maths with attitude

2nd Edition

Chance & Measurement Years 7 & 8

Charles Lovitt
Doug Williams

Mathematics Task Centre & Maths300
helping to create happy healthy cheerful productive inspiring classrooms

maths with attitude

2nd Edition

Chance & Measurement

Years 7 & 8

In this kit:
- *Hands-on problem solving tasks*
- *Detailed curriculum planning*

Access from Maths300:
- *Extensive lesson plans*
- *Software*

Doug Williams
Charles Lovitt

The **Maths With Attitude** series has been developed by The Task Centre Collective and is published by Black Douglas Professional Education Services.

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Maths With Attitude includes reference to hands-on problem solving tasks and Maths300 material.

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  happy, healthy, cheerful, productive, inspiring classrooms

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- to learners:

  stimulated students are creative and love to learn
- to pedagogy:

  the art of choosing teaching strategies to involve and interest all students
- to mathematics:

  mathematics is concrete, visual and makes sense
- to learning mathematics:

  all students can learn to work like a mathematician
- to teachers:

  the teacher is the most important resource in education
- to professional development:

  teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

Our Objective in Detail

What do we mean by creating:

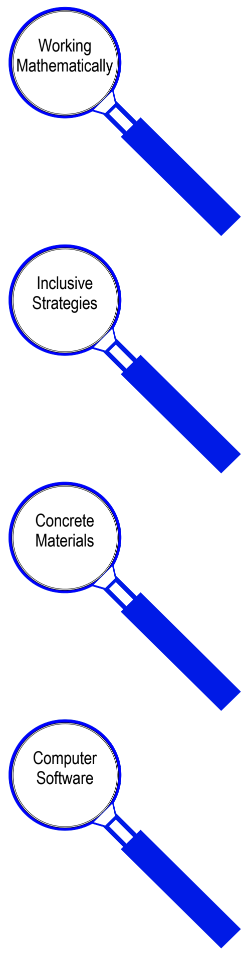
happy, healthy, cheerful, productive, inspiring classrooms

Happy…

means the elimination of the unnecessary fear of failure that hangs over so many students in their mathematics studies. Learning experiences *can* be structured so that all students see there is something in it for them and hence make a commitment to the learning. In so many 'threatening' situations, students see the impending failure and withhold their participation.

A phrase which describes the structure allowing all students to perceive something in it for them is *multiple entry points and multiple exit points*. That is, students can enter at a variety of levels, make progress and exit the problem having visibly achieved.

Healthy…



means *educationally healthy*. The learning environment should be a reflection of all that our community knows about how students learn. This translates into a rich array of teaching strategies that could and should be evident within the learning experience.

If we scrutinise the *exploration* through any lens, it should confirm to us that it is well structured or alert us to missed opportunities. For example, peering through a pedagogy lens we should see such features as:

- a story shell to embed the situation in a meaningful context
- significant active use of concrete materials
- a problem solving challenge which provides ownership for students
- small group work
- a strong visual component
- access to supportive software

Cheerful…

because we want 'happy' in the title twice!

Productive…

is the clear acknowledgment that students are working towards recognisable outcomes. They should know what these are and have guidelines to show they have either reached them or made progress. Teachers are accountable to these outcomes as well as to the quality of the learning environment.

Inspiring…

is about creating experiences that are uplifting or exalting; that actually *turn students on*. Experiences that make students feel great about themselves and empowered to act in meaningful ways.

Chance & Measurement Resources

To help you create

happy, healthy, cheerful, productive, inspiring classrooms

this kit contains

- 20 hands-on problem solving tasks from Mathematics Centre

and a Teachers' Manual which integrates the use of the tasks with

- 18 detailed lesson plans from Maths300

The kit offers **5 weeks** of Scope & Sequence planning in Chance & Measurement for *each* of Year 7 and Year 8. This is detailed in *Part 2: Planning Curriculum* which begins on Page 12. You are invited to map these weeks into your Year Planner.
Together, the four kits available for these levels provide 25 weeks of core curriculum in Working Mathematically (working like a mathematician).

Note: Membership of Maths300 is assumed.
The kit will be useful without it, but it will be much more useful with it.

Tasks

- 12 Counters
- Angle Estimation
- A Rectangle Of Squares
- Cat and Mouse
- Chocolate Chip Cookies
- Choosing Beads
- Counter Escape
- Duelling Dice
- How Many Things?
- Kids On Grids
- Matching Faces
- Pack The Box
- Playing With Objects
- Surface Area With Tricube
- The Frog Pond
- Triangle Area
- Triangle Perimeters
- Tube Toss
- Where Is The Rectangle?
- Win At The Fair

Part 2 of this manual introduces each task. The latest information can be found at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm

Maths300 Lessons

- Angle Estimation
- Area of a Circle
- Area of a Triangle
- Beetle Game
- Birth Month Paradox
- Cat and Mouse
- Chocolate Chip Cookies
- Circumference of a Circle
- Counter Escape
- Country Maps
- Duelling Dice
- Estimation Walks
- Highest Number
- Problem Dice
- Pulse Rates
- Temperature Graphs
- This Goes With This
- Win At The Fair

Lessons with Software

- Angle Estimation
- Area of a Triangle
- Beetle Game
- Birth Month Paradox
- Cat and Mouse
- Chocolate Chip Cookies
- Counter Escape
- Duelling Dice
- Highest Number
- Problem Dice
- Temperature Graphs
- Win At The Fair

Part 2 of this manual introduces each lesson. Full details can be found at:

- http://www.maths300.com

Working Like A Mathematician

Our attitude is:

all students can learn to work like a mathematician

What does a mathematician's work actually involve? Mathematicians have provided their answer on Page 8. In particular we are indebted to Dr. Derek Holton for the clarity of his contribution to this description.

Perhaps the most important aspect of Working Mathematically is the recognition that *knowledge is created by a community and becomes part of the fabric of that community*. Recognising, and engaging in, the process by which that knowledge is generated can help students to see themselves as able to work like a mathematician. Hence Working Mathematically is the framework of **Maths With Attitude.**

Skills, Strategies & Working Mathematically

A Working Mathematically curriculum places learning mathematical skills and problem solving strategies in their true context. Skills and strategies are the tools mathematicians employ in their struggle to solve problems. Lessons on skills or lessons on strategies are not an end in themselves.

- Our skill toolbox can be added to in the same way as the mechanic or carpenter adds tools to their toolbox. Equally, the addition of the tools is not for the sake of collecting them, but rather for the purpose of getting on with a job. A mathematician's job is to attempt to solve problems, not to collect tools that might one day help solve a problem.
- Our strategy toolbox has been provided through the collective wisdom of mathematicians from the past. All mathematical problems (and indeed life problems) that have ever been solved have been solved by the application of this concise set of strategies.

About Tasks

Our attitude is:

mathematics is concrete, visual and makes sense

Tasks are from Mathematics Task Centre. They are an invitation to two students to work like a mathematician (see Page 8).

The Task Centre concept began in Australia in the late 1970s as a collection of rich tasks housed in a special room, which came to be called a Task Centre. Since that time hundreds of Australian teachers, and, more recently, teachers from other countries, have adapted and modified the concept to work in their schools. For example, the special purpose room is no longer seen as an essential component, although many schools continue to opt for this facility.

A brief history of Task Centre development, considerable support for using tasks, for example Task Cameos, and a catalogue of all currently available tasks can be found at:

- http://www.mathematicscentre.com/taskcentre

Key principles are:

- A good task is the tip of an iceberg
- Each task has three lives
- Tasks involve students in the Working Mathematically process

The Task Centre Room or the Classroom?

There are good reasons for using the tasks in a special room which the students visit regularly. There are also different good reasons for keeping the tasks in classrooms. Either system can work well if staff are committed to a core curriculum built around learning to work like a mathematician.

- A task centre room creates a focus and presence for mathematics in the school. Tasks are often housed in clear plastic 'cake storer' type boxes. Display space can be more easily managed. The visual impact can be vibrant and purposeful.
- However, tasks can be more readily integrated into the curriculum if teachers have them at their finger tips in the classrooms. In this case tasks are often housed in press-seal plastic bags which take up less space and are more readily moved from classroom to classroom.

Tip of an Iceberg

The initial problem on the card can usually be solved in 10 to 20 minutes. The investigation iceberg which lies beneath may take many lessons (even a lifetime!). Tasks are designed so that the original problem reveals just the 'tip of the iceberg'. Task Cameos and Maths300 lessons help to dig deeper into the iceberg.

We are constantly surprised by the creative steps teachers and students take that lead us further into a task. No task is ever 'finished'.

> *Most tasks have many levels of entry and exit and therefore offer an on-going invitation to revisit them, and, importantly, multiple levels of success for students.*

Three Lives of a Task

This phrase, coined by a teacher, captures the full potential and flexibility of the tasks. Teachers say they like using them in three distinct ways:

1. As on the card, which is designed for two students.
2. As a whole class lesson involving all students, as supported by outlines in the Task Cameos and in detail through the Maths300 site.
3. Extended by an Investigation Guide (project), examples of which are included in both Task Cameos and Maths300.

The first life involves just the 'tip of the iceberg' of each task, but nonetheless provides a worthwhile problem solving challenge - one which 'demands' concrete materials in its solution. This is the invitation to work like a mathematician. Most students will experience some level of success and accomplishment in a short time.

The second life involves adapting the materials to involve the whole class in the investigation, in the first instance to model the work of a mathematician, but also to develop key outcomes or specific content knowledge. This involves choosing teaching craft to interest the students in the problem and then absorb them in it.

The third life challenges students to explore the 'rest of the iceberg' independently. Investigation Guides are used to probe aspects and extensions of the task and can be introduced into either the first or second life. Typically this involves providing suggestions for the direction the investigation might take. Students submit the 'story' of their work for 'portfolio assessment'. Typically a major criteria for assessment is application of the Working Mathematically process.

About Maths300

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

Maths300 is a subscription based web site. It is an attempt to collect and publish the 300 most 'interesting' maths lessons (K - 12).

- *Lessons have been successfully trialed in a range of classrooms.*
- *About one third of the lessons are supported by specially written software.*
- *Lessons are also supported by investigation sheets (with answers) and game boards where relevant.*
- *A 'living' Classroom Contributions section in each lesson includes the latest information from schools.*
- *The search engine allows teachers to find lessons by pedagogical feature, curriculum strand, content and year level.*
- *Lesson plans can be printed directly from the site.*
- *Each lesson supports teaches to model the Working Mathematically process.*

Modern internet facilities and computers allow teachers easy access to these lesson plans. Lesson plans need to be researched, reflected upon in the light of your own students and activated by collecting and organising materials as necessary.

Maths300 Software

Our attitude is:

> *stimulated students are creative and love to learn*

Pedagogically sound software is one feature likely to encourage enthusiastic learning and for that reason it has been included as an element in about one third of Maths300 lesson plans. The software is used to develop an investigation beyond its introduction and early exploration which is likely to include other pedagogical techniques such as concrete materials, physical involvement, estimation or mathematical conversation. The software is not the lesson plan. It is a feature of the lesson plan used at the teacher's discretion.

For school-wide use, the software needs to be downloaded from the site and installed in the school's network image. You will need to consult your IT Manager about these arrangements. It can also be downloaded to stand alone machines covered by the site licence, in particular a teacher's own laptop, from where it can be used with the whole class through a data projector.

Note:

- *Maths300 lessons and software may only be used by Maths300 members.*

Working Mathematically

First give me an interesting problem.

When mathematicians become interested in a problem they:

- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

Questions which help mathematicians learn more are:

- Can I check this another way?
- What happens if ...?
- How many solutions are there?
- How will I know when I have found them all?

When mathematicians have a problem they:

- Read & understand the problem.
- Plan a strategy to start the problem.
- Carry out their plan.
- Check the result.

A mathematician's strategy toolbox includes:

- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- Break a problem into smaller parts
- ...

If one way doesn't work, I just start again another way.

Professional Development Purpose

Our attitude is:

the teacher is the most important resource in education

> *We had our first study group on Monday. The session will be repeated again on Thursday. I had 15 teachers attend. We looked at the task Farmyard Friends (Task 129 from the Mathematics Task Centre). We extended it out like the questions from the companion Maths300 lesson suggested, and talked for quite a while about the concept of a factorial. This is exactly the type of dialog that I feel is essential for our elementary teachers to support the development of their math background. So anytime we can use the tasks to extend the teacher's math knowledge we are ahead of the game.*
> District Math Coordinator, Denver, Colorado

Research suggests that professional development most likely to succeed:

- is requested by the teachers
- takes place as close to the teacher's own working environment as possible
- takes place over an extended period of time
- provides opportunities for reflection and feedback
- enables participants to feel a substantial degree of ownership
- involves conscious commitment by the teacher
- involves groups of teachers rather than individuals from a school
- increases the participant's mathematical knowledge in some way
- uses the services of a consultant and/or critical friend

Maths With Attitude has been designed with these principles in mind. All the materials have been tried, tested and modified by teachers from a wide range of classrooms. We hope the resources will enable teacher groups to lead themselves further along the professional development road, and support systems to improve the learning outcomes for students K - 12.

With the support of Maths300 ETuTE, professional development can be a regular component of in-house professional development. See:

- http://www.mathematicscentre.com/taskcentre/resource.htm#etute

For external assistance with professional development, contact:

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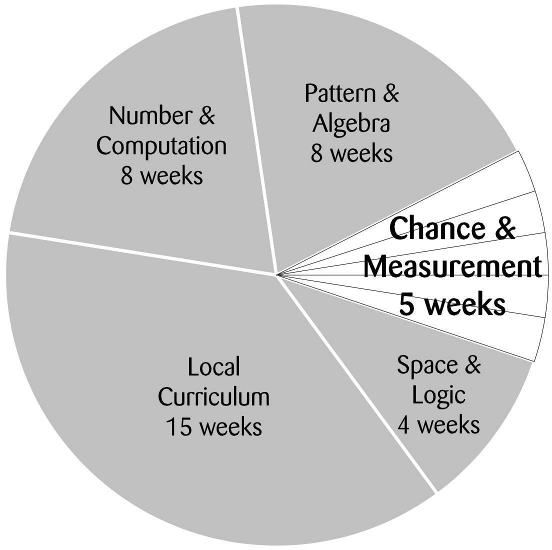
Part 2: Planning Curriculum

Curriculum Planners

Our attitude is:

learning is a personal journey stimulated by achievable challenge



Curriculum Planners:

- show one way these resources can be integrated into your weekly planning
- provide a starting point for those new to these materials
- offer a flexible structure for those more experienced

You are invited to map Planner weeks into your school year planner as the core of the curriculum.

Planners:

- detail each week lesson by lesson
- offer structures for using tasks and lessons
- are sequenced from lesson to lesson, week to week and year to year to 'grow' learning

Teachers and schools will map the material in their own way, but all will be making use of extensively trialed materials and pedagogy.

Using Resources

- Your kit contains 20 hands-on problem solving tasks and reference to relevant Maths300 lessons.
- Tasks are introduced in this manual and supported by the Task Cameos at:
  http://www.mathematicscentre.com/taskcentre/iceberg.htm
- Maths300 lessons are introduced in this manual and supported by detailed lesson plans at:
  http://www.maths300.com

In your preparation, please note:

- Planners assume 4 lessons per week of about 1 hour each.
- Planners are *not* prescribing a continuous block of work.
- Weeks can be interspersed with other learning; perhaps a **Maths With Attitude** week from a different strand.
- Weeks can sometimes be interchanged within the planner.
- Lessons can sometimes be interchanged within weeks.
- The four **Maths With Attitude** kits available at each year level offer 25 weeks of a Working Mathematically core curriculum.

A Way to Begin

- Glance over the Planner for your class. Skim through the comments for each task and lesson as it is named. This will provide an overview of the kit.
- Task Comments begin after the Planners. Lesson Comments begin after Task Comments. The index will also lead you to any task or lesson comments.
- Select your preferred starting week - usually Week 1.
- Now plan in detail by researching the comments and web support. Enjoy!

Research, Reflect, Activate

Curriculum Planner

Chance & Measurement: Year 7

<table>
<tr><th></th><th>Session 1</th><th>Session 2</th><th>Session 3</th><th>Session 4</th></tr>
<tr><td>Week 1</td><td colspan="3" rowspan="2">Measurement Unit: A practical two week introduction/ review/ extension of length, perimeter, circumference, angle measurement and area. Physical involvement, hands-on problem solving and estimation are key methodologies. The lessons are suitable for mixed ability classes and some teachers use these to 'get to know' their students in a fun way early in the new year.
Week 1 Lessons: Estimation Walks, Angle Estimation and Circumference of a Circle. The first two require either outdoor space, or an indoor area such as a gym.
Week 2 Lessons: Area of a Triangle and Area of a Circle. Both make powerful use of a visual/ conceptual approach that leads to evaluating areas using first principles thinking, rather than just applying a recipe. In each lesson a strong link is made to previous knowledge about evaluating the area of a rectangle. The software from Area of a Triangle works well with one computer and a data projector.</td><td rowspan="2">Tasks & Text: Divide the class into two halves. Use the 10 measurement tasks (see Page 16) with one half and related work from your text book with the other half. Area of a Triangle software can also be used to supply exercises.
In Week 2 swap the groups.</td></tr>
<tr><td>Week 2</td></tr>
<tr><td>Weeks 3 - 5</td><td colspan="4">Mixed Media Unit A: The Mixed Media teaching model is explained on Page 17. It respects the theory of multiple intelligences and therefore presents the content through a range of learning styles. It assumes ready access to computers for one third of the class. If these are not a fixture in the room, schools have adopted alternatives such as (a) making arrangements for students to visit computer sites within the school, or, (b) collecting an appropriate number of laptop computers (5 or 6 for a class of 30).
This unit offers considerable choice for both teachers and students and helps to develop independent learners within the content of probability (empirical and theoretical) and data recording and display. Intuitions are examined and extended. Short term variability is explored and compared to expected theoretical results. Several problem solving investigations are included and there is room for students to develop their literacy skills by preparing a report of their investigation(s). Software plays a large part in being able to gather sufficient data in real time.</td></tr>
</table>

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Curriculum Planner

Chance & Measurement: Year 8

<table>
<tr><th></th><th>Session 1</th><th>Session 2</th><th>Session 3</th><th>Session 4</th></tr>
<tr><td>Week 1</td><td colspan="3" rowspan="2">Measurement & Data Unit: In this two week unit the emphasis is on collection and display of data, and analysing and communicating the information it contains. Much of the information in the unit is personal data such as birth date, favourite things, and pulse rates. Some data derives from measurement.
Week 1 Lessons: Birth Month Paradox, This Goes With This.
Week 2 Lessons: Pulse Rates, Country Maps and Temperature Graphs.
Country Maps takes area measurement, explored in Year 7, into the realm of irregular areas. Temperature Graphs extends the geographic link to focus on interpreting graphs in a way that values students' personal knowledge. Using one computer and a data projector will support the presentation of both Birth Month Paradox and Temperature Graphs .</td><td rowspan="2">Tasks & Text: Divide the class into two halves. Use the 10 measurement tasks (see Page 16) with one half and related work from your text book with the other half. Also, worksheets can be printed from Temp. Graphs software.
In Week 2 swap the groups.</td></tr>
<tr><td>Week 2</td></tr>
<tr><td>Weeks
3 - 5</td><td colspan="4">Mixed Media Unit B: The Mixed Media teaching model is explained on Page 17. It respects the theory of multiple intelligences and therefore presents the content through a range of learning styles. It assumes ready access to computers for one third of the class. If these are not a fixture in the room, schools have adopted alternatives such as (a) making arrangements for students to visit computer sites within the school, or, (b) collecting an appropriate number of laptop computers (5 or 6 for a class of 30).
This unit offers considerable choice for both teachers and students and helps to refresh the chance and data work begun in a similar unit during Year 7. The resources for the unit are the same as for Year 7, but the range of choices means that each student can be involved in a new way. Some teachers find that students particularly enjoyed trying 'old' tasks again and, when ready, extended their interest to ones they had not yet explored. If the school develops the appropriate records, 'student output' in this unit can be compared with the corresponding Year 7 unit as an assessment indicator.</td></tr>
</table>

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Planning Notes

Enhancing Maths With Attitude

Resources to support learning to work like a mathematician are extensive and growing. There are more tasks and lessons available than have been included in this Chance & Measurement kit. You could use the following to enhance this kit.

Additional Tasks

- Task 25, In Between Time

  As time passes, both the minute hand and the hour hand move. This task first uses a thirty minute difference (embodying the idea of half of the hourly minute hand journey) to establish that students understand: (a) Time difference can be clockwise or anti-clockwise, and (b) If the minute hand moves half an hour, then the hour hand has moved half of its journey towards the next (or previous) hour. From this basis the students are challenged to set a position for one of the hands and explore where the other hand must, or sometimes could, be.

More information about these tasks may be available in the Task Cameo Library:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Additional Lessons

- Lesson 122, How Many People Can Stand?

  How many people can stand? It's a question which engineers and others often have to ask. In this lesson the question is asked in the context of a story shell where the classroom becomes the 'standing room only' section of a venue, for the purposes of encouraging estimation and calculation of number and area and a variety of approaches to solving the problem.
- Lesson 165, Surface Area With Tricubes

  The best text books introduce their work on surface area with glossy diagrams of cube-based structures. As pretty as they may be, they are not a substitute for concrete experience and kinaesthetically developed spatial perception. This lesson provides a hands-on problem solving introduction to surface area, base area and volume and in doing so replaces the drudgery of the text book with a willingness to practise and refine newly found skills.
- Lesson 166, Newspaper Cubes & Volume of a Room

  This lesson is about visualising volumes of cubes and cuboids. A conceptual notion is developed by making and counting cubes and this provides a firm basis for later understanding and application of the formula V=LxWxH. Additional benefits are experience of structural engineering features such as the need to use triangles to make objects rigid and consequent connections with students' real life experiences of building. The lesson makes the calculation of volume a richer experience than merely multiplying three numbers.

Keep in touch with new developments which enhance **Maths With Attitude** at:

- http://www.mathematicscentre.com/taskcentre/enhance.htm

Tasks By Content

The twenty tasks in this **Maths With Attitude** kit are grouped as follows:

Measurement

- Angle Estimation
- A Rectangle Of Squares
- How Many Things?
- Kids On Grids
- Pack The Box
- Playing With Objects
- Surface Area With Tricube
- Triangle Area
- Triangle Perimeters
- Where Is The Rectangle?

Chance & Data

- 12 Counters
- Cat and Mouse
- Chocolate Chip Cookies
- Choosing Beads
- Counter Escape
- Duelling Dice
- Matching Faces
- The Frog Pond
- Tube Toss
- Win At The Fair

Additional Materials

As stated, our attitude is that mathematics is concrete, visual and makes sense. We assume that all classrooms will have easy access to many materials beyond what we supply. For this unit you will need:

- A long tape, say 30m or more
- Calculators
- Cord, rope or string
- Counters
- Dice

Special Comments Year 7

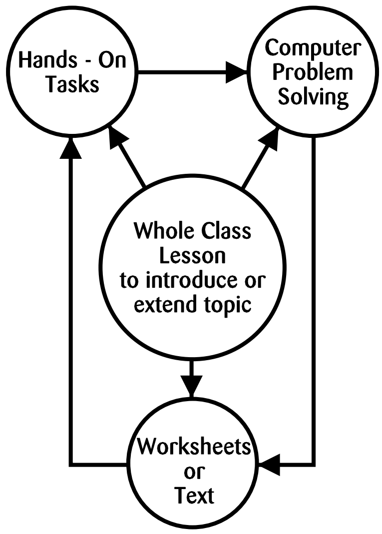
- Look ahead to Planner Week 1. You will need an outdoor space, or a large indoor space.
- Look ahead to Planner Week 2. The *Area of a Triangle* lesson works best if you have prepared sets of pieces on coloured paper and stored them in envelopes. Maths300 provides the masters for this printing.
- Look ahead to Planner Weeks 3 - 5. A Mixed Media Unit takes a little extra preparation in the beginning, but this is repaid during the unit because it runs for several days. Schools which have used the model find team preparation eases any burden, ie:
  - one teacher ensures the software station is prepared and runs effectively during the unit
  - one gets the tasks in order and keeps them that way during the unit
  - one selects appropriate material from the text and prepares the 'contract' sheet
  - one makes sure all teachers are resourced with whatever is necessary for the whole class lessons.
- If you are going to use Duelling Dice as a whole class lesson in the Mixed Media Unit, you will need to have a class set of the special cubes prepared in advance. This will take some time.

Special Comments Year 8

- Look ahead to Planner Weeks 1 & 2. You may need to book the use of a data projector.
- Look ahead to Planner Weeks 3 - 5. See comments for Year 7 re the same weeks.
- In the first week of the Mixed Media Unit you might also wish to use coloured cut-out pieces for the *Beetle Game* lesson. If you do, it will take some time to prepare a class set. It is not necessary to do this for the lesson to succeed, but it does add a concrete and colourful quality to the experience.

Mixed Media Unit

Mixed Media mathematics has been created as *one* structure which allows teachers to integrate problem solving tasks into the curriculum.



The design incorporates four different modes of learning into a structure which can be readily managed by one teacher, but which is enhanced when prepared and executed by a team.

A three week Mixed Media Unit includes:

- whole class lessons
- hands-on problem solving
- problem solving software
- skill practice worksheets (or text material)
- time to reflect on learning
- assessment opportunities

If this is the first time such a structure has been used in your classroom, it is a good idea to prepare the students in a manner which 'brings them into the experiment'.

A vital element of the process is to reflect on *what* is learned and *how* it is learned *before* the final assessment of the learning. Guidance with respect to assessment is also provided in this manual. In particular, the Pupil Self-Reflection information in the Assessment section Part 3 was designed by teachers who trialed the original Mixed Media units.

Mixed Media Unit A

Cat and Mouse is suggested as the whole class lesson for the first week. Print the board from the Maths300 lesson and collect a marker and a dice for each pair. The class lesson engages the students in an open-ended problem solving situation and also introduces the concept of expected value. When students move into their three Mixed Media groups the *Cat and Mouse* software is used to support the continuing investigation. The lesson plan provides a wealth of material from which teachers can prepare a sheet of focus questions to guide the investigation.

The tasks suggested for this Chance & Data unit are listed on Page 16.

Teachers have found that 'tucking one of these under the arm' and going into class to run a whole class investigation around it helps students to see the curriculum as more integrated. In whole class lessons teachers model how a mathematician works; when students open a task they are accepting the invitation to put the model into practice.

The third group in the rotation will be working for one period on probability and statistics work from your text. Students work in pairs on the tasks and the computers. Some teachers are happy for this collegiate approach to continue at the text station as well.

The second week, and perhaps the third, will need a new whole class lesson with software support and there are several to choose from on the Maths300 site. Of these, the lessons listed below have one or more Investigation Sheets supplied to guide the students further when it is their turn at the software station.

- Chocolate Chip Cookies
- Counter Escape
- Duelling Dice
- Highest Number
- Problem Dice
- Win At The Fair

Of course some may prefer to continue working on the *Cat and Mouse* investigation; the second investigation being an 'if you finish' standby. Also, it may be appropriate for students to continue some aspects of the whole class investigation during their period at the text station.

Teachers often find the third week needs to be flexible. For example, if the unit is going to have a strong assessment component based on the whole class investigation, the students may need time scheduled for report writing. In fact, it may be necessary to use the whole class lesson time this week to model report writing. Alternatively, teachers may be happy to continue the exploratory nature of the unit and thus introduce one more whole class lesson and follow up investigation from the set above.

In the preparation of this unit some teachers create a 'contract' sheet for students that sets out the expectations of the unit, for example:

- Participate in two whole class lessons.
- Keep a diary of your work on at least three tasks.
- Complete a written report of one software based investigation.
- Complete Exercises … from the text.

Student Publishing

It is inappropriate to simply expect students to publish a report of their investigation. We have to devote lesson time to teaching how to keep a journal while investigating and how to plan and present a report. The Recording & Publishing section of Mathematics Task Centre includes two different approaches to scaffolding this process with the class. Both include sample student work and suggest that a report can be presented in forms other than pencil and paper, for example PowerPoint. The links are titled 'Learning to Write a Maths Report' and 'Learning to Write a Maths Report 2' and can be found at:

- http://www.mathematicscentre.com/taskcentre/record.htm

Mixed Media Unit B

Beetle Game is suggested as the whole class lesson for the first week. With sufficient forward planning and the assistance of a teachers' aide or parent, class sets of laminated pieces can be prepared from the master in the Maths300 lesson. However, the lesson also runs effectively as a pencil and paper drawing.

The class lesson introduces several problem solving challenges and focuses on Stem & Leaf plots as a tool for data collection. When students move into their three Mixed Media groups the *Beetle Game* software is used to continue the investigation. The lesson plan provides a wealth of material from which teachers can prepare a sheet of focus questions to support students.

The remainder of the unit follows along the same lines as the unit above and uses the same resources. Teachers working together across Years 7 & 8 will be able to select a different set of lessons at each level to continue the students' learning related to probability and statistics.

Task Comments

- Tasks, lessons and unit plans prepare students for the more traditional skill practice lessons, which we invite you to weave into your curriculum. Teachers who have used practical, hands-on investigations as the focus of their curriculum, rather than focussing on the drill and practice diet of traditional mathematics, report success in referring to skill practice lessons as Toolbox Lessons. This links to the idea of a mathematician dipping into a toolbox to find and use skills to solve problems.

12 Counters

Students place 12 counters into 12 numbered rectangles in any way they wish, including using more than one counter in each rectangle. Counters are removed according to the sum of two dice.

At first students will probably place their counters at random, then, as they play, they work like a mathematician collecting and organising data (albeit internally) until they realise that the possible sums of two dice are not all equally likely. In fact it is not possible at all to remove a counter from Rectangle 1.

The game then becomes a problem solving challenge as they search for the 'best' way to place their counters to ensure a win in the lowest number of rolls. It may help students to see the 36 possibilities for the sum of two dice if the dice are different colours.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Angle Estimation

The special feature of this task is the Rotagrams which encourage estimation of angles. An angle is the shape made by two co-incident rays. Estimating or measuring an angle involves estimating or measuring the amount of turn between the rays. Without the Rotagram, the only tool normally available is the protractor. The focus with a protractor is on using it in the 'right way' to get the exact answer and with this implement the students don't get the opportunity to develop their visual image of the sizes of angles in the same way as with the Rotagram.

On a larger scale, students can be asked to estimate angles made by tying a fixed cord to a pole in the quadrangle and rotating another cord around the pole in relation to this fixed cord. This idea is explored in the companion Maths300 lesson, which includes software.

The Rotagram can also be fruitfully used in conjunction with the option Fraction Pie in the software of Maths300 Lesson 33, *Fraction Estimation*. Before using the software to show the requested fraction, students can be asked to estimate the fraction with the Rotagram. When the software shows the correct result, the Rotagram estimate can be easily checked by placing it against the screen. That is:

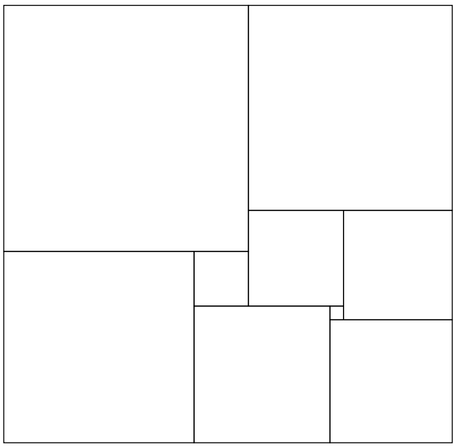
- Step 1: Guess the required fraction of a turn with the Rotagram.
- Step 2: Guess it with the software using the mouse.
- Step 3: Ask the software for the answer.
- Step 4: Check the Rotagram estimate against the answer.

A Rectangle Of Squares

Perhaps this task belongs in a Number & Computation unit, however the numbers involved do measure one side of each square and one part of the challenge is to calculate the area of the solution rectangle. Further, as students work with the task they are reinforcing the area of a rectangle as a visual image of a flat space being covered, rather than merely plugging numbers into an algebraic relationship.

In fact, understanding all the links between squares and their side length, and the perimeter and area of a rectangle, is vital if the hint on the card is to be applied efficiently. (Other strategies are possible too.)

- If the puzzle is to be solved, the area of the rectangle must be the sum of the areas of the squares.
- That sum can be calculated (1056) because we know the side lengths of the squares and we understand the concept of measuring area with squares.
- Combining these understandings leads to the calculation 1056 ÷ 33 to find the length of one other side of the rectangle.
- Then the perimeters of the squares have to be combined to find collections that equal 33 and 32. A solution is:



- It appears to be the only one, but an infinite set of solutions could be built from it by adding either a 32 or 33 square to the appropriate side and continuing to build.

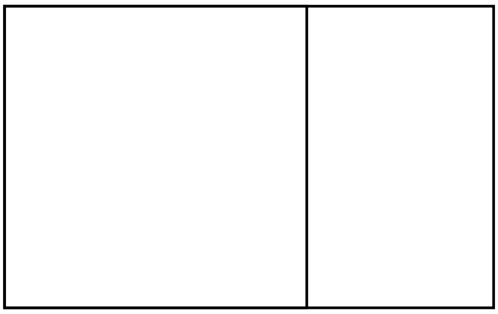
The task is supplied with one additional Size 1 square because this tiny piece is easily lost. Check that students don't try to solve the puzzle using both tiny squares.

An extension question might be to ask about rectangles created from a single square. Of course there are an infinite number that could be made, but the ones that have created considerable interest over thousands of years are the ones with a length to width ratio of $(1 + \sqrt{5})/2$ (approximately 1.618). This is the famous Golden Ratio.

- Ask students to mark the edge of one square on the left margin of a piece of graph paper. (Perhaps start with the 4-square.)
- Next they slowly rule in the top edge of the rectangle built from this line until it looks most pleasing to their eye.
- Complete the rectangle.
- Repeat for at least four other squares from the task and calculate the length to width ratio in each case.

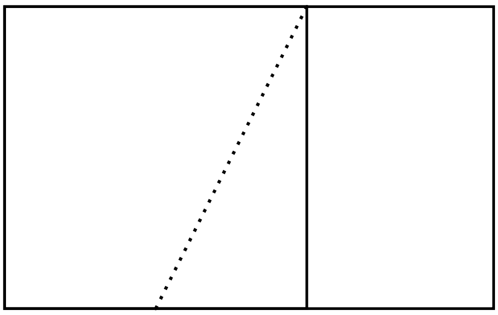
Most commonly, the ratio will approximate the one above. It is also worth calculating the length to width ratio of rectangles that do not appear as pleasing. How different are they from the Golden Ratio?

This investigation can be taken further by completing the square inside the pleasing rectangle.



What is the length to width ratio of the rectangle on the right? It turns out to be the Golden Ratio. A Golden Rectangle is one that infinitely creates new Golden Rectangles. To see that not all rectangles have this property, it is worth starting with others and dividing them into a square (based on the width) and a rectangle. Does the smaller rectangle thus created retain the length to width ratio of the original?

In fact, the Golden Rectangle generates from the (1, 2, $\sqrt{5}$) right triangle shown here. However, following that through requires knowledge of Pythagoras' Theorem and manipulation of surds which are not usually part of the curriculum at this level:



The construction procedure is:

- Rotate side length 2 anti-clockwise about the vertex at the right angle so that it lies along side length 1.
- Build the 2 x 2 square.
- Rotate the hypotenuse clockwise about its vertex on the base until it lies along side length 1.
- Complete the 'landscape' rectangle.
- The width of the smaller 'portrait' rectangle can now be calculated.
- This leads to discovering that the length to width ratio in both the 'landscape' rectangle and the smaller 'portrait' rectangle is the irrational form of the Golden Ratio as above.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Cat and Mouse

This is an intriguing game that explores the effect of board design and movement rules on who will succeed. Will the mouse get the cheese or will the cat get the mouse? What chance does the cat or the mouse have of succeeding? The students will need to work like a mathematician to answer this last question. Sufficient data will have to be collected before this probability question can be answered empirically. It is unlikely that two players will be able to run enough trials to do this on their own, so teachers can make an ongoing record on the class maths display board. Teachers might also like to place this information into a spreadsheet.

Another use of the task is to turn it into a kinaesthetic whole class investigation by quickly chalking the board design on the playground and asking a child to pretend to be the mouse. If twenty-five children each have a turn as the mouse, that's twenty-five pieces of data.

Yet another aspect is the analysis of the probabilities using both analytical and empirical approaches. Both approaches are part of the companion Maths300 lesson, which include software. The boards supplied with the task (and those in the software) were designed by primary school children. So, an extension of the task is to encourage students to design their own boards.

Chocolate Chip Cookies

Manufacturers of chocolate chip cookies are faced with the problem of how many chocolate chips to put into a mix so that the customers receive a 'guaranteed' minimum number in each cookie. It is simply not good marketing to have customers find cookies with no chocolate chips. However, achieving the minimum has to be balanced against how many chips can be afforded for each mix. The task gives students a chance to simulate this real world problem. The discussion and experimentation will help their intuitions about chance events to develop. As they collect data there will also be discussion and use of statistics such as the range of the data or the 'average' number of chips per cookie. There are extensive notes in the companion Maths300 lesson, which also includes software.

Choosing Beads

The task offers experience of sampling a mixed 'population' without replacement. Three colours of beads (A, B, C) are provided in a given ratio (15:12:4) and the initial challenge is:

> *What is the smallest number of beads you must select to be certain of having 3 the same colour?*

With eyes closed, students choose a bead, check its colour, choose again, check the colours and so on until three are the same colour.

Clearly choosing 3 beads is not enough. The three *might* be the same but we can't be *certain* that they will be. The card suggests an empirical beginning to the task and many students may need to do more experiments than the five that are suggested. As they do so, it may begin to occur to them that the chances of getting each colour is not what matters in this problem. Rather, what matters is the number already the same after each selection.

Selection 1:
Problem not solved - select again.
Selection 2:
Problem not solved - select again.
Selection 3:
Problem may be solved, but we can't be certain that it is. The possibilities are:
- 3 the same - which solves the problem
- 2 the same
- all different

Each of these last two possibilities has to be followed separately at the next step.

Selection 4a (starting with 2 the same out of 3):
- 3 the same and 1 different - which solves the problem
- 2 pairs of 2 the same
- 2 the same and 2 different others

Selection 4b (starting with all different)
- 2 the same and 2 different others

Again there are two possibilities to follow through at the next step.

Selection 5a (starting with 2 pairs of 2 the same):
- 3 the same and 1 pair different - which solves the problem
- 2 pairs of 2 the same and 1 different other

Selection 5b (starting with 2 the same and 2 different others)
- 3 the same and 2 different others - which solves the problem
- 2 pairs of 2 the same and 1 different other

Now there is only one possibility to check at the next step.

Selection 6 (starting with 2 pairs of 2 the same and 1 different other):
- 3 the same, 1 pair the same and 1 different other - which solves the problem
- 3 pairs of 2 the same

Again there is only one possibility to check at the next step.

Selection 7 (starting with 3 pairs of 2 the same)
- 3 the same and 2 pairs of different others - which solves the problem.

So the smallest number of beads that would have to be selected to be certain of three the same is 7. Did any of the students' trials take more than 7 beads?

Another question that might be explored by the more able students at this point is:

> *What is the probability that it will take to the seventh selection to get 3 the same?*

If the logic above is laid out as a tree diagram (albeit complex) and the different (and altering) probabilities connected to A, B and C at each stage are followed through, the question is answerable using a theoretical approach. However, the empirical approach opens up possibilities for whole class collection of data and thoughts about designing software to explore and record the investigation.

The problem card also opens another direction for investigation by identifying the variables in the original problem as:
- the number of colours in the container
- the number that have to be the same colour

Counter Escape

Three squares, six counters and permission to place counters into the squares in any arrangement. But where can they be placed so that all are removed in the least number of moves, given the dice roll rules governing removal from each square? This is an easy task to start, but to tackle the challenge requires recognition and discussion of the chances involved (perhaps at an intuitive level). As they play, students will develop and test hypotheses just like a mathematician. One question likely to arise is 'How much data is enough to test an hypothesis?'.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Duelling Dice

This is a fascinating task that will remind students of a playground game sometimes called 'Scissors, Rock & Paper'. Of course students can play this unusual dice game for fun and simply experience the unequal chances. However to answer the challenge on the card will require working like a mathematician. Once all the possibilities have been explored - it is a challenge in itself to decide what these are - the somewhat unexpected result is that no dice is 'best'. Whichever dice is chosen there is always one that can beat it.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

How Many Things?

The task could be seen as practical experience with ways of estimating number, so perhaps belongs in a number unit, but it also represents the type of problem faced by researchers who can only sample a population and then must make inferences about the whole population.

To strengthen the relationship of this task to the broader idea of sampling you could ask students to respond to challenges like:

> *You have been asked to find the average height of Australian females. Clearly you can't measure every person. Use your How Many Things? experience to help you design two alternative approaches to solving this problem.*

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

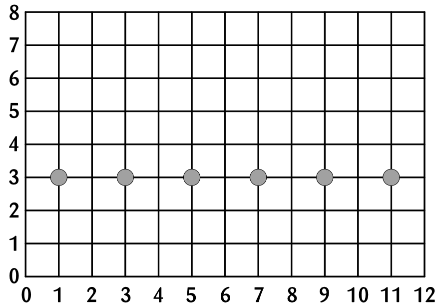
Kids On Grids

The measurement link in this task is the skill of locating objects in two dimensional space using an ordered pair of numbers. This will relate to mapping, reading street directories and creating and interpreting graphs. The task provides an opportunity to practise this skill in a practical hands-on manner. If you have a grid painted outside, the task can be modelled with the real children being real kids on grids.

However, the task is more than measurement. Kids can be placed on the grid in a random manner, but they can also be placed on the grid in a pattern. Creating a visual pattern by placing in this way is a doorway to graphical algebra. If there is a visual pattern there will always be a number pattern, and that pattern can be discovered by asking:

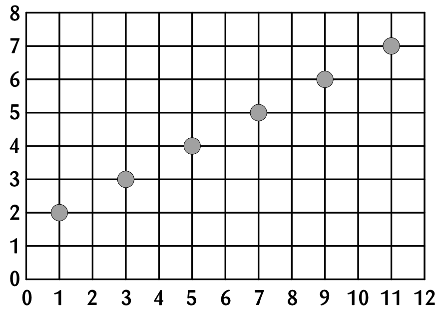
- *What is the same in all of the number pairs?*

For example if the kids were placed in this pattern:



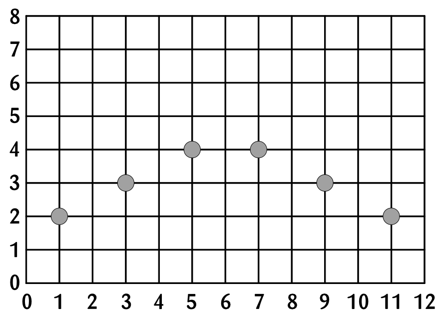
the number pairs would be (1, 3), (3, 3), (5, 3), (7, 3), (9, 3), (11, 3)
and what is the same is that the second number is always 3 ... $y = 3$.

If the kids were placed in this pattern:



the number pairs would be (1, 2), (3, 3), (5, 4), (7, 5), (9, 6), (11, 7)
and what is the same is that *twice the second number minus the first is always 3* ...
$2y - x = 3$.

If the kids were placed in this pattern:



the number pairs would be (1, 2), (3, 3), (5, 4), (7, 4), (9, 3), (11, 2) and the description this time might best be in two parts. For x less than or equal to 5, the rule is still *twice the second number minus the first is always 3*. What is the rule for x greater than or equal to 7?

Matching Faces

For most students this task is counter-intuitive. It would seem that the more faces, the more chance of making a match, but this is not so. In fact, the most likely result is exactly one match. The task encourages collecting an organised set of data in order to be able to make an hypothesis about the expected result. Further, it encourages the mathematician's question *What happens if…* because there is sufficient equipment to break the problem into smaller parts and try different numbers of faces and names.

Students might also begin to explore the theory of the problem by looking at the ways n objects can be arranged in N positions, which is equivalent to arranging the names with the faces. For example:

- 1 object in 1 position

A	Matches
a	1

- 2 objects in 2 positions

A	B	Matches
a	b	2
b	a	0

- 3 objects in 3 positions

A	B	C	Matches
a	b	c	3
a	c	b	1
b	a	c	1
b	c	a	0
c	a	b	0
c	b	a	1

- 4 objects in 4 positions

A	B	C	D	Matches
a	b	c	d	4
a	b	d	c	2
a	c	b	d	2
a	c	d	b	1
a	d	b	c	1
a	d	c	b	2
b	a	c	d	2
b	a	d	c	0
b	c	a	d	1
b	c	d	a	0
b	d	a	c	0
b	d	c	a	1
c	a	b	d	1
c	a	d	b	0
c	b	a	d	2
c	b	d	a	1
c	d	a	b	0
c	d	b	a	0
d	a	b	c	0
d	a	c	b	1
d	b	a	c	1
d	b	c	a	2
d	c	a	b	0
d	c	b	a	0

For 5 objects in 5 positions, there are 120 possibilities and these are quite a challenge to list. However the problem could readily be explored further if software were available to do the 'hack' work.

Such software could also be used to add data to the summary of the results so far which are:

Matches / Cases	0	1	2	3	4	5	Total no. of correct matches	Total no. of possible arrangements
1	0	1	-	-	-	-	1	1
2	1	0	1	-	-	-	2	2
3	2	3	0	1	-	-	6	6
4	9	8	6	0	1	-	24	24
5	?	?	?	?	?	?	120	120

Recording like this confirms that the expected number of matches is one match. However, there is also a tantalising hint of a pattern here. Some of the numbers in the 5 Row could be filled in with confidence. Students should also be able to explain why there are 0 matches in any row for the number one less than the number of objects. Perhaps they will see the suggestion of the triangle numbers in the diagonal just below the diagonal of zeros.

In summary, the task is, in the first instance, an experiment in probability. There are additional options that include exploring the reason for the counter-intuitive result by recording possibilities in a list, and further possible pattern investigations that develop from information in the listed data.

A mathematician is never finished with a problem; they are only finished for now.

Pack The Box

This spatial puzzle can be extremely frustrating, but there is no need to rush to a solution. If mathematicians are facing a real problem, they don't immediately know the answer. Many students will enjoy returning to the task time and again. The clue to its solution lies in counting cubes, which is itself the basis of measuring volume. The box can take 27 cubes. The blocks only total 24 cubes. So perhaps the problem can be best solved by deciding how to arrange the three empty spaces. Once the solution is found, the problem can be linked to **Kids On Grids** by asking:

> *Can you extend the method of identifying a point on a plane so it can be used to describe the position of the empty spaces in the solution?*

Or perhaps even further extended (if three pieces of information is considered enough data) by recognising the visual pattern in space and seeking the number pattern that links to it, in the same sense as this is described in **Kids On Grids**.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Playing With Objects

In the first instance the task encourages students to connect mathematical language on one set of cards with a feasible 'student language' description on the other. To that extent it perhaps belongs in a Space & Logic unit. However the task goes on to explore the volume relationships between the shapes by using rice as the unit. Students should keep a record of what they discover. (You may need to prepare this task by first cutting out the two sets of name cards provided.)

Surface Area With Tricube

Four tricubes can be made into a remarkable number of three dimensional objects. As the card suggests the volume of all of these is 12 cubes. It is the surface area and base area of each that varies. Students will be exploring these concepts of three dimensional measurement as they struggle to recreate each object from the given dimensions. The tricky ones of course are the ones with base area of zero. When students realise that a single tricube can be placed to form a triangular 'arch', they will be on the way to solving the last four challenges on the card.

It is possible to arrange four tricubes to make a larger tricube.

- What is the surface area and base area of this larger tricube?
- On square graph paper draw a looking down view of a Size 1 tricube. Now draw the Size 2 tricube and show how it is made up of single tricubes.
- Can you draw a Size 4 tricube and show how it is made from Size 1 tricubes? What is the surface area and base area of this Size 4 tricube?
- Can you predict the surface area and base area of the Size 8 tricube?

Technically of course the object made from 4 is not a new tricube. Rather it is one layer of new tricube which should really be 2 layers high if the proportions of the original tricube are to be maintained. That realisation opens a new investigation for which you would need a class set of Tricubes obtainable from Mathematics Centre. Trisquares, which are essentially two dimensional, are also available. See:

- http://www.mathematicscentre.com/taskcentre/resource.htm

The Frog Pond

This task combines basic arithmetic practice with intuitive probability. When the students begin to reason that both players have an equal chance of winning, the door is open to some *What if...* questions.

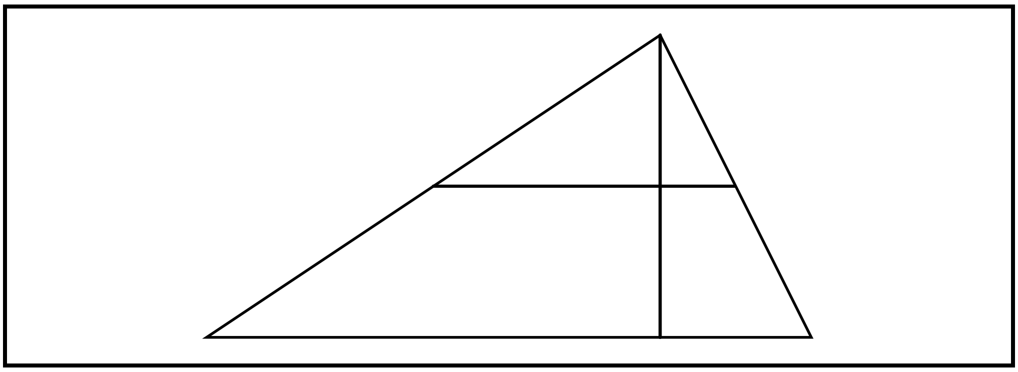
- *What if...* one player started with 5 frogs and the other started with 4?
- *What if...* the number of frogs was the same but one player had to miss a turn if they rolled 1 OUT (or IN)?
- *What if...* to win you had to empty the pond exactly?
- *What if...* you were asked to design your own game like this? How would you make it interesting and challenging?

Find more information about this task in the Task Cameo Library at:

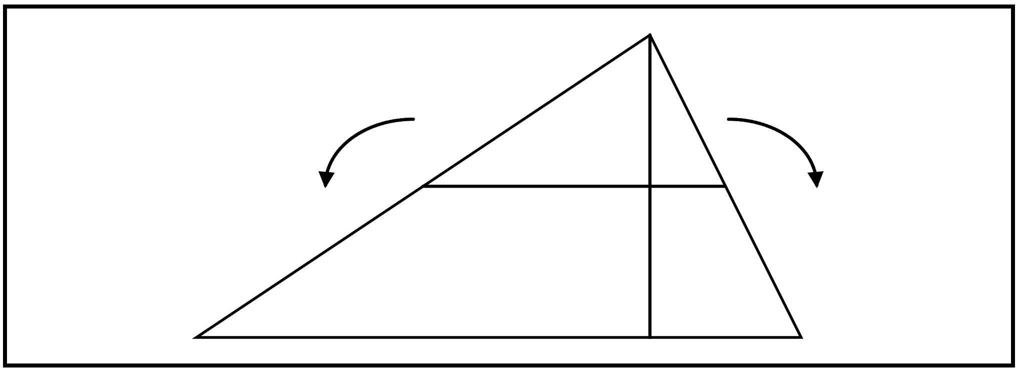
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Triangle Area

The first problem for some students will be making a triangle from the pieces. This is not necessarily an easy spatial task. In each case the pieces have been made by dropping a perpendicular from the apex, then, half way down the perpendicular, drawing a line parallel to the base.



From here, each of the top pieces rotates to make the rectangle.



The task helps students work towards the understanding that because:

- the area of the triangle and the rectangle are the same, and
- the base of the triangle is the same as the base of the rectangle, and
- the height of the rectangle is half the height of the triangle

then:

Area of a triangle
= Area of a rectangle with the same base as the triangle, but half its height.

This is an important discovery because mathematicians do know how to find the area of a rectangle. There are extensive notes about this task in the companion Maths300 lesson, *Triangle Area*, which includes software.

Triangle Perimeters

Within a broad problem solving situation, students explore perimeter at a concrete level consistent with the concept of counting equal units. The larger problem involves fixing the area of a shape (ie: fixing the number of triangle tiles) and investigating how perimeter varies.

If students are challenged to find *all* the five and six triangle shapes - a challenge that grows from Questions 2 & 3 - they would also be engaged in a serious spatial problem.

To expand the problem further in the measurement direction ask questions like:

What would happen to area formulas if the standard unit of measure had been an equilateral triangle?

In part the answer is easy. The area of an equilateral triangle would be $A = S^2$ where S is the side length. Rhombi and parallelograms might not be too difficult either, but what about other shapes? Following this thinking through mirrors what mathematicians had to do to develop our common formulas based on the square as the measurement unit. It also helps to confirm that area is not about formulas - they are the shortcut for counting. Area is about counting units of tessellated shape covering a flat space. Finding ways to do that efficiently for a range of shapes requires considerable visual thinking. It also requires application of the mathematician's strategy of relating new problems to known information.

Tube Toss

This task can be approached by experiment (empirically) and through theory. For example, Question 1 explores a win if the 3 lands face up. Students could do 100 trials to guide them in a decision about pay out, or they could consider the possibilities.

In this case the 3 has a one in two chance of occurring because the relevant outcome of the experiment is either a 3 up or not. The other discs don't matter. So, if it costs $1 per game to play and you paid out $2 for a win, you would expect no profit or loss.

It is important in the development of understanding about chance that the students do carry out the 100 trials to compare the 1 in 2 theory with the short term variation that appears in the experimental approach. For example in one set of 55 trials we carried out, 3 UP scored 25 and 3 DOWN scored 30. Would the scores have become more equal after 100 trials? Is it more instructive to keep a running record of the changing *percentage* of each possibility compared to the total number of experiments?

Question 2 investigates a win if either a 1 or a 2 or both show face up. The 3 doesn't matter, so the sample space for the experiment is:

1	2
Y	Y
Y	N
N	Y
N	N

Y = 'yes', face up and N = 'no', face down

Therefore, in 4 games the player can expect to win 3 times. So, if you charge $1 to play and expect to make neither profit nor loss, you would pay $\$1.33^1/_3$ for a win. Technically this amount isn't possible, because the smallest Australian coin is 5¢, so there is room here to discuss how the game could be promoted so actual currency could be used. Again, it is worth exploring how many trials are enough for the experimental data to fall into line with this theory.

Question 3 involves all three numbers. The player wins with a total of 3 or more.

If 0 indicates all discs face down, the sample space for this experiment is:

0	1	2	3	1+2	1+3	2+3	1+2+3
N	N	N	Y	Y	Y	Y	Y

Therefore, in 8 games the player can expect to win 5 times. So, if you charge $1 to play and expect to make neither profit nor loss, you would pay $1.60 cents for a win.

An extension to the task is to ask students to design their own rules for a win and explore the break even payments.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Where Is The Rectangle?

In essence this task applies the mathematician's strategy of working backwards. Taking the example on the card, if you know the area of the rectangle is 12, the rectangle could still be 3 x 4 or 6 x 2. At least, on this geoboard, it can't be 12 x 1. Of course this thinking doesn't come into play until you find a starting corner by 'guess, check and improve'.

The task involves multiple aspects of mathematics:

- Recognition and application of the ordered pair convention (and therefore it links to the task **Kids On Grids**).
- Recording data.
- Application of problem solving strategies.
- Making and testing hypotheses.
- Factors, multiples and primes.
- Area and length measurement.

It is also a task students enjoy returning to so they can try the challenge again.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Win At The Fair

Here is another fairground game situation like **Tube Toss**. This time though the designer of the game hasn't considered the chances involved. It becomes clear (usually before 100 trials) that the designer is paying out more than is being received. The challenge then is to 'fix' the game to either break even, or make a reasonable profit. The recording sheet you may need for this task is at the end of this manual. There are extensive notes in the companion Maths300 lesson, which also includes software that allows students to experiment with 'fixes'.

Lesson Comments

- These comments introduce you to each Maths300 lesson. The complete plan is easily accessed through the lesson library available to members at:
  http://www.maths300.com
  where they are listed alphabetically by lesson name.

Angle Estimation

This lesson takes students outside to estimate angles made physically by tying cords to a pole and rotating. When students first make an estimate of this sort they are not just testing their knowledge of the relative sizes of angles. They are also making a personal commitment to the mini-challenge which stimulates a desire to 'know the right answer'. In this way, teaching craft and content are bound together in a lesson that uses physical involvement and software to support the learning.

The lesson involves recognising an angle as the shape made by two line segments and includes both informal and formal measurement of the amount of turning between the arms of the angle. The lesson also makes a great partner to Lesson 33, Fraction Estimation, from the Maths300 site through Option 3 of the *Fraction Estimation* software.

Area of a Circle

A circle has an area of 40 sq. cm. What is its radius?

This typical school mathematics exercise is usually done by solving an equation, almost as a recipe. However, a branch of mathematics known as 'Discrete Mathematics' offers students a quite different approach to working with such problems. The approach uses a first principles iterative 'guess and check' method with a calculator, rather than the traditional algebraic method. It also visually links the area of the circle to the area of its 'corner square' so when the students are calculating they can 'see' what the numbers mean.

Comments from trial schools include:

- *This is one of the most satisfying lessons I know. The reason I say this is the considerable success the lesson offers all those students who usually struggle with the abstractions of algebraic processes.*
- *Everyone understood and could articulate what they were doing and why.*
- *Everyone felt successful.*
- *Doesn't take too long.*

Area of a Triangle

If you know how to calculate the area of a rectangle, then the area of a triangle is easy, because all triangles can be turned into rectangles!

This lesson is firmly aimed at the concept or first principles level, the challenge being to convert an irregular shape, namely a triangle, into the more familiar and regular rectangle. In doing this, emphasis is given to visualisation skills, concrete materials and the graphic animation capacity of computer software. The lesson does require the printing and cutting of some printed material for each group. Masters are supplied.

Beetle Game

In this lesson a popular old parlour game gains new life as a probability based investigation. Two players race to complete their beetle by rolling dice. Each must first roll a six to get a body before they can start to build the other parts of the beetle. Great fun to play, but consider the mathematical investigations that can grow from it. What is the average game length (number of rolls) to complete the beetle? What is the least number of rolls? What is the greatest number? These questions can be attacked through first hand data (empirically), through computer simulation and theoretically, thereby opening the learning door for a broad range of students.

If you decide to use the very colourful Beetle provided in the lesson you will need to access a colour printer in advance and may also need to arrange for lamination. It is suggested on the Beetle Board that students can do the cutting to prepare the game and the pieces can be packed in press-seal bags to make a class set. Of course, students can play the game simply by drawing the Beetle on scrap paper.

> *I found using the cut-outs for the demonstration and then letting the students use them for the first few games was very effective. Later they just drew the beetles on paper.*

Software is provided to extend the investigation.

Birth Month Paradox

Five people meet at a party; what is the chance that they share the same birth month? Simple logic might argue that 5 is less than half of the 12 months available, so the chances are less than 50%. The surprising answer is that the chances are about 62%, significantly more than 50%. This counter-intuitive result is the feature of an investigation which involves estimation, personal data, mathematical modelling and a computer simulation.

The problem is a variation of a classic problem usually studied in senior probability classes. The classic problem relates to birth days not birth months. For example, there are 36 people in a room at a party, what is the chance that two of them share exactly the same birth day (eg: July 19th)? Again, simple reasoning suggests that 36 is about 1/10 of the 365 days in a year, so the probability or chance should be 1 in 10? The surprising result is that the real chance is over 80% likely.

The usual solution method for this problem is application of a known formula. However, beginning with birth month rather than birth day and presenting an investigation rather than an exercise, can result in an interesting learning experience for many year levels.

This lesson plan is supported by software.

Cat and Mouse

A simple appealing game about a mouse moving around a board. Depending on dice rolls, the mouse either 'wins the cheese' or gets 'sent to the cat'. The game turns into a rich investigation involving many probability and data concepts. Then a computer simulation allows the investigation to be extended to other boards and even deeper probability analysis.

Chocolate Chip Cookies

In this lesson the challenge of preparing chocolate chip cookies with at least a minimum number of yummy chocolate chips is unashamedly used to encourage construction of a simulation, and explore the uses of the statistics which describe and limit it. Apart from mathematical outcomes there is the important realisation for students of the way mathematics is used in manufacturing industries.

Software supports the exploration, and you are strongly encouraged to use real cookies as the stimulus for the lesson. You may even want to do some research in your own time first on the brand that offers the most useful cookies for the classroom experiment. Easily definable chunky ones seem to make the search for chips more efficient for the students.

Circumference of a Circle

This lesson elegantly lifts learning about the circumference of a circle out of the text book page and makes it a real experience. Students develop a deeper understanding of the formula for the circumference, and for the meanings of related geometric language, by being physically involved in stepping out circles which they create. They discover that the circumference is about 6 times the radius, which makes 'pi' more meaningful and less 'magic'.

Counter Escape

We have three 'boxes' (A, B, C) and 3 counters. Any number of counters (up to 3) can be placed in any of the boxes. They are removed from a box by the roll of a dice according to the rules that set up unequal probabilities.

> *What is the best way to place the counters to remove them all in the least number of rolls?*

Simple to state, simple to set up yet suitable for both primary and secondary students. The best placement strategy is debatable and the computer simulation allows students to investigate the problem empirically. More advanced students can tackle the theoretical analysis.

Country Maps

People carry visual mental maps of their own country. Such maps are constructed from personal experiences over a long period of time. But how accurate are they? This lesson encourages students to peer into these personal visual constructs and to check them against reality. In doing so, the concepts of ratio and area are explored. The use of estimation, visual learning, problem solving, small group discussion and links to geography all act to make this a powerful and interesting lesson.

From a professional development point of view, the lesson treatment contrasts strongly with 'text-book' treatment of ratios. It is in analysing these contrasts that significant debate about effective teaching techniques can develop.

The lesson is written around an Australian context - Victoria is given the area of one unit, how big are the other states? - but can be easily adapted for any regional area or country. Maps are provided for both Australia and the United States. These can be printed for the students and as overhead transparencies for the teacher.

Duelling Dice

Two players have four unusual dice - the faces have some numbers that are non-standard for a dice and on some of them numbers are repeated. Teachers will need to prepare these dice in advance from wooden cubes.

Each player selects one of the colours. Then both players roll their dice - the higher number wins a point. Ignoring draws, the first to reach 7 points wins the game.

- Which is the better dice to choose?

This task is a classic 'tip of an iceberg'. Exploring any two dice provides an unexpected and counter intuitive result, but what if three dice were used, then which is the best? ... or ... What if all four dice are rolled?

These additional problems open up an even richer array of possibilities, which can all be pursued through:

- playing the game, or
- computer simulation, or
- theoretical analysis.

Software is provided to extend the investigation.

Estimation Walks

The lesson is built around each pair of students walking from a starting point to a spot they estimate is a given distance away. The estimate is checked with a tape. The lesson produces data about how close the estimates are to the measured distances. The lesson plan explores ways of representing the data and introduces or refreshes the statistic of average (or mean). There is a strong link to work on decimals both in the calculation of the average and in the representation of lengths as decimal fractions of a metre.

Highest Number

A very rich lesson which teachers report engages students of all ability levels. It starts out as a simple, well-known place value game with cards, but develops into many other learning outcomes, including statistics and probability. The collection of whole class data is a highlight and includes the use of stem and leaf graphs. As the lesson develops, there are several opportunities to enhance the lesson further through the software simulation.

Of all the ten features of the lesson suggested by teachers as being vital to its development (see Overview page on site), the one with the strongest feedback is that the lesson has a non-threatening quality. This is perhaps because it is easy to get started and there are multiple levels of success. Teachers report using the lesson successfully from Year 2 to Year 12.

Problem Dice

Problem Dice focuses on the concept of a fair game and it offers opportunity to explore the students' possible misconceptions of fairness. Pairs take turns to roll two dice and find the difference. The rules of the opening game are Player A wins a point if the difference is 0, 1, 2 and Player B wins if the difference is 3, 4, 5. Students may expect this to be a fair situation because each player 'owns' three numbers. Playing several games suggests otherwise and this leads to investigating further to find a fair game. The investigation is supported by software.

Pulse Rates

Students learn how to measure pulse, learn a recognised fitness test and use the test to collect data that builds a picture of their fitness and the fitness of others. Collecting data, organising it graphically and learning to interpret graphs are the mathematical objectives. The lesson is an example of how a text book exercise can be tweaked to expose the Working Mathematically process. The lesson also offers an alternative assessment approach and the opportunity to use spreadsheets.

Temperature Graphs

At first glance, this lesson doesn't seem dramatically different from traditional practice. However, it does contain several elegant and significant features which contrast somewhat with that traditional approach.

Students in small groups are given a number of graphs representing the average temperatures of various cities over a one-year period. They are challenged as a group to match each graph to a city and to justify their choice. The lesson actively uses a relevant context (regional geography), problem solving and co-operative group work to develop graph reading skills. This approach directly contrasts with a traditional 'text-book' approach.

Extensions using computer support allow the teacher and student to explore a range of similar challenges. The software has an option to enter data (which would have to be obtained from the web, or the weather bureau) for cities relevant to the local country or a country being studied in geography.

This Goes With This

The lesson uses the students' own survey data to demonstrate links between vulgar fractions and percent, and strip graphs, circle graphs and pie charts. The simplicity of the demonstration often results in an *Oh is that how they figure it out?* response. Rarely is there as powerful an illustration which makes important mathematical concepts and their integration so clear and understandable. The meaning of percentage when applied to a pie chart becomes a highly visual moment.

If the data used is the students' personal contribution, then the students own bodies can represent their piece of data. They can become that data on a bar graph, strip graph or circle graph and thereby are kinaesthetically involved in the lesson.

Win At The Fair

The lesson begins as a 'fun' game, but once sufficient data has been collected it becomes obvious that the 'owner' of the game is losing money. The students 'help out' the owner by changing the board or the rules to find a game that makes a fair profit. The lesson can be approached empirically, but there is also plenty of room for theoretical investigation. The game board is vital in this lesson, so you will need to allow time to print copies. Some schools print onto card, laminate and make a class set of resources that can be reused each year.

Part 3:
Value
Adding

The Poster Problem Clinic

Maths With Attitude kits offer several models for building a Working Mathematically curriculum around tasks. Each kit uses a different model, so across the range of 16 kits, teachers' professional learning continues and students experience variety. The Poster Problem Clinic is an additional model. It can be used to lead students into working with tasks, or it can be used in a briefer form as a opening component of each task session.

> *I was apprehensive about using tasks when it seemed such a different way of working. I felt my children had little or no experience of problem solving and I wanted to prepare them to think more deeply. The Clinic proved a perfect way in.*

Careful thought needs to be given to management in such lessons. One approach to getting the class started on the tasks and giving it a sense of direction and purpose is to start with a whole class problem. Usually this is displayed on a poster that all can see, perhaps in a Maths Corner. Another approach is to print a copy for each person. A Poster Problem Clinic fosters class discussion and thought about problem solving strategies.

Starting the lesson this way also means that just prior to liberating the students into the task session, they are all together to allow the teacher to make any short, general observations about classroom organisation, or to celebrate any problem solving ideas that have arisen.

One teacher describes the session like this:

> *I like starting with a class problem - for just a few minutes - it focuses the class attention, and often allows me to introduce a particular strategy that is new or needs emphasis.*

It only takes a short time to introduce a poster and get some initial ideas going. The class discussion develops a way of thinking. It allows class members to hear, and learn from their peers, about problem solving strategies that work for them.

> *If we don't collectively solve the problem in 5 minutes, I will leave the problem 'hanging' and it gives a purpose to the class review session at the end.*
> *Sometimes I require everyone to work out and write down their solution to the whole class problem. The staggered finishing time for this allows me to get organised and help students get started on tasks without being besieged.*
> *I try to never interrupt the task session, but all pupils know we have a five minute review session at the end to allow them to comment on such things as an activity they particularly liked. We often close then with an agreed answer to our whole class problem.*

A Clinic in Action

The aims of the regular clinic are:

- to provide children with the opportunity to learn a variety of strategies
- to familiarise children with a process for solving problems.

The following example illustrates a structure which many teachers have found successful when running a clinic.

Preparation

For each session teachers need:

- a Strategy Board as below
- a How To Solve A Problem chart as below
- to choose a suitable problem and prepare it as a poster
- to organise children into groups of two or three.

The Strategy Board can be prepared in advance as a reference for the children, or may be developed *with* the children as they explore problem solving and suggest their own versions of the strategies.

The problem can be chosen from

- a book
- the task collection
- prepared collections such as Professor Morris Puzzles which can be viewed at: http://www.mathematicscentre.com/taskcentre/resource.htm#profmorr

The example which follows is from the task collection. The teacher copied it onto a large sheet of paper and asked some children to illustrate it. *The teacher also changed the number of sheep to sixty* to make the poster a little different from the one in the task collection.

The Strategy Board and the How To Solve A Problem chart can be used in any maths activity and are frequently referred to in Maths300 lessons.

The Clinic

The poster used for this example session is:

> Eric the Sheep is lining up to be shorn before the hot summer ahead. There are sixty [60] sheep in front of him. Eric can't be bothered waiting in the queue properly, so he decides to sneak towards the front.
>
> Every time one [1] sheep is taken to be shorn, Eric <u>then</u> sneaks past two [2] sheep. How many sheep will be shorn <u>before</u> Eric?

This Poster Problem Clinic approach is also extensively explored in Maths300 Lesson 14, *The Farmer's Puzzle*.

Strategy Board

DO I KNOW A SIMILAR PROBLEM?	ACT IT OUT
GUESS, CHECK AND IMPROVE	DRAW A PICTURE OR GRAPH
TRY A SIMPLER PROBLEM	MAKE A MODEL
WRITE AN EQUATION	LOOK FOR A PATTERN
MAKE A LIST OR TABLE	TRY ALL POSSIBILITIES
WORK BACKWARDS	SEEK AN EXCEPTION
BREAK INTO SMALLER PARTS	...

How To Solve A Problem

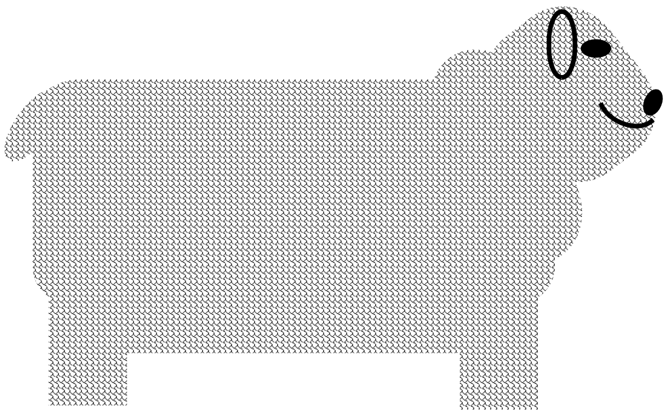
SEE & UNDERSTAND	Do I understand what the problem is asking? Discuss
PLANNING	Select a strategy from the board. Plan how you intend solving the problem.
DOING IT	Try out your idea.
CHECK IT	Did it work out? If so reflect on the activity. If not, go back to step one.

Step 1

- Tell the children that we are at Stage 1 of our four stage plan ... **See & Understand** ... Point to it! Read the problem with the class. Discuss the problem and clarify any misunderstandings.
- If children do not clearly understand what the problem is asking, they will not cope with the next stage. A good way of finding out if a child understands a problem is for her/him to retell it.
- Allow time for questions - approximately 3 to 5 minutes.

Step 2

- Tell the children that we are at Stage 2 of our four stage plan ... **Planning**. In their groups children select one or more strategies from the Strategy Board and discuss/organise how to go about solving the problem.
- Without guidance, children will often skip this step and go straight to Doing It. It is vital to emphasise that this stage is simply planning, not solving, the problem.
- After about 3 minutes, ask the children to share their plans.



Plan 1

Well we're drawing a picture and sort of making a model.

Can you give me more information please Brigid?

We're putting 60 crosses on our paper for sheep and the pen top will be Eric. Then Claire will circle one from that end, and I will pass two crosses with my pen top.

Plan 2

Our strategy is Guess and Check.

That's good Nick, but how are you going to check your guess?

Oh, we're making a model.

Go on …

> *John's getting MAB smalls to be sheep and I'm getting a domino to be Eric and the chalk box to be the shed for shearing.*

Plan 3

> *We are doing it for 3 sheep then 4 sheep then 5 sheep and so on. Later we will look at 60.*

Great so you are going to try a simpler problem, make a table and look for a pattern.

This sharing of strategies is invaluable as it provides children who would normally feel lost in this type of activity with an opportunity to listen to their peers and make sense out of strategy selection. Note that such children are not given the answer. Rather they are assisted with understanding the power of selecting and applying strategies.

Step 3

- Tell the children that we are at Stage 3 of our four stage plan … **Doing It**. Children collect what they need and carry out their plan.

Step 4

- Tell the children that we are at Stage 4 of our four stage plan … **Check It**. Come together as a class for groups to share their findings. Again emphasis is on strategies.

> *We used the drawing strategy, but we changed while we were doing it because we saw a pattern.*

So Jake, you used the Look For A Pattern strategy. What was it?

> *We found that when Eric passed 10 sheep, 5 had been shorn, so 20 sheep meant 10 had been shorn … and that means when Eric passes 40 sheep, 20 were shorn and that makes the 60 altogether.*

Great Jake. How would you work out the answer for 59 sheep or 62 sheep?

Sharing time is also a good opportunity to add in a strategy which no one may have used. For example:

> *Maybe we could've used the Number Sentence strategy, ie: 1 sheep goes to be shorn and Eric passes two sheep. That's 3 sheep, so perhaps, 60 divided into groups of 3, or 60 ÷ 3 gives the answer.*

Round off the lesson by referring to the Working Mathematically chart. There will be many opportunities to compliment the students on working like a mathematician.

Curriculum Planning Stories

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

In more than a decade of using tasks and many years of using the detailed whole class lessons of Maths300, teachers have developed several models for integrating tasks and whole class lessons. Some of those stories are retold here. Others can be found at:

- http://www.mathematicscentre.com/taskcentre/plans.htm

Story 1: Threading

> *Educational research caused me a dilemma. It tells us that students construct their own learning and that this process takes time. My understanding of the history of mathematics told me that certain concepts, such as place value and fractions, took thousands of years for mathematicians to understand. The dilemma was being faced with a textbook that expected students to 'get it' in a concentrated one, two or three week block of work and then usually not revisit the topic again until the next academic year.*

A Working Mathematically curriculum reflects the need to provide time to learn in a supportive, non-threatening environment and...

> *When I was involved in a Calculating Changes PD program I realised that:*
>
> - *choosing rich and revisitable activities, which are familiar in structure but fresh in challenge each time they are used, and*
> - *threading them through the curriculum over weeks for a small amount of time in each of several lessons per week*
>
> *resulted in deeper learning, especially when partnered with purposeful discussion and recording.*

Calculating Changes:

- http://www.mathematicscentre.com/calchange

Story 2: Your turn

Some teachers are making extensive use of a partnership between the whole class lessons of Maths300 and small group work with the tasks. Setting aside a lesson for using the tasks in the way they were originally designed now seems to have more meaning, as indicated by this teacher's story:

> *When I was thinking about helping students learn to work like a mathematician, my mind drifted to my daughter learning to drive. She*

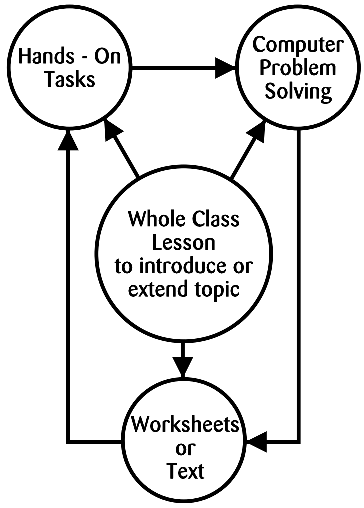
needed me to model how to do it and then she needed lots of opportunity to try it for herself.

That's when the idea clicked of using the Maths300 lessons as a model and the tasks as a chance for the students to have their turn to be a mathematician.

The Maths300 lessons illustrate how other teachers have modelled the process, so I felt I could do it too. Now the process is always on display on the wall or pasted inside the student's journal.

A session just using the tasks had seemed a bit like play time before this. Now I see it as an integral part of learning to work mathematically.

Story 3: Mixed Media



It was our staff discussion on Gardner's theory of Multiple Intelligences that led us into creating mixed media units. That and the access you have provided to tasks and Maths300 software.

We felt challenged to integrate these resources into our syllabus. There was really no excuse for a text book diet that favours the formal learners. We now often use four different modes of learning in the work station structure shown. It can be easily managed by one teacher, but it is better when we plan and execute it together.

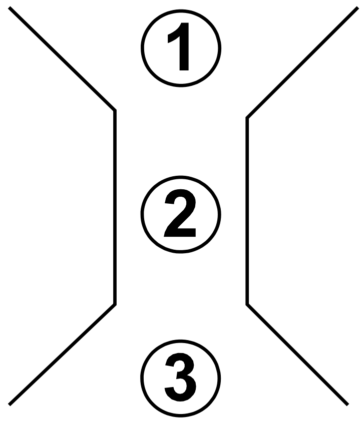
Story 4: Replacement Unit

We started meeting with the secondary school maths teachers to try to make transition between systems easier for the students. After considerable discussion we contracted a consultant who suggested that school might look too much the same across the transition when the students were hoping for something new. On the other hand our experience suggested that there needed to be some consistency in the way teachers worked.

We decided to 'bite the bullet' and try a hands-on problem solving unit in one strand. We selected two menus of twenty hands-on tasks, one for the primary and one for the secondary, that became the core of the unit. We deliberately overlapped some tasks that we knew were very rich and added some new ones for the high school.

Class lessons and investigation sheets were used to extend the tasks, within a three week model.

It is important to note that although these teachers structured a 3 week unit for the students, they strongly advised an additional *Week Zero* for staff preparation. The units came to be called Replacement Units.



Week Zero - Planning

Staff familiarise themselves with the material and jointly plan the unit. This is not a model that can be 'planned on the way to class'.

> *Getting together turned out to be great professional development for our group.*

Week 1 - Introduction

Students explore the 20 tasks listed on a printed menu:

- students explore the tip of the task, as on the card
- students move from task to task following teacher questioning that suggests there is more to the task than the tip
- in discussion with students, teachers gather informal assessment information that guides lesson planning for the following week.

> *We gave the kids an 'encouragement talk' first about joining us in an experiment in ways of learning maths and then gave out the tasks. The response was intelligent and there was quite a buzz in the room.*

Week 2 - Formalisation

> *It was good for both us and the students that the lessons in this week were a bit more traditional. However, they weren't text book based. We used whole class lessons based on the tasks they had been exploring and taught the Working Mathematically process, content and report writing.*
>
> *Assessment was via standard teacher-designed tests, quizzes and homework.*

Week 3 - Investigations

> *We were most delighted with Week 3. Each student chose one task from the menu and carried out an in-depth investigation into the iceberg guided by an investigation sheet. They had to publish a report of their investigation and we were quite surprised at the outcomes. It was clear that the first two weeks had lifted the image of mathematics from 'boring repetition' to a higher level of intellectual activity.*

Story 5: Curriculum shift

> *I think our school was like many others. The syllabus pattern was 10 units of three weeks each through the year. We had drifted into that through a text book driven curriculum and we knew the students weren't responding.*

Our consultant suggested that there was sameness about the intellectual demands of this approach which gave the impression that maths was the pursuit of skills. We agreed to select two deeper investigations to add to each unit. It took some time and considerable commitment, but we know that we have now made a curriculum shift. We are more satisfied and so are the students.

The principles guiding this shift were:

- Agree

  The 20 particular investigations for the year are agreed to by all teachers. If, for example, Cube Nets *is decided as one of these, then all the teachers are committed to present this within its unit.*

- Publish

  The investigations are written into the published syllabus. Students and parents are made aware of their existence and expect them to occur.

- Commit

  Once agreed, teachers are required to present the chosen investigations. They are not a negotiable 'extra'.

- Value

  The investigations each illustrate an explicit form of the Working Mathematically process. This is promoted to students, constantly referenced and valued.

- Assess

  The process provides students with scaffolding for their written reports and is also known by them as the criteria for assessment. (See next page.)

- Report

  The assessment component features within the school reporting structure.

A Final Comment

Including investigations has become policy.

Why? Because to not do so is to offer a diminished learning experience.

The investigative process ranks equally with skill development and needs to be planned for, delivered, assessed and reported.

Perhaps most of all we are grateful to our consultant because he was prepared to begin where we were. We never felt as if we had to throw out the baby and the bath water.

Assessment

Our attitude is:

stimulated students are creative and love to learn

Regardless of the way you use your **Maths With Attitude** resource, a variety of procedures can be employed to assess this learning.

Where these assessment procedures are applied to task sessions and involve written responses from students, teachers will need to be careful that the writing does not become too onerous. Students who get bogged down in doing the writing may lose interest in doing the tasks.

In addition to the ideas below, useful references are:

- http://www.mathematicscentre.com/taskcentre/assess.htm
- http://www.mathematicscentre.com/taskcentre/report.htm

The first offers several methods of assessment with examples and the second is a detailed lesson plan to support students to prepare a Maths Report.

Journal Writing

Journal writing is a way of determining whether the task or lesson has been understood by the student. The pupil can comment on such things as:

- What I learned in this task.
- What strategies I/we tried (refer to the Strategy Board).
- What went wrong.
- How I/we fixed it.
- Jottings - ie: any special thoughts or observations

Some teachers may prefer to have the page folded vertically, so that children's reflective thoughts can be recorded adjacent to critical working.

Assessment Form

An assessment form uses questions to help students reflect upon specific issues related to a specific task.

Anecdotal Records

Some teachers keep ongoing records about how students are tackling the tasks. These include jottings on whether students were showing initiative, whether they were working co-operatively, whether they could explain ideas clearly, whether they showed perseverance.

Checklists

A simple approach is to create a checklist based on the Working Mathematically process. Teachers might fill it in following questioning of individuals, or the students may fill it in and add comments appropriately.

Pupil Self-Reflection

Many theorists value and promote metacognition, the notion that learning is more permanent if pupils deliberately and consciously analyse their own learning. The deliberate teaching strategy of oral questioning and the way pupils record their work is an attempt to manifest this philosophy in action. The alternative is the tempting 'butterfly' approach which is to madly do as many activities as possible, mostly superficially, in the mistaken belief that quantity equates to quality.

> *I had to work quite hard to overcome previously entrenched habits of just getting the answer, any answer, and moving on to the next task.*

Thinking about *what* was learned *how* it was learned consolidates and adds to the learning.

When it follows an extensive whole class investigation, a reflection lesson such as this helps to shift entrenched approaches to mathematics learning. It is also an important component of the assessment process. On the one hand it gives you a lot of real data to assist your assessment. On the other it prepares the students for any formal assessment which you may choose to round off a unit.

Introduction

Ask students to recall what was done during the unit or lesson by asking a few individuals to say what *they* did, eg:

> *What did you do or learn that was new?*
> *What can you now do/understand that is new?*
> *What do you know now that you didn't know 1 (2, 3, ...) lesson ago?*

Continuing Discussion

Get a few ideas from the first students you ask, then:

- organise 5 -10 minute buzz groups of three or four students to chat together with one person to act as a recorder. These groups address the same questions as above.
- have a reporting session, with the recorder from each group telling the class about the group's ideas.

Student comments could be recorded on the board, perhaps in three groups.

Ideas & Facts	Maths Skills	Process (learning) Skills

If you need more questions to probe deeper and encourage more thought about process, try the following:

> *What new things did you do that were part of* how *you learned?*
> *Who uses this kind of knowledge and skill in their work?*

Student Recording

Hand out the REFLECTION sheet (next page) and ask students to write their own reflection about what they did, based on the ideas shared by the class. Collect these for interest and, possibly, assessment information.

REFLECTION

me looking at me learning

NAME: CLASS:

Working With Parents

Balancing Problem Solving with Basic Skill Practice

Many schools find that parents respond well to an evening where they have an opportunity to work with the tasks and perhaps work a task together as a 'whole class'. Resourced by the materials in this kit, teachers often feel quite confident to run these practical sessions. Comments from parents like:

> *I wish I had learnt maths like this.*

are very supportive. Letting students 'host' the evening is an additional benefit to the home/school relationship.

The 4½ Minute Talk

Charles Lovitt has considerable experience working with parents and has developed a crisp, parent-friendly talk which he shares below. Many others have used it verbatim with great success.

Why the Four and a Half Minute Talk?

When talking with parents about Problem Solving or the meaning of the term Working Mathematically, I have often found myself in the position, after having promoted inquiry based or investigative learning, of the parents saying:

> *Well - that's all very well - BUT ...*

at which stage they often express their concern for basic (meaning arithmetic) skill development.

The weakness of my previous attempts has been that I have been unable to reassure parents that problem solving does not mean sacrificing our belief in the virtues of such basic skill development.

One of the unfortunate perceptions about problem solving is that if a student is engaged in it, then somehow they are not doing, or it may be at the expense of, important skill based work.

This Four and a Half Minute Talk to parents is an attempt to express my belief that basic skill practice and problem solving development can be closely intertwined and not seen as in some way mutually exclusive.

(I'm still somewhat uncomfortable using the expression 'basic skills' in the above way as I am certain that some thinking, reasoning, strategy and communication skills are also 'basic'.)

Another aspect of the following 'talk' is that, as teachers put more emphasis on including investigative problem solving into their courses, a question arises about the source of suitable tasks.

This talk argues that we can learn to create them for ourselves by 'tweaking' the closed tasks that heavily populate out existing text exercises, and hence not be dependent on external suppliers. (Even better if students begin to create such opportunities for themselves.)

The Talk

In preparation, write the following graphic on the board:

CLOSED	OPEN	EXTENDED INVESTIGATION
		How many solutions exist? How do you know you have found them all?

I would like to show you what teachers are beginning to do to achieve some of the thinking and reasoning and communication skills we hope students will develop. I would like to show you three examples.

Example One: *6 + 5 = ?*

I write this question under the 'closed' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 ___		How many solutions exist? How do you know you have found them all?

And I ask:

What is the answer to this question?

I then explain that:

We often ask students many closed questions such as 6 + 5 = ?

The only response the students can tell us is "The answer is 11." ... and as a reward for getting it correct we ask another twenty questions just like it.

What some teachers are doing is trying to *tweak* the question and ask it a different way, for example:

I have two counting numbers that add to 11. What might the numbers be?

[Counting numbers = positive whole numbers including zero]

I write this under the ‘open’ label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 ___	? + ? 11	How many solutions exist? How do you know you have found them all?

What is the answer to the question now?

At this stage it becomes apparent there are several solutions:

The question is now a bit more open than it was before, allowing students to tell you things like 8 + 3, or 10 + 1, or 11 + 0 etc.

Let's see what happens if the teacher 'tweaks' it even further with the investigative challenge *or* extended investigation question:

How many solutions are there altogether?

and more importantly, and with greater emphasis on the second question:

How could you convince someone else that you have found them all?

Now the original question is definitely different - it still involves the skills of addition but now also involves thinking, reasoning and problem solving skills, strategy development and particularly communication skills.

Young students will soon tell you the answer is ‘six different ones’, but they must also confront the communication and reasoning challenge of convincing you that there are only six and no more.

Example Two: *Finding Averages*

Again, as I go through this example, I write it into the diagram on the board in the relevant sections.

The CLOSED question is: *11, 12, 13 - find the average*

Tweaking this makes it an OPEN question and it becomes:

I have three counting numbers whose average is 12. What might the numbers be?

Students will often say:

10, 12, 14 … or 9, 12, 15 … or even 12, 12, 12

After realising there are many answers, you can tweak it some more and turn it into an EXTENDED INVESTIGATION:

How many solutions exist? … AND …

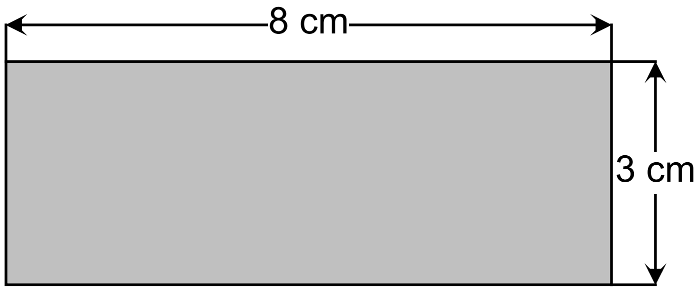
How do you know you have found them all?

Now the question is of a quite different nature. It still involves the arithmetic skill, but has something else as well - and that something else is the thinking, reasoning and communication skills necessary to find all of the combinations and convince someone else that you have done so.

By the time a student announces, with confidence, there are 127 different ways (which there are) that student will have engaged in all of these aspects, ie: the skill of calculating averages, (and some combination number theory) as well as significant strategy and reasoning experiences.

Example Three: *Finding the Area of a Rectangle*

A typical CLOSED question is:



Find the area. Find the perimeter.

The OPEN question is:

A rectangle has 24 squares inside:

What might its length and width be?

What might its perimeter be?

The EXTENDED INVESTIGATION version is:

Given they are whole number lengths, how many different rectangles are there? … AND …

How do you know you have found them all?

In summary, mathematics teachers are trying to convert *some* (not all) of the many closed questions that populate our courses and 'push' them towards the investigation direction. In doing so, we keep the skills we obviously value, but also activate the thinking, reasoning and justification skills we hope students will also develop.

This sequence of three examples hopefully shows two major features:

- That skills and problem solving can 'live alongside each other' and be developed concurrently.
- That the process of creating open-ended investigations can be done by anyone - just go to any source of closed questions and try 'tweaking' them as above. If it only worked for one question per page it would still provide a very large supply of investigations.

In terms of the effect of the talk on parents, I have usually found them to be reassured that we are not compromising important skill development (and nor do we want to). The only debate then becomes whether the additional skills of thinking, reasoning and communication are also desirable.

I've also been told that parents appreciate it because of the essential simplicity of the examples - no complicated theoretical jargon.

A Working Mathematically Curriculum

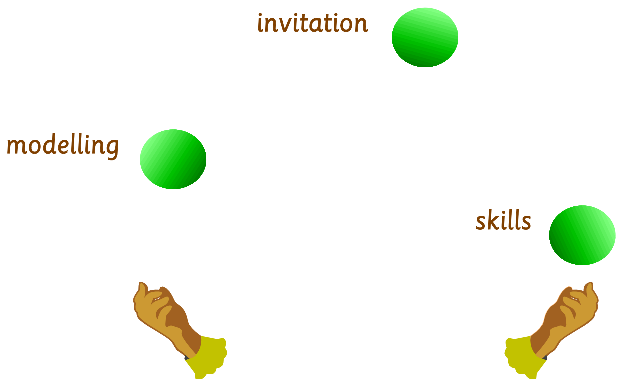
An Investigative Approach to Learning

The aim of a Working Mathematically curriculum is to help students learn to work like a mathematician. This process is detailed earlier (Page 8) in a one page document which becomes central to such a curriculum.

The change of emphasis brings a change of direction which *implies and requires* a balance between:

- the process of being a mathematician, and
- the development of skills needed to be a *successful* mathematician.

This journey is not two paths. It is one path made of two interwoven threads in the same way as DNA, the building block of life, is one compound made of two interwoven coils. To achieve a Working Mathematically curriculum teachers need to balance three components.



The task component of **Maths With Attitude** offers each pair of students an invitation to work like a mathematician.

The Maths300 component of **Maths With Attitude** assists teachers to model working like a mathematician.

Content skills are developed in context. They *are* important, but it is the application of skills within the process of Working Mathematically that has developed, and is developing, the human community's mathematical knowledge.

A focus for the Working Mathematically teacher is to help students develop mathematical skills in the context of problem posing and solving.

> *We are all 'born' with the same size mathematical toolbox, in the same way as I can own the same size toolbox as my motor mechanic. However, my motor mechanic has many more tools in her box than I and she has had more experience than I using them in context. Someone has helped her learn to use those tools while crawling under a car.*

Afzal Ahmed, Professor of Mathematics at Chichester, UK, once quipped:

> *If teachers of mathematics had to teach soccer, they would start off with a lesson on kicking the ball, follow it with lessons on trapping the ball and end with a lesson on heading the ball. At no time would they play a game of football.*

Such is not the case when teaching a Working Mathematically curriculum.

Elements of a Working Mathematically Curriculum

Working Mathematically is a K - 12 experience offering a balanced curriculum structured around the components below.

Hands-on Problem Solving Play

Mathematicians don't know the answer to a problem when they start it. If they did, it wouldn't be a problem. They have to play around with it. Each task invites students to play with mathematics 'like a mathematician'.

Skill Development

A mathematician needs skills to solve problems. Many teachers find it makes sense to students to place skill practice in the context of *Toolbox Lessons* which *help us better use the Working Mathematically Process* (Page 8).

Focus on Process

This is what mathematicians do; engage in the problem solving process.

Strategy Development

Mathematicians also make use of a strategy toolbox. These strategies are embedded in Maths300 lessons, but may also have a separate focus. Poster Problem Clinics are a useful way to approach this component.

Concept Development

A few major concepts in mathematics took centuries for the human race to develop and apply. Examples are place value, fractions and probability. In the past students have been expected to understand such concepts after having 'done' them for a two week slot. Typically they were not visited again until the next year. A Working Mathematically curriculum identifies these concepts and regularly 'threads' them through the curriculum.

Planning to Work Mathematically

The class, school or system that shifts towards a Working Mathematically curriculum will no longer use a curriculum document that looks like a list of content skills. The document would be clear in:

- choosing genuine problems to initiate investigation
- choosing a range of best practice teaching strategies to interest a wider range of students
- practising skills for the purpose of problem solving

Some teachers have found the planning template on the next page assists them to keep this framework at the forefront of their planning. It can be used to plan single lessons, or units built of several lessons. There are examples from schools in the Curriculum & Planning section of Maths300 and a Word document version of the template.

Unit Planning Page

Class

Topic

Pedagogy	Problem Solving In this topic how will I engage my students in the Working Mathematically process?	Skills
How do I create an environment where students know what they are doing and why they have accepted the challenge?		Does the challenge identify skills to practise? Are there other skills to practise in preparation for future problem solving?

Notes

As a general guide:

- Find a problem(s) to solve related to the topic.
- Choose the best teaching craft likely to engage the learners.
- Where possible link skill practice to the problem solving process.

More on Professional Development

For many teachers there will be new ideas within **Maths With Attitude**, such as unit structures, views of how students learn, teaching strategies, classroom organisation, assessment techniques and use of concrete materials. It is anticipated (and expected) that as teachers explore the material in their classrooms they will meet, experiment with and reflect upon these ideas with a view to long term implications for the school program and for their own personal teaching.

Being explored 'on-the-job' so to speak, in the teacher's own classroom, makes the professional development more meaningful and practical for the teacher. This is also a practical and economic alternative for a local authority.

Strategic Use by Systems

From Years 3 - 10, **Maths With Attitude** is designed as a professional development vehicle by schools or clusters or systems because it carries a variety of sound educational messages. They might choose **Maths With Attitude** because:

- It can be used to highlight how investigative approaches to mathematics can be built into balanced unit plans without compromising skill development and without being relegated to the margins of a syllabus as something to be done only after 'the real' content has been covered.
- It can be used to focus on how a balance of concept, skill and application work can all be achieved within the one manageable unit structure.
- It can be used to show how a variety of assessment practices can be used concurrently to build a picture of student progress.
- It can be used to focus on transition between primary and secondary school by moving towards harmony and consistency of approach.
- It can be used to raise and continue debate about the pedagogy (art of teaching) that supports deeper mathematical learning for a wider range of students.

Teachers in Years K - 2 are similarly encouraged in professional growth through **Working Mathematically with Infants**, which derives from Calculating Changes, a network of teachers enhancing children's number skills from Years K - 6.

In supporting its teachers by supplying these resources in conjunction with targeted professional development over time, a system can fuel and encourage classroom-based debate on improving outcomes. There is evidence that by exploring alternative teaching strategies and encouraging curriculum shift towards Working Mathematically, learners improve and teachers are more satisfied. For more detail visit Research & Stories at:

- http://www.mathematicscentre.com/taskcentre/do.htm

We would be happy to discuss professional development with system leaders.

Web Reference

The starting point for all aspects of learning to work like a mathematician, including Calculating Changes, and the teaching craft which encourages it is:

- http://www.mathematicscentre.com/

Appendix: Recording Sheets

Win At The Fair: On-going Results Sheet

Sheet Number ....

One row per pair. Enter your initials in the 'Tally for...' box.

	20¢	50¢	$1	$2	$3	$4	$5
Tally for							
Your totals							
Totals so far							

One row per pair. Enter your initials in the 'Tally for...' box.

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