maths with attitude

2nd Edition

Chance & Measurement Years 5 & 6

Charles Lovitt
Doug Williams

Mathematics Task Centre & Maths300
helping to create happy healthy cheerful productive inspiring classrooms

Chance & Measurement

Years 5 & 6

In this kit:

- *Hands-on problem solving tasks*
- *Detailed curriculum planning*

Access from Maths300:

- *Extensive lesson plans*
- *Software*

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Maths With Attitude includes reference to hands-on problem solving tasks and Maths300 material.

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- To support teachers, schools and systems wanting to create:

  happy, healthy, cheerful, productive, inspiring classrooms

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  learning is a personal journey stimulated by achievable challenge
- to learners:

  stimulated students are creative and love to learn
- to pedagogy:

  the art of choosing teaching strategies to involve and interest all students
- to mathematics:

  mathematics is concrete, visual and makes sense
- to learning mathematics:

  all students can learn to work like a mathematician
- to teachers:

  the teacher is the most important resource in education
- to professional development:

  teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

Our Objective in Detail

What do we mean by creating:

happy, healthy, cheerful, productive, inspiring classrooms

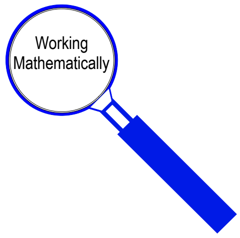
Happy…

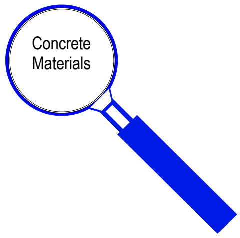
means the elimination of the unnecessary fear of failure that hangs over so many students in their mathematics studies. Learning experiences *can* be structured so that all students see there is something in it for them and hence make a commitment to the learning. In so many 'threatening' situations, students see the impending failure and withhold their participation.

A phrase which describes the structure allowing all students to perceive something in it for them is *multiple entry points and multiple exit points*. That is, students can enter at a variety of levels, make progress and exit the problem having visibly achieved.

Healthy…

means *educationally healthy*. The learning environment should be a reflection of all that our community knows about how students learn. This translates into a rich array of teaching strategies that could and should be evident within the learning experience.





If we scrutinise the *exploration* through any lens, it should confirm to us that it is well structured or alert us to missed opportunities. For example, peering through a pedagogy lens we should see such features as:

- a story shell to embed the situation in a meaningful context
- significant active use of concrete materials
- a problem solving challenge which provides ownership for students
- small group work
- a strong visual component
- access to supportive software

Cheerful…

because we want 'happy' in the title twice!

Productive…

is the clear acknowledgment that students are working towards recognisable outcomes. They should know what these are and have guidelines to show they have either reached them or made progress. Teachers are accountable to these outcomes as well as to the quality of the learning environment.

Inspiring…

is about creating experiences that are uplifting or exalting; that actually *turn students on*. Experiences that make students feel great about themselves and empowered to act in meaningful ways.

Chance & Measurement Resources

To help you create

happy, healthy, cheerful, productive, inspiring classrooms

this kit contains

- 20 hands-on problem solving tasks from Mathematics Centre

and a Teachers' Manual which integrates the use of the tasks with

- 10 detailed lesson plans from Maths300

The kit offers **5 weeks** of Scope & Sequence planning in Chance & Measurement for *each* of Year 5 and Year 6. This is detailed in *Part 2: Planning Curriculum* which begins on Page 12. You are invited to map these weeks into your Year Planner.
Together, the four kits available for these levels provide 25 weeks of core curriculum in Working Mathematically (working like a mathematician).

Note: Membership of Maths300 is assumed.
The kit will be useful without it, but it will be much more useful with it.

Tasks

- 64 = 65
- Angle Estimation
- Brick Walls
- Cat and Mouse
- Chocolate Chip Cookies
- Final Eight
- Game Show
- Highest Number 2
- Kids On Grids
- Matching Faces
- Photo Angles
- Planets
- Red & Black Card Game
- Scale Drawing
- Surface Area With Tricube
- Take A Chance
- Time Swing
- Triangle Area
- Volume Line Up
- Where Is The Rectangle?

Part 2 of this manual introduces each task. The latest information can be found at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm

Maths300 Lessons

- Beetle Game
- Country Maps
- Duelling Dice
- Feet-uring Mathematics
- Greedy Pig
- Have A Hexagon
- Planets
- Red & Black Card Game
- Temperature Graphs
- This Goes With This

Lessons with Software

- Beetle Game
- Duelling Dice
- Greedy Pig
- Have A Hexagon
- Temperature Graphs

Part 2 of this manual introduces each lesson. Full details can be found at:

- http://www.maths300.com

Working Like A Mathematician

Our attitude is:

all students can learn to work like a mathematician

What does a mathematician's work actually involve? Mathematicians have provided their answer on Page 8. In particular we are indebted to Dr. Derek Holton for the clarity of his contribution to this description.

Perhaps the most important aspect of Working Mathematically is the recognition that *knowledge is created by a community and becomes part of the fabric of that community*. Recognising, and engaging in, the process by which that knowledge is generated can help students to see themselves as able to work like a mathematician. Hence Working Mathematically is the framework of **Maths With Attitude.**

Skills, Strategies & Working Mathematically

A Working Mathematically curriculum places learning mathematical skills and problem solving strategies in their true context. Skills and strategies are the tools mathematicians employ in their struggle to solve problems. Lessons on skills or lessons on strategies are not an end in themselves.

- **Our skill toolbox** can be added to in the same way as the mechanic or carpenter adds tools to their toolbox. Equally, the addition of the tools is not for the sake of collecting them, but rather for the purpose of getting on with a job. A mathematician's job is to attempt to solve problems, not to collect tools that might one day help solve a problem.
- **Our strategy toolbox** has been provided through the collective wisdom of mathematicians from the past. All mathematical problems (and indeed life problems) that have ever been solved have been solved by the application of this concise set of strategies.

About Tasks

Our attitude is:

mathematics is concrete, visual and makes sense

Tasks are from Mathematics Task Centre. They are an invitation to two students to work like a mathematician (see Page 8).

The Task Centre concept began in Australia in the late 1970s as a collection of rich tasks housed in a special room, which came to be called a Task Centre. Since that time hundreds of Australian teachers, and, more recently, teachers from other countries, have adapted and modified the concept to work in their schools. For example, the special purpose room is no longer seen as an essential component, although many schools continue to opt for this facility.

A brief history of Task Centre development, considerable support for using tasks, for example Task Cameos, and a catalogue of all currently available tasks can be found at:

- http://www.mathematicscentre.com/taskcentre

Key principles are:

- A good task is the tip of an iceberg
- Each task has three lives
- Tasks involve students in the Working Mathematically process

The Task Centre Room or the Classroom?

There are good reasons for using the tasks in a special room which the students visit regularly. There are also different good reasons for keeping the tasks in classrooms. Either system can work well if staff are committed to a core curriculum built around learning to work like a mathematician.

- A task centre room creates a focus and presence for mathematics in the school. Tasks are often housed in clear plastic 'cake storer' type boxes. Display space can be more easily managed. The visual impact can be vibrant and purposeful.
- However, tasks can be more readily integrated into the curriculum if teachers have them at their finger tips in the classrooms. In this case tasks are often housed in press-seal plastic bags which take up less space and are more readily moved from classroom to classroom.

Tip of an Iceberg

The initial problem on the card can usually be solved in 10 to 20 minutes. The investigation iceberg which lies beneath may take many lessons (even a lifetime!). Tasks are designed so that the original problem reveals just the 'tip of the iceberg'. Task Cameos and Maths300 lessons help to dig deeper into the iceberg.

We are constantly surprised by the creative steps teachers and students take that lead us further into a task. No task is ever 'finished'.

> *Most tasks have many levels of entry and exit and therefore offer an on-going invitation to revisit them, and, importantly, multiple levels of success for students.*

Three Lives of a Task

This phrase, coined by a teacher, captures the full potential and flexibility of the tasks. Teachers say they like using them in three distinct ways:

1. As on the card, which is designed for two students.
2. As a whole class lesson involving all students, as supported by outlines in the Task Cameos and in detail through the Maths300 site.
3. Extended by an Investigation Guide (project), examples of which are included in both Task Cameos and Maths300.

The first life involves just the 'tip of the iceberg' of each task, but nonetheless provides a worthwhile problem solving challenge - one which 'demands' concrete materials in its solution. This is the invitation to work like a mathematician. Most students will experience some level of success and accomplishment in a short time.

The second life involves adapting the materials to involve the whole class in the investigation, in the first instance to model the work of a mathematician, but also to develop key outcomes or specific content knowledge. This involves choosing teaching craft to interest the students in the problem and then absorb them in it.

The third life challenges students to explore the 'rest of the iceberg' independently. Investigation Guides are used to probe aspects and extensions of the task and can be introduced into either the first or second life. Typically this involves providing suggestions for the direction the investigation might take. Students submit the 'story' of their work for 'portfolio assessment'. Typically a major criteria for assessment is application of the Working Mathematically process.

About Maths300

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

Maths300 is a subscription based web site. It is an attempt to collect and publish the 300 most 'interesting' maths lessons (K - 12).

- *Lessons have been successfully trialed in a range of classrooms.*
- *About one third of the lessons are supported by specially written software.*
- *Lessons are also supported by investigation sheets (with answers) and game boards where relevant.*
- *A 'living' Classroom Contributions section in each lesson includes the latest information from schools.*
- *The search engine allows teachers to find lessons by pedagogical feature, curriculum strand, content and year level.*
- *Lesson plans can be printed directly from the site.*
- *Each lesson supports teaches to model the Working Mathematically process.*

Modern internet facilities and computers allow teachers easy access to these lesson plans. Lesson plans need to be researched, reflected upon in the light of your own students and activated by collecting and organising materials as necessary.

Maths300 Software

Our attitude is:

> *stimulated students are creative and love to learn*

Pedagogically sound software is one feature likely to encourage enthusiastic learning and for that reason it has been included as an element in about one third of Maths300 lesson plans. The software is used to develop an investigation beyond its introduction and early exploration which is likely to include other pedagogical techniques such as concrete materials, physical involvement, estimation or mathematical conversation. The software is not the lesson plan. It is a feature of the lesson plan used at the teacher's discretion.

For school-wide use, the software needs to be downloaded from the site and installed in the school's network image. You will need to consult your IT Manager about these arrangements. It can also be downloaded to stand alone machines covered by the site licence, in particular a teacher's own laptop, from where it can be used with the whole class through a data projector.

Note:

- *Maths300 lessons and software may only be used by Maths300 members.*

Working Mathematically

First give me an interesting problem.

When mathematicians become interested in a problem they:

- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

Questions which help mathematicians learn more are:

- Can I check this another way?
- What happens if …?
- How many solutions are there?
- How will I know when I have found them all?

When mathematicians have a problem they:

- Read & understand the problem.
- Plan a strategy to start the problem.
- Carry out their plan.
- Check the result.

A mathematician's strategy toolbox includes:

- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- Break a problem into smaller parts
- …

If one way doesn't work, I just start again another way.

Professional Development Purpose

Our attitude is:

the teacher is the most important resource in education

> *We had our first study group on Monday. The session will be repeated again on Thursday. I had 15 teachers attend. We looked at the task Farmyard Friends (Task 129 from the Mathematics Task Centre). We extended it out like the questions from the companion Maths300 lesson suggested, and talked for quite a while about the concept of a factorial. This is exactly the type of dialog that I feel is essential for our elementary teachers to support the development of their math background. So anytime we can use the tasks to extend the teacher's math knowledge we are ahead of the game.*
> District Math Coordinator, Denver, Colorado

Research suggests that professional development most likely to succeed:

- is requested by the teachers
- takes place as close to the teacher's own working environment as possible
- takes place over an extended period of time
- provides opportunities for reflection and feedback
- enables participants to feel a substantial degree of ownership
- involves conscious commitment by the teacher
- involves groups of teachers rather than individuals from a school
- increases the participant's mathematical knowledge in some way
- uses the services of a consultant and/or critical friend

Maths With Attitude has been designed with these principles in mind. All the materials have been tried, tested and modified by teachers from a wide range of classrooms. We hope the resources will enable teacher groups to lead themselves further along the professional development road, and support systems to improve the learning outcomes for students K - 12.

With the support of Maths300 ETuTE, professional development can be a regular component of in-house professional development. See:

- http://www.mathematicscentre.com/taskcentre/resource.htm#etute

For external assistance with professional development, contact:

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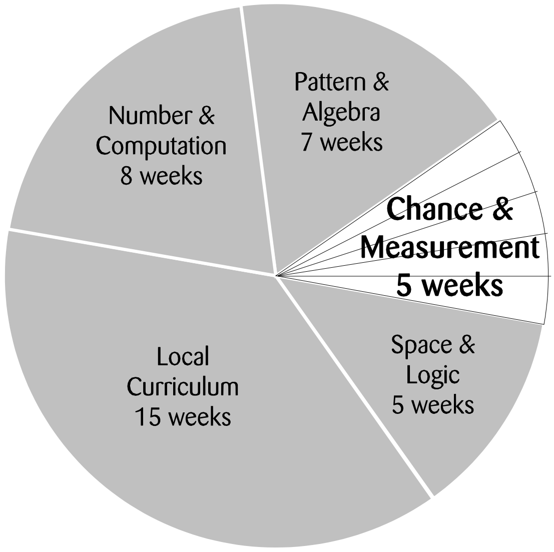
Part 2: Planning Curriculum

Curriculum Planners

Our attitude is:

learning is a personal journey stimulated by achievable challenge



Curriculum Planners:

- show one way these resources can be integrated into your weekly planning
- provide a starting point for those new to these materials
- offer a flexible structure for those more experienced

You are invited to map Planner weeks into your school year planner as the core of the curriculum.

Planners:

- detail each week lesson by lesson
- offer structures for using tasks and lessons
- are sequenced from lesson to lesson, week to week and year to year to 'grow' learning

Teachers and schools will map the material in their own way, but all will be making use of extensively trialed materials and pedagogy.

Using Resources

- Your kit contains 20 hands-on problem solving tasks and reference to relevant Maths300 lessons.
- Tasks are introduced in this manual and supported by the Task Cameos at:
  http://www.mathematicscentre.com/taskcentre/iceberg.htm
- Maths300 lessons are introduced in this manual and supported by detailed lesson plans at:
  http://www.maths300.com

In your preparation, please note:

- Planners assume 4 lessons per week of about 1 hour each.
- Planners are *not* prescribing a continuous block of work.
- Weeks can be interspersed with other learning; perhaps a **Maths With Attitude** week from a different strand.
- Weeks can sometimes be interchanged within the planner.
- Lessons can sometimes be interchanged within weeks.
- The four **Maths With Attitude** kits available at each year level offer 25 weeks of a Working Mathematically core curriculum.

A Way to Begin

- Glance over the Planner for your class. Skim through the comments for each task and lesson as it is named. This will provide an overview of the kit.
- Task Comments begin after the Planners. Lesson Comments begin after Task Comments. The index will also lead you to any task or lesson comments.
- Select your preferred starting week - usually Week 1.
- Now plan in detail by researching the comments and web support. Enjoy!

Research, Reflect, Activate

Curriculum Planner

Chance & Measurement: Year 5

<table>
<tr><th></th><th>Session 1</th><th>Session 2</th><th>Session 3</th><th>Session 4</th></tr>
<tr><td>Week 1</td><td colspan="3">Integrated Curriculum Unit: The mathematical focus of Feet-uring Mathematics is measurement and data presentation. However, the lesson can be the source of a much more extensive unit that includes art, language and social science objectives. The lesson provides a pair of bare feet (on a reproducible page) and the students build these into a character.</td><td rowspan="5">Tasks Day: Pairs choose from the 20 tasks. Each task is an invitation to work like a mathematician.

One session each week. The Working Mathematically process (p.8) offers guidance. Students keep a journal.

The WM process is pasted to the inside cover of the journal.

Journal entries are of diary/note form with date, sketches etc.

Students may be required to develop one investigation into a more extensive report, perhaps for assessment.</td></tr>
<tr><td>Week 2</td><td colspan="3">Whole Class Investigation: Greedy Pig is an easy to state easy to start game situation that is often played for the arithmetic skill practice it provides. However, this lesson takes the game much further by looking at the total score after five rounds of the game and investigating strategies to maximise this score. The lesson involves the display of data and use of statistics to analyse data.</td></tr>
<tr><td>Week 3</td><td colspan="3">Whole Class Investigation: Country Maps and Temperature Graphs can become part of a unit linked to geography studies. Mathematically Country Maps compares areas and involves decimals.</td></tr>
<tr><td>Week 4</td><td colspan="3">Skill Development: Temperature Graphs presents skills related to average and data display in the context of comparing temperature measurement in different cities. Software adds to the possibilities of the lesson.</td></tr>
<tr><td>Week 5</td><td colspan="3">Whole Class Investigation: Duelling Dice is an experiment in probability that relates to the playground game Scissors, Rock, Paper. The specially prepared dice (stickers on wooden cubes) suggest counter-intuitive results that are investigated using mathematics related to data display, long run frequency and statistical inference. Software is provided to assist with the analysis.</td></tr>
</table>

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Curriculum Planner

Chance & Measurement: Year 6

	Session 1	Session 2	Session 3	Session 4
Week 1	**Whole Class Investigation:** The string and cards provided in the **Planets** task is all you need to create the whole class lesson. The length of the string is proportional to the length of the solar system and this classroom-size model allows investigation of the relative distances of each planet from the sun. The answer suggests further investigation and other projects.			**Tasks Day:** Pairs choose from the 20 tasks. Each task is an invitation to work like a mathematician.
Week 2	**Skill Development:** *This Goes With This* uses the students' own data to explore various forms of graphical representation. The links between strip graphs, circle graphs and pie charts are explored, as is the link between vulgar fractions and percentages. The lesson has a highly visual component and can include whole body involvement. Statistics such as range, mode, median and mean can also be included.			One session each week. The Working Mathematically process (p.8) offers guidance. Students keep a journal.
Week 3	**Whole Class Investigation:** *Red & Black Card Game* is a game of chance that leads students into the arithmetic of positive and negative numbers. A problem solving twist brings in statistics and data analysis.			The WM process is pasted to the inside cover of the journal.
Week 4	**Whole Class Investigation:** *Have A Hexagon* is a game that students usually feel has a counter-intuitive twist. Exploration and attempted correction of the unfairness in the game creates a major investigation.			Journal entries are of diary/note form with date, sketches etc.
Week 5	**Whole Class Investigation:** *Beetle Game* is sometimes played by families and may be familiar to students. The rolls of the dice are used to make a beetle according to a set of rules. Probability based investigations develop from the game, for example, average game length, least number of rolls and greatest number of rolls. The companion software allows thousands of games to be played in real time.			Students may be required to develop one investigation into a more extensive report, perhaps for assessment.

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Planning Notes

Enhancing Maths With Attitude

Resources to support learning to work like a mathematician are extensive and growing. There are more tasks and lessons available than have been included in this Chance & Measurement kit. You could use the following to enhance this kit.

Additional Tasks

- Task 72, Farmyard
  A spatial challenge with aspects of area. A farmer wants to divide a field shaped like an L into smaller areas which are all the same size and shape.
- Task 88, Rice, Rice, Rice
  Set in a story shell, the challenge is to estimate grains of rice without actually counting their number. This is equivalent to the problems faced by many people in practical employment - for example a bricklayer estimating the number of bricks to order to clad a house. Such problems are often guided by 'rules of thumb'. In this task students are experimenting with possibilities so there is no right or wrong answer.
- Task 241, Sicherman Dice
  It's easy enough to work out the possible results when two usual cube dice are rolled and the numbers added. It's also not too much of a challenge to work out the distribution of those sums and the associated probability of each outcome. But what happens if I asked you to find other cube dice that could be rolled and summed to get the same probabilities? And what happens if I restrict the dice faces to non-zero whole numbers? These investigations are the focus of Sicherman Dice.

More information about these tasks may be available in the Task Cameo Library:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Additional Lessons

- Lesson 122, How Many People Can Stand?
  How many people can stand? It's a question which engineers and others often have to ask. In this lesson the question is asked in the context of a story shell where the classroom becomes the 'standing room only' section of a venue, for the purposes of encouraging estimation and calculation of number and area and a variety of approaches to solving the problem.
- Lesson 126, Make A Moke
  Played in a similar way to Beetle Game (Lesson 121), Make A Moke is used to challenge student misconceptions about probability related to a cube dice. In particular the lesson focuses on the misconception that 'the six never comes up for me' and explores the reality behind this idea both empirically and theoretically.
- Lesson 132, 64 = 65
  We all know 64 does not equal 65! But this jigsaw puzzle visually suggests that it is true. This paradox puzzle is one of a genre of 'missing square puzzles'. Cut the 4 pieces from an 8x8 frame and the area is clearly 64. Place the same pieces into a 13x5 frame and the area now appears to be 65. Where did the extra square come from? The search for,

and explanation of, the extra square involves many different mathematical tools and suits many year levels. However, noticing that the key numbers involved (3, 5, 8 and 13) are successive terms of the Fibonacci sequence is the starting point for an extended investigation.

- Lesson 159, Chances With Crosses

  Place the digits 1 to 9 in the shape of a 'plus' sign. Now shuffle the digits until the horizontal and vertical arms have the same total. This number puzzle is titled Crosses and is the basis of Lesson 112. However, Chances With Crosses explores a different part of the iceberg of this task. Suppose the digits are placed at random, what is the chance they will form a solution to the puzzle? The results are quite counter-intuitive when compared to students' initial expectations. The companion software allows the chances to be explored at significantly greater depth and allows the empirical results to be compared with theoretical calculations.
- Lesson 165, Surface Area with Tricubes

  The best text books introduce their work on surface area with glossy diagrams of cube-based structures. As pretty as they may be, they are not a substitute for concrete experience and kinaesthetically developed spatial perception. This lesson provides a hands-on problem solving introduction to surface area, base area and volume and in doing so replaces the drudgery of the text book with a willingness to practise and refine newly found skills.
- Lesson 166, Newspaper Cubes & the Volume of a Room

  This lesson is about visualising volumes of cubes and cuboids. A conceptual notion is developed by making and counting cubes and this provides a firm basis for later understanding and application of the formula V=LxWxH. Additional benefits are experience of structural engineering features such as the need to use triangles to make objects rigid and consequent connections with students' real life experiences of building. The lesson makes the calculation of volume a richer experience than merely multiplying three numbers.
- Lesson 170, Take A Chance

  This lesson is based around a card game which involves risking counters at each play. The game situation initiates interest and the requirement to risk counters is a measure of student understanding of the chances involved. As they experience the game and make judgements, each student develops a notion of 'good chance'. Challenging students to state their clues for this 'good chance' opens a range of possible explorations, including whole class investigation, small group work guided by an Investigation Sheet or a major project.
- Lesson 175, Dice Cricket

  This game simulates real limited-over cricket formats such as one-day cricket or Twenty20. Played as a game between two students, the context provides a rich array of mathematical ideas from a simple starting point. The lesson involves collecting and analysing class data which allows analysis of several aspects of the mathematics. Using the computer simulation of the game, long-term patterns can be explored, and empirical results compared with theoretical expectations.
- Lesson 180, Maths of Lotto

Gambling, in all forms, is a significant social issue. This lesson focuses on the Lotto-style game and takes the approach that if players better understand their chances they are likely to make better choices about how much of their income to commit to the chance of winning. The game is presented as a whole class investigation involving probability, combination theory, statistics and working mathematically. Companion software extends the problem solving possibilities.

- Lesson 183, Snakes & Ladders

  This famous childhood game originated in ancient India around 2 CE and is riddled with opportunities for mathematical investigation. After playing, exploring and analysing the game, students can take on the role of 'mathematical game board designers' and produce their own game boards to meet desired specifications. Using a computer, students can add snakes or ladders wherever they choose and analyse the effects of doing so. Hypotheses can be created and tested, both theoretically and empirically.

Keep in touch with new developments which enhance **Maths With Attitude** at:

- http://www.mathematicscentre.com/taskcentre/enhance.htm

Additional Materials

As stated, our attitude is that mathematics is concrete, visual and makes sense. We assume that all classrooms will have easy access to many materials beyond what we supply. For this unit you will need:

- Wooden cubes
- Packs of playing cards
- Dice

Special Comments Year 5

- Look ahead to Planner Weeks 3 & 4. These lessons could be part of a unit in geography or a similar subject. Check the lessons in advance and compare them with your curriculum in other subjects, so that you have enough time to prepare the integrated unit should you choose to use them in this way.
- Look ahead to Planner Week 5. You will need to prepare the coloured cubes in the manner described in the lesson.

Special Comments Year 6

- Look ahead to Planner Week 5. You may wish to print the Beetle pieces in colour on card or transparency plastic. Students can do the cutting.

Task Comments

- Tasks, lessons and unit plans prepare students for the more traditional skill practice lessons, which we invite you to weave into your curriculum. Teachers who have used practical, hands-on investigations as the focus of their curriculum, rather than focussing on the drill and practice diet of traditional mathematics, report success in referring to skill practice lessons as Toolbox Lessons. This links to the idea of a mathematician dipping into a toolbox to find and use skills to solve problems.

64 = 65

This is one of a number of Disappearing Square puzzles that can be found in the literature. It simply isn't possible that a square of 64 square units can turn into a rectangle of 65. How has the extra square appeared?

There are two levels of answer to this, but in Years 5 & 6 it is probably most appropriate to accept answers along the lines of the pieces of the rectangle 'not quite lining up'. If the perimeter of the rectangle is laid against straight edges so that it is 'square' there is a long thin parallelogram in the middle ... and you can guess the area of this parallelogram.

The unit square in the problem is actually 2 centimetres each side. A 13 x 5 rectangle of these unit squares has been provided on a worksheet at the end of this manual so students can line up the rectangle in this way.

The other level of answer to the explanation of the extra square relies on Pythagoras' Theorem and Trigonometry so the students may meet this puzzle again in later years.

An extension of the problem comes from noticing a link in the key numbers of the problem. The square is 8 x 8 and the rectangle is 13 x 5. The key numbers are 5, 8 and 13. A student might notice that the first two sum to the third. It is not much data, but it might lead to investigating whether 3 other numbers might exist that could generate this 'extra square phenomenon'.

- One might consider asking whether ?, 5 and 8 could be the three numbers. The ? would then have to be 3 if the sum of the first two is to equal the third.
- In the original problem it was the middle number that was the square. The rectangle was formed from the other two.
- If 3, 5 and 8 are to work, the square would be 25 square units and the rectangle 24. Hmmm, one square different. It might be possible to do a construction to create a new 'extra square phenomenon'.
- If this worked, the next smaller set of numbers would be ?, 3, 5, implying the set 2, 3 and 5.
- In this case the square would be 9 square units and the rectangle 10. But this time the rectangle is bigger than the square. In the previous set of numbers it was smaller, and in the original problem it was bigger. Hmmm.
- Continuing the exploration leads to the sets 1, 2 and 3 and 1, 1 and 2. Of course we could also investigate larger sets than the original 5, 8 and 13; such as 8, 13 and 21.

Looking back over the numbers generated in this way, ie: 1, 1, 2, 3, 5, 8, 13, 21..., identifies Fibonacci's sequence ... and therein lies an additional wealth of possibilities.

Angle Estimation

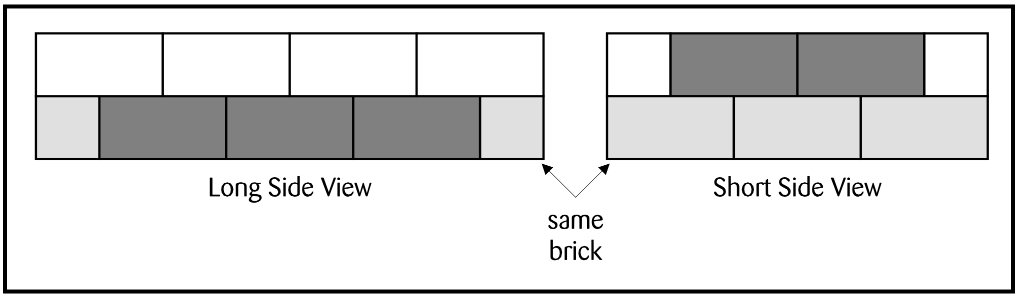
The special feature of this task is the Rotagrams which allow estimation of angles. An angle is the shape made by two co-incident rays. Estimating or measuring an angle involves estimating or measuring the amount of turn between the two rays. Without the Rotagram, the only tool normally available is the protractor, and with that implement the students don't get the opportunity to develop their spatial conception of the sizes of angles in the same way as is encouraged by estimation.

The Rotagram can also be fruitfully used in conjunction with the option Fraction Pie in the software of Maths300 Lesson 33, *Fraction Estimation*. Before using the software to show the requested fraction, students can be asked to estimate the fraction on the Rotagram. When the software shows the correct result, the Rotagram estimate can be easily checked by placing it against the screen.

Also, Lesson 133, *Angle Estimation*, offers lots of opportunity to use Rotagrams. However, it begins on a larger scale. Students are asked to estimate angles made by tying a fixed cord to a pole in the quadrangle and rotating another cord around the pole in relation to this fixed cord.

Brick Walls

It may be a long time since you have played with joining bricks like this. Try building the pictured wall yourself. Arranging the first and second layers so they lock is a neat construction technique that mirrors the way real buildings are made.



The task involves aspects of perimeter, area and volume. It also depends on seeing the rectangular brick as the unit, rather than the cube which would be more familiar. Real bricks are usually made this way and purchased per brick rather than per cube. Why is this? Students could investigate the likely advantages of building with rectangular rather than square bricks.

The answers to the questions on the card are:

- 24 bricks
- 24 bricks (if only to the top of the wall)
- 32 bricks (an additional 'cube' - half brick - is required in each direction because the wall is one 'cube' wide, so one extra brick in each direction on each layer.)

There is an algebraic aspect to this task which involves exploring the next new wall, then the next, then the next, and so on, and being able to predict the number of bricks needed for any level of expanded perimeter.

However, within this measurement context, the students could:

- study and report on the use of bricks in buildings within the school grounds.
- find out trade secrets from a bricklayer, for example, how do they know how many bricks to order for a new house?
- if more Little Architect Bricks, or equivalent, are available, explore building other models, perhaps ones which also have internal brick walls.

Cat and Mouse

This is an intriguing game that explores the effect of board design and movement rules on who will succeed, cat or mouse. Will the mouse get the cheese or will the cat get the mouse? What chance does the cat or the mouse have of succeeding? The students will need to work like a mathematician to answer this last question. Sufficient data will have to be collected before this probability question can be answered empirically. It is unlikely that two players will be able to run enough trials to do this on their own, so teachers can make an ongoing record on the class maths display board. Teachers might also like to use the data to introduce students to the use of a spreadsheet.

Another use of the task is to turn it into a kinaesthetic whole class investigation by quickly chalking the board design on the playground and using a child as the mouse. If twenty-five children each have a turn as the mouse, that's twenty-five pieces of data.

Yet another aspect, for use in senior classes, is the analysis of the probabilities using both analytical and empirical approaches.

The two boards supplied with the task were designed by primary school children. This fact points to an extension of the task. There are extensive notes about the task in its Maths300 companion lesson.

Chocolate Chip Cookies

Manufacturers of chocolate chip cookies are faced with the problem of how many chocolate chips to put into a mix so that the customers receive a 'guaranteed' minimum number in each cookie. It is simply not good marketing to have customers find cookies with no chocolate chips. However, achieving the minimum has to be balanced against how many chips can be afforded for each mix. The task gives students a chance to simulate this real world problem. The discussion and experimentation will help their intuitions about chance events to develop. As they collect data there will also be informal discussion of statistics such as the range of the data or the 'average' number of chips per cookie. There are extensive notes about the task in its Maths300 companion lesson.

Final Eight

Every team sport has to find a way to fairly decide its overall annual champion each year. Mathematicians have spent many years designing finals systems to achieve this end. This task is an example of one way to run the final series in a league that includes eight teams in the final play off.

Mathematicians have to model, or simulate, these real life situations. They need to gather and analyse data about the possible outcomes of alternative structures before they can recommend a particular one. It is an empirical approach that solves such problems, rather than a theoretical one.

Such approaches are underpinned with assumptions and, in the fun of collecting the data in this task, it should not be forgotten that the assumption is each team in any match has an equal chance of winning.

The structure in this task has been used by the Australian Football League (AFL) to decide its premier team. Was it fair, or did the finishing position at the end of the season load the chances of winning the flag? That investigation is the main thrust of the task. Working through the experimentation puts the students in the same position as professional mathematicians. The children are reminded several times to record the position of each team at the end of the season. That can then be compared to the team that becomes the Premiers. However, you may have to remind them again of the sense of this recording, because they can easily get lost in the fun of 'running' the finals.

Extensions

- A Final Four

  The league from which the AFL is derived once used a final system that only included the top four teams on the ladder. Investigate the chance of becoming premiers from each finishing position in this system:

  Round 1

  1st vs 2nd and 3rd vs 4th. The winner of 1st and 2nd had a bye in Round 2. The loser of 3rd and 4th was eliminated.

  Round 2

  Loser (1 & 2) vs Winner (3 & 4). Loser is eliminated.

  Round 3

  Winner (1 & 2 - Round 1) vs Winner Round 2. Winner of this match is the Premiership team and the loser is Runner-up.

- Suppose that one team in the final series 'had the edge', ie: there was not an equal chance of winning when playing this team. How would that affect the data gathered in the task? One way to model this is to complete Question 1 as indicated, record this Final Eight, then put the team names in a 'hat' and randomly choose one to be the team with the edge. Now use only one dice to play each match. If a match includes this team, then 1, 2, 3, 4 wins for the team with the edge and only 5 or 6 wins for the other team. If a match does not include this team, then 1, 2, 3 wins for Team A and 4, 5, 6 wins for Team B.
- There are two Maths300 lessons called Sporting Finals. One is for the AFL and one for the game of rugby as played in the NRL.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Game Show

This task is likely to cause lots of discussion. Everyone thinks they understand what the chances are in this game, which came from a real life US TV show, but many people are inaccurate in their reasoning. When the TV show was first broadcast in the United States any number of people wrote to the public media to advise the contestant whether or not to change their mind. Even the mathematicians among them failed to agree on the correct theory to analyse the problem.

Students at this level have access to their personal intuitive understanding of probability and are invited to use this to begin their journey with the problem. They are also in a position to repeat the activity in an ordered way comparing the number of successes when they do change their choice with the number of successes when they don't. If this empirical approach is carried out by many pairs and the results collated, a picture grows of which is the better action to take. At a later time in their journey to learn to work like a mathematician, students can investigate the theory that supports this experimental result. This task has a companion Maths300 lesson which includes software.

Highest Number 2

This game of chance is more than it seems. Certainly it offers a review of place value, but in the longer term an investigation of strategies to decide where to place the cards involves deeper thought.

Drawn First	**Placement Strategy**
1	Place in Ones. Everything else beats it.
2	Place in Ones. Only one other number (1) could turn up that would be better in this position.
3	Place in Ones. There are only 2 chances in 8 that a card will turn up that would be better in this position.
4	If this is placed in the Tens, there are 5 chances out of 8 that a bigger number will turn up for the Hundreds. There are 3 chances out of 8 that a smaller number will come up and that will go in the Ones.
5	If this is placed in the Tens, there are 4 chances out of 8 that a bigger number will turn up for the Hundreds. There are 4 chances out of 8 that a smaller number will come up and that will go in the Ones. This is a hard one to decide.
6	If this is placed in the Tens, there are 3 chances out of 8 that the next number will be bigger. If it is placed in the Hundreds there are three numbers out of 8 that could beat it and would be better in the hundreds. Might place a 6 in the hundreds ... but there are two more rolls to come ...
7	If this is placed in the Tens, there are 2 chances out of 8 that the next number will be bigger. If it is placed in the Hundreds there are two numbers out of 8 that could beat it and would be better in the hundreds. Possibly better to place a 7 in the hundreds. ... but there are two more rolls to come ...
8	Place in Hundreds. There is only one number that would be better in this place; that's 1 chance out of 8 of getting a nine in the next roll ... but if that doesn't happen, then one chance in 7 on the third roll.
9	Place in Hundreds.

One approach to examining these strategies is to make a list, as a mathematician would, of the possible first numbers and consider where they could be placed to increase the chances of making the higher number. An example of student thinking is shown in the table opposite. However, considerable complexity is added because the game still has two more rolls to decide the remaining numbers.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

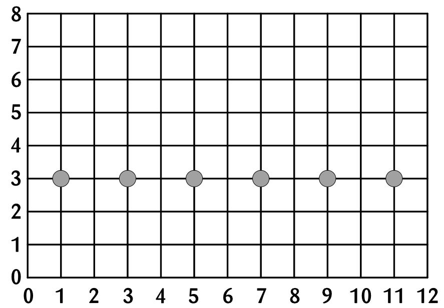
Kids On Grids

The measurement link in this task is the skill of locating objects in two dimensional space using an ordered pair of numbers. This will relate to mapping, reading street directories and creating and interpreting graphs. The task provides an opportunity to practise this skill in a practical hands-on manner. If you have a grid painted on the playground the task can be modelled with the real children.

However, the task is more than measurement. Kids can be placed on the grid in a random manner, but they can also be placed on the grid in a pattern. Creating a visual pattern by placing in this way is a doorway to graphical algebra. If there is a visual pattern there will always be a number pattern, and that pattern can be discovered by asking:

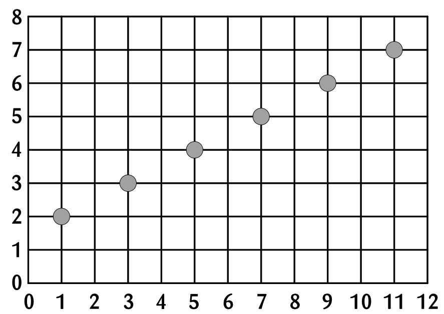
- *What is the same in all of the number pairs?*

For example if the kids were placed in this pattern:



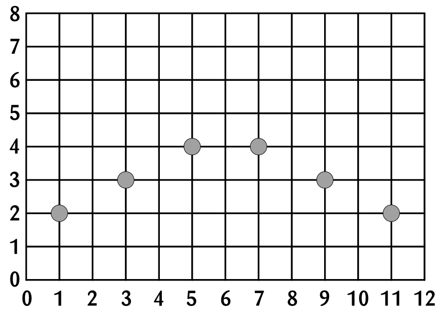
the number pairs would be (1, 3), (3, 3), (5, 3), (7, 3), (9, 3), (11, 3)
...and what is the same is that the second number is always 3, ie: y = 3.

If the kids were placed in this pattern:



the number pairs would be (1, 2), (3, 3), (5, 4), (7, 5), (9, 6), (11, 7)
and what is the same is that *twice the second number minus the first is always 3*, ie: $2y - x = 3$.

If the kids were placed in this pattern:



the number pairs would be (1, 2), (3, 3), (5, 4), (7, 4), (9, 3), (11, 2)
and the description this time might best be in two parts. For x less than or equal to 5, the rule is still *twice the second number minus the first is always 3*. What is the rule for x greater than or equal to 7?

Matching Faces

For most students this task is counter-intuitive. It would seem that the more faces, the more chance of making a match, but this is not so. In fact, the most likely result is exactly one match. The task encourages collecting an organised set of data in order to be able to make an hypothesis about the expected result. Further, it encourages the mathematician's question *What happens if...* because there is sufficient equipment to break the problem into smaller parts and try different numbers of faces and names.

Students might also begin to explore the theory of the problem by looking at the ways *n* objects can be arranged in *N* positions, which is equivalent to arranging the names with the faces, for example, as shown on the next page.

For 5 objects in 5 positions, there are 120 possibilities and these are quite a challenge to list. However the problem could readily be explored further if software were available to do the 'hack' work.

Such software could also be used to add data to the summary of the results so far, which appears on Page 26. Recording like this confirms that the expected number of matches is one match. However, there is also a tantalising hint of a pattern.

- 1 object in 1 position

A	Matches
a	1

- 2 objects in 2 positions

A	B	Matches
a	b	2
b	a	0

- 3 objects in 3 positions

A	B	C	Matches
a	b	c	3
a	c	b	1
b	a	c	1
b	c	a	0
c	a	b	0
c	b	a	1

- 4 objects in 4 positions

A	B	C	D	Matches
a	b	c	d	4
a	b	d	c	2
a	c	b	d	2
a	c	d	b	1
a	d	b	c	1
a	d	c	b	2
b	a	c	d	2
b	a	d	c	0
b	c	a	d	1
b	c	d	a	0
b	d	a	c	0
b	d	c	a	1
c	a	b	d	1
c	a	d	b	0
c	b	a	d	2
c	b	d	a	1
c	d	a	b	0
c	d	b	a	0
d	a	b	c	0
d	a	c	b	1
d	b	a	c	1
d	b	c	a	2
d	c	a	b	0
d	c	b	a	0

Matches Cases	0	1	2	3	4	5
1	0	1	-	-	-	-
2	1	0	1	-	-	-
3	2	3	0	1	-	-
4	9	8	6	0	1	-
5	?	?	?	?	?	?

Total no. of correct matches	Total no. of possible arrangements
1	1
2	2
6	6
24	24
120	120

Some of the numbers in Row 5 could be filled in with confidence. Students should also be able to explain why there are 0 matches in any row for the number one less than the number of objects. Perhaps they will see the suggestion of the triangle numbers in the diagonal just below the diagonal of zeros.

In summary, the task is, in the first instance, an experiment in probability. There are additional options that include exploring the reason for the counter-intuitive result by recording possibilities in a list, and further possible pattern investigations that develop from interrogating the list.

A mathematician is never finished with a problem; they are only finished for now.

Photo Angles

The great value of this task is that at all levels from K to 12 it generates mathematical discussion. Reconstructing the physical situation from the photo is the first level of talk. This involves measurement concepts based around the unit square on the grid.

Previous experience with shadows encourages discussion of where the light might have been placed and concepts related to distance and angle come into the conversation. Students usually make their hands into a lens and look through it as they move around the grid to match the camera angle.

This informal level of investigation is encouraged to develop further by the inclusion of a tape measure. Frequently, students who have explored the problem at the first level then ask each other *What do we use this tape for?*. In the primary school one practical way to follow this up is to use lamps or torches to try to recreate the position of the light. Then its position can be measured.

But measured from where? Students will need to enter a new phase of discussion to choose a reference point and then work out how to tell someone else this position in three dimensional space. That might be achieved in one of two ways:

- A direct line measurement from the reference to the light source which will also require specifying the angle of the string.
- A system of right angle left/right, forward/backward, up/down measurements making an ordered triple, in the same way as an ordered pair of numbers specifies a point in two dimensions.

Using the school digital camera in a similar experimental way can help to determine its position when the photo was taken, and again, this point can be measured.

At an older age students could use trigonometry knowledge to determine the position of the light and the camera; or create the 3D equation of the lines and solve these as simultaneous equations.

Planets

The measurements on the card are taken at a moment in time and represent an average. The movement of the planets is more complex than suggested by the table; for example not all the planets orbit the earth in the same plane, but occasionally they are in an alignment that allows the image of the string to be valid. Some students with an interest in astronomy may wish to discuss such details.

One of the exciting things about this task is that although it deals with very large numbers, judicious application of estimation (rounding off) and multiplication go a long way towards uncovering the wonder of the creation that resides within our solar system. The string is about 6 metres long. The solar system is about 6,000 million kilometres from the Sun to Pluto. This sets a scale:

- 1 metre = 1,000 million kilometres (roughly)

Half of the length to Pluto is about 3000 million kilometres and remarkably Uranus is quite close to that position. Half of the distance between Uranus and Pluto is 4500 million kilometres, and again a planet, Neptune, is there. Continuing this proportional reasoning helps to find a position for each of the other planets, with the only break from the pattern occurring between Mars and Jupiter. It was, in fact, this break from the pattern which led astronomers to look more carefully in this region and eventually discover the asteroids, which by some judgements are the fragments of a destroyed planet.

An important question to a mathematician is *Can I check it another way?*. In this case the placement of the planet cards along the string can be checked by application of the scale in a more precise manner, namely:

- 1 metre = 1,000 million kilometres (roughly)
- 1 centimetre = 10 million kilometres
- 1 millimetre = 1 million kilometres

So on this scale, Mercury would be 5.8 centimetres from the Sun, Venus would be 10.9cm and Earth would be 15cm. That is a very small proportion of the total 6m distance, and yet Earth is the only place in the Universe where we are sure there is life.

An extension of the task develops from the realisation that the cards are all the same size and therefore don't truly represent the diameters of the planets. Research in an encyclopedia will reveal the diameters of the planets and the students can work out what these would become if the solar system was only as big as the string.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Red & Black Card Game

At the same time as it introduces students to the arithmetic of positive and negative numbers, this chance task provides data for a statistical investigation. If the challenge on the card is to be answered on the basis of evidence, then the students will need to experiment, collect and organise data and make and test hypotheses based on the evidence. They will be working like a mathematician as they do this.

Find more information about this task in the Task Cameo Library at:

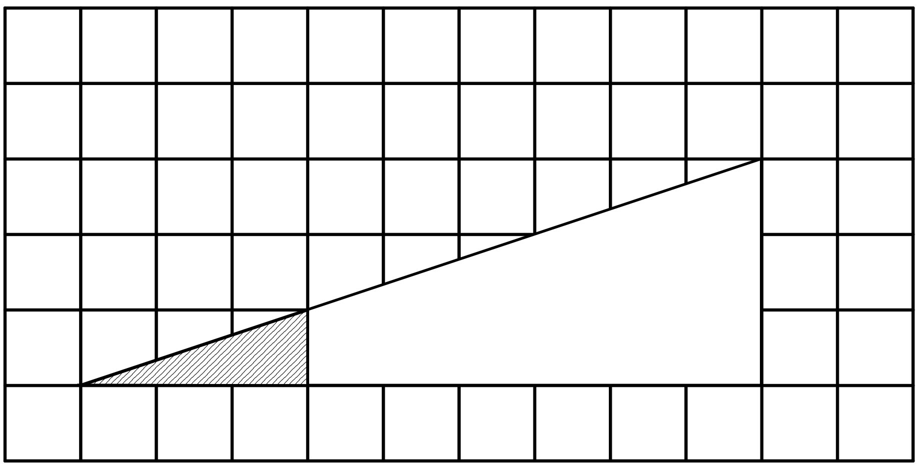
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Scale Drawing

This task is intended as a straightforward introduction to the topic of scale drawing. Partners should be able to convince each other that their drawings are an exact scaled up (or down) version of the original. This will involve reference to the number of units in each side and each part of the house.

Measuring in centimetres and comparing the scales on the two grids will lead to the realisation that if a length is doubled in both directions (horizontally and vertically) then the area is four times bigger. What would happen to the area if an object's lengths were scaled up by a factor of 3, 4, 5, ...? Students can experiment further with graph paper to explore this question. Other questions that could be asked are:

- What happens to the angles of a shape when it is scaled up or down?
- Can you create a grid with a scale factor of 2 horizontally, but a scale factor of 1 vertically? Can you predict how the house would change on that grid?
- What about a grid with a scale factor of 1 horizontally and 2 vertically?
- Create other distorted houses.
- How is scale drawing applied in the world? Make a class scrap book or display.
- Draw two right-angled triangles with a common angle. Scale the larger one by a factor of 3. An example is shown below. Compare corresponding side lengths in the two triangles. What do you notice?



- Compare any two side lengths in the one triangle. Now compare the corresponding side lengths in the other triangle. What do you notice?

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Surface Area With Tricube

Four tricubes can be made into a remarkable number of three dimensional objects. As the card suggests the volume of all of these is 12 cubes. It is the surface area and base area of each that varies.

Students will be exploring these concepts of three dimensional measurement as they struggle to recreate each object from the given dimensions. The tricky ones of course are the ones with a base area of zero. When students realise that a single tricube can be placed to form a triangular arch, they will be on the way to solving the last four challenges on the card.

It is possible to arrange four tricubes to make a larger tricube.

- What is the surface area and base area of this larger tricube?
- On square graph paper draw a looking down view of a Size 1 tricube. Now draw the Size 2 tricube and show how it is made up of single tricubes.
- Can you draw a Size 4 tricube and show how it is made from Size 1 tricubes? What is the surface area and base area of this Size 4 tricube?
- Can you predict the surface area and base area of the Size 8 tricube?

Take A Chance

This task allows students to develop an understanding of chance events. Consequently, hopefully, they will then be less likely to be exploited in social gaming situations. The basic mathematics behind the task is that if the odds are in the players favour they risk counters. If the odds are against them, they should refuse to risk counters, or perhaps, minimise the risk to 1 disc.

The chances can be analysed systematically as follows:

- If the two cards exposed are an Ace and a King, then what are the chances that the next card lies between an Ace and a King? There are 50 cards left, 44 of which lie between an Ace and a King, and six[6] cards do not. So the chances of 'winning' are very high at 44/50 so the player should risk 3 tokens or discs.
- Similarly any situation can be analysed in this way. If the two exposed cards are a 7 and a Jack, then of the remaining 50 cards, only 12 are between a 7 and a Jack, so the chance of winning is 12/50 and the chance of losing is 38/50. So the player should only risk one token, if they choose to risk any at all. Similarly for any situation where the number of cards between the exposed cards is 4 such as Ace and 5, 2 and 6, 3 and 7, 4 and 8 etc.

So what is the situation where the game is close to fair? To be fair, the chances of winning and losing must be the same which would be 25/50. Is such a situation possible?

If the number of cards between is 6, which is the case if the exposed cards are Ace and 8, 2 and 9, 3 and 10, 4 and Jack, 5 and Queen or 6 and King, then there are 6 numbers (which means 24 cards) which are winners. Hence the chances of winning are 24/50 and the chance of losing is 26/50. Since you have a little more chance of losing than winning, then the best strategy may be to minimise the risk to 1 token. Hence the overall optimal strategy could be:

- If the number of cards between is 7 or more, risk three tokens.
- If the number of cards between is 6 or less, risk one token.

Or it could be:

- If the number of cards between is 7 or more, risk three tokens.
- If the number of cards between is 6 or less, risk zero tokens.

Apart from making this one decision, there is no middle ground. Students often think that there might be a situation where betting two tokens is 'best', but the optimal strategy is 'if the odds are in your favour', then you should bet the maximum, if the odds are against you, you should bet the minimum.

Interestingly, in social gaming within our community, all the games are biased against the player, so the optimal strategy is to bet the minimum which is ZERO.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Time Swing

This task is built around the mathematician's question *What happens if...?* In this case:

- What happens if we change the length of the pendulum?
- What happens if we change the mass on the end?
- What happens if we change the angle through which the pendulum is first raised?

It involves a considerable amount of time measurement in seconds, the application of average and measurement of the length of the pendulum. It can develop into an extensive investigation and is easily adapted to a whole class situation. Some students will make their pendulums by slipping the nut onto the string and doubling the string to capture it at the centre. Others will want to tie their string from the highest point in the classroom.

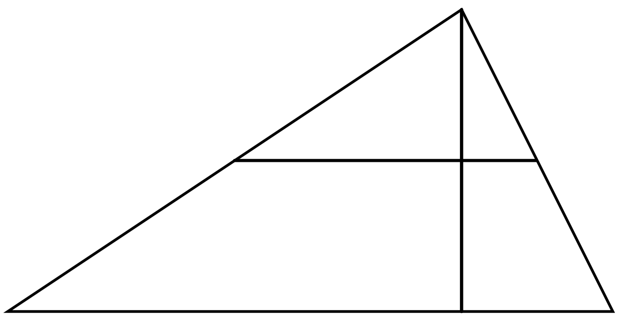
Students could research pendulums in popular science books and gather examples (if only in photo form) of pendulums in clocks, metronomes and other practical situations. In later years they are likely to meet a formula linking the variables in a pendulum situation and may relate it to their own earlier experiments.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Triangle Area

The first problem for some students will be making a triangle from the pieces. This is not necessarily an easy spatial task. In each case the pieces have been made by dropping a perpendicular from the apex, then, half way down the perpendicular, drawing a line parallel to the base.

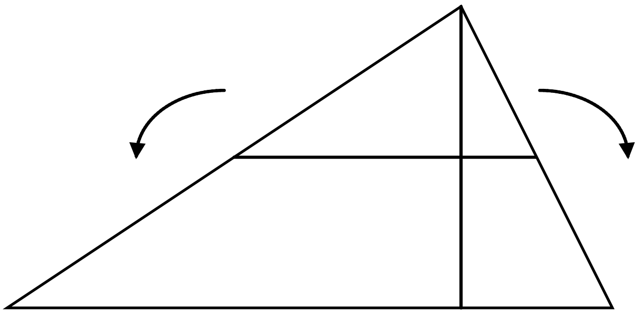


From here, each of the top pieces rotates to make the rectangle.

The task helps students work towards the understanding that because:

- the area of the triangle and the rectangle are the same, and
- the base of the triangle is the same as the base of the rectangle, and

- the height of the rectangle is half the height of the triangle



then:

Area of a triangle

= Area of a rectangle with the same base

as the triangle, but half its height.

This is an important discovery because mathematicians do know how to find the area of a rectangle.

There is a short video of a Year 5 student discovering the transformation at:

- http://www.mathematicscentre.com/mathematicscentre/cubetube.htm

At the same link there is an Investigation Guide that will support students to explore this task further.

Volume Line Up

The value of this task is the extensive range of three dimensional objects supplied with the task. It is unusual for students to be able to explore such a variety so the task is a full one that encourages extensive experimentation and reporting.

Students will discuss and discover relationships between the objects, for example, the cone is so much taller than the hemisphere, but they appear to hold the same amount of rice. Also it seems that it takes three fills of the square-based pyramid to fill the cube.

The task encourages estimation before measurement and suggests that capacity is the word used when measuring the inside of an object with a fluid.

Extensions

- Volume of large spaces is normally measured by cubes that are one metre in each direction. Ask students to estimate and check the volume of the classroom.

Where Is The Rectangle?

In essence this task applies the mathematician's strategy of working backwards. Taking the example on the card, if you know the area of the rectangle is 12, the rectangle could still be 3 by 4 or 6 by 2. On this geoboard, it can't be 12 by 1. Of course this thinking doesn't come into play until you find a starting corner by guess check and improve.

The task involves multiple mathematics:

- Recognition and application of the ordered pair convention
- Recording data
- Application of problem solving strategies
- Making and testing hypotheses
- Factors, multiples and primes
- Area and length measurement

It is also a task students enjoy returning to so they can try the challenge again.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Lesson Comments

- These comments introduce you to each Maths300 lesson. The complete plan is easily accessed through the lesson library available to members at:
  http://www.maths300.com
  where they are listed alphabetically by lesson name.

Beetle Game

In this lesson a popular old parlour game gains new life as a probability based investigation. Two players race to complete their beetle by rolling dice. Each must first roll a six to get a body before they can start to build the other parts of the beetle. Great fun to play, but consider the mathematical investigations that can grow from it. What is the average game length (number of rolls) to complete the beetle? What is the least number of rolls? What is the greatest number? These questions can be attacked through first hand data (ie: empirically), through computer simulation and theoretically, thereby opening the learning door for a broad range of students.

If you decide to use the very colourful Beetle provided in the lesson you will need to access a colour printer in advance and may also need to arrange for lamination. It is suggested on the Beetle Board that the students can do the cutting to prepare the game and the pieces can be packed in press-seal bags to make a class set. Of course, students can play the game simply by drawing the Beetle on scrap paper.

> *I found using the cut-outs for the demonstration and then letting the students use them for the first few games was very effective. Later they just drew the beetles on paper.*

Software is provided to extend the investigation.

Country Maps

People carry visual mental maps of their own country. Such maps are constructed from personal experiences over a long period of time. But how accurate are they? This lesson encourages students to peer into these personal visual constructs and to check them against reality. In doing so, the concept of ratios and area are explored. The use of estimation, visual learning, problem solving, small group discussion and links to geography all act to make this a powerful and interesting lesson.

From a professional development point of view, the lesson treatment contrasts strongly with 'text-book' treatment of the topic of ratios. It is in analysing these contrasts that significant debate about effective teaching techniques can develop.

The lesson is written around an Australian context - Victoria is given the area of one unit, how big are the other states? - but can be easily adapted for any regional area or country. Maps are provided for both Australia and the United States. These can be printed as sheets for the students and as overhead transparencies or electronic whiteboard slides for the teacher.

Duelling Dice

Two players have four unusual dice - the faces have some numbers that are non-standard for a dice and on some of them numbers are repeated. Teachers will need to prepare these dice in advance from wooden cubes.

Each player selects one of the colours. Then both players roll their dice - the higher number wins a point. Ignoring draws, the first to 7 points wins the game.

- Which is the better dice to choose?

This task is a classic 'tip of an iceberg'. Exploring any two dice provides an unexpected and counter intuitive result, but what if three dice were used, then which is the best? ... or ... What if all four dice are rolled?

These additional problems open up an even richer array of possibilities, which can all be pursued through:

- playing the game, or
- computer simulation, or
- theoretical analysis.

Software is provided to extend the investigation.

Feet-uring Mathematics

The lesson provides a reproducible sheet which is a pair of bare feet. This may seem an unusual starting point for a series of mathematics lessons, but perhaps that is exactly what captures the students initial interest. First the class discusses characters who might 'belong' to the feet. Then groups draw their selected one in large size. These characters have a length (height) and the estimation, measurement and comparison starts from there. The data that comes from the measurement can be displayed and that opens further opportunities for the development of mathematical skills.

Beyond all this, the lesson can be the source of cross-curriculum activity in art, language and a range of other areas. There are many specific suggestions for this at the end of the lesson plan.

Greedy Pig

This popular class dice game has, with small adjustments, been extended into a whole class investigation. The class stands up, the teacher rolls a dice and students score whatever is rolled. This is repeated, with students aggregating their scores. They decide when to sit down and hence keep their score to that stage. However, if the number 2 is rolled while they are standing up their entire score for that round is zero. Playing five rounds makes one game. The investigative question then becomes *When is the best time to sit down?*

The lesson includes:

- mental arithmetic
- informal probability experiences such as the contribution of short term variability to the long run theoretical probability of a dice number coming up one in six rolls
- stem and leaf graphs to display data
- statistics used to analyse and compare the data.

All of this happens within the problem solving context above, and software allows thousands of games of a particular strategy to be played in real time. The software also introduces the data display technique of box and whisker plots. There is ample opportunity for the students to write a report on their investigation. This could include screen captures of their various software experiments.

Have A Hexagon

The game board supplied is three hexagons, each divided into six sections. The numbers which have been placed in the sections are the possible products of two dice. Players select the hexagon they think will have all its products come up first. The activity is easy to play empirically, but it is harder to explain the counter intuitive result. The software broadens the investigative opportunities. The software also enhances the lesson with its graphic presentation and by generating results, but in addition it offers opportunity for exploring *What happens if...?* questions such as *Can we alter the playing board to make a fair game?*.

As a game, students learn a lot about intuitive probability and have the opportunity to practise some of their times tables, but it is in analysing the long run results that the full potential of the task becomes apparent.

Planets

The wonders hidden in the creation of the solar system are revealed in this whole class lesson which uses a classroom size scale to map the planets in relation to the sun. There is a pattern in the distances of the planets from the sun, and it is estimation, mental strategies and proportional reasoning that reveal it, rather than number-perfect calculation. As a result it allows students who may be 'computation negative' an opportunity to participate. The distances are provided for printing as an overhead transparency, poster, or electronic whiteboard display.

The lesson can flow into a further comparison of the diameters of the planets and may develop into a science unit. Some teachers enlarge the scale and use the school grounds to build the model. Few remain untouched by the visual image of the magnificence of creation, and the apparently insignificant place of themselves within it.

Red & Black Card Game

Two students each have half a deck of number or playing cards. The total starts at zero. Players takes turns to play their top card and add it to the total. Red cards are negative, black cards positive. Ace counts as 1. Court cards (J, Q, K) count as 10. In order to win, one player is trying to make the running total exceed +15 (ie: 15 in the black direction), the other player -15 (15 in the red direction).

One pack between four works if one pair uses hearts and spades and the other uses diamonds and clubs. If playing cards are unavailable use numeral cards printed in two colours.

This simple card game provides basic practice in integer arithmetic and exposure to probability and statistics concepts through an interesting problem solving tweak. Students are asked to design a variation of the game which (on average) lasts for their chosen number of turns. The process of doing this addresses learning outcomes in language, art and technology as well as mathematics.

Temperature Graphs

At first glance, this lesson doesn't seem dramatically different from traditional practice. However, it does contain several elegant and significant features which contrast somewhat with that traditional approach.

Students in small groups are given a number of graphs representing the average temperatures of various cities over a one-year period. They are challenged as a group to match each graph to a city and to justify their choice. The lesson actively uses a relevant context (regional geography), problem solving and co-operative group work to develop graph reading skills. This approach directly contrasts with a traditional 'text-book' approach.

Extensions using computer support allow the teacher and student to explore a range of similar challenges. The software has an option to enter data (which would have to be obtained from the web, or the weather bureau) for cities relevant to the local country or a country being studied in geography.

This Goes With This

The lesson uses the students' own survey data to demonstrate links between vulgar fractions and percent, and strip graphs, circle graphs and pie charts. The simplicity of the demonstration often results in an *Oh is that how they figure it out?* response. Rarely is there as powerful an illustration which makes important mathematical concepts and their integration so clear and understandable. The meaning of percentage when applied to a pie chart becomes a highly visual moment.

If the data used is the students' personal contribution, then the students own bodies can represent their piece of data. They can become that data on a bar graph, strip graph or circle graph and thereby be physically involved in the lesson.

With the understanding gained through this lesson, the class can collect, display, analyse and criticise examples of graphs in the media. In this way a broader unit can be developed.

The lesson plan includes excellent classroom contributions from very different classrooms.

Part 3:
Value
Adding

The Poster Problem Clinic

Maths With Attitude kits offer several models for building a Working Mathematically curriculum around tasks. Each kit uses a different model, so across the range of 16 kits, teachers' professional learning continues and students experience variety. The Poster Problem Clinic is an additional model. It can be used to lead students into working with tasks, or it can be used in a briefer form as a opening component of each task session.

> *I was apprehensive about using tasks when it seemed such a different way of working. I felt my children had little or no experience of problem solving and I wanted to prepare them to think more deeply. The Clinic proved a perfect way in.*

Careful thought needs to be given to management in such lessons. One approach to getting the class started on the tasks and giving it a sense of direction and purpose is to start with a whole class problem. Usually this is displayed on a poster that all can see, perhaps in a Maths Corner. Another approach is to print a copy for each person. A Poster Problem Clinic fosters class discussion and thought about problem solving strategies.

Starting the lesson this way also means that just prior to liberating the students into the task session, they are all together to allow the teacher to make any short, general observations about classroom organisation, or to celebrate any problem solving ideas that have arisen.

One teacher describes the session like this:

> *I like starting with a class problem - for just a few minutes - it focuses the class attention, and often allows me to introduce a particular strategy that is new or needs emphasis.*

It only takes a short time to introduce a poster and get some initial ideas going. The class discussion develops a way of thinking. It allows class members to hear, and learn from their peers, about problem solving strategies that work for them.

> *If we don't collectively solve the problem in 5 minutes, I will leave the problem 'hanging' and it gives a purpose to the class review session at the end.*
> *Sometimes I require everyone to work out and write down their solution to the whole class problem. The staggered finishing time for this allows me to get organised and help students get started on tasks without being besieged.*
> *I try to never interrupt the task session, but all pupils know we have a five minute review session at the end to allow them to comment on such things as an activity they particularly liked. We often close then with an agreed answer to our whole class problem.*

A Clinic in Action

The aims of the regular clinic are:

- to provide children with the opportunity to learn a variety of strategies
- to familiarise children with a process for solving problems.

The following example illustrates a structure which many teachers have found successful when running a clinic.

Preparation

For each session teachers need:

- a Strategy Board as below
- a How To Solve A Problem chart as below
- to choose a suitable problem and prepare it as a poster
- to organise children into groups of two or three.

The Strategy Board can be prepared in advance as a reference for the children, or may be developed *with* the children as they explore problem solving and suggest their own versions of the strategies.

The problem can be chosen from

- a book
- the task collection
- prepared collections such as Professor Morris Puzzles which can be viewed at: http://www.mathematicscentre.com/taskcentre/resource.htm#profmorr

The example which follows is from the task collection. The teacher copied it onto a large sheet of paper and asked some children to illustrate it. *The teacher also changed the number of sheep to sixty* to make the poster a little different from the one in the task collection.

The Strategy Board and the How To Solve A Problem chart can be used in any maths activity and are frequently referred to in Maths300 lessons.

The Clinic

The poster used for this example session is:

> Eric the Sheep is lining up to be shorn before the hot summer ahead. There are sixty [60] sheep in front of him. Eric can't be bothered waiting in the queue properly, so he decides to sneak towards the front.
>
> Every time one [1] sheep is taken to be shorn, Eric <u>then</u> sneaks past two [2] sheep. How many sheep will be shorn <u>before</u> Eric?

This Poster Problem Clinic approach is also extensively explored in Maths300 Lesson 14, *The Farmer's Puzzle*.

Strategy Board

DO I KNOW A SIMILAR PROBLEM?	ACT IT OUT
GUESS, CHECK AND IMPROVE	DRAW A PICTURE OR GRAPH
TRY A SIMPLER PROBLEM	MAKE A MODEL
WRITE AN EQUATION	LOOK FOR A PATTERN
MAKE A LIST OR TABLE	TRY ALL POSSIBILITIES
WORK BACKWARDS	SEEK AN EXCEPTION
BREAK INTO SMALLER PARTS	...

How To Solve A Problem

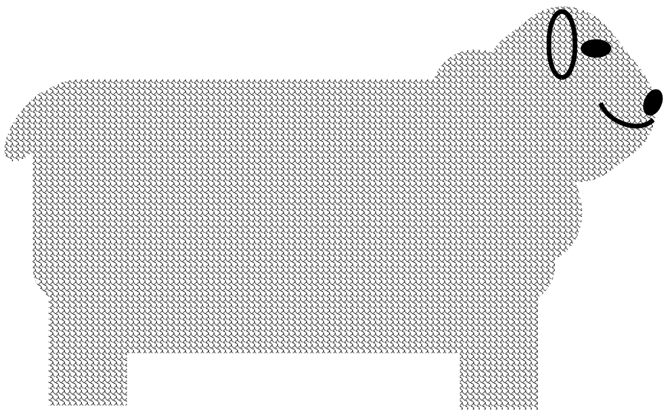
SEE & UNDERSTAND	Do I understand what the problem is asking? Discuss
PLANNING	Select a strategy from the board. Plan how you intend solving the problem.
DOING IT	Try out your idea.
CHECK IT	Did it work out? If so reflect on the activity. If not, go back to step one.

Step 1

- Tell the children that we are at Stage 1 of our four stage plan ... **See & Understand** ... Point to it! Read the problem with the class. Discuss the problem and clarify any misunderstandings.
- If children do not clearly understand what the problem is asking, they will not cope with the next stage. A good way of finding out if a child understands a problem is for her/him to retell it.
- Allow time for questions - approximately 3 to 5 minutes.

Step 2

- Tell the children that we are at Stage 2 of our four stage plan ... **Planning**. In their groups children select one or more strategies from the Strategy Board and discuss/organise how to go about solving the problem.
- Without guidance, children will often skip this step and go straight to Doing It. It is vital to emphasise that this stage is simply planning, not solving, the problem.
- After about 3 minutes, ask the children to share their plans.



Plan 1

Well we're drawing a picture and sort of making a model.

Can you give me more information please Brigid?

We're putting 60 crosses on our paper for sheep and the pen top will be Eric. Then Claire will circle one from that end, and I will pass two crosses with my pen top.

Plan 2

Our strategy is Guess and Check.

That's good Nick, but how are you going to check your guess?

Oh, we're making a model.

Go on ...

> *John's getting MAB smalls to be sheep and I'm getting a domino to be Eric and the chalk box to be the shed for shearing.*

Plan 3

> *We are doing it for 3 sheep then 4 sheep then 5 sheep and so on. Later we will look at 60.*

Great so you are going to try a simpler problem, make a table and look for a pattern.

This sharing of strategies is invaluable as it provides children who would normally feel lost in this type of activity with an opportunity to listen to their peers and make sense out of strategy selection. Note that such children are not given the answer. Rather they are assisted with understanding the power of selecting and applying strategies.

Step 3

- Tell the children that we are at Stage 3 of our four stage plan ... **Doing It**. Children collect what they need and carry out their plan.

Step 4

- Tell the children that we are at Stage 4 of our four stage plan ... **Check It**. Come together as a class for groups to share their findings. Again emphasis is on strategies.

> *We used the drawing strategy, but we changed while we were doing it because we saw a pattern.*

So Jake, you used the Look For A Pattern strategy. What was it?

> *We found that when Eric passed 10 sheep, 5 had been shorn, so 20 sheep meant 10 had been shorn ... and that means when Eric passes 40 sheep, 20 were shorn and that makes the 60 altogether.*

Great Jake. How would you work out the answer for 59 sheep or 62 sheep?

Sharing time is also a good opportunity to add in a strategy which no one may have used. For example:

> *Maybe we could've used the Number Sentence strategy, ie: 1 sheep goes to be shorn and Eric passes two sheep. That's 3 sheep, so perhaps, 60 divided into groups of 3, or 60 ÷ 3 gives the answer.*

Round off the lesson by referring to the Working Mathematically chart. There will be many opportunities to compliment the students on working like a mathematician.

Curriculum Planning Stories

Our attitude is:

teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

In more than a decade of using tasks and many years of using the detailed whole class lessons of Maths300, teachers have developed several models for integrating tasks and whole class lessons. Some of those stories are retold here. Others can be found at:

- http://www.mathematicscentre.com/taskcentre/plans.htm

Story 1: Threading

Educational research caused me a dilemma. It tells us that students construct their own learning and that this process takes time. My understanding of the history of mathematics told me that certain concepts, such as place value and fractions, took thousands of years for mathematicians to understand. The dilemma was being faced with a textbook that expected students to 'get it' in a concentrated one, two or three week block of work and then usually not revisit the topic again until the next academic year.

A Working Mathematically curriculum reflects the need to provide time to learn in a supportive, non-threatening environment and...

When I was involved in a Calculating Changes PD program I realised that:

- *choosing rich and revisitable activities, which are familiar in structure but fresh in challenge each time they are used, and*
- *threading them through the curriculum over weeks for a small amount of time in each of several lessons per week*

resulted in deeper learning, especially when partnered with purposeful discussion and recording.

Calculating Changes:

- http://www.mathematicscentre.com/calchange

Story 2: Your turn

Some teachers are making extensive use of a partnership between the whole class lessons of Maths300 and small group work with the tasks. Setting aside a lesson for using the tasks in the way they were originally designed now seems to have more meaning, as indicated by this teacher's story:

When I was thinking about helping students learn to work like a mathematician, my mind drifted to my daughter learning to drive. She *needed me to model how to do it and then she needed lots of opportunity to try it for herself.*

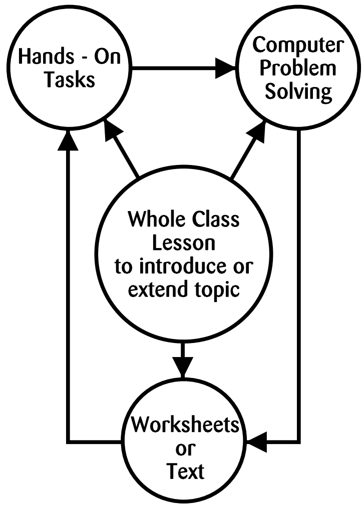
That's when the idea clicked of using the Maths300 lessons as a model and the tasks as a chance for the students to have their turn to be a mathematician.

The Maths300 lessons illustrate how other teachers have modelled the process, so I felt I could do it too. Now the process is always on display on the wall or pasted inside the student's journal.

A session just using the tasks had seemed a bit like play time before this. Now I see it as an integral part of learning to work mathematically.

Story 3: Mixed Media

It was our staff discussion on Gardner's theory of Multiple Intelligences that led us into creating mixed media units. That and the access you have provided to tasks and Maths300 software.



We felt challenged to integrate these resources into our syllabus. There was really no excuse for a text book diet that favours the formal learners. We now often use four different modes of learning in the work station structure shown. It can be easily managed by one teacher, but it is better when we plan and execute it together.

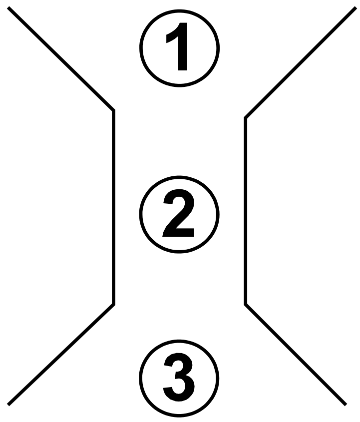
Story 4: Replacement Unit

We started meeting with the secondary school maths teachers to try to make transition between systems easier for the students. After considerable discussion we contracted a consultant who suggested that school might look too much the same across the transition when the students were hoping for something new. On the other hand our experience suggested that there needed to be some consistency in the way teachers worked.

We decided to 'bite the bullet' and try a hands-on problem solving unit in one strand. We selected two menus of twenty hands-on tasks, one for the primary and one for the secondary, that became the core of the unit. We deliberately overlapped some tasks that we knew were very rich and added some new ones for the high school.

Class lessons and investigation sheets were used to extend the tasks, within a three week model.

It is important to note that although these teachers structured a 3 week unit for the students, they strongly advised an additional *Week Zero* for staff preparation. The units came to be called Replacement Units.



Week Zero - Planning

Staff familiarise themselves with the material and jointly plan the unit. This is not a model that can be 'planned on the way to class'.

> *Getting together turned out to be great professional development for our group.*

Week 1 - Introduction

Students explore the 20 tasks listed on a printed menu:

- students explore the tip of the task, as on the card
- students move from task to task following teacher questioning that suggests there is more to the task than the tip
- in discussion with students, teachers gather informal assessment information that guides lesson planning for the following week.

> *We gave the kids an 'encouragement talk' first about joining us in an experiment in ways of learning maths and then gave out the tasks. The response was intelligent and there was quite a buzz in the room.*

Week 2 - Formalisation

> *It was good for both us and the students that the lessons in this week were a bit more traditional. However, they weren't text book based. We used whole class lessons based on the tasks they had been exploring and taught the Working Mathematically process, content and report writing.*
>
> *Assessment was via standard teacher-designed tests, quizzes and homework.*

Week 3 - Investigations

> *We were most delighted with Week 3. Each student chose one task from the menu and carried out an in-depth investigation into the iceberg guided by an investigation sheet. They had to publish a report of their investigation and we were quite surprised at the outcomes. It was clear that the first two weeks had lifted the image of mathematics from 'boring repetition' to a higher level of intellectual activity.*

Story 5: Curriculum shift

> *I think our school was like many others. The syllabus pattern was 10 units of three weeks each through the year. We had drifted into that through a text book driven curriculum and we knew the students weren't responding.*

Our consultant suggested that there was sameness about the intellectual demands of this approach which gave the impression that maths was the pursuit of skills. We agreed to select two deeper investigations to add to each unit. It took some time and considerable commitment, but we know that we have now made a curriculum shift. We are more satisfied and so are the students.

The principles guiding this shift were:

- Agree

  The 20 particular investigations for the year are agreed to by all teachers. If, for example, Cube Nets *is decided as one of these, then all the teachers are committed to present this within its unit.*

- Publish

  The investigations are written into the published syllabus. Students and parents are made aware of their existence and expect them to occur.

- Commit

  Once agreed, teachers are required to present the chosen investigations. They are not a negotiable 'extra'.

- Value

  The investigations each illustrate an explicit form of the Working Mathematically process. This is promoted to students, constantly referenced and valued.

- Assess

  The process provides students with scaffolding for their written reports and is also known by them as the criteria for assessment. *(See next page.)*

- Report

  The assessment component features within the school reporting structure.

A Final Comment

Including investigations has become policy.

Why? Because to not do so is to offer a diminished learning experience.

The investigative process ranks equally with skill development and needs to be planned for, delivered, assessed and reported.

Perhaps most of all we are grateful to our consultant because he was prepared to begin where we were. We never felt as if we had to throw out the baby and the bath water.

Assessment

Our attitude is:

stimulated students are creative and love to learn

Regardless of the way you use your **Maths With Attitude** resource, a variety of procedures can be employed to assess this learning.

Where these assessment procedures are applied to task sessions and involve written responses from students, teachers will need to be careful that the writing does not become too onerous. Students who get bogged down in doing the writing may lose interest in doing the tasks.

In addition to the ideas below, useful references are:

- http://www.mathematicscentre.com/taskcentre/assess.htm
- http://www.mathematicscentre.com/taskcentre/report.htm

The first offers several methods of assessment with examples and the second is a detailed lesson plan to support students to prepare a Maths Report.

Journal Writing

Journal writing is a way of determining whether the task or lesson has been understood by the student. The pupil can comment on such things as:

- What I learned in this task.
- What strategies I/we tried (refer to the Strategy Board).
- What went wrong.
- How I/we fixed it.
- Jottings - ie: any special thoughts or observations

Some teachers may prefer to have the page folded vertically, so that children's reflective thoughts can be recorded adjacent to critical working.

Assessment Form

An assessment form uses questions to help students reflect upon specific issues related to a specific task.

Anecdotal Records

Some teachers keep ongoing records about how students are tackling the tasks. These include jottings on whether students were showing initiative, whether they were working co-operatively, whether they could explain ideas clearly, whether they showed perseverance.

Checklists

A simple approach is to create a checklist based on the Working Mathematically process. Teachers might fill it in following questioning of individuals, or the students may fill it in and add comments appropriately.

Pupil Self-Reflection

Many theorists value and promote metacognition, the notion that learning is more permanent if pupils deliberately and consciously analyse their own learning. The deliberate teaching strategy of oral questioning and the way pupils record their work is an attempt to manifest this philosophy in action. The alternative is the tempting 'butterfly' approach which is to madly do as many activities as possible, mostly superficially, in the mistaken belief that quantity equates to quality.

> *I had to work quite hard to overcome previously entrenched habits of just getting the answer, any answer, and moving on to the next task.*

Thinking about *what* was learned *how* it was learned consolidates and adds to the learning.

When it follows an extensive whole class investigation, a reflection lesson such as this helps to shift entrenched approaches to mathematics learning. It is also an important component of the assessment process. On the one hand it gives you a lot of real data to assist your assessment. On the other it prepares the students for any formal assessment which you may choose to round off a unit.

Introduction

Ask students to recall what was done during the unit or lesson by asking a few individuals to say what *they* did, eg:

> *What did you do or learn that was new?*
> *What can you now do/understand that is new?*
> *What do you know now that you didn't know 1 (2, 3, ...) lesson ago?*

Continuing Discussion

Get a few ideas from the first students you ask, then:

- organise 5 -10 minute buzz groups of three or four students to chat together with one person to act as a recorder. These groups address the same questions as above.
- have a reporting session, with the recorder from each group telling the class about the group's ideas.

Student comments could be recorded on the board, perhaps in three groups.

<u>Ideas & Facts</u>   <u>Maths Skills</u>   <u>Process (learning) Skills</u>

If you need more questions to probe deeper and encourage more thought about process, try the following:

> *What new things did you do that were part of* how *you learned?*
> *Who uses this kind of knowledge and skill in their work?*

Student Recording

Hand out the REFLECTION sheet (next page) and ask students to write their own reflection about what they did, based on the ideas shared by the class. Collect these for interest and, possibly, assessment information.

REFLECTION

me looking at me learning

NAME: CLASS:

Working With Parents

Balancing Problem Solving with Basic Skill Practice

Many schools find that parents respond well to an evening where they have an opportunity to work with the tasks and perhaps work a task together as a 'whole class'. Resourced by the materials in this kit, teachers often feel quite confident to run these practical sessions. Comments from parents like:

> *I wish I had learnt maths like this.*

are very supportive. Letting students 'host' the evening is an additional benefit to the home/school relationship.

The 4½ Minute Talk

Charles Lovitt has considerable experience working with parents and has developed a crisp, parent-friendly talk which he shares below. Many others have used it verbatim with great success.

Why the Four and a Half Minute Talk?

When talking with parents about Problem Solving or the meaning of the term Working Mathematically, I have often found myself in the position, after having promoted inquiry based or investigative learning, of the parents saying:

> *Well - that's all very well - BUT ...*

at which stage they often express their concern for basic (meaning arithmetic) skill development.

The weakness of my previous attempts has been that I have been unable to reassure parents that problem solving does not mean sacrificing our belief in the virtues of such basic skill development.

One of the unfortunate perceptions about problem solving is that if a student is engaged in it, then somehow they are not doing, or it may be at the expense of, important skill based work.

This Four and a Half Minute Talk to parents is an attempt to express my belief that basic skill practice and problem solving development can be closely intertwined and not seen as in some way mutually exclusive.

(I'm still somewhat uncomfortable using the expression 'basic skills' in the above way as I am certain that some thinking, reasoning, strategy and communication skills are also 'basic'.)

Another aspect of the following 'talk' is that, as teachers put more emphasis on including investigative problem solving into their courses, a question arises about the source of suitable tasks.

This talk argues that we can learn to create them for ourselves by 'tweaking' the closed tasks that heavily populate out existing text exercises, and hence not be dependent on external suppliers. (Even better if students begin to create such opportunities for themselves.)

The Talk

In preparation, write the following graphic on the board:

CLOSED	OPEN	EXTENDED INVESTIGATION
		How many solutions exist? How do you know you have found them all?

I would like to show you what teachers are beginning to do to achieve some of the thinking and reasoning and communication skills we hope students will develop. I would like to show you three examples.

Example One: *6 + 5 = ?*

I write this question under the 'closed' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 ___		How many solutions exist? How do you know you have found them all?

And I ask:

What is the answer to this question?

I then explain that:

We often ask students many closed questions such as 6 + 5 = ?

The only response the students can tell us is "The answer is 11." … and as a reward for getting it correct we ask another twenty questions just like it.

What some teachers are doing is trying to *tweak* the question and ask it a different way, for example:

I have two counting numbers that add to 11. What might the numbers be?

[Counting numbers = positive whole numbers including zero]

I write this under the 'open' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 ___	? + ? 11	How many solutions exist? How do you know you have found them all?

What is the answer to the question now?

At this stage it becomes apparent there are several solutions:

The question is now a bit more open than it was before, allowing students to tell you things like 8 + 3, or 10 + 1, or 11 + 0 etc.

Let's see what happens if the teacher 'tweaks' it even further with the investigative challenge *or* extended investigation question:

How many solutions are there altogether?

and more importantly, and with greater emphasis on the second question:

How could you convince someone else that you have found them all?

Now the original question is definitely different - it still involves the skills of addition but now also involves thinking, reasoning and problem solving skills, strategy development and particularly communication skills.

Young students will soon tell you the answer is 'six different ones', but they must also confront the communication and reasoning challenge of convincing you that there are only six and no more.

Example Two: *Finding Averages*

Again, as I go through this example, I write it into the diagram on the board in the relevant sections.

The CLOSED question is: *11, 12, 13 - find the average*

Tweaking this makes it an OPEN question and it becomes:

I have three counting numbers whose average is 12. What might the numbers be?

Students will often say:

10, 12, 14 … or 9, 12, 15 … or even 12, 12, 12

After realising there are many answers, you can tweak it some more and turn it into an EXTENDED INVESTIGATION:

How many solutions exist? … AND …

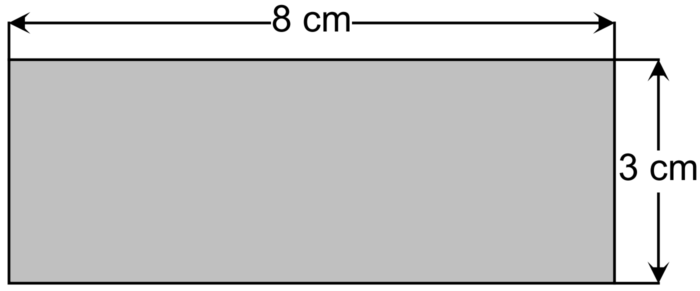
How do you know you have found them all?

Now the question is of a quite different nature. It still involves the arithmetic skill, but has something else as well - and that something else is the thinking, reasoning and communication skills necessary to find all of the combinations and convince someone else that you have done so.

By the time a student announces, with confidence, there are 127 different ways (which there are) that student will have engaged in all of these aspects, ie: the skill of calculating averages, (and some combination number theory) as well as significant strategy and reasoning experiences.

Example Three: *Finding the Area of a Rectangle*

A typical CLOSED question is:



Find the area. Find the perimeter.

The OPEN question is:

A rectangle has 24 squares inside:

What might its length and width be?

What might its perimeter be?

The EXTENDED INVESTIGATION version is:

Given they are whole number lengths, how many different rectangles are there? … AND …

How do you know you have found them all?

In summary, mathematics teachers are trying to convert *some* (not all) of the many closed questions that populate our courses and 'push' them towards the investigation direction. In doing so, we keep the skills we obviously value, but also activate the thinking, reasoning and justification skills we hope students will also develop.

This sequence of three examples hopefully shows two major features:

- That skills and problem solving can 'live alongside each other' and be developed concurrently.
- That the process of creating open-ended investigations can be done by anyone - just go to any source of closed questions and try 'tweaking' them as above. If it only worked for one question per page it would still provide a very large supply of investigations.

In terms of the effect of the talk on parents, I have usually found them to be reassured that we are not compromising important skill development (and nor do we want to). The only debate then becomes whether the additional skills of thinking, reasoning and communication are also desirable.

I've also been told that parents appreciate it because of the essential simplicity of the examples - no complicated theoretical jargon.

A Working Mathematically Curriculum

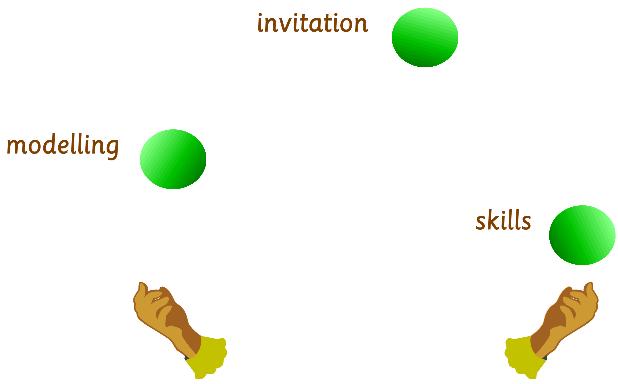
An Investigative Approach to Learning

The aim of a Working Mathematically curriculum is to help students learn to work like a mathematician. This process is detailed earlier (Page 8) in a one page document which becomes central to such a curriculum.

The change of emphasis brings a change of direction which *implies and requires* a balance between:

- the process of being a mathematician, and
- the development of skills needed to be a *successful* mathematician.

This journey is not two paths. It is one path made of two interwoven threads in the same way as DNA, the building block of life, is one compound made of two interwoven coils. To achieve a Working Mathematically curriculum teachers need to balance three components.



The task component of **Maths With Attitude** offers each pair of students an invitation to work like a mathematician.

The Maths300 component of **Maths With Attitude** assists teachers to model working like a mathematician.

Content skills are developed in context. They *are* important, but it is the application of skills within the process of Working Mathematically that has developed, and is developing, the human community's mathematical knowledge.

A focus for the Working Mathematically teacher is to help students develop mathematical skills in the context of problem posing and solving.

> *We are all 'born' with the same size mathematical toolbox, in the same way as I can own the same size toolbox as my motor mechanic. However, my motor mechanic has many more tools in her box than I and she has had more experience than I using them in context. Someone has helped her learn to use those tools while crawling under a car.*

Afzal Ahmed, Professor of Mathematics at Chichester, UK, once quipped:

> *If teachers of mathematics had to teach soccer, they would start off with a lesson on kicking the ball, follow it with lessons on trapping the ball and end with a lesson on heading the ball. At no time would they play a game of football.*

Such is not the case when teaching a Working Mathematically curriculum.

Elements of a Working Mathematically Curriculum

Working Mathematically is a K - 12 experience offering a balanced curriculum structured around the components below.

Hands-on Problem Solving Play

Mathematicians don't know the answer to a problem when they start it. If they did, it wouldn't be a problem. They have to play around with it. Each task invites students to play with mathematics 'like a mathematician'.

Skill Development

A mathematician needs skills to solve problems. Many teachers find it makes sense to students to place skill practice in the context of *Toolbox Lessons* which *help us better use the Working Mathematically Process* (Page 8).

Focus on Process

This is what mathematicians do; engage in the problem solving process.

Strategy Development

Mathematicians also make use of a strategy toolbox. These strategies are embedded in Maths300 lessons, but may also have a separate focus. Poster Problem Clinics are a useful way to approach this component.

Concept Development

A few major concepts in mathematics took centuries for the human race to develop and apply. Examples are place value, fractions and probability. In the past students have been expected to understand such concepts after having 'done' them for a two week slot. Typically they were not visited again until the next year. A Working Mathematically curriculum identifies these concepts and regularly 'threads' them through the curriculum.

Planning to Work Mathematically

The class, school or system that shifts towards a Working Mathematically curriculum will no longer use a curriculum document that looks like a list of content skills. The document would be clear in:

- choosing genuine problems to initiate investigation
- choosing a range of best practice teaching strategies to interest a wider range of students
- practising skills for the purpose of problem solving

Some teachers have found the planning template on the next page assists them to keep this framework at the forefront of their planning. It can be used to plan single lessons, or units built of several lessons. There are examples from schools in the Curriculum & Planning section of Maths300 and a Word document version of the template.

Unit Planning Page

Class

Topic

Pedagogy	Problem Solving	Skills
	In this topic how will I engage my students in the Working Mathematically process?	
How do I create an environment where students know what they are doing and why they have accepted the challenge?		Does the challenge identify skills to practise? Are there other skills to practise in preparation for future problem solving?

Notes

As a general guide:

- Find a problem(s) to solve related to the topic.
- Choose the best teaching craft likely to engage the learners.
- Where possible link skill practice to the problem solving process.

More on Professional Development

For many teachers there will be new ideas within **Maths With Attitude**, such as unit structures, views of how students learn, teaching strategies, classroom organisation, assessment techniques and use of concrete materials. It is anticipated (and expected) that as teachers explore the material in their classrooms they will meet, experiment with and reflect upon these ideas with a view to long term implications for the school program and for their own personal teaching.

Being explored 'on-the-job' so to speak, in the teacher's own classroom, makes the professional development more meaningful and practical for the teacher. This is also a practical and economic alternative for a local authority.

Strategic Use by Systems

From Years 3 - 10, **Maths With Attitude** is designed as a professional development vehicle by schools or clusters or systems because it carries a variety of sound educational messages. They might choose **Maths With Attitude** because:

- It can be used to highlight how investigative approaches to mathematics can be built into balanced unit plans without compromising skill development and without being relegated to the margins of a syllabus as something to be done only after 'the real' content has been covered.
- It can be used to focus on how a balance of concept, skill and application work can all be achieved within the one manageable unit structure.
- It can be used to show how a variety of assessment practices can be used concurrently to build a picture of student progress.
- It can be used to focus on transition between primary and secondary school by moving towards harmony and consistency of approach.
- It can be used to raise and continue debate about the pedagogy (art of teaching) that supports deeper mathematical learning for a wider range of students.

Teachers in Years K - 2 are similarly encouraged in professional growth through **Working Mathematically with Infants**, which derives from Calculating Changes, a network of teachers enhancing children's number skills from Years K - 6.

In supporting its teachers by supplying these resources in conjunction with targeted professional development over time, a system can fuel and encourage classroom-based debate on improving outcomes. There is evidence that by exploring alternative teaching strategies and encouraging curriculum shift towards Working Mathematically, learners improve and teachers are more satisfied. For more detail visit Research & Stories at:

- http://www.mathematicscentre.com/taskcentre/do.htm

We would be happy to discuss professional development with system leaders.

Web Reference

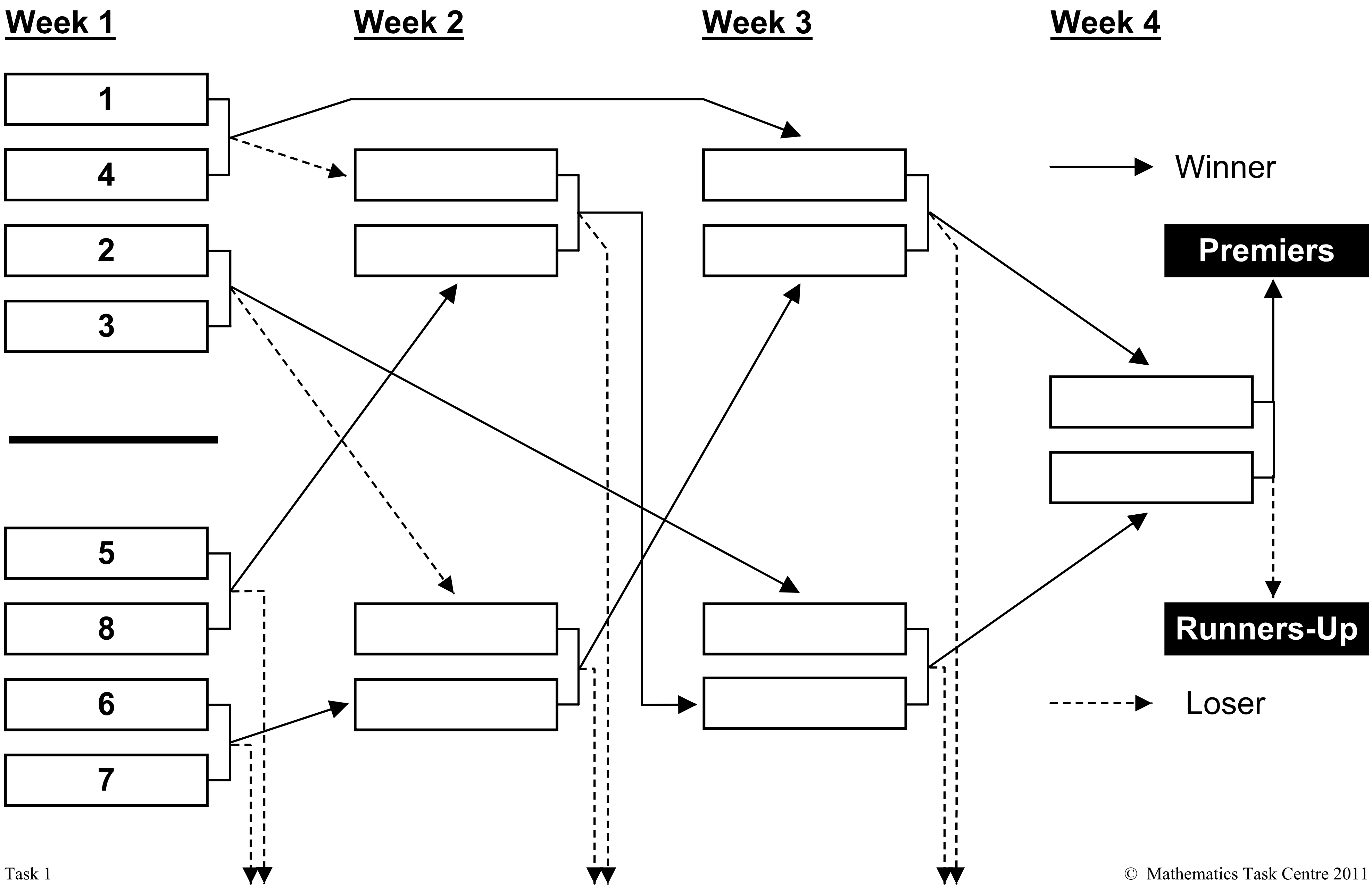
The starting point for all aspects of learning to work like a mathematician, including Calculating Changes, and the teaching craft which encourages it is:

- http://www.mathematicscentre.com/mathematicscentre

Appendix: Recording Sheets

64 = 65

FINAL EIGHT


Week 1
Week 2
Week 3
Week 4
1
4
2
3
5
8
6
7
Winner
Premiers
Runners-Up
Loser