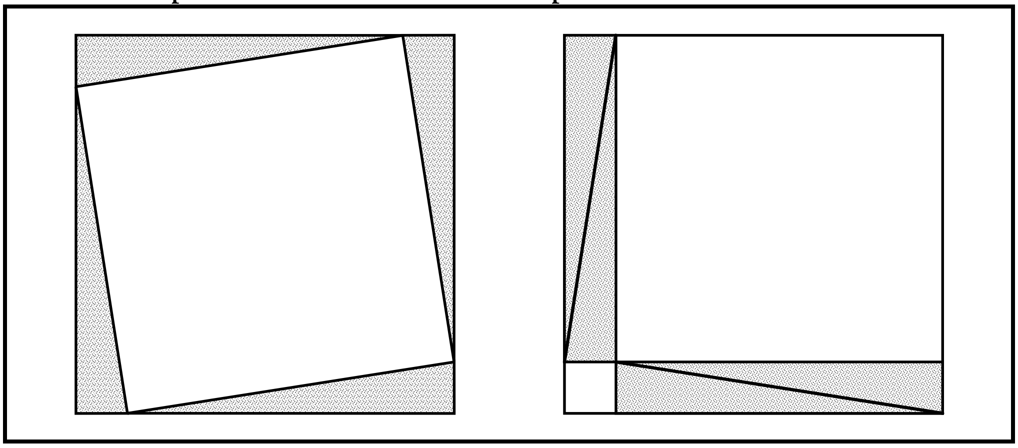
5. …to cut the pieces of the puzzle.



Finally, therefore, to construct the dissection for any right angled triangle, use the length of the hypotenuse to determine one of the sides of the 2:1 rectangle that begins the sequence above. There is a considerable amount of measurement (or use of the tools in a drawing software) involved in challenging students to create the dissection for a right triangle of their choice.

Pythagoras 2

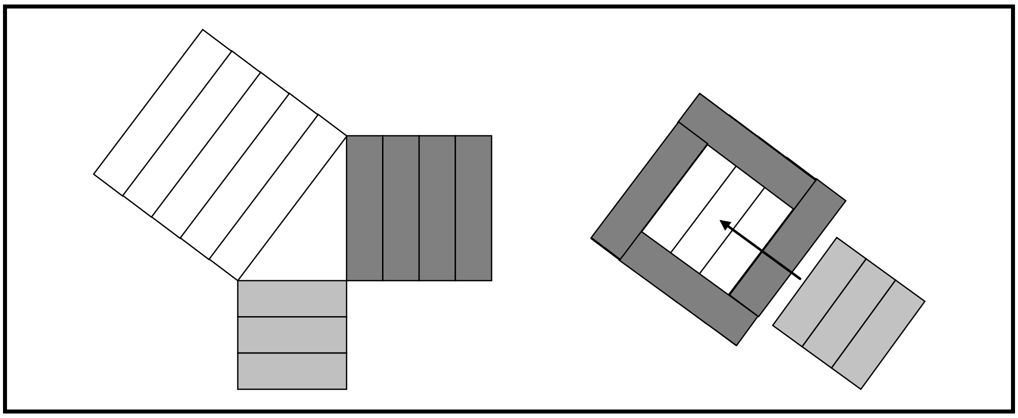
This demonstration of Pythagoras' Theorem is, in some ways, more sophisticated. It only involves translation and it links to the algebraic representation of the theorem as $a^2 + b^2 = c^2$. An example of how to transform the pieces is:



The key to this Pythagoras demonstration is the white space. The amount hasn't changed. In the first drawing it is represented by a square built on the hypotenuse. In the second, it is represented by the sum of the squares built on the other sides.

Pythagoras Rods

The challenge in this demonstration of Pythagoras' Theorem is to find more than one way to rearrange the rods to show that the area of the square built on the hypotenuse is equal to the sum of the areas of the squares built on the other two sides. One way is:



Looking for others is an application of the mathematician's question: *Can I check it another way?*. The material in the task also allows students to try the Pythagoras Hypothesis (that's all it really is at the moment) with a different right angled triangle, and with a non-right angled triangle. This latter suggests that the relationship only exists in right angled triangles - indeed, it is their defining property.

This task, more so than the other two Pythagoras tasks also raises a question which bothered mathematicians for 2000 years. We can build a square on each side of a right triangle because by knowing one length, we know the square which can grow from it. The rods show us that for a right angled triangle the algebraic form of the demonstration is $a^2 + b^2 = c^2$, where c is the length of the hypotenuse.

Equally, if we know a length, we can build a cube on that length, and if the students have sufficient numbers of each rod, they can try it with this material. Will the cube built on the hypotenuse have the same volume as the sum of the cubes built on the other two sides? Algebraically, are there any a, b, c such that:

$$a^3 + b^3 = c^3$$

And, whether or not there are solutions for powers of three, why not ask for powers of 4 or 5 or 6 or..., even if we can't imagine the geometric structure they represent?

What puzzled mathematicians for all that time was that although no one could find any a, b, c sets which satisfied any power other than 2, on the other hand, no one could prove that there weren't any.

No one that is until Pierre de Fermat in 1637 claimed to have proved that the hypothesis could only work for powers of 2. Trouble was that he died without revealing his proof; in fact, he died without revealing the proof of many of his published suppositions. Later, mathematicians checked all of his discoveries and found them to be true. The last one to be proven was the proposition that, in essence, Pythagoras (in this generalised sense) only works for powers of 2. This is what gave the extended Pythagoras hypothesis the name Fermat's Last Theorem.

Fermat's Last Theorem remained the mathematicians' Achilles Heel for another 350 years until Andrew Wiles, in 1994, presented the 100 page proof which is now accepted by mathematicians.

So, what happens if you do try Pythagoras' Theorem for powers other than 2? It doesn't work. However, the question can be understood by students of this age, and it opens the door to a study of mathematical history which can bring real mathematicians from past and present into the classroom.

There is a book, a documentary video and a Broadway musical based around Wiles solution and the history which led to it.

- *Singh Simon (1997)* **Fermat's Last Theorem***, obtainable through Simon's site: http://www.simonsingh.net*
- *Singh, Simon,* **Fermat's Enigma** *(in USA), Walker and Company, New York*
- *The documentary video, which preceded the book, is now available on Google Videos and can be accessed through Simon's site.*
- *The Broadway musical is a theatrical presentation of the essence of Andrew Wiles' struggle to solve Fermat's Last Theorem. It has extensive possibilities for cross-curricula study in, at least, mathematics and drama. The title is* **Fermat's Last Tango** *and it is available as video or DVD from Clay Mathematics Institute at: http://www.claymath.org/publications/Fermats_Last_Tango*

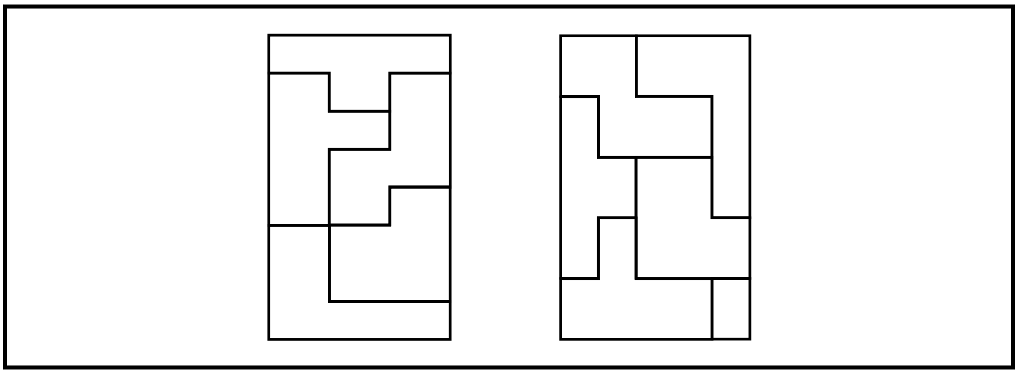
In addition there is a huge amount of information related to the problem, the history of its solution, and the lives of the mathematicians involved which is available through the Web. This offers many opportunities for assignment work.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Rectangle Nightmare

This puzzle was designed by Geoff Giles, a well known Scottish mathematics educator. It is one of the genre of 'disappearing square' puzzles, but it has a cunning twist, where, by transformation, the square becomes a rectangle. Again, the dissection has strong links to Fibonacci numbers.
The shapes can be fitted into the frame with or without the rectangle as shown. Variations are possible.



At the first level of analysis there is a very narrow space between the shapes in the first solution and the frame in which they fit. That space must be the same as the space taken up by the additional rectangle. However, a deeper analysis of how this task was designed can be the source of much more investigation. Geoff's paper about the task has been added to the cameo for Task 84, **Rectangle Nightmare**, so it is inappropriate to reproduce it here. It is strongly recommended that you print it from:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Same Or Different

The task requires the students to carry out experiments and collect sufficient data to be able to empirically decide which combinations of blocks are fair. One appeal of the game is that the results are quite 'unexpected and counter-intuitive'. The theoretical confirmation of the fairness of each game is within the ability of students of this age. The exciting aspect of the task is that (with computer support) students can be encouraged to see a number pattern (Triangle Numbers) developing in the sets of blocks which make fair games. Deeper analysis of the theory related to fair games shows algebraically, and quite surprisingly, that this non-random pattern has to be, even though the experiment is a random event. In fact, the sets of blocks which produce fair games are governed by the discriminant of a quadratic which, in its turn comes from a Diophantine Equation.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Take A Chance

Dealing with the last part of the task first, the answer is 364, but the interest is in the methods students choose to arrive at this. A mathematician would expect to check their answer another way (see Working Mathematically, Page 8), so it is appropriate to expect students to provide the answer in at least two ways.

However, for this unit, the main interest in the task is the game and the application of intuitive and theoretical probability which it encourages. When the students make statements like *I have a good chance of winning this one.*, push them for an explanation. You can encourage this type of discussion by asking students to explain the 'betweens' on which they would definitely bet.

Pressing further, you can set up particular cases from zero betweens to 11 betweens and ask students for their intuitive response to the number of counters they would risk. These responses can be compared to the equivalent theoretical result. For example if the end cards are 4 and 10, there are 20 cards of the remaining 50 which could win. That's a 2 in 5 chance of winning. How many students are prepared to take that chance and how many counters are they prepared to bet if they do? You might consider an assignment asking students to investigate and comment upon all the possible 'betweens'.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

The Hole In The Triangle

Again we have a task which demonstrates why a mathematician cannot rely on how something looks to be sure of the result. Our eyes deceive us once more, just as in **64 = 65**, **Cross & Square**, and **Rectangle Nightmare**. Again conservation of area provides the paradox of one square apparently being 'created', and in this case the paradox is made more apparent by the contrasting colours of the pieces, which distract us from the truth.

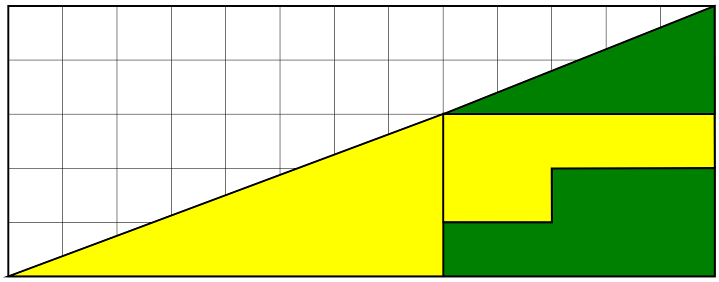


Diagram 1

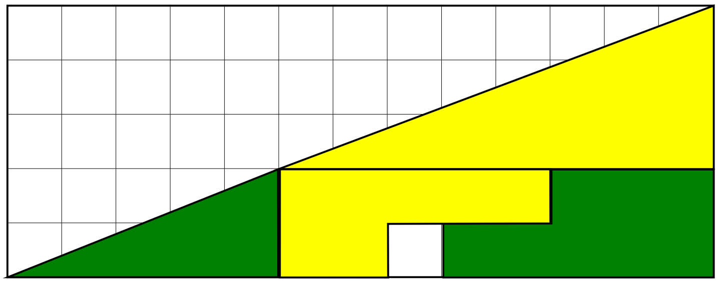


Diagram 2

Where has the additional square come from? Closer inspection of what seems to be a straight hypotenuse of the largest triangle shows that it is made from two pieces which only *appear* to have the same slope.

Darker Triangle: $^{Rise}/_{Run} = {}^{2}/_{5} = 0.4$
Lighter Triangle: $^{Rise}/_{Run} = {}^{3}/_{8} = 0.375$
Largest Triangle: $^{Rise}/_{Run} = {}^{5}/_{13} = 0.3846...$ (if it were actually made):

So, in fact, in neither of the arrangements do the four pieces make the bounding triangle which they seem to make. Further, the slope of the darker triangle is steeper than that of the largest triangle and that of the lighter triangle is less. However this is still not sufficient knowledge to explain the extra square.

Diagram 1

- Use the bottom left corner of the grid as (0,0).
- Assume the darker and lighter triangles are correctly placed on the grid intersection where they meet at: (8, 3).
- Is (8, 3) on the hypotenuse of the largest triangle?
- Co-ordinate geometry tells us the equation of this hypotenuse is: $y = {}^{5}/_{13}\, x$, so if x = 8 in this equation the y value is just more than 3.
- So the darker and lighter triangles run *inside* the line of the hypotenuse and form two sides of the parallelogram described in **64 = 65**.

Diagram 2

- Use the bottom left corner of the grid as (0,0).
- Assume the darker and lighter triangles are correctly placed on the grid intersection where they meet at: (5, 2).
- Is (5, 2) on the hypotenuse of the largest triangle?
- Co-ordinate geometry tells us the equation of this hypotenuse is: $y = 5/13\, x$, so if x = 5 in this equation the y value is just less than 2.
- So the darker and lighter triangles run *outside* the line of the hypotenuse and form the other two sides of the parallelogram described in **64 = 65**.

The 13 x 5 grid is the basis of both tasks. We know the parallelogram in **64 = 65** is one unit. However, since we are only dealing with half the rectangle from **64 = 65** in the current task, the comparison can only explain half a unit of area.

The other half comes from the actual difference in area between the four pieces and the largest triangle.

Four pieces:
    7 + 8 + 5 + 12 = 32 square units.
Largest Triangle
    ½ (13 x 5) = 32.5 square units

So:

- In Diagram 1 the two triangle pieces 'push back' into the larger triangle by half a square unit compared to the hypotenuse of the largest triangle and 'cover up' the half unit difference in area.
- In Diagram 2 the two triangle pieces 'pull out' beyond the larger triangle by half a square compared to its hypotenuse and 'stretch out' an extra half unit above the half unit difference in area.
- Leaving behind a unit square created from two half squares - ½ + ½ = 1.

What a wonderful task! Within the capacity of students of this age to understand, it gives meaning to:

- conservation of area
- understanding and application of gradient
- co-ordinate geometry
- area of a triangle
- number theory - Fibonacci is at the basis of the whole thing of course
- concept of proof

Time Swing

Students may have had the opportunity to try this experiment in science. However the task is equally valid in mathematics and it is included in this unit for both its measurement components - length and time - and the need to collect, organise, display and interpret data. In this case, not data from a chance event, but data from an experiment governed by a formula from Physics. The task sets the experiment in a real-life context and offers several problems to solve.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

What's In The Bag?

This task models sampling techniques as they are applied in the 'real world' in opinion polls, market research or biological surveys. The mathematical decisions involved require application of the probability theory and an understanding of concepts such as sample space, sample point, and event.

Lesson Comments

- These comments introduce you to each Maths300 lesson. The complete plan is easily accessed through the lesson library available to members at:
  http://www.maths300.com
  where they are listed alphabetically by lesson name.

Biggest Volume

Cut out a 10 by 20cm piece of grid paper. Cut a small square from each corner and fold the remainder into a box. What size cut out square gives the maximum volume for the box? This classic problem is usually given in Year 11 or 12 because by then students have the calculus tools to perform the algebraic exercise.

However in this lesson it is presented as an extended investigation, which after initial concrete experience, is pursued using an empirical approach supported by software. Interestingly, the problem is now well within the abilities of students at much younger year levels and introduces them to pre-calculus ideas.

Cylinder Volumes

The full title of the lesson is *Cylinder Volumes and the Pacific Ocean*. Students are surprised to find that it is possible to fit the Pacific Ocean into a piece of A4 paper.

Use two A4 pieces of paper. Roll one along the long side edge to form a 'squat' cylinder and the other along the short side to form a 'tall' cylinder.

- *Which cylinder holds the greater volume?*

This is a clever lesson because of the simplicity of the challenge, the easy to get materials (A4 paper and cubes) and the somewhat unexpected results. It can be used with a wide range of abilities and at various levels of sophistication. The task is well known but is usually presented as an exercise at Year 11 or 12 level. However, here it is presented as an investigation, with emphasis on concrete materials, small group work and the contribution from computer software. The lesson provides a practical introduction to pre-calculus ideas.

Dice Differences

This lesson is an interesting extension of the *Problem Dice* investigation which is included in the Chance & Measurement kit for Years 7 & 8. The fantasy story context of a 'prisoner release' scheme seems to add both a structure and motivation. When the difference between two dice are calculated, there are 6 possible outcomes, from 0 to 5. The non-equal probabilities of each of these provides the underlying theory.

A major aspect of the investigation is that it is very easy to get started, but finding and proving the 'best' answer is very challenging. Hence the lesson can be effectively used at this level because it requires planning an extensive set of experiments and collecting and recording data to support a proposition about the best strategy. The computer simulation provides a tool for quickly gathering data to either confirm or disprove students' theories.

Dice Footy

Roll two dice in a simulated game of Aussie Rules football. A student rolls 4 and then a 1 which means 4 goals (G), 1 behind (B) for a total of 25 points (Pts). But this is just the first quarter. Play all four quarters in a competition against a partner and the scores are similar to the real thing. This simple dice simulation has the fun of a game, but also involves many number, chance and data concepts. Collecting class data allows analysis of several aspects of the mathematics. By computerising the model, long term patterns can be explored, and empirical results can be compared with theoretical expectations.

Famous Mathematicians

Swap cards or novelties are often used as promotional items in snack foods and similar products and many students have been involved in trying to collect complete sets. In the context of learning more about famous mathematicians, students explore a mathematical model that investigates the time it might take to collect a complete set. Students' initial perceptions are contrasted with the empirical results and the assumptions in the model are examined.

A valuable spin-off of the lesson is the investigation of the lives of mathematicians depicted on the cards and posters. Mathematics becomes more real as these mathematicians from past and present become more 'known' to the students.

First Down The Mountain

This activity is most intriguing. The 'game' setting provides a rich contextual resource from which many investigative opportunities arise. It is easy to play empirically but much harder to explain the counter intuitive results that develop. The computer, apart from the graphic presentation and generating results for analysis, adds a great facility to explore and check theoretical aspects.

As a game, students can learn a lot about intuitive probability, but it is in analysing the long-run results (with the aid of a computer simulation) that the full potential of the investigation becomes apparent. Most senior students (and teachers) argue intuitively that to reach the line before the finish line (ie: one step from the finish) is equally likely for each marker. Then, since they are all equally likely to reach this line, the next and last step to the finish line advantages the '7' since this is then the most likely outcome. But the long term results show clearly that this is not the case. The theoretical justification of this outcome requires considerable effort.

Game Show

Suppose you are a contestant in a TV quiz show, and you are given a choice of three boxes, one of which contains a prize. After you pick a box, the compère opens one of the other boxes, **which they know to be empty**, and says "Would you like to change your mind?"

This becomes the puzzle: *Should the contestant change their mind or stay with their original guess?*

This apparently simple event on a real TV quiz show generated enormous interest from viewers (including mathematicians) who wrote in their thousands to tell a newspaper columnist - Marilyn vos Savant - that her explanation about probability was wrong. However she was correct, and all those Ph. D. critics were red faced. The class experiment recreates the TV quiz scenario, is supported by a software simulation and develops the correct answer. Explaining the answer seriously challenges students' logic and descriptive abilities.

Look for Marilyn's article and some of the original criticism at:

- http://marilynvossavant.com/game-show-problem/

Greedy Pig

The class stands and the teacher rolls a dice. Students score the number rolled. The first two rolls are free, but beyond that if a 2 is rolled and a student is still standing their score is immediately zero and they have to sit. As the game progresses, each student decides when to sit (and hence disengage from the game) so they get the best possible non-zero score. After x rolls? After collecting y points?

Given the game may be an 'old friend' this is a perfect opportunity to ask, *Now that we know more mathematics, what can we learn from it this time?*. There are many aspects of data collection, presentation and analysis which inform decisions about 'quitting' strategies. These aspects are directly course related at this level and include stem and leaf plots, box and whisker plots, mean, median and mode. Further, students can be led into the theoretical analysis of some strategies.

Have A Hexagon

The game board is three hexagons, each divided into six sections. The numbers printed in the sections are the possible products of two dice. Players select the hexagon they think will have all its products come up first. The activity is easy to play empirically, but it is harder to explain the counter intuitive result. The software adds to the investigative opportunities and enhances the lesson with its graphic presentation and by generating results. In addition it offers opportunity for exploring *What happens if...?* questions.

As a game, students learn a lot about intuitive probability, but it is in analysing the long run results that the full potential of the task becomes apparent.

Newspaper Pathways

Take one sheet from a newspaper and place it on the floor. It doesn't cover much ground. But suppose this was the start of a pathway and the next newspaper sheet was placed end to end with it. Suppose we continue the path using all the sheets from this newspaper, how long do you think the path would be?

Suppose the pathway was extended even further by placing end to end all the sheets from all the copies of this newspaper printed today. Where do you predict the path would end?

Students engage in estimation, problem solving, measurement and calculation as an idea forms of the quantity of waste newspaper in the world. Social issues related to recycling of waste grow naturally from this manageable introduction. Extensions include two and three dimensional mathematics when the editions of a newspaper required to cover the area of your capital city, or to wrap up the earth are explored.

Planets

The wonders hidden in the creation of the solar system are revealed in this whole class lesson which uses a classroom size scale to map the planets in relation to the sun.

There is a pattern in the distances of the planets from the sun, and it is estimation, mental strategies and proportional reasoning that reveal it, rather than number-perfect calculation. As a result it allows students who may be 'computation negative' an opportunity to participate.

Few remain untouched by the visual image of the magnificence of creation, and the apparently insignificant place of themselves within it.

Pythagoras & Other Polygons

It is often surprising to students that Pythagoras' Theorem holds true for polygons other than squares on the sides of a right triangle. Indeed it holds for any regular polygon - even for semi-circles. The investigation is about exploring and proving this theory.

Radioactivity

This lesson addresses the hugely important current social issue of the development of Uranium mines and the problems of radioactive waste. Radioactive waste involves the concept of a half-life and exponential decay functions. All radioactive material is described in terms of its half-life.

Arising from this community concern the students 'pretend' to be uranium atoms and model the decay process. A computer simulation then provides an investigative tool to explore the underlying concepts of 'half-life' and exponential decay. Students discover just how long some of this material can stay in the environment.

The mathematical aspects of this lesson should be seen as supporting a joint study of the topic with other subject disciplines to bring out the full range issues.

Same Or Different

This lesson models the task of the same name which the students have probably tackled. There is a bag containing 4 blocks, 2 red and 2 blue. Two students are playing a game in which they both reach in and draw one block from the bag.

- Player A wins a point if the colours are the SAME.
- Player B wins a point if the colours are DIFFERENT.
- Is the game fair for both players?

Playing this game many times shows the game is not fair. This creates the environment for the challenge of the lesson, which is *What combination of blocks in the bag will make for a fair game?*

The lesson starts simply, by playing the (2, 2) game, but then turns into an extended investigation which includes the elements of concrete materials, a game context, a problem solving challenge, a computer simulation, and the unexpected result of multiple solutions. The analysis of the results exposes several 'big' statistics and probability concepts and is governed by the discriminant of a quadratic equation.

However, despite this 'serious' mathematics, the game could be played by Year 2 students and it is interesting to ponder that its solution is not only dependent on an understanding of the properties of odd and even numbers, but, it can be represented by a model young children might discover when playing with blocks.

Sporting Finals

The final series for any sport is designed to end the season and find a worthy champion. Many complex structures have been designed to achieve this. They are mathematical and logical and arranged to be as fair as possible for all the teams involved. This lesson engages students in a simulation of the 'final eight' used by the AFL (Australian Football League). In playing the simulation, students both learn about the final structure and the underlying mathematics. Clearly the lesson is best run in the time just preceding the final series. To this end it has been placed in a Focus on Football week which can be slotted into the curriculum when appropriate. In this week, the lesson is partnered with *Dice Footy*.

The Grubby Pages Effect

Before the advent of calculators, people used a book of logarithm tables for calculations. Over 100 years ago, an astronomer, Simon Newcomb, noticed that these books were dirtier or grubbier near the front, indicating that over time, the users were more frequently looking up numbers with a lower first digit. This curiosity became known as the 'grubby pages' effect. However, at that time no one thought it had any real significance.

In 1938 Frank Benford rediscovered the pattern, and found the underlying law which is that the digits occur in proportion to $\log(1 + {}^{1}/_{n})$. However he did not explain why this should be so, nor did he deduce any application.

Then, in 1996, a mathematician named Ted Hill established the underlying logic behind the pattern. It was at this stage that people realised the possibilities of Benford's Law having many applications, such as detecting fraud.

Unusually, the lesson begins with homework, so you need to plan to introduce it in the session before. Students need to:

- Pick up any magazine or newspaper or printed material.
- Open it up at any page.
- Randomly place a finger anywhere on the page.
- Find the nearest 'number' that is mentioned or printed.
- Collect the first digit of that number.
- Do this 50 times.

The returns provide the data for a fascinating class lesson.

Trigonometry Walk

They say seeing is believing; but if you just 'see' a table of trig values, what can you believe? Perhaps that this is second hand data which you can use but you don't have to understand.

On the other hand, if you could not only see, but 'feel' where those numbers come from, the level of engagement in the learning is likely to change - a deeper understanding is likely to develop. This lesson adopts the Unit Circle model for developing trigonometric values and through physical involvement and software emphasises a visually based understanding of sine, cosine, tangent and their measurements.

What's In The Bag?

Secretly one player puts ten mixed coloured cubes in a bag. Without looking in the bag, a second player is allowed to take a sample of four cubes. The cubes are put back in the bag and the sampling repeated twice more. How many cubes of each colour are likely to be in the bag?

The lesson involves sampling techniques and mathematically-based decisions equivalent to the way statistics are used in the 'real world'.

Part 3:
Value
Adding

The Poster Problem Clinic

Maths With Attitude kits offer several models for building a Working Mathematically curriculum around tasks. Each kit uses a different model, so across the range of 16 kits, teachers' professional learning continues and students experience variety. The Poster Problem Clinic is an additional model. It can be used to lead students into working with tasks, or it can be used in a briefer form as a opening component of each task session.

> *I was apprehensive about using tasks when it seemed such a different way of working. I felt my children had little or no experience of problem solving and I wanted to prepare them to think more deeply. The Clinic proved a perfect way in.*

Careful thought needs to be given to management in such lessons. One approach to getting the class started on the tasks and giving it a sense of direction and purpose is to start with a whole class problem. Usually this is displayed on a poster that all can see, perhaps in a Maths Corner. Another approach is to print a copy for each person. A Poster Problem Clinic fosters class discussion and thought about problem solving strategies.

Starting the lesson this way also means that just prior to liberating the students into the task session, they are all together to allow the teacher to make any short, general observations about classroom organisation, or to celebrate any problem solving ideas that have arisen.

One teacher describes the session like this:

> *I like starting with a class problem - for just a few minutes - it focuses the class attention, and often allows me to introduce a particular strategy that is new or needs emphasis.*

It only takes a short time to introduce a poster and get some initial ideas going. The class discussion develops a way of thinking. It allows class members to hear, and learn from their peers, about problem solving strategies that work for them.

> *If we don't collectively solve the problem in 5 minutes, I will leave the problem 'hanging' and it gives a purpose to the class review session at the end.*
> *Sometimes I require everyone to work out and write down their solution to the whole class problem. The staggered finishing time for this allows me to get organised and help students get started on tasks without being besieged.*
> *I try to never interrupt the task session, but all pupils know we have a five minute review session at the end to allow them to comment on such things as an activity they particularly liked. We often close then with an agreed answer to our whole class problem.*

A Clinic in Action

The aims of the regular clinic are:

- to provide children with the opportunity to learn a variety of strategies
- to familiarise children with a process for solving problems.

The following example illustrates a structure which many teachers have found successful when running a clinic.

Preparation

For each session teachers need:

- a Strategy Board as below
- a How To Solve A Problem chart as below
- to choose a suitable problem and prepare it as a poster
- to organise children into groups of two or three.

The Strategy Board can be prepared in advance as a reference for the children, or may be developed *with* the children as they explore problem solving and suggest their own versions of the strategies.

The problem can be chosen from

- a book
- the task collection
- prepared collections such as Professor Morris Puzzles which can be viewed at: http://www.mathematicscentre.com/taskcentre/resource.htm#profmorr

The example which follows is from the task collection. The teacher copied it onto a large sheet of paper and asked some children to illustrate it. *The teacher also changed the number of sheep to sixty* to make the poster a little different from the one in the task collection.

The Strategy Board and the How To Solve A Problem chart can be used in any maths activity and are frequently referred to in Maths300 lessons.

The Clinic

The poster used for this example session is:

> Eric the Sheep is lining up to be shorn before the hot summer ahead. There are sixty [60] sheep in front of him. Eric can't be bothered waiting in the queue properly, so he decides to sneak towards the front.
>
> Every time one [1] sheep is taken to be shorn, Eric <u>then</u> sneaks past two [2] sheep. How many sheep will be shorn <u>before</u> Eric?

This Poster Problem Clinic approach is also extensively explored in Maths300 Lesson 14, *The Farmer's Puzzle*.

Strategy Board

DO I KNOW A SIMILAR PROBLEM?	ACT IT OUT
GUESS, CHECK AND IMPROVE	DRAW A PICTURE OR GRAPH
TRY A SIMPLER PROBLEM	MAKE A MODEL
WRITE AN EQUATION	LOOK FOR A PATTERN
MAKE A LIST OR TABLE	TRY ALL POSSIBILITIES
WORK BACKWARDS	SEEK AN EXCEPTION
BREAK INTO SMALLER PARTS	...

How To Solve A Problem

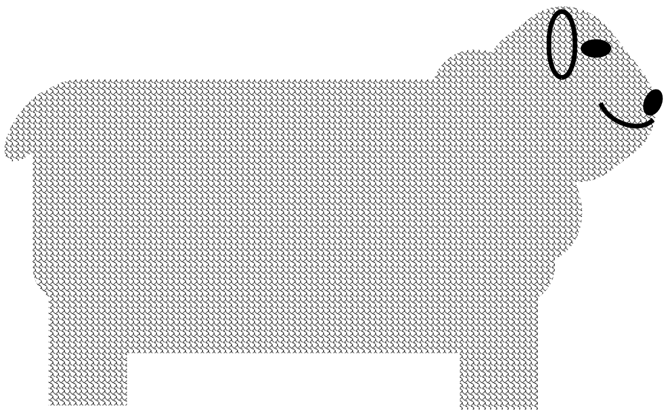
SEE & UNDERSTAND	Do I understand what the problem is asking? Discuss
PLANNING	Select a strategy from the board. Plan how you intend solving the problem.
DOING IT	Try out your idea.
CHECK IT	Did it work out? If so reflect on the activity. If not, go back to step one.

Step 1

- Tell the children that we are at Stage 1 of our four stage plan ... **See & Understand** ... Point to it! Read the problem with the class. Discuss the problem and clarify any misunderstandings.
- If children do not clearly understand what the problem is asking, they will not cope with the next stage. A good way of finding out if a child understands a problem is for her/him to retell it.
- Allow time for questions - approximately 3 to 5 minutes.

Step 2

- Tell the children that we are at Stage 2 of our four stage plan ... **Planning**. In their groups children select one or more strategies from the Strategy Board and discuss/organise how to go about solving the problem.
- Without guidance, children will often skip this step and go straight to Doing It. It is vital to emphasise that this stage is simply planning, not solving, the problem.
- After about 3 minutes, ask the children to share their plans.



Plan 1

Well we're drawing a picture and sort of making a model.

Can you give me more information please Brigid?

We're putting 60 crosses on our paper for sheep and the pen top will be Eric. Then Claire will circle one from that end, and I will pass two crosses with my pen top.

Plan 2

Our strategy is Guess and Check.

That's good Nick, but how are you going to check your guess?

Oh, we're making a model.

Go on ...

> *John's getting MAB smalls to be sheep and I'm getting a domino to be Eric and the chalk box to be the shed for shearing.*

Plan 3

> *We are doing it for 3 sheep then 4 sheep then 5 sheep and so on. Later we will look at 60.*

Great so you are going to try a simpler problem, make a table and look for a pattern.

This sharing of strategies is invaluable as it provides children who would normally feel lost in this type of activity with an opportunity to listen to their peers and make sense out of strategy selection. Note that such children are not given the answer. Rather they are assisted with understanding the power of selecting and applying strategies.

Step 3

- Tell the children that we are at Stage 3 of our four stage plan ... **Doing It**. Children collect what they need and carry out their plan.

Step 4

- Tell the children that we are at Stage 4 of our four stage plan ... **Check It**. Come together as a class for groups to share their findings. Again emphasis is on strategies.

> *We used the drawing strategy, but we changed while we were doing it because we saw a pattern.*

So Jake, you used the Look For A Pattern strategy. What was it?

> *We found that when Eric passed 10 sheep, 5 had been shorn, so 20 sheep meant 10 had been shorn ... and that means when Eric passes 40 sheep, 20 were shorn and that makes the 60 altogether.*

Great Jake. How would you work out the answer for 59 sheep or 62 sheep?

Sharing time is also a good opportunity to add in a strategy which no one may have used. For example:

> *Maybe we could've used the Number Sentence strategy, ie: 1 sheep goes to be shorn and Eric passes two sheep. That's 3 sheep, so perhaps, 60 divided into groups of 3, or 60 ÷ 3 gives the answer.*

Round off the lesson by referring to the Working Mathematically chart. There will be many opportunities to compliment the students on working like a mathematician.

Curriculum Planning Stories

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

In more than a decade of using tasks and many years of using the detailed whole class lessons of Maths300, teachers have developed several models for integrating tasks and whole class lessons. Some of those stories are retold here. Others can be found at:

- http://www.mathematicscentre.com/taskcentre/plans.htm

Story 1: Threading

> *Educational research caused me a dilemma. It tells us that students construct their own learning and that this process takes time. My understanding of the history of mathematics told me that certain concepts, such as place value and fractions, took thousands of years for mathematicians to understand. The dilemma was being faced with a textbook that expected students to 'get it' in a concentrated one, two or three week block of work and then usually not revisit the topic again until the next academic year.*

A Working Mathematically curriculum reflects the need to provide time to learn in a supportive, non-threatening environment and...

> *When I was involved in a Calculating Changes PD program I realised that:*
>
> - *choosing rich and revisitable activities, which are familiar in structure but fresh in challenge each time they are used, and*
> - *threading them through the curriculum over weeks for a small amount of time in each of several lessons per week*
>
> *resulted in deeper learning, especially when partnered with purposeful discussion and recording.*

Calculating Changes:

- http://www.mathematicscentre.com/calchange

Story 2: Your turn

Some teachers are making extensive use of a partnership between the whole class lessons of Maths300 and small group work with the tasks. Setting aside a lesson for using the tasks in the way they were originally designed now seems to have more meaning, as indicated by this teacher's story:

> *When I was thinking about helping students learn to work like a mathematician, my mind drifted to my daughter learning to drive. She*

needed me to model how to do it and then she needed lots of opportunity to try it for herself.

That's when the idea clicked of using the Maths300 lessons as a model and the tasks as a chance for the students to have their turn to be a mathematician.

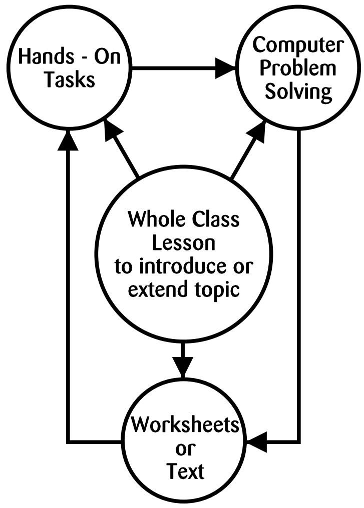
The Maths300 lessons illustrate how other teachers have modelled the process, so I felt I could do it too. Now the process is always on display on the wall or pasted inside the student's journal.

A session just using the tasks had seemed a bit like play time before this. Now I see it as an integral part of learning to work mathematically.

Story 3: Mixed Media

It was our staff discussion on Gardner's theory of Multiple Intelligences that led us into creating mixed media units. That and the access you have provided to tasks and Maths300 software.

We felt challenged to integrate these resources into our syllabus. There was really no excuse for a text book diet that favours the formal learners. We now often use four different modes of learning in the work station structure shown. It can be easily managed by one teacher, but it is better when we plan and execute it together.



Story 4: Replacement Unit

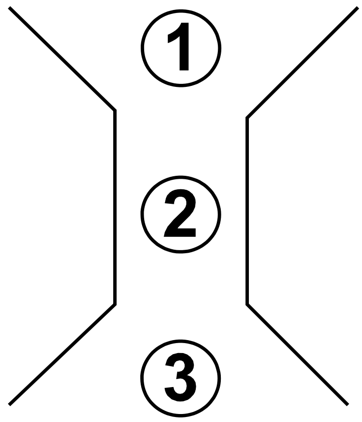
We started meeting with the secondary school maths teachers to try to make transition between systems easier for the students. After considerable discussion we contracted a consultant who suggested that school might look too much the same across the transition when the students were hoping for something new. On the other hand our experience suggested that there needed to be some consistency in the way teachers worked.

We decided to 'bite the bullet' and try a hands-on problem solving unit in one strand. We selected two menus of twenty hands-on tasks, one for the primary and one for the secondary, that became the core of the unit. We deliberately overlapped some tasks that we knew were very rich and added some new ones for the high school.

Class lessons and investigation sheets were used to extend the tasks, within a three week model.

It is important to note that although these teachers structured a 3 week unit for the students, they strongly advised an additional *Week Zero* for staff preparation. The units came to be called Replacement Units.

Week Zero - Planning



Staff familiarise themselves with the material and jointly plan the unit. This is not a model that can be 'planned on the way to class'.

> *Getting together turned out to be great professional development for our group.*

Week 1 - Introduction

Students explore the 20 tasks listed on a printed menu:

- students explore the tip of the task, as on the card
- students move from task to task following teacher questioning that suggests there is more to the task than the tip
- in discussion with students, teachers gather informal assessment information that guides lesson planning for the following week.

> *We gave the kids an 'encouragement talk' first about joining us in an experiment in ways of learning maths and then gave out the tasks. The response was intelligent and there was quite a buzz in the room.*

Week 2 - Formalisation

> *It was good for both us and the students that the lessons in this week were a bit more traditional. However, they weren't text book based. We used whole class lessons based on the tasks they had been exploring and taught the Working Mathematically process, content and report writing.*
>
> *Assessment was via standard teacher-designed tests, quizzes and homework.*

Week 3 - Investigations

> *We were most delighted with Week 3. Each student chose one task from the menu and carried out an in-depth investigation into the iceberg guided by an investigation sheet. They had to publish a report of their investigation and we were quite surprised at the outcomes. It was clear that the first two weeks had lifted the image of mathematics from 'boring repetition' to a higher level of intellectual activity.*

Story 5: Curriculum shift

> *I think our school was like many others. The syllabus pattern was 10 units of three weeks each through the year. We had drifted into that through a text book driven curriculum and we knew the students weren't responding.*

Our consultant suggested that there was sameness about the intellectual demands of this approach which gave the impression that maths was the pursuit of skills. We agreed to select two deeper investigations to add to each unit. It took some time and considerable commitment, but we know that we have now made a curriculum shift. We are more satisfied and so are the students.

The principles guiding this shift were:

- Agree

  The 20 particular investigations for the year are agreed to by all teachers. If, for example, Cube Nets *is decided as one of these, then all the teachers are committed to present this within its unit.*

- Publish

  The investigations are written into the published syllabus. Students and parents are made aware of their existence and expect them to occur.

- Commit

  Once agreed, teachers are required to present the chosen investigations. They are not a negotiable 'extra'.

- Value

  The investigations each illustrate an explicit form of the Working Mathematically process. This is promoted to students, constantly referenced and valued.

- Assess

  The process provides students with scaffolding for their written reports and is also known by them as the criteria for assessment. (See next page.)

- Report

  The assessment component features within the school reporting structure.

A Final Comment

Including investigations has become policy.

Why? Because to not do so is to offer a diminished learning experience.

The investigative process ranks equally with skill development and needs to be planned for, delivered, assessed and reported.

Perhaps most of all we are grateful to our consultant because he was prepared to begin where we were. We never felt as if we had to throw out the baby and the bath water.

Assessment

Our attitude is:

stimulated students are creative and love to learn

Regardless of the way you use your **Maths With Attitude** resource, a variety of procedures can be employed to assess this learning.

Where these assessment procedures are applied to task sessions and involve written responses from students, teachers will need to be careful that the writing does not become too onerous. Students who get bogged down in doing the writing may lose interest in doing the tasks.

In addition to the ideas below, useful references are:

- http://www.mathematicscentre.com/taskcentre/assess.htm
- http://www.mathematicscentre.com/taskcentre/report.htm

The first offers several methods of assessment with examples and the second is a detailed lesson plan to support students to prepare a Maths Report.

Journal Writing

Journal writing is a way of determining whether the task or lesson has been understood by the student. The pupil can comment on such things as:

- What I learned in this task.
- What strategies I/we tried (refer to the Strategy Board).
- What went wrong.
- How I/we fixed it.
- Jottings - ie: any special thoughts or observations

Some teachers may prefer to have the page folded vertically, so that children's reflective thoughts can be recorded adjacent to critical working.

Assessment Form

An assessment form uses questions to help students reflect upon specific issues related to a specific task.

Anecdotal Records

Some teachers keep ongoing records about how students are tackling the tasks. These include jottings on whether students were showing initiative, whether they were working co-operatively, whether they could explain ideas clearly, whether they showed perseverance.

Checklists

A simple approach is to create a checklist based on the Working Mathematically process. Teachers might fill it in following questioning of individuals, or the students may fill it in and add comments appropriately.

Pupil Self-Reflection

Many theorists value and promote metacognition, the notion that learning is more permanent if pupils deliberately and consciously analyse their own learning. The deliberate teaching strategy of oral questioning and the way pupils record their work is an attempt to manifest this philosophy in action. The alternative is the tempting 'butterfly' approach which is to madly do as many activities as possible, mostly superficially, in the mistaken belief that quantity equates to quality.

> *I had to work quite hard to overcome previously entrenched habits of just getting the answer, any answer, and moving on to the next task.*

Thinking about *what* was learned *how* it was learned consolidates and adds to the learning.

When it follows an extensive whole class investigation, a reflection lesson such as this helps to shift entrenched approaches to mathematics learning. It is also an important component of the assessment process. On the one hand it gives you a lot of real data to assist your assessment. On the other it prepares the students for any formal assessment which you may choose to round off a unit.

Introduction

Ask students to recall what was done during the unit or lesson by asking a few individuals to say what *they* did, eg:

> *What did you do or learn that was new?*
> *What can you now do/understand that is new?*
> *What do you know now that you didn't know 1 (2, 3, ...) lesson ago?*

Continuing Discussion

Get a few ideas from the first students you ask, then:

- organise 5 -10 minute buzz groups of three or four students to chat together with one person to act as a recorder. These groups address the same questions as above.
- have a reporting session, with the recorder from each group telling the class about the group's ideas.

Student comments could be recorded on the board, perhaps in three groups.

<u>Ideas & Facts</u> <u>Maths Skills</u> <u>Process (learning) Skills</u>

If you need more questions to probe deeper and encourage more thought about process, try the following:

> *What new things did you do that were part of* how *you learned?*
> *Who uses this kind of knowledge and skill in their work?*

Student Recording

Hand out the REFLECTION sheet (next page) and ask students to write their own reflection about what they did, based on the ideas shared by the class. Collect these for interest and, possibly, assessment information.

REFLECTION

me looking at me learning

NAME: CLASS:

Working With Parents

Balancing Problem Solving with Basic Skill Practice

Many schools find that parents respond well to an evening where they have an opportunity to work with the tasks and perhaps work a task together as a 'whole class'. Resourced by the materials in this kit, teachers often feel quite confident to run these practical sessions. Comments from parents like:

> *I wish I had learnt maths like this.*

are very supportive. Letting students 'host' the evening is an additional benefit to the home/school relationship.

The 4½ Minute Talk

Charles Lovitt has considerable experience working with parents and has developed a crisp, parent-friendly talk which he shares below. Many others have used it verbatim with great success.

Why the Four and a Half Minute Talk?

When talking with parents about Problem Solving or the meaning of the term Working Mathematically, I have often found myself in the position, after having promoted inquiry based or investigative learning, of the parents saying:

> *Well - that's all very well - BUT ...*

at which stage they often express their concern for basic (meaning arithmetic) skill development.

The weakness of my previous attempts has been that I have been unable to reassure parents that problem solving does not mean sacrificing our belief in the virtues of such basic skill development.

One of the unfortunate perceptions about problem solving is that if a student is engaged in it, then somehow they are not doing, or it may be at the expense of, important skill based work.

This Four and a Half Minute Talk to parents is an attempt to express my belief that basic skill practice and problem solving development can be closely intertwined and not seen as in some way mutually exclusive.

(I'm still somewhat uncomfortable using the expression 'basic skills' in the above way as I am certain that some thinking, reasoning, strategy and communication skills are also 'basic'.)

Another aspect of the following 'talk' is that, as teachers put more emphasis on including investigative problem solving into their courses, a question arises about the source of suitable tasks.

This talk argues that we can learn to create them for ourselves by 'tweaking' the closed tasks that heavily populate out existing text exercises, and hence not be dependent on external suppliers. (Even better if students begin to create such opportunities for themselves.)

The Talk

In preparation, write the following graphic on the board:

CLOSED	OPEN	EXTENDED INVESTIGATION
		How many solutions exist? How do you know you have found them all?

I would like to show you what teachers are beginning to do to achieve some of the thinking and reasoning and communication skills we hope students will develop. I would like to show you three examples.

Example One: *6 + 5 = ?*

I write this question under the 'closed' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
$\begin{array}{r} 6 \\ \underline{+\ 5} \\ \_\_ \end{array}$		How many solutions exist? How do you know you have found them all?

And I ask:

What is the answer to this question?

I then explain that:

We often ask students many closed questions such as 6 + 5 = ?

The only response the students can tell us is "The answer is 11." … and as a reward for getting it correct we ask another twenty questions just like it.

What some teachers are doing is trying to *tweak* the question and ask it a different way, for example:

I have two counting numbers that add to 11. What might the numbers be?

[Counting numbers = positive whole numbers including zero]

I write this under the ‘open’ label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 ___	? + ? 11	How many solutions exist? How do you know you have found them all?

What is the answer to the question now?

At this stage it becomes apparent there are several solutions:

The question is now a bit more open than it was before, allowing students to tell you things like 8 + 3, or 10 + 1, or 11 + 0 etc.

Let's see what happens if the teacher 'tweaks' it even further with the investigative challenge *or* extended investigation question:

How many solutions are there altogether?

and more importantly, and with greater emphasis on the second question:

How could you convince someone else that you have found them all?

Now the original question is definitely different - it still involves the skills of addition but now also involves thinking, reasoning and problem solving skills, strategy development and particularly communication skills.

Young students will soon tell you the answer is ‘six different ones’, but they must also confront the communication and reasoning challenge of convincing you that there are only six and no more.

Example Two: *Finding Averages*

Again, as I go through this example, I write it into the diagram on the board in the relevant sections.

The CLOSED question is: *11, 12, 13 - find the average*

Tweaking this makes it an OPEN question and it becomes:

I have three counting numbers whose average is 12. What might the numbers be?

Students will often say:

10, 12, 14 … or 9, 12, 15 … or even 12, 12, 12

After realising there are many answers, you can tweak it some more and turn it into an EXTENDED INVESTIGATION:

How many solutions exist? … AND …

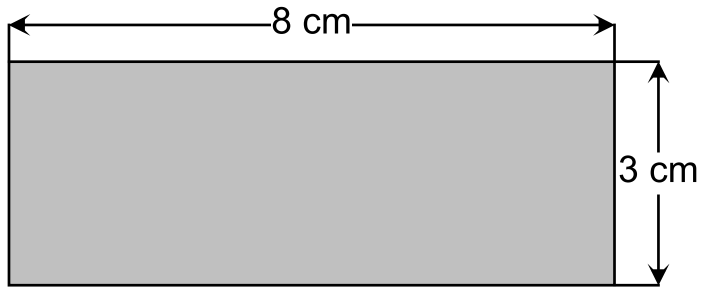
How do you know you have found them all?

Now the question is of a quite different nature. It still involves the arithmetic skill, but has something else as well - and that something else is the thinking, reasoning and communication skills necessary to find all of the combinations and convince someone else that you have done so.

By the time a student announces, with confidence, there are 127 different ways (which there are) that student will have engaged in all of these aspects, ie: the skill of calculating averages, (and some combination number theory) as well as significant strategy and reasoning experiences.

Example Three: *Finding the Area of a Rectangle*

A typical CLOSED question is:



Find the area. Find the perimeter.

The OPEN question is:

A rectangle has 24 squares inside:

What might its length and width be?

What might its perimeter be?

The EXTENDED INVESTIGATION version is:

Given they are whole number lengths, how many different rectangles are there? ... AND ...

How do you know you have found them all?

In summary, mathematics teachers are trying to convert *some* (not all) of the many closed questions that populate our courses and 'push' them towards the investigation direction. In doing so, we keep the skills we obviously value, but also activate the thinking, reasoning and justification skills we hope students will also develop.

This sequence of three examples hopefully shows two major features:

- That skills and problem solving can 'live alongside each other' and be developed concurrently.
- That the process of creating open-ended investigations can be done by anyone - just go to any source of closed questions and try 'tweaking' them as above. If it only worked for one question per page it would still provide a very large supply of investigations.

In terms of the effect of the talk on parents, I have usually found them to be reassured that we are not compromising important skill development (and nor do we want to). The only debate then becomes whether the additional skills of thinking, reasoning and communication are also desirable.

I've also been told that parents appreciate it because of the essential simplicity of the examples - no complicated theoretical jargon.

A Working Mathematically Curriculum

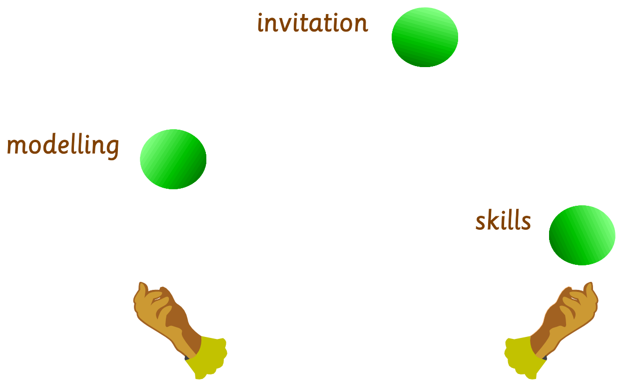
An Investigative Approach to Learning

The aim of a Working Mathematically curriculum is to help students learn to work like a mathematician. This process is detailed earlier (Page 8) in a one page document which becomes central to such a curriculum.

The change of emphasis brings a change of direction which *implies and requires* a balance between:

- the process of being a mathematician, and
- the development of skills needed to be a *successful* mathematician.

This journey is not two paths. It is one path made of two interwoven threads in the same way as DNA, the building block of life, is one compound made of two interwoven coils. To achieve a Working Mathematically curriculum teachers need to balance three components.



The task component of **Maths With Attitude** offers each pair of students an invitation to work like a mathematician.

The Maths300 component of **Maths With Attitude** assists teachers to model working like a mathematician.

Content skills are developed in context. They *are* important, but it is the application of skills within the process of Working Mathematically that has developed, and is developing, the human community's mathematical knowledge.

A focus for the Working Mathematically teacher is to help students develop mathematical skills in the context of problem posing and solving.

> *We are all 'born' with the same size mathematical toolbox, in the same way as I can own the same size toolbox as my motor mechanic. However, my motor mechanic has many more tools in her box than I and she has had more experience than I using them in context. Someone has helped her learn to use those tools while crawling under a car.*

Afzal Ahmed, Professor of Mathematics at Chichester, UK, once quipped:

> *If teachers of mathematics had to teach soccer, they would start off with a lesson on kicking the ball, follow it with lessons on trapping the ball and end with a lesson on heading the ball. At no time would they play a game of football.*

Such is not the case when teaching a Working Mathematically curriculum.

Elements of a Working Mathematically Curriculum

Working Mathematically is a K - 12 experience offering a balanced curriculum structured around the components below.

Hands-on Problem Solving Play

Mathematicians don't know the answer to a problem when they start it. If they did, it wouldn't be a problem. They have to play around with it. Each task invites students to play with mathematics 'like a mathematician'.

Skill Development

A mathematician needs skills to solve problems. Many teachers find it makes sense to students to place skill practice in the context of *Toolbox Lessons* which *help us better use the Working Mathematically Process* (Page 8).

Focus on Process

This is what mathematicians do; engage in the problem solving process.

Strategy Development

Mathematicians also make use of a strategy toolbox. These strategies are embedded in Maths300 lessons, but may also have a separate focus. Poster Problem Clinics are a useful way to approach this component.

Concept Development

A few major concepts in mathematics took centuries for the human race to develop and apply. Examples are place value, fractions and probability. In the past students have been expected to understand such concepts after having 'done' them for a two week slot. Typically they were not visited again until the next year. A Working Mathematically curriculum identifies these concepts and regularly 'threads' them through the curriculum.

Planning to Work Mathematically

The class, school or system that shifts towards a Working Mathematically curriculum will no longer use a curriculum document that looks like a list of content skills. The document would be clear in:

- choosing genuine problems to initiate investigation
- choosing a range of best practice teaching strategies to interest a wider range of students
- practising skills for the purpose of problem solving

Some teachers have found the planning template on the next page assists them to keep this framework at the forefront of their planning. It can be used to plan single lessons, or units built of several lessons. There are examples from schools in the Curriculum & Planning section of Maths300 and a Word document version of the template.

Unit Planning Page

Class

Topic

Pedagogy	Problem Solving In this topic how will I engage my students in the Working Mathematically process?	Skills
How do I create an environment where students know what they are doing and why they have accepted the challenge?		Does the challenge identify skills to practise? Are there other skills to practise in preparation for future problem solving?

Notes

As a general guide:

- Find a problem(s) to solve related to the topic.
- Choose the best teaching craft likely to engage the learners.
- Where possible link skill practice to the problem solving process.

More on Professional Development

For many teachers there will be new ideas within **Maths With Attitude**, such as unit structures, views of how students learn, teaching strategies, classroom organisation, assessment techniques and use of concrete materials. It is anticipated (and expected) that as teachers explore the material in their classrooms they will meet, experiment with and reflect upon these ideas with a view to long term implications for the school program and for their own personal teaching.

Being explored 'on-the-job' so to speak, in the teacher's own classroom, makes the professional development more meaningful and practical for the teacher. This is also a practical and economic alternative for a local authority.

Strategic Use by Systems

From Years 3 - 10, **Maths With Attitude** is designed as a professional development vehicle by schools or clusters or systems because it carries a variety of sound educational messages. They might choose **Maths With Attitude** because:

- It can be used to highlight how investigative approaches to mathematics can be built into balanced unit plans without compromising skill development and without being relegated to the margins of a syllabus as something to be done only after 'the real' content has been covered.
- It can be used to focus on how a balance of concept, skill and application work can all be achieved within the one manageable unit structure.
- It can be used to show how a variety of assessment practices can be used concurrently to build a picture of student progress.
- It can be used to focus on transition between primary and secondary school by moving towards harmony and consistency of approach.
- It can be used to raise and continue debate about the pedagogy (art of teaching) that supports deeper mathematical learning for a wider range of students.

Teachers in Years K - 2 are similarly encouraged in professional growth through **Working Mathematically with Infants**, which derives from Calculating Changes, a network of teachers enhancing children's number skills from Years K - 6.

In supporting its teachers by supplying these resources in conjunction with targeted professional development over time, a system can fuel and encourage classroom-based debate on improving outcomes. There is evidence that by exploring alternative teaching strategies and encouraging curriculum shift towards Working Mathematically, learners improve and teachers are more satisfied. For more detail visit Research & Stories at:

- http://www.mathematicscentre.com/taskcentre/do.htm

We would be happy to discuss professional development with system leaders.

Web Reference

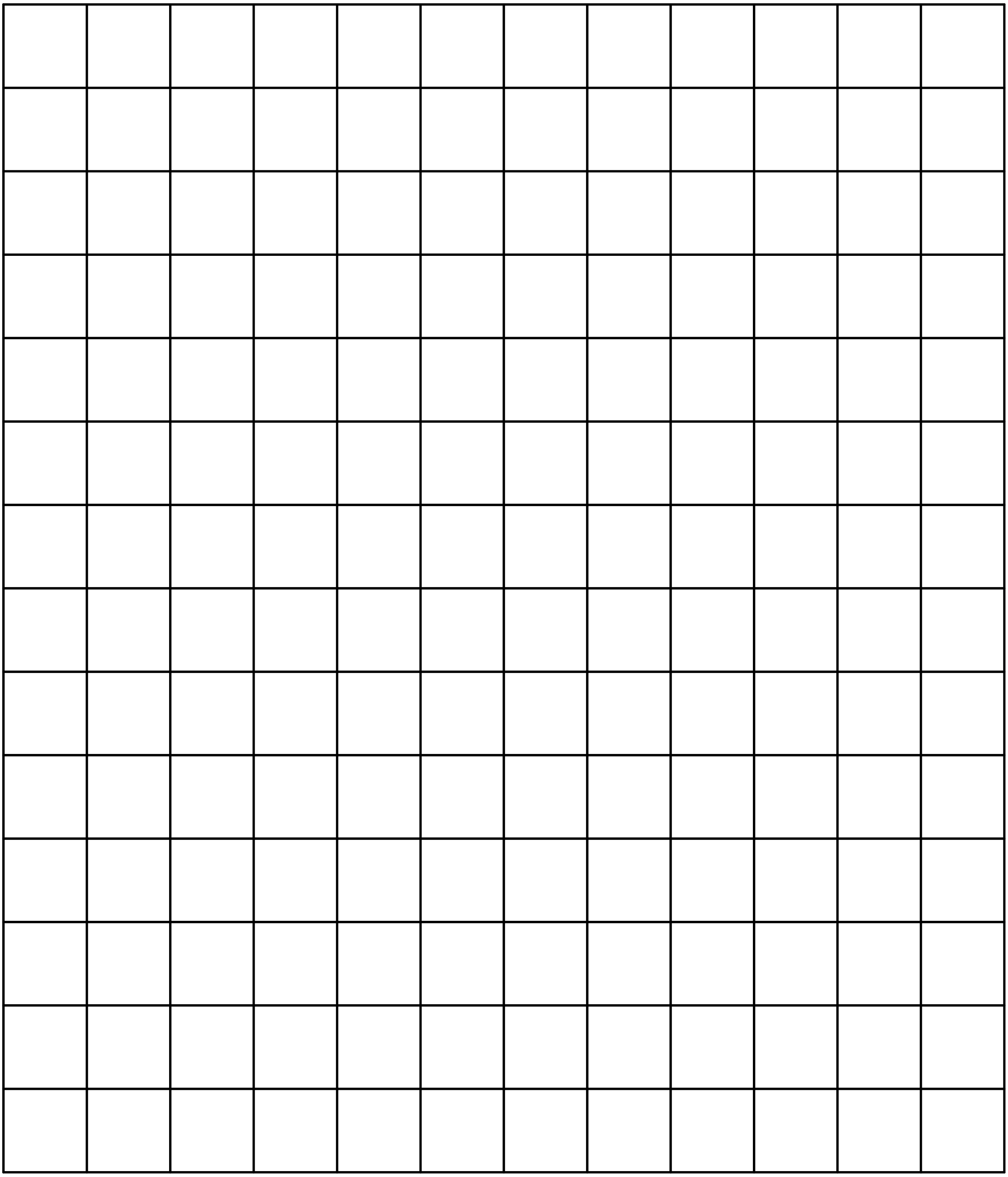
The starting point for all aspects of learning to work like a mathematician, including Calculating Changes, and the teaching craft which encourages it is:

- http://www.mathematicscentre.com

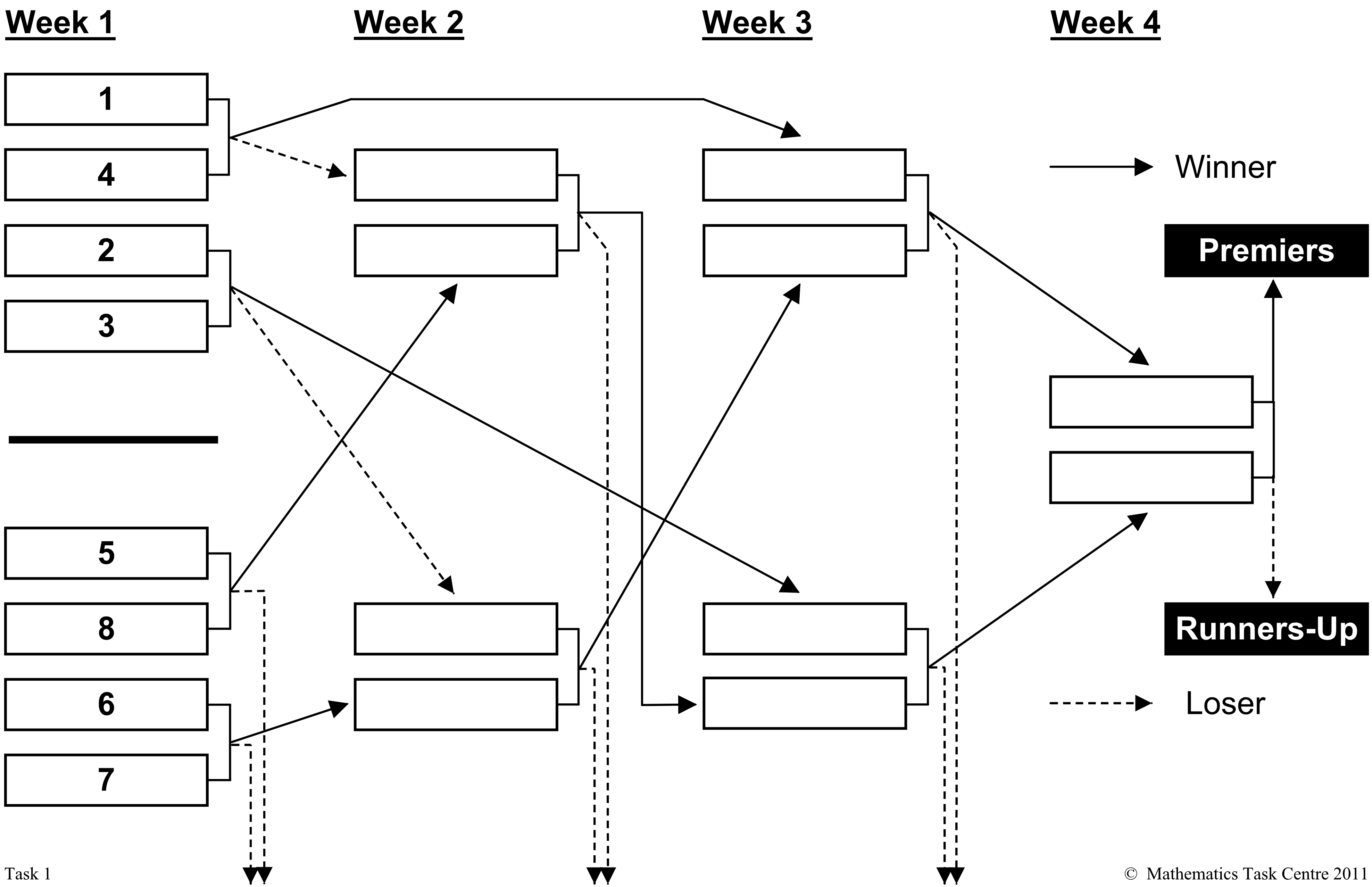
Appendix: Recording Sheets

64 = 65

Cross & Square Area Sheet



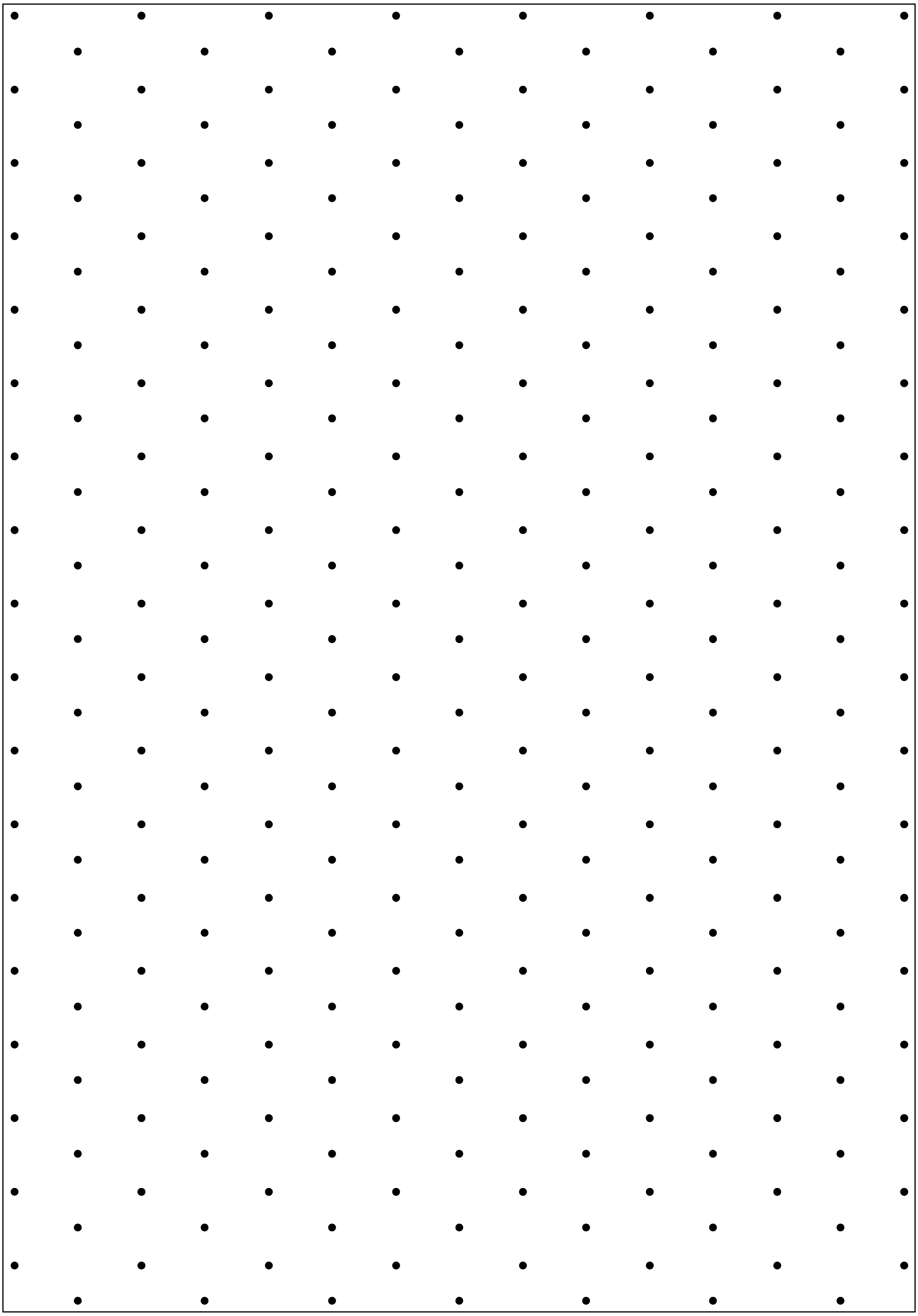
FINAL EIGHT



Week 1
Week 2
Week 3
Week 4
1
4
2
3
5
8
6
7
Winner
Premiers
Runners-Up
Loser

Growing Tricubes Recording Sheet

Reproducible Page



Names: .... Class: ....