maths with attitude

2nd Edition

Space & Logic Years 7 & 8

Charles Lovitt
Doug Williams

Mathematics Task Centre & Maths300
helping to create happy healthy cheerful productive inspiring classrooms

maths with attitude

2nd Edition

Space & Logic

Years 7 & 8

In this kit:

- *Hands-on problem solving tasks*
- *Detailed curriculum planning*

Access from Maths300:

- *Extensive lesson plans*
- *Software*

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Maths With Attitude includes reference to hands-on problem solving tasks and Maths300 material.

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- To support teachers, schools and systems wanting to create:
  happy, healthy, cheerful, productive, inspiring classrooms

Our Attitude

- to learning:
  learning is a personal journey stimulated by achievable challenge
- to learners:
  stimulated students are creative and love to learn
- to pedagogy:
  the art of choosing teaching strategies to involve and interest all students
- to mathematics:
  mathematics is concrete, visual and makes sense
- to learning mathematics:
  all students can learn to work like a mathematician
- to teachers:
  the teacher is the most important resource in education
- to professional development:
  teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

Our Objective in Detail

What do we mean by creating:

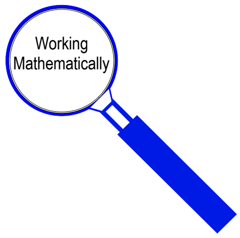
happy, healthy, cheerful, productive, inspiring classrooms

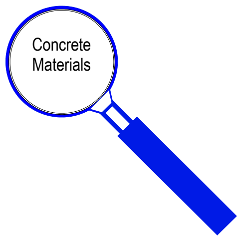
Happy…

means the elimination of the unnecessary fear of failure that hangs over so many students in their mathematics studies. Learning experiences *can* be structured so that all students see there is something in it for them and hence make a commitment to the learning. In so many 'threatening' situations, students see the impending failure and withhold their participation.

A phrase which describes the structure allowing all students to perceive something in it for them is *multiple entry points and multiple exit points*. That is, students can enter at a variety of levels, make progress and exit the problem having visibly achieved.

Healthy…





means *educationally healthy*. The learning environment should be a reflection of all that our community knows about how students learn. This translates into a rich array of teaching strategies that could and should be evident within the learning experience.

If we scrutinise the *exploration* through any lens, it should confirm to us that it is well structured or alert us to missed opportunities. For example, peering through a pedagogy lens we should see such features as:

- a story shell to embed the situation in a meaningful context
- significant active use of concrete materials
- a problem solving challenge which provides ownership for students
- small group work
- a strong visual component
- access to supportive software

Cheerful…

because we want 'happy' in the title twice!

Productive…

is the clear acknowledgment that students are working towards recognisable outcomes. They should know what these are and have guidelines to show they have either reached them or made progress. Teachers are accountable to these outcomes as well as to the quality of the learning environment.

Inspiring…

is about creating experiences that are uplifting or exalting; that actually *turn students on*. Experiences that make students feel great about themselves and empowered to act in meaningful ways.

Space & Logic Resources

To help you create

happy, healthy, cheerful, productive, inspiring classrooms

this kit contains

- 20 hands-on problem solving tasks from Mathematics Centre

and a Teachers' Manual which integrates the use of the tasks with

- 8 detailed lesson plans from Maths300

The kit offers **4 weeks** of Scope & Sequence planning in Space and Logic for *each* of Year 7 and Year 8. This is detailed in *Part 2: Planning Curriculum* which begins on Page 12. You are invited to map these weeks into your Year Planner. Together, the four kits available for these levels provide 25 weeks of core curriculum in Working Mathematically (working like a mathematician).

Note: Membership of Maths300 is assumed.
The kit will be useful without it, but it will be much more useful with it.

Tasks

- Back to Back Building
- Calendar
- Crossing The Desert
- Dividing Shapes
- Famous Mathematicians
- Football Ladder
- Hearts & Loops
- Human Moves Monster
- In The Bag
- Keith's Kubes
- Knight Swap
- Leading The Blind
- Making Triangles
- Mirror Patterns 1
- Paving Views
- Racetrack
- Reflections
- Six Square Puzzle
- Wallpaper Patterns
- Who Owns The Monkey?

Part 2 of this manual introduces each task. The latest information can be found at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm

Maths300 Lessons

- Cube Nets
- Football Ladder
- Knight's Tour
- Nim
- Police Line Up
- Red To Blue
- Spirolaterals
- String Shapes

Lessons with Software

- Knight's Tour
- Spirolaterals

Part 2 of this manual introduces each lesson. Full details can be found at:

- http://www.maths300.com

Working Like A Mathematician

Our attitude is:

all students can learn to work like a mathematician

What does a mathematician's work actually involve? Mathematicians have provided their answer on Page 8. In particular we are indebted to Dr. Derek Holton for the clarity of his contribution to this description.

Perhaps the most important aspect of Working Mathematically is the recognition that *knowledge is created by a community and becomes part of the fabric of that community*. Recognising, and engaging in, the process by which that knowledge is generated can help students to see themselves as able to work like a mathematician. Hence Working Mathematically is the framework of **Maths With Attitude.**

Skills, Strategies & Working Mathematically

A Working Mathematically curriculum places learning mathematical skills and problem solving strategies in their true context. Skills and strategies are the tools mathematicians employ in their struggle to solve problems. Lessons on skills or lessons on strategies are not an end in themselves.

- **Our skill toolbox** can be added to in the same way as the mechanic or carpenter adds tools to their toolbox. Equally, the addition of the tools is not for the sake of collecting them, but rather for the purpose of getting on with a job. A mathematician's job is to attempt to solve problems, not to collect tools that might one day help solve a problem.
- **Our strategy toolbox** has been provided through the collective wisdom of mathematicians from the past. All mathematical problems (and indeed life problems) that have ever been solved have been solved by the application of this concise set of strategies.

About Tasks

Our attitude is:

mathematics is concrete, visual and makes sense

Tasks are from Mathematics Task Centre. They are an invitation to two students to work like a mathematician (see Page 8).

The Task Centre concept began in Australia in the late 1970s as a collection of rich tasks housed in a special room, which came to be called a Task Centre. Since that time hundreds of Australian teachers, and, more recently, teachers from other countries, have adapted and modified the concept to work in their schools. For example, the special purpose room is no longer seen as an essential component, although many schools continue to opt for this facility.

A brief history of Task Centre development, considerable support for using tasks, for example Task Cameos, and a catalogue of all currently available tasks can be found at:

- http://www.mathematicscentre.com/taskcentre

Key principles are:
- A good task is the tip of an iceberg
- Each task has three lives
- Tasks involve students in the Working Mathematically process

The Task Centre Room or the Classroom?

There are good reasons for using the tasks in a special room which the students visit regularly. There are also different good reasons for keeping the tasks in classrooms. Either system can work well if staff are committed to a core curriculum built around learning to work like a mathematician.
- A task centre room creates a focus and presence for mathematics in the school. Tasks are often housed in clear plastic 'cake storer' type boxes. Display space can be more easily managed. The visual impact can be vibrant and purposeful.
- However, tasks can be more readily integrated into the curriculum if teachers have them at their finger tips in the classrooms. In this case tasks are often housed in press-seal plastic bags which take up less space and are more readily moved from classroom to classroom.

Tip of an Iceberg

The initial problem on the card can usually be solved in 10 to 20 minutes. The investigation iceberg which lies beneath may take many lessons (even a lifetime!). Tasks are designed so that the original problem reveals just the 'tip of the iceberg'. Task Cameos and Maths300 lessons help to dig deeper into the iceberg.

We are constantly surprised by the creative steps teachers and students take that lead us further into a task. No task is ever 'finished'.

> *Most tasks have many levels of entry and exit and therefore offer an on-going invitation to revisit them, and, importantly, multiple levels of success for students.*

Three Lives of a Task

This phrase, coined by a teacher, captures the full potential and flexibility of the tasks. Teachers say they like using them in three distinct ways:
1. As on the card, which is designed for two students.
2. As a whole class lesson involving all students, as supported by outlines in the Task Cameos and in detail through the Maths300 site.
3. Extended by an Investigation Guide (project), examples of which are included in both Task Cameos and Maths300.

The first life involves just the 'tip of the iceberg' of each task, but nonetheless provides a worthwhile problem solving challenge - one which 'demands' concrete materials in its solution. This is the invitation to work like a mathematician. Most students will experience some level of success and accomplishment in a short time.

The second life involves adapting the materials to involve the whole class in the investigation, in the first instance to model the work of a mathematician, but also to develop key outcomes or specific content knowledge. This involves choosing teaching craft to interest the students in the problem and then absorb them in it.

The third life challenges students to explore the 'rest of the iceberg' independently. Investigation Guides are used to probe aspects and extensions of the task and can be introduced into either the first or second life. Typically this involves providing suggestions for the direction the investigation might take. Students submit the 'story' of their work for 'portfolio assessment'. Typically a major criteria for assessment is application of the Working Mathematically process.

About Maths300

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

Maths300 is a subscription based web site. It is an attempt to collect and publish the 300 most 'interesting' maths lessons (K - 12).

- *Lessons have been successfully trialed in a range of classrooms.*
- *About one third of the lessons are supported by specially written software.*
- *Lessons are also supported by investigation sheets (with answers) and game boards where relevant.*
- *A 'living' Classroom Contributions section in each lesson includes the latest information from schools.*
- *The search engine allows teachers to find lessons by pedagogical feature, curriculum strand, content and year level.*
- *Lesson plans can be printed directly from the site.*
- *Each lesson supports teaches to model the Working Mathematically process.*

Modern internet facilities and computers allow teachers easy access to these lesson plans. Lesson plans need to be researched, reflected upon in the light of your own students and activated by collecting and organising materials as necessary.

Maths300 Software

Our attitude is:

> *stimulated students are creative and love to learn*

Pedagogically sound software is one feature likely to encourage enthusiastic learning and for that reason it has been included as an element in about one third of Maths300 lesson plans. The software is used to develop an investigation beyond its introduction and early exploration which is likely to include other pedagogical techniques such as concrete materials, physical involvement, estimation or mathematical conversation. The software is not the lesson plan. It is a feature of the lesson plan used at the teacher's discretion.

For school-wide use, the software needs to be downloaded from the site and installed in the school's network image. You will need to consult your IT Manager about these arrangements. It can also be downloaded to stand alone machines covered by the site licence, in particular a teacher's own laptop, from where it can be used with the whole class through a data projector.

Note:

- *Maths300 lessons and software may only be used by Maths300 members.*

Working Mathematically

First give me an interesting problem.

When mathematicians become interested in a problem they:

- *Play with the problem to collect & organise data about it.*
- *Discuss & record notes and diagrams.*
- *Seek & see patterns or connections in the organised data.*
- *Make & test hypotheses based on the patterns or connections.*
- *Look in their strategy toolbox for problem solving strategies which could help.*
- *Look in their skill toolbox for mathematical skills which could help.*
- *Check their answer and think about what else they can learn from it.*
- *Publish their results.*

Questions which help mathematicians learn more are:

- *Can I check this another way?*
- *What happens if ...?*
- *How many solutions are there?*
- *How will I know when I have found them all?*

When mathematicians have a problem they:

- *Read & understand the problem.*
- *Plan a strategy to start the problem.*
- *Carry out their plan.*
- *Check the result.*

A mathematician's strategy toolbox includes:

- *Do I know a similar problem?*
- *Guess, check and improve*
- *Try a simpler problem*
- *Write an equation*
- *Make a list or table*
- *Work backwards*
- *Act it out*
- *Draw a picture or graph*
- *Make a model*
- *Look for a pattern*
- *Try all possibilities*
- *Seek an exception*
- *Break a problem into smaller parts*
- *...*

If one way doesn't work, I just start again another way.

Professional Development Purpose

Our attitude is:

the teacher is the most important resource in education

> *We had our first study group on Monday. The session will be repeated again on Thursday. I had 15 teachers attend. We looked at the task Farmyard Friends (Task 129 from the Mathematics Task Centre). We extended it out like the questions from the companion Maths300 lesson suggested, and talked for quite a while about the concept of a factorial. This is exactly the type of dialog that I feel is essential for our elementary teachers to support the development of their math background. So anytime we can use the tasks to extend the teacher's math knowledge we are ahead of the game.*
> District Math Coordinator, Denver, Colorado

Research suggests that professional development most likely to succeed:

- is requested by the teachers
- takes place as close to the teacher's own working environment as possible
- takes place over an extended period of time
- provides opportunities for reflection and feedback
- enables participants to feel a substantial degree of ownership
- involves conscious commitment by the teacher
- involves groups of teachers rather than individuals from a school
- increases the participant's mathematical knowledge in some way
- uses the services of a consultant and/or critical friend

Maths With Attitude has been designed with these principles in mind. All the materials have been tried, tested and modified by teachers from a wide range of classrooms. We hope the resources will enable teacher groups to lead themselves further along the professional development road, and support systems to improve the learning outcomes for students K - 12.

With the support of Maths300 ETuTE, professional development can be a regular component of in-house professional development. See:

- http://www.mathematicscentre.com/taskcentre/resource.htm#etute

For external assistance with professional development, contact:

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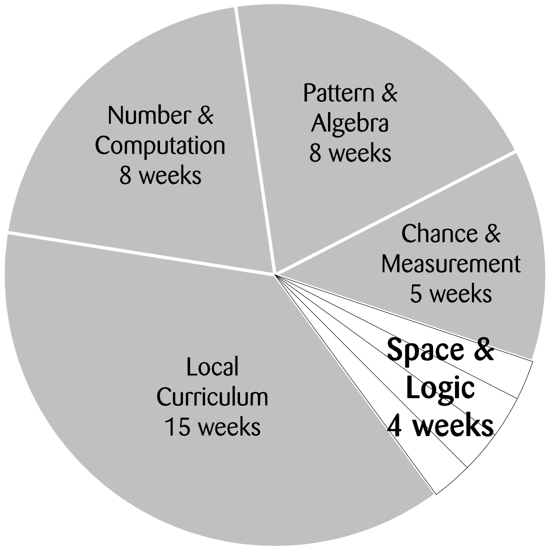
Part 2: Planning Curriculum

Curriculum Planners

Our attitude is:

learning is a personal journey stimulated by achievable challenge



Curriculum Planners:

- show one way these resources can be integrated into your weekly planning
- provide a starting point for those new to these materials
- offer a flexible structure for those more experienced

You are invited to map Planner weeks into your school year planner as the core of the curriculum.

Planners:

- detail each week lesson by lesson
- offer structures for using tasks and lessons
- are sequenced from lesson to lesson, week to week and year to year to 'grow' learning

Teachers and schools will map the material in their own way, but all will be making use of extensively trialed materials and pedagogy.

Using Resources

- Your kit contains 20 hands-on problem solving tasks and reference to relevant Maths300 lessons.
- Tasks are introduced in this manual and supported by the Task Cameos at:
  http://www.mathematicscentre.com/taskcentre/iceberg.htm
- Maths300 lessons are introduced in this manual and supported by detailed lesson plans at:
  http://www.maths300.com

In your preparation, please note:

- Planners assume 4 lessons per week of about 1 hour each.
- Planners are *not* prescribing a continuous block of work.
- Weeks can be interspersed with other learning; perhaps a **Maths With Attitude** week from a different strand.
- Weeks can sometimes be interchanged within the planner.
- Lessons can sometimes be interchanged within weeks.
- The four **Maths With Attitude** kits available at each year level offer 25 weeks of a Working Mathematically core curriculum.

A Way to Begin

- Glance over the Planner for your class. Skim through the comments for each task and lesson as it is named. This will provide an overview of the kit.
- Task Comments begin after the Planners. Lesson Comments begin after Task Comments. The index will also lead you to any task or lesson comments.
- Select your preferred starting week - usually Week 1.
- Now plan in detail by researching the comments and web support. Enjoy!

Research, Reflect, Activate

Curriculum Planner

Space & Logic: Year 7

<table>
<tr><th></th><th>Session 1</th><th>Session 2</th><th>Session 3</th><th>Session 4</th></tr>
<tr><td>Week 1</td><td colspan="3">Whole Class Investigation & Group Work: String Shapes is a great lesson to use in the early weeks of a new year. It involves going outside and investigating properties of 2D shapes on a large scale using loops of string. This leads into text-based work on the same topic.</td><td rowspan="4">Tasks Day: Pairs choose from the 20 tasks. Each task is an invitation to work like a mathematician.

One session each week. The Working Mathematically process (p.9) offers guidance. Students keep a journal.

Journal entries are of diary/note form with date, sketches etc.

Students may be required to develop one investigation into a more extensive report, perhaps for assessment.</td></tr>
<tr><td>Week 2</td><td colspan="3">Whole Class Investigation: Spirolaterals begins with a kinaesthetic approach that can be used outside or in the gym. Students investigate pathways (spirolaterals) formed by certain rules and look for patterns. The physical introduction develops into further investigation using graph paper and the companion software. and contributes to the development of spatial awareness.</td></tr>
<tr><td>Week 3</td><td colspan="3">Whole Class Investigation & Group Work: Football Ladder is a logic puzzle that begins with the class investigating one set of clues. The challenge is for students to create a similar puzzle of their own.</td></tr>
<tr><td>Week 4</td><td colspan="3">Whole Class Investigation: Cube Nets sets the scene for visiting 3D shapes and their properties. This lesson is totally reliant on the concrete materials described, so forward planning to have this available will be necessary. The lesson can be used in conjunction with the common lesson of cutting and folding card to make a cube from a net, however, it offers much more than that.</td></tr>
</table>

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.
- In those regions with four terms you might consider using one of these weeks each term. Alternatively, use them as a four week block.
- The Task Centre also offers a Mixed Media unit titled 'Points of View' which complements this MWA kit. See Planning Notes, Page 15.

Curriculum Planner

Space & Logic: Year 8

<table>
<tr><th></th><th>Session 1</th><th>Session 2</th><th>Session 3</th><th>Session 4</th></tr>
<tr><td>Week 1</td><td colspan="3">Integrated Curriculum Unit: Police Line Up can be investigated at many levels depending on the difficulty of the clues chosen. Having completed the initial puzzle together, students are challenged to create similar puzzles of their own. Adding the twist '...that shows me the most interesting mathematics you know' produces a wide range of results. The lesson is also a source of inspiration for teachers building cross-curriculum units in the middle years.</td><td rowspan="4">Tasks Day: Pairs choose from the 20 tasks. Each task is an invitation to work like a mathematician. One session each week.

The Working Mathematically process (p.9) offers guidance. Students keep a journal.

Journal entries are of diary/note form with date, sketches etc.

Students may be required to develop one investigation into a more extensive report, perhaps for assessment.</td></tr>
<tr><td>Week 2</td><td colspan="3">Whole Class Investigation: Knights Tour seems to have an in-built intrigue which captures most students of this age. Using graph paper and software, the students investigate the possibility of taking a knight for a tour around a chess board and landing once on every square. The number of solutions surprises.</td></tr>
<tr><td>Week 3</td><td colspan="3">Whole Class Investigation: At first Nim seems like a parlour game, but seeking strategies to ensure a win leads to extensive application of the Working Mathematically process and makes links to algebra.</td></tr>
<tr><td>Week 4</td><td colspan="3">Whole Class Investigation: Red To Blue is a problem that is easy to state and easy to start, but opens into a broad investigation. Asking the mathematician's question, What happens if..?. extends the problem into pattern and algebra. Use of a 'whole body' introduction and further development through concrete materials makes the problem ideal for mixed ability classes.</td></tr>
</table>

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.
- In those regions with four terms you might consider using one of these weeks each term. Alternatively, use them as a four week block.
- The Task Centre also offers a Mixed Media unit titled 'Points of View' which complements this MWA kit. See Planning Notes, Page 15.

Planning Notes

Enhancing Maths With Attitude

Resources to support learning to work like a mathematician are extensive and growing. There are more tasks and lessons available than have been included in this Space & Logic kit. You could use the following to enhance this kit.

Additional Tasks

- Task 50, Flight Departures
  An observer gives clues about the take off order of four planes at an airport. The challenge is to decide the actual order in which the planes took off.
- Task 96, Networks
  A game with laminated cardboard tiles which encourages thinking ahead in a spatial situation that is complicated by a growing network of lines. A player can win either by placing the piece which reaches the Finish square, or by placing a piece that forces their opponent to lose.
- Task 153, Knight Protectors
  A famous chess-based puzzle requiring significant patience that results in a beautiful rotationally symmetric solution. The challenge is to place the minimum number of knights on a chess board so that every square is either occupied or attacked.

More information about these tasks may be available in the Task Cameo Library:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Keep in touch with new developments which enhance **Maths With Attitude** at:

- http://www.mathematicscentre.com/taskcentre/enhance.htm

Additional Materials

As stated, our attitude is that mathematics is concrete, visual and makes sense. We assume that all classrooms will have easy access to many materials beyond what we supply. For this unit you will need:

- Loops of cord, string or rope of about 20m on the basis of one loop per 6 students for the *String Shapes* lesson

However, in Year 7 you *will* need special equipment called 3d Geoshapes (square pieces only) for the *Cube Nets* lesson. You can find 3d Geoshapes through the Mathematics Centre Resources link at.

- http://www.mathematicscentre.com/taskcentre/resource.htm#3dgeo

The following kit is *not* essential to this MWA unit, but it is a first class companion resource that will develop and extend the students' spatial skills. It too has been prepared within the framework of students learning to work like a mathematician.

Points of View: A Mixed Media unit from The Task Centre Collective which provides ten tasks, Maths300-style software titled *Building Views*, and an extensive manual. The manual contains a broad range of supportive information from trial schools and a variety of investigation sheets. The content relates to the representation of three dimensional objects in two dimensions. It includes

work on nets, plan and elevation views and isometric drawing, and therefore develops beyond the content of **Maths With Attitude** Space & Logic 7 & 8.
Nine of the ten tasks in Points of View are independent of the ones in this MWA unit. Only the task Keith's Kubes appears in both.

This kit is available to non-members of Maths300.

There is more information in the Integrating Tasks link of Mathematics Task Centre

- http://www.mathematicscentre.com/taskcentre/plans.htm#mm

Special Comments Year 7

Samples of posters produced by students of Wade High school where this kit was used as a four week unit to begin Year 7 can be found at:

- http://www.mathematicscentre.com/taskcentre/wade1.htm

- *String Shapes* in Planner Week 1 needs the string loops mentioned above. This series of lessons also requires printing and preparation of booklets (supplied with the lesson). The lesson is either run outside, or in the gymnasium. In either case you will have to plan ahead to make use of these school facilities.
- *Spirolaterals* in Planner Week 2 will also need to be run outside or in the gymnasium. This lesson also has a software component, so you will need to plan access to computers for some parts of this week.
- *Football Ladder* in Planner Week 3 will require some preparation of print materials supplied with the lesson. A colour printer will produce a great result in this case and, if sufficient time is allowed for laminating in addition to printing, the faculty with have an excellent, permanent resource.
- *Cube Nets* in Planner Week 4 will require the 3d Geoshape squares mentioned above. They are essential.

Special Comments Year 8

- *Police Line Up* in Planner Week 1 requires selecting and printing clue cards supplied with the lesson. If you choose to use the extensive extra material in the lesson you will also need felt markers, perhaps from the art room, and cards for students to draw on.
- *Knights Tour*, in Planner Week 2 is a software based lesson so may involve organising computer use. For the concrete introduction, you will also need counters to move as knights in chess.
- *Nim* in Planner Week 3 will require a large number of counters or equivalent material. Poly Plug works extremely well and is very easy to manage.
- *Red to Blue* in Planner Week 4 may be enhanced by counters with a different colour on each side, or Poly Plug. However, the concrete material can just as easily be ripped up pieces of paper with R written on one side and B written on the other.

Find Poly Plug information at:

- http://www.mathematicscentre.com/taskcentre/polyplug.htm

Task Comments

- Tasks, lessons and unit plans prepare students for the more traditional skill practice lessons, which we invite you to weave into your curriculum. Teachers who have used practical, hands-on investigations as the focus of their curriculum, rather than focussing on the drill and practice diet of traditional mathematics, report success in referring to skill practice lessons as Toolbox Lessons. This links to the idea of a mathematician dipping into a toolbox to find and use skills to solve problems.

Back To Back Building

Student A makes a 3D object from a collection of linking cubes. Student B is sitting back to back with Student A and can't see what is made. Student B has the same colours as Student A and is challenged to recreate the original object by following the oral instructions of the first student. The objective is to become increasingly precise in using mathematical language. Students also experience transformations of objects in space which is informal groundwork for 3D geometry.

Is it possible to design a 3D co-ordinate system that would uniquely describe the position of each cube in space?

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Calendar

Students seem to enjoy this task because it is something with which they are familiar. The first thing they usually discuss is the number of days in each month. The task is perceived as non-threatening because the trial and improve approach means that if something doesn't work it can simply be rubbed out and replaced.

After a while students realise that some numbers have to be on both cubes in order to make dates like 11th and 22nd. Once they have found a way to label the cubes, they usually enjoy verbalising and demonstrating their solution.

Some extension questions are:

- Can your cubes be used to show numbers other than those needed for the days in all the months?
- Can you design a set of cubes to make all the possible six figure dates, ie: 3rd April 2006 would be 030406?

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Crossing The Desert

This task is sure to generate discussion. There are interpretations of the problem that are important to explore. Perhaps the most relevant reason for encouraging the discussion is that a mathematician needs to understand, in fact in many cases clarify and refine, the problem before attempting a solution. Usually this process involves 'talking it through' with colleagues. In this sense the problem encourages justification of answers against agreed criteria, rather than production of *the* solution, or sequence of solutions.

Discussion will identify some or all of the following aspects of the problem:

- Travelling to the oasis requires 8 days food each way per person and one more food portion for Day 9.
- That is 17 food portions per person, so 34 for two people if they both reach the oasis and return. Therefore the 24 food portions allowed are not enough for both travellers to reach the oasis and return.
- One person *will* need 17 food portions (the Messenger) so there are 7 others to 'play with'.
- We could assume that each person eats at noon.
- We could assume that the message is delivered 'in seconds' and the Messenger then turns for home. They don't spend a day resting at the oasis and therefore only need one portion of food on Day 9.
- In a similar way, we could also assume that on the day one person (the Companion) turns around, they also only need one portion of food.

Since the card suggests burying food, students usually try that option in conjunction with the trial and improve strategy. However burying alone doesn't reveal a solution; although burying on Day 5 comes close and an interpretation based on this potential solution is made below.

The key is to realise that the Companion needs food for their own return journey, could bury some for the Messenger to use on the way back and could *give* some to the Messenger to make their total back up to the maximum of 12.

This understanding leads to the following solutions:

	Day	**Give**	**Bury**
Solution 1	3	3	4
Solution 2	4	4	1
Solution 3	4	3	2
Solution 4	4	2	3
Solution 5	4	1	4

Notes

- Solution 1 gets the messenger home with food to spare.
- It could be reasoned that more solutions are possible if you allow progressive burial and/or giving. For example in Solution 5, the four buried

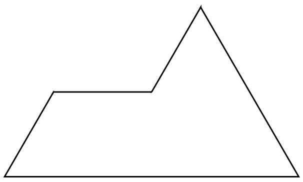
portions could be buried 1 on each day, or 2 on Day 2 and 2 on Day 4 and so on. Similarly the giving of 1 could occur on any of the four days. Students can make the decision whether these are seen as merely variations, or as new solutions. It really doesn't matter which. What matters is the reasoned discussion and agreement should the issue arise.

- Equally the students could debate the merits of this potential Bury Only solution. *If* you accept that in real life the food you eat on Day N provides the energy to travel on Day (N + 1), then they could travel together to Day 5. The Companion would need only 3 pieces of food for the return journey because the third of these would provide the energy to travel home on the last day (Day 1 on the board). The Companion's remaining 4 food portions would be buried. The Messenger would continue the journey, and be able to return to Day 5 on the strength of the food eaten the day before. The four buried portions would be sufficient (consistent with this interpretation) to complete the return journey. Again the real issue is not whether this is a solution, but rather whether it is a solution in the context of this interpretation.
- Newcomb Secondary College assisted with the design of the reproducible board provided at the end of this manual.

Find more information about this task in the Task Cameo Library at:

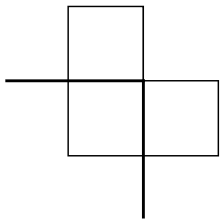
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Dividing Shapes

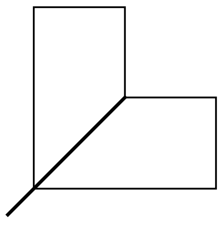


This task helps students learn to visualise the way shapes are made up. It is an important skill. For example, there is no simple formula for finding the area of a Sphinx, but this irregular shape can be partitioned in several ways to make pieces for which area formulae are known. For example:

- 6 small equilateral triangles
- 1 large equilateral triangle and a rhombus
- 1 small equilateral triangle and an isos. trap.
- 2 isosceles trapeziums

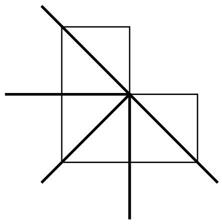


Using this approach is applying the mathematician's strategies of breaking a problem into smaller parts and looking for opportunities to apply previous knowledge.

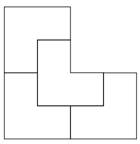


Dividing Shapes encourages this way of thinking in the more familiar situation of a square-based shape. The first three questions are 'warm-ups'. Possible answers are shown

Students might notice a connection between the required shapes and the rotation of a match around the concave angle of the tricube shapes.



The solution to the main challenge relates this task back to Task 166, Sphinx.



Just as a tricube can be made from four tricubes, so a sphinx can be made from four sphinxes.

If you have enough additional tricubes, students could explore:

- Tessellating tricubes in 2 dimensions.
- Tessellating tricubes in 3 dimensions. The new shape formed in the task is not technically a tricube. To maintain its 'cube' nature, it would need to be made of 8 tricubes - 2 layers of the arrangement shown.

Tricubes are listed in the Resources section of Mathematics Centre:

- http://www.mathematicscentre.com/taskcentre/resource.htm

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Famous Mathematicians

We would not expect students to become authors without knowing something of the life and work of famous writers. Similarly if students are learning to work like a mathematician, then it is appropriate to come into contact with the lives and work of professionals in the field. This task starts that process. The word search is engaging and once solved, students use the results to choose three mathematicians whom they are expected to research further. It is no accident that half of the mathematicians listed are female.

The world wide web is a wonderful tool for turning this task into an extensive project. Students can begin simply by entering their chosen names into a search engine such as Google. There are many sites devoted to the history of mathematics and the lives of mathematicians. For example, a significant site listing the work of women mathematicians is:

- http://www.agnesscott.edu/lriddle/women/women.htm

The following brief excerpt indicates that mathematicians are real people living real lives which are just as interesting as any other famous person.

> *Besides her extensive work in mathematics, Grace Chisholm Young completed all the requirements for a medical degree except the internship, learned six languages, and taught each of her six children a musical instrument. She wrote one of the first books for children on reproduction. Called "Bimbo and the Frogs," the book was a very tasteful yet scientific explanation that did not talk down to children.*
>
> http://www.agnesscott.edu/lriddle/women/young.htm

To focus research and raise the intellectual level above merely collecting and recording facts, students might be asked to write to themes such as:

- Use your three mathematicians as examples to compare the professional difficulties faced by male and female mathematicians.
- Your three mathematicians have made significant contributions to society. Discuss.

With the involvement of the history, language and technology departments, there is great potential for this task to grow into an integrated cross-curriculum unit.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Find more information about the companion Maths300 lesson (Lesson 124) which links the task to the mathematics of collecting a set of promotional cards like the ones sometimes offered in snack and fast food products at:

- http://www.maths300.com

Football Ladder

It doesn't matter whether students have an interest or not in this code of football. The puzzle is based in the language and logic of the clues and is not dependent on knowledge of the code. The solution (from the top) is:

> Hawthorn, Fremantle, Essendon, Carlton, Brisbane, Sydney, Adelaide, Melbourne, Collingwood, Geelong, Richmond, Kangaroos, West Coast, St. Kilda, Port Adelaide, Bulldogs.

Once solved, an obvious extension is to create and trial a similar puzzle based on the student's chosen sport.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Find more information about the companion Maths300 lesson (Lesson 69) which extends the puzzle within the Australian Football code at:

- http://www.maths300.com

Hearts & Loops

There is no need to ask about the solution of this task. Some day, someone will solve it and the problem will become how to prevent the successful person from telling everyone else. Of course, the problem isn't considered 'done' until the person can take it apart *and* put it back together.

Actually, in some ways, it doesn't seem to matter if the solution does travel the class 'grapevine'. Most students want to try for themselves, and, in fact, frequently revisit the puzzle just to make sure that they can still do it. Another challenge that can be added for those who do solve it is to *ask* the successful student to tell someone else how to do it *provided* the doer is the only one to hold the puzzle *and* the teller either sits on their own hands or keeps them behind their back.

Mathematically the task is developing spatial visualisation, and, through the extension, mathematical language. There is also a link to topology. Clearly, since the pieces can be taken apart, at least one of them must be an open curve (like a classic doughnut ring with a bite out of it). There is whole branch of mathematics that derives from identifying closed and open curves, and it includes study of such recreational pastimes as the Möbius Strip and the Königsberg Bridge Problem.

A hint that sometimes helps when a student becomes too frustrated is to suggest they think in terms of turning a key in a lock.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Human Moves Monster

Students soon learn that they need to record their moves so that they aren't repeating the same ones without success. Graph paper is handy for this. For the most part students approach this problem using the trial, record and improve strategy. The task can be difficult and perhaps that makes it all the more satisfying when all animals are captured.

A	◺ B	C	D	E
F	◺ G ●	◺ H ●	I ●	J
K	◺ L ●	M ●	◺ N ●	O
P	◺ Q ●	◺ R ●	◺ S ●	◺ T
U	V	W	X	Y

One solution for the 'black dots' part of the task is 10 moves, as follows:

Begin on P

P --> M (capture)

M--> T (no capture)

T --> I (capture)

I --> L (capture)

L --> S (capture)

S --> H (capture)

H --> Q (capture)

Q --> N (capture)

N --> G (capture)

G --> R (capture)

One solution for the 'white triangles' part of the task is 11 moves as follows:

Begin on I

I --> L (capture)

L --> S (capture)

S --> H (capture)

H --> Q (capture)

Q --> N (capture)

N --> G (capture)

G --> R (capture)

R --> I (no capture)

I --> T (capture)

T --> I (no capture)

I --> B (capture)

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

In The Bag

A fascinating series of spatial challenges that depend on the tactile/kinaesthetic sense rather than the visual sense. Students can exit this task after any question and feel successful, then re-enter it at another time and try a higher level of difficulty. The most difficult puzzle is hard to get into the frame even when the pieces can be seen. Any student who can do it in the bag has great command of their tactile skills and visual memory.

The task can be related to

- some of the work done by astronauts in outer space
- operating 'robot arms' inside containers of radioactive material
- doctors instructing a robot to operate while viewing a monitor on the other side of the room.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

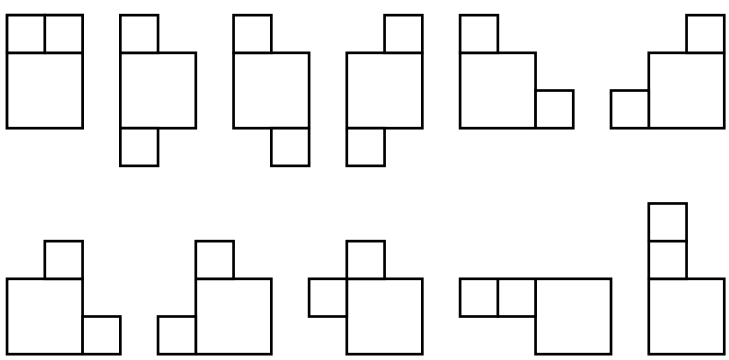
Keith's Kubes

This open-ended problem has many solutions, shown below. It reinforces and develops the ideas in Back To Back Building. The essence of the task is, given a square prism made from four cubes, how can two other cubes be attached to make different solids? The solutions are drawn or represented in any way chosen by the student.

In the answers we have used a convention for defining what is 'the same'. Constructions that can be made identical by a turning on a vertical axis (as if they were houses rotated on a block of land) are said to be the same.

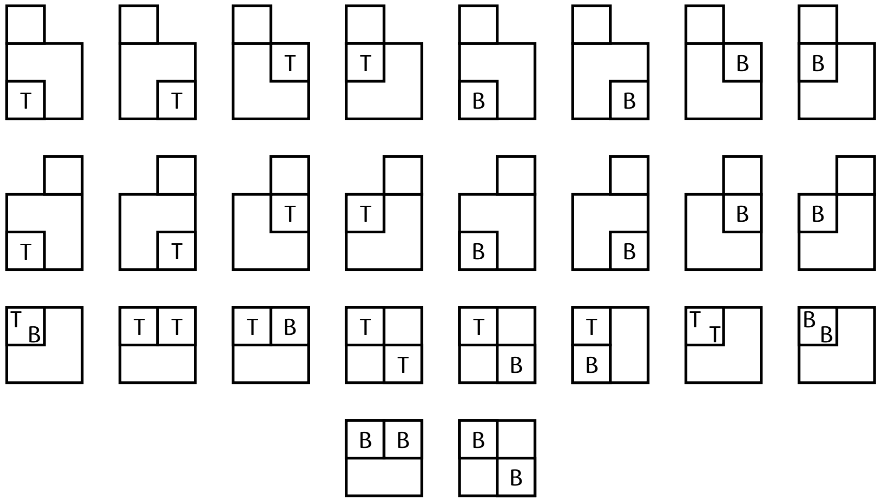
Answers

Question 2



Question 3

The convention used is that T shows a cube on top of the set of four and B shows a cube below it.



Knight Swap

This is another task that uses the L-shaped movement of the knight. **Like Human Moves Monster**, this puzzle has also been adapted from the game of Chess. Once you know the solution described here, your biggest problem will be to control your desire to tell it to the students.

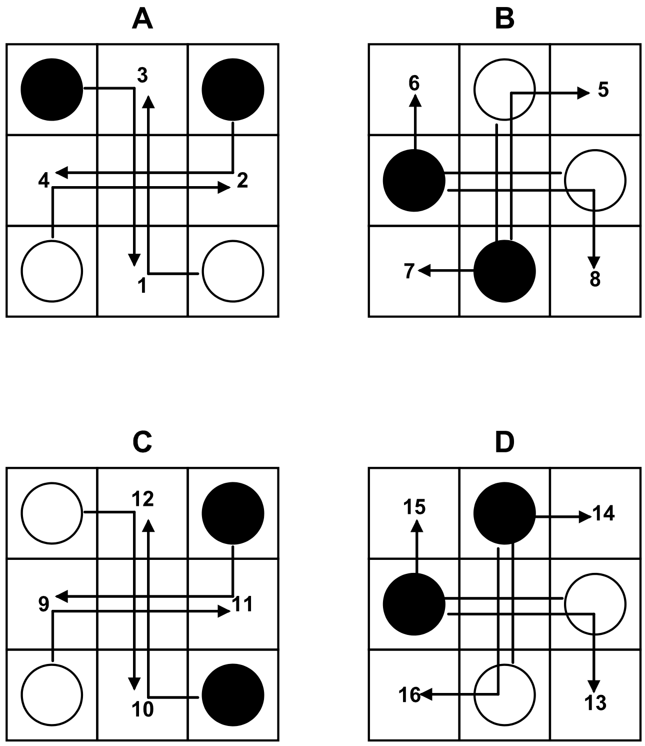
In fact, if you don't struggle with this problem yourself before you read the solution, you will miss out on the thrill of solving it. That is exactly what you don't want the students to miss, and it takes practice to develop your teaching craft in a way that develops enough support for those who are frustrated, but doesn't steal from them the chance of personal solution.

'Sleeping on it' can work with this problem. The trial and improve approach used by most people to begin the problem is actually collecting data which the mind can work on when you are asleep and develop into an enlightened approach in the morning. This aspect of problem solving is embedded in our culture in dictums like:

- sleep on it
- it will be all right in the morning
- try something different for a while then come back to it

which are based on the actual experiences of many people - perhaps even yourself. However, in part because of the artificial time table placed on learning by school structures, opportunities that encourage this approach are not easy to find, even though it is an approach used by mathematicians.

Knight Swap does provide one such opportunity. It makes no difference when **Knight Swap** is solved. There is no examination demand that requires the solution to be known now. Solving it or not, doesn't affect advancement in any content area. But solving it *yourself* may affect the way you feel about yourself as a problem solver and therefore directly influence your belief in your ability to solve problems across a range of content areas.



When you, or the students, are ready for a hint, consider these:

- None of the pieces can ever land in the middle square, so the problem is about how the knights move around the perimeter.
- 'Around the perimeter' ... hmmm ... perhaps there is an aspect of rotation in the solution. After all, the problem could be solved by rotating the board, but unfortunately that is not a 'legal' move. So, if we can't rotate the board, perhaps we can rotate the counters.
- The hint on the card says each piece moves four times, perhaps that implies that each piece has to visit each side of the board.

Thoughts like this, in combination with the trial, record and improve strategy, lead to a solution as shown.

- In each stage the knights are shown in their starting position.
- The numbers show where each knight finishes at the end of each stage.

Perhaps the best way to experience the rotational symmetry embedded in this task is to use four students on a large grid and ask them to describe their relationship to the person in front and behind as each stage is completed.

Leading The Blind

Language, shape and space intersect again in this task. One player has to guide the other player, who has their eyes closed, to match a shape to its frame. The need for more and more precise mathematical language becomes clear. There are many opportunities to introduce language that can help the students refine their efforts.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Making Triangles

In this task the only thing that matters is the colours in the triangle. The order of the colours doesn't matter. For example R-**B**-Y is the same as Y-**B**-R. In this description, the bold colour is thought of as the side placed horizontally in front of you on the table as a base.

The task uses criteria to sort and classify. Perhaps the students will use the criteria of the types of triangles as suggested by the card. Perhaps they will focus on the number of each colour and come back to the triangle types later. Some students begin quite randomly and it is heartening to see them reconsider this approach and try a more systematic strategy. Mathematical conversation is a key feature of the task and one of the high points for teachers is when students realise that some triangles belong to more than one category. For example all equilateral triangles are also isosceles.

The students are asked to draw each triangle, but the purpose of this is to reinforce that a mathematician records notes and diagrams as they investigate a problem. So, if they choose to use a code rather than lines and colours that is fine.

One way to approach an organised search is to start with the smallest rod, Red, and make all the possible 3-side, 2-side and 1-side red triangles, then to repeat this approach for Yellow, Green and Blue, being careful not to build any that have already been made.

Red as Base
- 3 sides: R-**R**-R
- 2 sides: R-**R**-Y ... R-**R**-G ... R-**R**-B
- 1 side: Y-**R**-G ... Y-**R**-B ... G-**R**-B (other 2-side situations occur below)

Yellow as Base
- 3 sides: Y-**Y**-Y
- 2 sides: Y-**Y**-R ... Y-**Y**-G ... Y-**Y**-B
- 1 side: G-**Y**-B (other possibilities have been made)

Green as Base
- 3 sides: G-**G**-G
- 2 sides: G-**G**-R ... G-**G**-Y ... G-**G**-B
- 1 side: All possibilities have been made.

Blue as Base
- 3 sides: B-**B**-B
- 2 sides: B-**B**-R ... B-**B**-Y ... B-**B**-G
- 1 side: All possibilities have been made.

There are 20 different triangles in all. Four are Equilateral, 16 are Isosceles (which includes the 4 equilateral) and 4 are scalene. One of the scalene ones (G-**R**-B) is also Right Angled.

Extensions to the task could include:
- Exploring the possibilities if using a fifth colour.
- Exploring the possibilities of making quadrilaterals.
- Investigating how you know without making the shape whether three sticks will make a triangle.

Find more information about this task in the Task Cameo Library at:
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Mirror Patterns 1

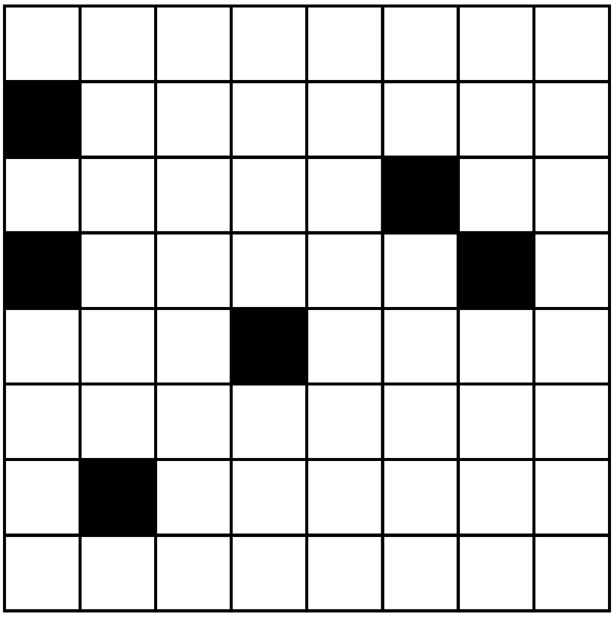
As engaging to most students as playing with a kaleidoscope. The task can open the door to other activities with mirrors. Lots of informal experience with the properties of reflection which will prepare the students for mathematics related to angles and their measurement.

Find more information about this task in the Task Cameo Library at:
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

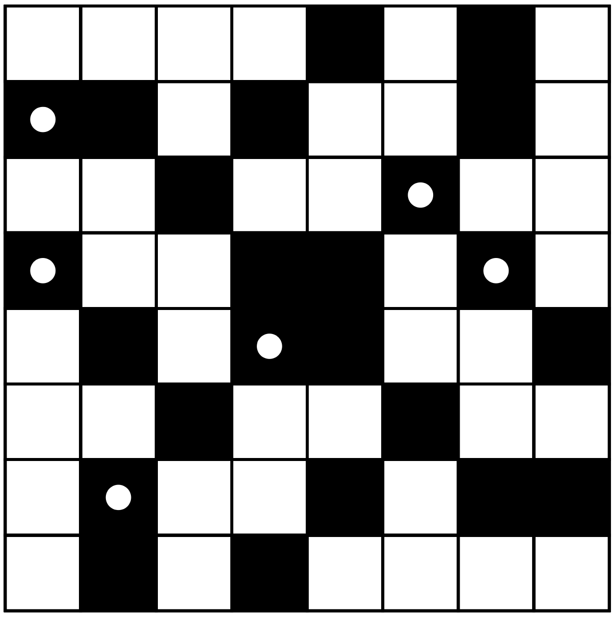
Paving Views

Rotational symmetry is involved in this task, and in **Knight Swap** and **Wallpaper Patterns**. Perhaps this is an under-rated aspect of mathematics curricula. More time seems to be spent on line symmetry than rotational symmetry and yet rotational symmetry has just as much application in, for example, art and design in the world. The story shell surrounding this task even suggests as much.

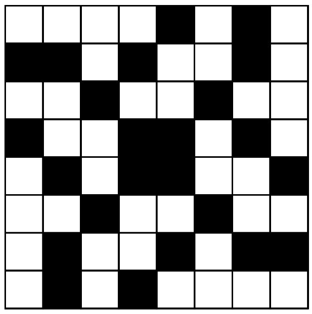
The courtyard begins like this:



The courtyard ends like this:



18 tiles have been added. The white spots show the original tiles. This is the minimum solution.



Without the white spots the rotational symmetry is clearer, as in the inset picture. The courtyard can be viewed from all four sides and will look the same. But what clues could the students use to develop this solution?

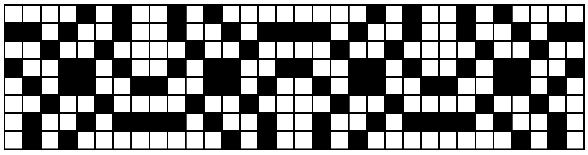
- Choose one side - say the left half.
- Imagine this rotated about the centre to become the bottom half.
- Which squares need a black tile added? (Remember some of the black ones needed may already be there.)
- Which squares are already black when you get to the bottom half? These will tell you which additional squares on the left half should have been black to start with.
- Now rotate the new bottom half to become the right half. Fill in the blacks and also fill backwards again, this time into the bottom half first, then back again to the left half.
- Rotate to the top half and repeat, filling back three times as necessary.
- Check the whole visually from each side.

This approach combines the mathematician's strategies of *breaking a problem into parts* and *working backwards*. Of course, there will other ways students will examine this problem and reach the same conclusion.

Extensions

- Use a drawing program or the table facilities of Word to make an electronic form of the courtyard as a record of the solution.
- Students design their own rotationally symmetric courtyard.
- Use the electronic form as a tile to create new designs, eg:

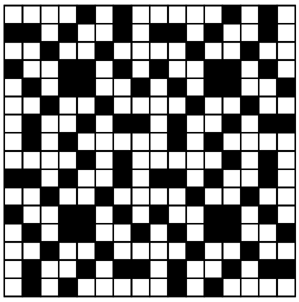
The 8x8 tile at the left end is the solution to the task. It has been reflected in a vertical line through its right side. This second tile has been reflected again the same way. Then the third has been reflected the same way to create the fourth.



This could be a tiled section of a railway platform. Children enter at the left end and exit at the right, stepping only on black. How many different pathways can they take?

In this case the original 8x8 solution tile is in the top left. It has been translated right, then the pair has been translated down.

- *Use this grid to make a Crossnumber or a Crossword to demonstrate your knowledge of mathematics?*



Racetrack

This time reflective symmetry engages the body's kinaesthetic sense in an unexpected way. There are many levels of difficulty within the task so there is something in the task for everyone. You will find students will often want to return to the challenge.

It may be worth relating this task to the truck driver's challenge of reversing into a space using mirrors only. Students can come close to modelling this by holding the task mirror in front of them as if it were the central or side mirror of a vehicle, then, starting at the classroom doorway, try to navigate themselves safely to the far back corner of the room by looking in the mirror only.

The worksheet you may need for this task is at the end of this manual.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Reflections

The objective of the task is to provide further experience with the properties of reflection. In particular the students are likely to develop, or confirm, the idea that each point and its reflection are equidistant from the mirror.

Extensions include:

- Asking the students to make the initials of their name and its reflection.

- Shifting the task to graph paper to encourage further experiment with reflection - perhaps with two mirror lines.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Six Square Puzzle

Match stick puzzles help to develop spatial perception and logical reasoning. This task offers only a selection from the many found in literature and builds them into a theme by using the same starting point in each case. Students will be using visualisation and reasoning strategies such as *What happens if...?* and *Breaking a problem into parts*, or *Working backwards*, to solve the puzzles.

The solutions are listed below, but perhaps you could generate additional interest in the task by declaring that you don't know any of the solutions, but you will name each one after the pair that discovers it. You can turn this into a whole class activity by using popsticks (which are easier to manage because of their size). These solutions can be recorded in a class Match Puzzles Book (or electronic folder) and students can also add new puzzles to this record.

Redhead Matches, available throughout Australia, sometimes carry matchstick puzzles on the back. The company now offers on line Match Stick puzzle software at:

- http://www.redheads.com.au/games.php

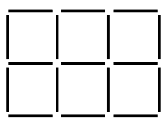
and a large number of other challenges and craft work as well.

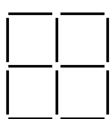
You will also find a collection of puzzles (with solutions) on the web at:

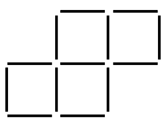
- www.jimloy.com/puzz/match.htm

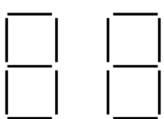
as part of Jim Loy's Puzzle Page, which has a far broader collection of mathematically based challenges than just these.

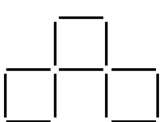
Solutions

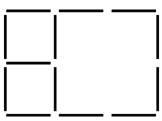
The start: 

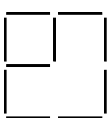
Solution 1: 

Solution 2: 

Solution 3: 

Solution 4: 

Solution 5: 

Solution 6: 

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

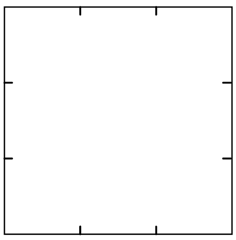
Wallpaper Patterns

Along with **Paving Views** this task offers an introduction to transformation geometry. In this case the introduction is through techniques actually used in the design of wallpaper and in other art, such as that of M. C. Escher.

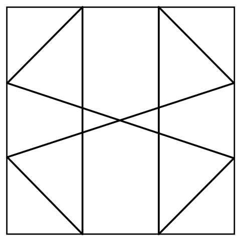
The task includes two distinct sets of 16 tiles. The card doesn't actually state this, but it will be necessary for students to sort their cards into piles to efficiently tackle the task. The rules suggested are starting points for investigation. The students are expected to explore further. The solutions to the task card and student-created patterns can be recorded with a digital camera. This will be even more relevant if it is the habit to record problem solving in an electronic journal. There may be different interpretations of some of the pattern making instructions, so no 'answers' are given here. If the student can justify their result with a reasoned interpretation of the rule, then they have been successful.

Extensions

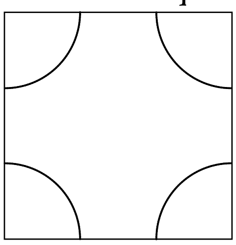
- What happens if a tile is rotated before being placed in the next square? ...and how does the result differ if you also change the rule for placing the next tile?
- Ask the students to creatively join the dots on a tile whose edges have been divided into thirds like this:



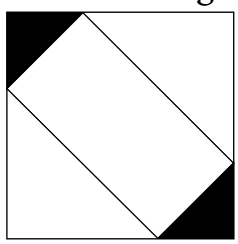
Cards cut to 6cm or 9cm squares work well.



Students can use straight lines...

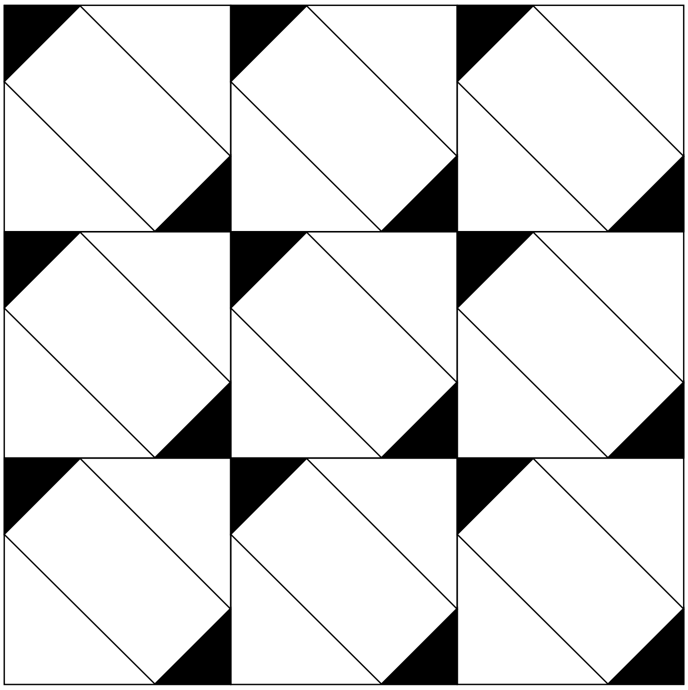


or curves, or a combination of both...



or add areas of colour.

Once students have chosen a design they like, they make several copies of it and create wallpaper patterns by rule. Again, this work can overlap with technology objectives by using a drawing package to produce results such as the design on the next page which has been created by simple translation of the last tile shown above. Results can be quite stunning and when the idea turns into a project, the students work makes an excellent display.



Who Owns The Monkey?

The Malaysians do - but the challenge is in convincing yourself that this is the only conclusion the clues allow. The challenge is not easy and it is important that the students have an opportunity to struggle with it. It is not necessary that the problem is solved in one sitting. However, it is usually a popular task and frequently students want to keep working on it until the solution appears.

Once it is solved, the next question is *How did the designer come up with the problem?*. The most likely answer is that (s)he came up with the story line, set out the cards and then invented the clues. This realisation suggests the possibility of the students making up a similar problem of their own. They could begin by laying the cards out in any order based on the houses and constructing their own set of clues. This is not an easy exercise. There needs to be just enough clues to make the problem interesting and solvable, but the clues must not conflict. The students would need to draft their clues and trial them on others before 'releasing' them to the public.

Alternatively, students could create their own set of cards and clues not dependent at all on the materials in the task. Story shells might be:

- Who Owns The Footy Shorts?
- Who's Hobby Is Camping?
- Who Gets The Dessert?

Designing the puzzle, inventing the clues and validating that users find it interesting and challenging is a major project that can simultaneously address mathematics, language and technology outcomes.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Lesson Comments

- These comments introduce you to each Maths300 lesson. The complete plan is easily accessed through the lesson library available to members at:
  http://www.maths300.com
  where they are listed alphabetically by lesson name.

Cube Nets

Almost all teachers have asked students to construct a cube from its net; usually asking all students to start from the same T-shaped net. That is a great lesson for the measurement toolbox, but this *Cube Nets* lesson investigates all the different nets of a cube and illustrates so much more of the Working Mathematically process. However, it is equipment specific. It is one case where only this equipment makes the lesson possible. So, you need access to plenty of 3d Geoshape squares, or their equivalent, before you can use this lesson. You can find 3d Geoshapes through the Resources link on Mathematics Centre As a teacher comments in the introduction to the lesson:

> *The construction lesson was a regular part of my curriculum until I discovered 3d Geoshapes. What was a standard piece of good practice, becomes so much richer with this equipment. I take the Cube Nets task out of the hands-on activities kit and convert it to a class lesson with the 3d Geoshapes.*

With the manipulative available, the data becomes visual - an opportunity for students to see that data doesn't necessarily mean numbers - and the students see ways of classifying the various nets. Using this mathematician's skill of sorting and classifying, the students are able to attack the deeper questions:

- How many nets of a cube are there?
- How do we know when we have found them all?

At many stages in the lesson, the teacher is able to draw a parallel with the way a professional mathematician works. This puts the teacher in the position of being able to congratulate and celebrate a student's efforts as being consistent with the way a mathematician works. Students learn that they *can* work mathematically.

The lesson also opens the door to exploring other nets. 3d Geoshapes can also be purchased in triangles, pentagons, hexagons and other shapes that make such exploration very practical.

Football Ladder

The context of the well known Australian Football League (AFL) provides the setting for this language and logic puzzle. The major aspect is the provision of concrete materials (cards) which make the task accessible to, and popular with, almost all students. Content outcomes are numerical order, awareness of a range of problem solving strategies and correct use of spatial language. Another feature is a kinaesthetic option where the class members can 'act out' the ladder positions. The third puzzle also offers arithmetic skill practice related to the way points are assigned for a win, loss or draw.

Although the particular sport may not be relevant to your classroom culture, after using the Lesson Plan as it is, the students can be challenged to prepare similar problems based on local sporting codes. Alternatively, the plan provides you with the model to prepare a relevant puzzle of your own.

An extension of the lesson also offers an excellent way to introduce selections and arrangements (permutations and combinations). The teacher (or the students) can choose the conditions in the problem to manage the difficulty of the challenges.

Knight's Tour

Take a chess piece (the Knight) for a 'walk' on a chess board and land on every square exactly once. This spatial puzzle, with its origins in the game of Chess, is most engaging at many year levels.

> *I was delighted to discover that the lessons from Maths300 work equally successfully with Scottish students as with Australian students and am still amazed at how quickly a disruptive class can be calmed by presenting them with the investigation, Knight's Tour!*

Knight's Tour has a compelling appeal that encourages students to 'stick at it'. While obviously a spatial puzzle, it is the class statistics that help place the learning focus firmly on problem solving and strategy development. The various options of the companion software add significant power to the search. The extensions and developments described and explored in the lesson make it clear that there is a great deal of content here for the student learning to work like a mathematician.

> *An additional advantage of the lesson is the prolonged periods of 'enjoyable' silence which it frequently creates in the classroom.*

Nim

This is one version of a famous two person strategy game. Counters are arranged in 4 rows with 1, 3, 5 & 7 counters in each row. Players take turns to remove one or more counters from one row. The player who has to remove the last counter loses the game. So, on any turn, any number of counters may be taken (including the whole row), but they may only be taken from one row.

The purpose of the task is to encourage problem solving strategies:

- looking ahead
- reasoning ('what-if')
- working backwards

It is almost essential for students to use counters; at least at first. They make the problem 'real'. The lesson may be guided by the investigation sheet supplied with the Maths300 lesson and there are a number of extensions suggested.

Police Line Up

Sometimes you are awed by the way creative teachers build on each other's ideas to turn a standard text book exercise into a wonderfully rich lesson sequence. This language and logic puzzle, which gives clues to identify one character in a police line up, is one of those examples.

- First it is lifted from the text book page by the provision of concrete materials (cards) which make the task accessible to, and popular with, virtually all students.
- Another teacher has added a kinaesthetic dimension by enlarging the cards so the students can 'become' the characters by wearing the cards around their necks.
- Another has added a puzzle-creating aspect which invites the students to display 'all they know about mathematics'. Still another teacher can see the value of using different sets of clues.
- Once combination theory is added, the problem reaches to the top levels of secondary school.

> *I've always liked these puzzles but found over the years that only about 25% of my students enjoyed them when they had to do them by pencil and paper using a grid. But when I went to the trouble of providing character cards, all of a sudden 100% of the students became involved. The lesson is so easy to run and all students seem to get involved. It has a non-threatening quality about its structure.*

Red To Blue

Four students stand facing the class. The class instructs all except one (ie: any three at a time) to turn around 180 degrees. Using this rule, is it possible to make all four turn their backs to the class? A challenge that is simple to express, easy to begin and rich with possibilities for a wide range of abilities and ages.

The kinaesthetic whole class introduction is followed by checking the logic with concrete materials at a personal/small group level. This sets the problem up for later exploration of other cases such as *What if there were five or six or seven or … students?*. However, before that, the problem is an excellent springboard for examining the number of different strategies a mathematician could use to try to solve this problem. The thrust of the lesson at this stage is therefore to highlight the value of the Strategy Toolbox.

Exploring the *What if…?* cases leads to a generalisation about all cases that have the rule of turning all except one each time, ie: the turning (n - 1) case.

Of course this answer only opens the door to another series of questions, ie: *What happens if the rule is turn (n - 2) or (n - 3) or …?*.

So, much of the excitement of this lesson, especially for the teacher, is that an easy-to-start logic puzzle that seems to engage almost all students, is a pathway to number and algebra. Several content outcomes from the one challenge could be seen as a powerful use of curriculum time.

Spirolaterals

A simple set of instructions for 'walking' lines on a grid offers a stimulating introduction to a fascinating array of geometric shapes. If you can't use an outside area, then an assembly hall or multi-purpose room will work. The physical involvement in this lesson is an important construct in the learning and leads well into the pencil and paper exploration of the extended challenge.

One set of rules will produce a spirolateral of Order 3. But what happens if the rule is changed? And what if we change the grid from square to isometric (triangular)?

Exploring these questions converts the activity into an extended investigation involving many number and geometric patterns. The openness of the investigation invites many levels of challenge - however the main one would be the generalisation:

Predict the shape or pattern for any set of instructions.

So again we have an engaging problem in shape and space that becomes a springboard into number and algebra.

Maths300 software allows theories about the shapes and the relationship between the numbers involved to be tested. Try hard to resist the temptation to use the software too soon.

String Shapes

Imagine 4 students in a group with a long brightly coloured loop of string. The group is asked to form the string into a square. Having made what they think is a square; the challenge is to devise a test to see if they have indeed made what they think they have. This physical or kinaesthetic involvement sets the scene for learning the properties and names of a collection of geometric shapes in an atmosphere of co-operative, small group, problem solving.

In this way, the focus of this lesson is on the properties of polygons and it provides so many options that you may wish to select the types of polygons that students explore. The loops of string involve the students in whole body learning as a sort of human geoboard. Teachers find the discussion that develops in this group situation is far richer and more purposeful than attempting to explore the properties of these shapes from a text book page. On the other hand the text book exercises make much more sense if they follow the String Shapes experience.

This is a lesson that requires lots of space. So if you are going outside, plan ahead for the weather. If not, you will need a gymnasium or multi-purpose room.

Part 3: Value Adding

The Poster Problem Clinic

Maths With Attitude kits offer several models for building a Working Mathematically curriculum around tasks. Each kit uses a different model, so across the range of 16 kits, teachers' professional learning continues and students experience variety. The Poster Problem Clinic is an additional model. It can be used to lead students into working with tasks, or it can be used in a briefer form as a opening component of each task session.

I was apprehensive about using tasks when it seemed such a different way of working. I felt my children had little or no experience of problem solving and I wanted to prepare them to think more deeply. The Clinic proved a perfect way in.

Careful thought needs to be given to management in such lessons. One approach to getting the class started on the tasks and giving it a sense of direction and purpose is to start with a whole class problem. Usually this is displayed on a poster that all can see, perhaps in a Maths Corner. Another approach is to print a copy for each person. A Poster Problem Clinic fosters class discussion and thought about problem solving strategies.

Starting the lesson this way also means that just prior to liberating the students into the task session, they are all together to allow the teacher to make any short, general observations about classroom organisation, or to celebrate any problem solving ideas that have arisen.

One teacher describes the session like this:

I like starting with a class problem - for just a few minutes - it focuses the class attention, and often allows me to introduce a particular strategy that is new or needs emphasis.

It only takes a short time to introduce a poster and get some initial ideas going. The class discussion develops a way of thinking. It allows class members to hear, and learn from their peers, about problem solving strategies that work for them.

If we don't collectively solve the problem in 5 minutes, I will leave the problem 'hanging' and it gives a purpose to the class review session at the end.
Sometimes I require everyone to work out and write down their solution to the whole class problem. The staggered finishing time for this allows me to get organised and help students get started on tasks without being besieged.
I try to never interrupt the task session, but all pupils know we have a five minute review session at the end to allow them to comment on such things as an activity they particularly liked. We often close then with an agreed answer to our whole class problem.

A Clinic in Action

The aims of the regular clinic are:

- to provide children with the opportunity to learn a variety of strategies
- to familiarise children with a process for solving problems.

The following example illustrates a structure which many teachers have found successful when running a clinic.

Preparation

For each session teachers need:

- a Strategy Board as below
- a How To Solve A Problem chart as below
- to choose a suitable problem and prepare it as a poster
- to organise children into groups of two or three.

The Strategy Board can be prepared in advance as a reference for the children, or may be developed *with* the children as they explore problem solving and suggest their own versions of the strategies.

The problem can be chosen from

- a book
- the task collection
- prepared collections such as Professor Morris Puzzles which can be viewed at: http://www.mathematicscentre.com/taskcentre/resource.htm#profmorr

The example which follows is from the task collection. The teacher copied it onto a large sheet of paper and asked some children to illustrate it. *The teacher also changed the number of sheep to sixty* to make the poster a little different from the one in the task collection.

The Strategy Board and the How To Solve A Problem chart can be used in any maths activity and are frequently referred to in Maths300 lessons.

The Clinic

The poster used for this example session is:

> Eric the Sheep is lining up to be shorn before the hot summer ahead. There are sixty [60] sheep in front of him. Eric can't be bothered waiting in the queue properly, so he decides to sneak towards the front.
>
> Every time one [1] sheep is taken to be shorn, Eric <u>then</u> sneaks past two [2] sheep. How many sheep will be shorn <u>before</u> Eric?

This Poster Problem Clinic approach is also extensively explored in Maths300 Lesson 14, *The Farmer's Puzzle*.

Strategy Board

DO I KNOW A SIMILAR PROBLEM?	ACT IT OUT
GUESS, CHECK AND IMPROVE	DRAW A PICTURE OR GRAPH
TRY A SIMPLER PROBLEM	MAKE A MODEL
WRITE AN EQUATION	LOOK FOR A PATTERN
MAKE A LIST OR TABLE	TRY ALL POSSIBILITIES
WORK BACKWARDS	SEEK AN EXCEPTION
BREAK INTO SMALLER PARTS	...

How To Solve A Problem

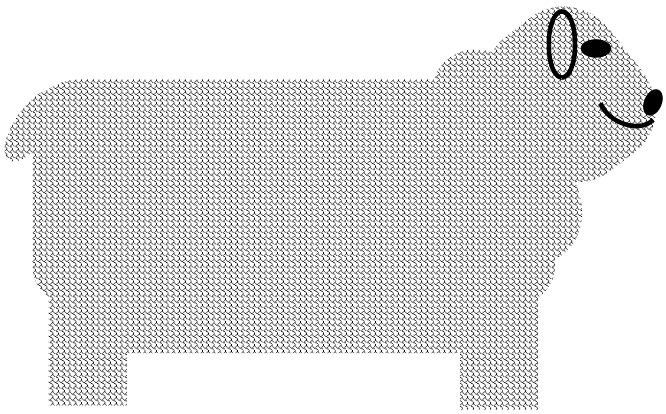
SEE & UNDERSTAND	Do I understand what the problem is asking? Discuss
PLANNING	Select a strategy from the board. Plan how you intend solving the problem.
DOING IT	Try out your idea.
CHECK IT	Did it work out? If so reflect on the activity. If not, go back to step one.

Step 1

- Tell the children that we are at Stage 1 of our four stage plan ... **See & Understand** ... Point to it! Read the problem with the class. Discuss the problem and clarify any misunderstandings.
- If children do not clearly understand what the problem is asking, they will not cope with the next stage. A good way of finding out if a child understands a problem is for her/him to retell it.
- Allow time for questions - approximately 3 to 5 minutes.

Step 2

- Tell the children that we are at Stage 2 of our four stage plan ... **Planning**. In their groups children select one or more strategies from the Strategy Board and discuss/organise how to go about solving the problem.
- Without guidance, children will often skip this step and go straight to Doing It. It is vital to emphasise that this stage is simply planning, not solving, the problem.
- After about 3 minutes, ask the children to share their plans.



Plan 1

> *Well we're drawing a picture and sort of making a model.*

Can you give me more information please Brigid?

> *We're putting 60 crosses on our paper for sheep and the pen top will be Eric. Then Claire will circle one from that end, and I will pass two crosses with my pen top.*

Plan 2

> *Our strategy is Guess and Check.*

That's good Nick, but how are you going to check your guess?

> *Oh, we're making a model.*

Go on ...

> *John's getting MAB smalls to be sheep and I'm getting a domino to be Eric and the chalk box to be the shed for shearing.*

Plan 3

> *We are doing it for 3 sheep then 4 sheep then 5 sheep and so on. Later we will look at 60.*

Great so you are going to try a simpler problem, make a table and look for a pattern.

This sharing of strategies is invaluable as it provides children who would normally feel lost in this type of activity with an opportunity to listen to their peers and make sense out of strategy selection. Note that such children are not given the answer. Rather they are assisted with understanding the power of selecting and applying strategies.

Step 3

- Tell the children that we are at Stage 3 of our four stage plan ... **Doing It**. Children collect what they need and carry out their plan.

Step 4

- Tell the children that we are at Stage 4 of our four stage plan ... **Check It**. Come together as a class for groups to share their findings. Again emphasis is on strategies.

> *We used the drawing strategy, but we changed while we were doing it because we saw a pattern.*

So Jake, you used the Look For A Pattern strategy. What was it?

> *We found that when Eric passed 10 sheep, 5 had been shorn, so 20 sheep meant 10 had been shorn ... and that means when Eric passes 40 sheep, 20 were shorn and that makes the 60 altogether.*

Great Jake. How would you work out the answer for 59 sheep or 62 sheep?

Sharing time is also a good opportunity to add in a strategy which no one may have used. For example:

> *Maybe we could've used the Number Sentence strategy, ie: 1 sheep goes to be shorn and Eric passes two sheep. That's 3 sheep, so perhaps, 60 divided into groups of 3, or 60 ÷ 3 gives the answer.*

Round off the lesson by referring to the Working Mathematically chart. There will be many opportunities to compliment the students on working like a mathematician.

Curriculum Planning Stories

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

In more than a decade of using tasks and many years of using the detailed whole class lessons of Maths300, teachers have developed several models for integrating tasks and whole class lessons. Some of those stories are retold here. Others can be found at:

- http://www.mathematicscentre.com/taskcentre/plans.htm

Story 1: Threading

> *Educational research caused me a dilemma. It tells us that students construct their own learning and that this process takes time. My understanding of the history of mathematics told me that certain concepts, such as place value and fractions, took thousands of years for mathematicians to understand. The dilemma was being faced with a textbook that expected students to 'get it' in a concentrated one, two or three week block of work and then usually not revisit the topic again until the next academic year.*

A Working Mathematically curriculum reflects the need to provide time to learn in a supportive, non-threatening environment and...

> *When I was involved in a Calculating Changes PD program I realised that:*
>
> - *choosing rich and revisitable activities, which are familiar in structure but fresh in challenge each time they are used, and*
> - *threading them through the curriculum over weeks for a small amount of time in each of several lessons per week*
>
> *resulted in deeper learning, especially when partnered with purposeful discussion and recording.*

Calculating Changes:

- http://www.mathematicscentre.com/calchange

Story 2: Your turn

Some teachers are making extensive use of a partnership between the whole class lessons of Maths300 and small group work with the tasks. Setting aside a lesson for using the tasks in the way they were originally designed now seems to have more meaning, as indicated by this teacher's story:

> *When I was thinking about helping students learn to work like a mathematician, my mind drifted to my daughter learning to drive. She*

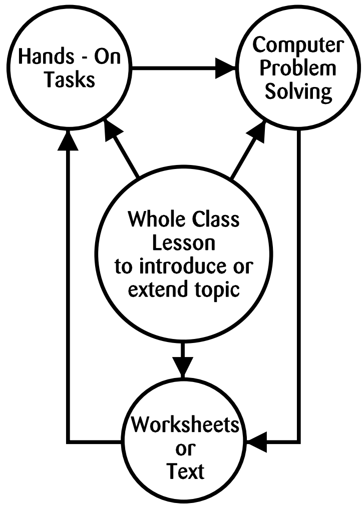
needed me to model how to do it and then she needed lots of opportunity to try it for herself.

That's when the idea clicked of using the Maths300 lessons as a model and the tasks as a chance for the students to have their turn to be a mathematician.

The Maths300 lessons illustrate how other teachers have modelled the process, so I felt I could do it too. Now the process is always on display on the wall or pasted inside the student's journal.

A session just using the tasks had seemed a bit like play time before this. Now I see it as an integral part of learning to work mathematically.

Story 3: Mixed Media



It was our staff discussion on Gardner's theory of Multiple Intelligences that led us into creating mixed media units. That and the access you have provided to tasks and Maths300 software.

We felt challenged to integrate these resources into our syllabus. There was really no excuse for a text book diet that favours the formal learners. We now often use four different modes of learning in the work station structure shown. It can be easily managed by one teacher, but it is better when we plan and execute it together.

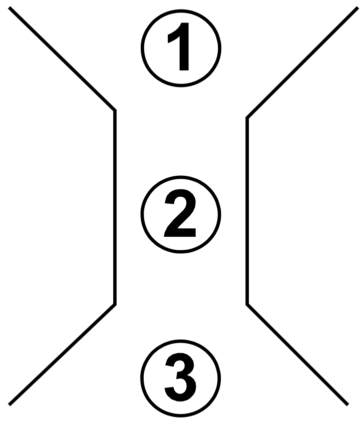
Story 4: Replacement Unit

We started meeting with the secondary school maths teachers to try to make transition between systems easier for the students. After considerable discussion we contracted a consultant who suggested that school might look too much the same across the transition when the students were hoping for something new. On the other hand our experience suggested that there needed to be some consistency in the way teachers worked.

We decided to 'bite the bullet' and try a hands-on problem solving unit in one strand. We selected two menus of twenty hands-on tasks, one for the primary and one for the secondary, that became the core of the unit. We deliberately overlapped some tasks that we knew were very rich and added some new ones for the high school.

Class lessons and investigation sheets were used to extend the tasks, within a three week model.

It is important to note that although these teachers structured a 3 week unit for the students, they strongly advised an additional *Week Zero* for staff preparation. The units came to be called Replacement Units.



Week Zero - Planning

Staff familiarise themselves with the material and jointly plan the unit. This is not a model that can be 'planned on the way to class'.

> *Getting together turned out to be great professional development for our group.*

Week 1 - Introduction

Students explore the 20 tasks listed on a printed menu:

- students explore the tip of the task, as on the card
- students move from task to task following teacher questioning that suggests there is more to the task than the tip
- in discussion with students, teachers gather informal assessment information that guides lesson planning for the following week.

> *We gave the kids an 'encouragement talk' first about joining us in an experiment in ways of learning maths and then gave out the tasks. The response was intelligent and there was quite a buzz in the room.*

Week 2 - Formalisation

> *It was good for both us and the students that the lessons in this week were a bit more traditional. However, they weren't text book based. We used whole class lessons based on the tasks they had been exploring and taught the Working Mathematically process, content and report writing.*
>
> *Assessment was via standard teacher-designed tests, quizzes and homework.*

Week 3 - Investigations

> *We were most delighted with Week 3. Each student chose one task from the menu and carried out an in-depth investigation into the iceberg guided by an investigation sheet. They had to publish a report of their investigation and we were quite surprised at the outcomes. It was clear that the first two weeks had lifted the image of mathematics from 'boring repetition' to a higher level of intellectual activity.*

Story 5: Curriculum shift

> *I think our school was like many others. The syllabus pattern was 10 units of three weeks each through the year. We had drifted into that through a text book driven curriculum and we knew the students weren't responding.*

Our consultant suggested that there was sameness about the intellectual demands of this approach which gave the impression that maths was the pursuit of skills. We agreed to select two deeper investigations to add to each unit. It took some time and considerable commitment, but we know that we have now made a curriculum shift. We are more satisfied and so are the students.

The principles guiding this shift were:

- Agree

  The 20 particular investigations for the year are agreed to by all teachers. If, for example, Cube Nets *is decided as one of these, then all the teachers are committed to present this within its unit.*

- Publish

  The investigations are written into the published syllabus. Students and parents are made aware of their existence and expect them to occur.

- Commit

  Once agreed, teachers are required to present the chosen investigations. They are not a negotiable 'extra'.

- Value

  The investigations each illustrate an explicit form of the Working Mathematically process. This is promoted to students, constantly referenced and valued.

- Assess

  The process provides students with scaffolding for their written reports and is also known by them as the criteria for assessment. (See next page.)

- Report

  The assessment component features within the school reporting structure.

A Final Comment

Including investigations has become policy.

Why? Because to not do so is to offer a diminished learning experience.

The investigative process ranks equally with skill development and needs to be planned for, delivered, assessed and reported.

Perhaps most of all we are grateful to our consultant because he was prepared to begin where we were. We never felt as if we had to throw out the baby and the bath water.

Assessment

Our attitude is:

stimulated students are creative and love to learn

Regardless of the way you use your **Maths With Attitude** resource, a variety of procedures can be employed to assess this learning.

Where these assessment procedures are applied to task sessions and involve written responses from students, teachers will need to be careful that the writing does not become too onerous. Students who get bogged down in doing the writing may lose interest in doing the tasks.

In addition to the ideas below, useful references are:

- http://www.mathematicscentre.com/taskcentre/assess.htm
- http://www.mathematicscentre.com/taskcentre/report.htm

The first offers several methods of assessment with examples and the second is a detailed lesson plan to support students to prepare a Maths Report.

Journal Writing

Journal writing is a way of determining whether the task or lesson has been understood by the student. The pupil can comment on such things as:

- What I learned in this task.
- What strategies I/we tried (refer to the Strategy Board).
- What went wrong.
- How I/we fixed it.
- Jottings - ie: any special thoughts or observations

Some teachers may prefer to have the page folded vertically, so that children's reflective thoughts can be recorded adjacent to critical working.

Assessment Form

An assessment form uses questions to help students reflect upon specific issues related to a specific task.

Anecdotal Records

Some teachers keep ongoing records about how students are tackling the tasks. These include jottings on whether students were showing initiative, whether they were working co-operatively, whether they could explain ideas clearly, whether they showed perseverance.

Checklists

A simple approach is to create a checklist based on the Working Mathematically process. Teachers might fill it in following questioning of individuals, or the students may fill it in and add comments appropriately.

Pupil Self-Reflection

Many theorists value and promote metacognition, the notion that learning is more permanent if pupils deliberately and consciously analyse their own learning. The deliberate teaching strategy of oral questioning and the way pupils record their work is an attempt to manifest this philosophy in action. The alternative is the tempting 'butterfly' approach which is to madly do as many activities as possible, mostly superficially, in the mistaken belief that quantity equates to quality.

> *I had to work quite hard to overcome previously entrenched habits of just getting the answer, any answer, and moving on to the next task.*

Thinking about *what* was learned *how* it was learned consolidates and adds to the learning.

When it follows an extensive whole class investigation, a reflection lesson such as this helps to shift entrenched approaches to mathematics learning. It is also an important component of the assessment process. On the one hand it gives you a lot of real data to assist your assessment. On the other it prepares the students for any formal assessment which you may choose to round off a unit.

Introduction

Ask students to recall what was done during the unit or lesson by asking a few individuals to say what *they* did, eg:

> *What did you do or learn that was new?*
> *What can you now do/understand that is new?*
> *What do you know now that you didn't know 1 (2, 3, ...) lesson ago?*

Continuing Discussion

Get a few ideas from the first students you ask, then:

- organise 5 -10 minute buzz groups of three or four students to chat together with one person to act as a recorder. These groups address the same questions as above.
- have a reporting session, with the recorder from each group telling the class about the group's ideas.

Student comments could be recorded on the board, perhaps in three groups.

<u>Ideas & Facts</u> <u>Maths Skills</u> <u>Process (learning) Skills</u>

If you need more questions to probe deeper and encourage more thought about process, try the following:

> *What new things did you do that were part of* how *you learned?*
> *Who uses this kind of knowledge and skill in their work?*

Student Recording

Hand out the REFLECTION sheet (next page) and ask students to write their own reflection about what they did, based on the ideas shared by the class. Collect these for interest and, possibly, assessment information.

REFLECTION

me looking at me learning

NAME: CLASS:

Working With Parents

Balancing Problem Solving with Basic Skill Practice

Many schools find that parents respond well to an evening where they have an opportunity to work with the tasks and perhaps work a task together as a 'whole class'. Resourced by the materials in this kit, teachers often feel quite confident to run these practical sessions. Comments from parents like:

> *I wish I had learnt maths like this.*

are very supportive. Letting students 'host' the evening is an additional benefit to the home/school relationship.

The 4½ Minute Talk

Charles Lovitt has considerable experience working with parents and has developed a crisp, parent-friendly talk which he shares below. Many others have used it verbatim with great success.

Why the Four and a Half Minute Talk?

When talking with parents about Problem Solving or the meaning of the term Working Mathematically, I have often found myself in the position, after having promoted inquiry based or investigative learning, of the parents saying:

> *Well - that's all very well - BUT ...*

at which stage they often express their concern for basic (meaning arithmetic) skill development.

The weakness of my previous attempts has been that I have been unable to reassure parents that problem solving does not mean sacrificing our belief in the virtues of such basic skill development.

One of the unfortunate perceptions about problem solving is that if a student is engaged in it, then somehow they are not doing, or it may be at the expense of, important skill based work.

This Four and a Half Minute Talk to parents is an attempt to express my belief that basic skill practice and problem solving development can be closely intertwined and not seen as in some way mutually exclusive.

(I'm still somewhat uncomfortable using the expression 'basic skills' in the above way as I am certain that some thinking, reasoning, strategy and communication skills are also 'basic'.)

Another aspect of the following 'talk' is that, as teachers put more emphasis on including investigative problem solving into their courses, a question arises about the source of suitable tasks.

This talk argues that we can learn to create them for ourselves by 'tweaking' the closed tasks that heavily populate out existing text exercises, and hence not be dependent on external suppliers. (Even better if students begin to create such opportunities for themselves.)

The Talk

In preparation, write the following graphic on the board:

CLOSED	OPEN	EXTENDED INVESTIGATION
		How many solutions exist? How do you know you have found them all?

I would like to show you what teachers are beginning to do to achieve some of the thinking and reasoning and communication skills we hope students will develop. I would like to show you three examples.

Example One: *6 + 5 = ?*

I write this question under the 'closed' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
$\begin{array}{r} 6 \\ \underline{+\ 5} \\ \underline{\quad} \end{array}$		How many solutions exist? How do you know you have found them all?

And I ask:

What is the answer to this question?

I then explain that:

We often ask students many closed questions such as 6 + 5 = ?

The only response the students can tell us is "The answer is 11." ... and as a reward for getting it correct we ask another twenty questions just like it.

What some teachers are doing is trying to *tweak* the question and ask it a different way, for example:

I have two counting numbers that add to 11. What might the numbers be?

[Counting numbers = positive whole numbers including zero]

I write this under the 'open' label on the diagram:

CLOSED	**OPEN**	**EXTENDED INVESTIGATION**
$\begin{array}{r} 6 \\ \underline{+\ 5} \\ \underline{\quad} \end{array}$	$\begin{array}{r} ? \\ \underline{+\ ?} \\ \underline{11} \end{array}$	How many solutions exist? How do you know you have found them all?

What is the answer to the question now?

At this stage it becomes apparent there are several solutions:

The question is now a bit more open than it was before, allowing students to tell you things like 8 + 3, or 10 + 1, or 11 + 0 etc.

Let's see what happens if the teacher 'tweaks' it even further with the investigative challenge *or* extended investigation question:

How many solutions are there altogether?

and more importantly, and with greater emphasis on the second question:

How could you convince someone else that you have found them all?

Now the original question is definitely different - it still involves the skills of addition but now also involves thinking, reasoning and problem solving skills, strategy development and particularly communication skills.

Young students will soon tell you the answer is 'six different ones', but they must also confront the communication and reasoning challenge of convincing you that there are only six and no more.

Example Two: *Finding Averages*

Again, as I go through this example, I write it into the diagram on the board in the relevant sections.

The CLOSED question is: *11, 12, 13 - find the average*

Tweaking this makes it an OPEN question and it becomes:

I have three counting numbers whose average is 12. What might the numbers be?

Students will often say:

10, 12, 14 … or 9, 12, 15 … or even 12, 12, 12

After realising there are many answers, you can tweak it some more and turn it into an EXTENDED INVESTIGATION:

How many solutions exist? … AND …

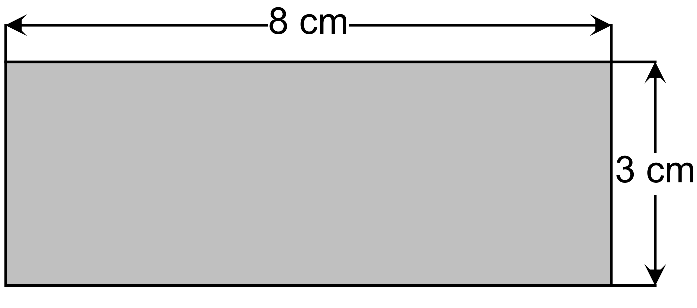
How do you know you have found them all?

Now the question is of a quite different nature. It still involves the arithmetic skill, but has something else as well - and that something else is the thinking, reasoning and communication skills necessary to find all of the combinations and convince someone else that you have done so.

By the time a student announces, with confidence, there are 127 different ways (which there are) that student will have engaged in all of these aspects, ie: the skill of calculating averages, (and some combination number theory) as well as significant strategy and reasoning experiences.

Example Three: *Finding the Area of a Rectangle*

A typical CLOSED question is:



Find the area. Find the perimeter.

The OPEN question is:

A rectangle has 24 squares inside:

What might its length and width be?

What might its perimeter be?

The EXTENDED INVESTIGATION version is:

Given they are whole number lengths, how many different rectangles are there? … AND …

How do you know you have found them all?

In summary, mathematics teachers are trying to convert *some* (not all) of the many closed questions that populate our courses and 'push' them towards the investigation direction. In doing so, we keep the skills we obviously value, but also activate the thinking, reasoning and justification skills we hope students will also develop.

This sequence of three examples hopefully shows two major features:

- That skills and problem solving can 'live alongside each other' and be developed concurrently.
- That the process of creating open-ended investigations can be done by anyone - just go to any source of closed questions and try 'tweaking' them as above. If it only worked for one question per page it would still provide a very large supply of investigations.

In terms of the effect of the talk on parents, I have usually found them to be reassured that we are not compromising important skill development (and nor do we want to). The only debate then becomes whether the additional skills of thinking, reasoning and communication are also desirable.

I've also been told that parents appreciate it because of the essential simplicity of the examples - no complicated theoretical jargon.

A Working Mathematically Curriculum

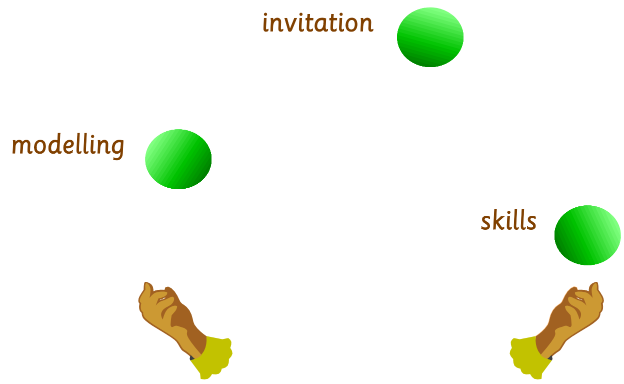
An Investigative Approach to Learning

The aim of a Working Mathematically curriculum is to help students learn to work like a mathematician. This process is detailed earlier (Page 8) in a one page document which becomes central to such a curriculum.

The change of emphasis brings a change of direction which *implies and requires* a balance between:

- the process of being a mathematician, and
- the development of skills needed to be a *successful* mathematician.

This journey is not two paths. It is one path made of two interwoven threads in the same way as DNA, the building block of life, is one compound made of two interwoven coils. To achieve a Working Mathematically curriculum teachers need to balance three components.



The task component of **Maths With Attitude** offers each pair of students an invitation to work like a mathematician.

The Maths300 component of **Maths With Attitude** assists teachers to model working like a mathematician.

Content skills are developed in context. They *are* important, but it is the application of skills within the process of Working Mathematically that has developed, and is developing, the human community's mathematical knowledge.

A focus for the Working Mathematically teacher is to help students develop mathematical skills in the context of problem posing and solving.

> *We are all 'born' with the same size mathematical toolbox, in the same way as I can own the same size toolbox as my motor mechanic. However, my motor mechanic has many more tools in her box than I and she has had more experience than I using them in context. Someone has helped her learn to use those tools while crawling under a car.*

Afzal Ahmed, Professor of Mathematics at Chichester, UK, once quipped:

> *If teachers of mathematics had to teach soccer, they would start off with a lesson on kicking the ball, follow it with lessons on trapping the ball and end with a lesson on heading the ball. At no time would they play a game of football.*

Such is not the case when teaching a Working Mathematically curriculum.

Elements of a Working Mathematically Curriculum

Working Mathematically is a K - 12 experience offering a balanced curriculum structured around the components below.

Hands-on Problem Solving Play

Mathematicians don't know the answer to a problem when they start it. If they did, it wouldn't be a problem. They have to play around with it. Each task invites students to play with mathematics 'like a mathematician'.

Skill Development

A mathematician needs skills to solve problems. Many teachers find it makes sense to students to place skill practice in the context of *Toolbox Lessons* which *help us better use the Working Mathematically Process* (Page 8).

Focus on Process

This is what mathematicians do; engage in the problem solving process.

Strategy Development

Mathematicians also make use of a strategy toolbox. These strategies are embedded in Maths300 lessons, but may also have a separate focus. Poster Problem Clinics are a useful way to approach this component.

Concept Development

A few major concepts in mathematics took centuries for the human race to develop and apply. Examples are place value, fractions and probability. In the past students have been expected to understand such concepts after having 'done' them for a two week slot. Typically they were not visited again until the next year. A Working Mathematically curriculum identifies these concepts and regularly 'threads' them through the curriculum.

Planning to Work Mathematically

The class, school or system that shifts towards a Working Mathematically curriculum will no longer use a curriculum document that looks like a list of content skills. The document would be clear in:

- choosing genuine problems to initiate investigation
- choosing a range of best practice teaching strategies to interest a wider range of students
- practising skills for the purpose of problem solving

Some teachers have found the planning template on the next page assists them to keep this framework at the forefront of their planning. It can be used to plan single lessons, or units built of several lessons. There are examples from schools in the Curriculum & Planning section of Maths300 and a Word document version of the template.

Unit Planning Page

Class

Topic

Pedagogy	Problem Solving In this topic how will I engage my students in the Working Mathematically process?	Skills
How do I create an environment where students know what they are doing and why they have accepted the challenge?		Does the challenge identify skills to practise? Are there other skills to practise in preparation for future problem solving?

Notes

As a general guide:

- Find a problem(s) to solve related to the topic.
- Choose the best teaching craft likely to engage the learners.
- Where possible link skill practice to the problem solving process.

More on Professional Development

For many teachers there will be new ideas within **Maths With Attitude**, such as unit structures, views of how students learn, teaching strategies, classroom organisation, assessment techniques and use of concrete materials. It is anticipated (and expected) that as teachers explore the material in their classrooms they will meet, experiment with and reflect upon these ideas with a view to long term implications for the school program and for their own personal teaching.

Being explored 'on-the-job' so to speak, in the teacher's own classroom, makes the professional development more meaningful and practical for the teacher. This is also a practical and economic alternative for a local authority.

Strategic Use by Systems

From Years 3 - 10, **Maths With Attitude** is designed as a professional development vehicle by schools or clusters or systems because it carries a variety of sound educational messages. They might choose **Maths With Attitude** because:

- It can be used to highlight how investigative approaches to mathematics can be built into balanced unit plans without compromising skill development and without being relegated to the margins of a syllabus as something to be done only after 'the real' content has been covered.
- It can be used to focus on how a balance of concept, skill and application work can all be achieved within the one manageable unit structure.
- It can be used to show how a variety of assessment practices can be used concurrently to build a picture of student progress.
- It can be used to focus on transition between primary and secondary school by moving towards harmony and consistency of approach.
- It can be used to raise and continue debate about the pedagogy (art of teaching) that supports deeper mathematical learning for a wider range of students.

Teachers in Years K - 2 are similarly encouraged in professional growth through **Working Mathematically with Infants**, which derives from Calculating Changes, a network of teachers enhancing children's number skills from Years K - 6.

In supporting its teachers by supplying these resources in conjunction with targeted professional development over time, a system can fuel and encourage classroom-based debate on improving outcomes. There is evidence that by exploring alternative teaching strategies and encouraging curriculum shift towards Working Mathematically, learners improve and teachers are more satisfied. For more detail visit Research & Stories at:

- http://www.mathematicscentre.com/taskcentre/do.htm

We would be happy to discuss professional development with system leaders.

Web Reference

The starting point for all aspects of learning to work like a mathematician, including Calculating Changes, and the teaching craft which encourages it is:

- http://www.mathematicscentre.com/mathematicscentre

Appendix: Recording Sheets

Crossing The Desert

	Desert View Caravan Park & Kiosk	
Day 1	Start 	Day 1
Day 2		Day 2
Day 3		Day 3
Day 4		Day 4
Day 5		Day 5
Day 6		Day 6
Day 7		Day 7
Day 8		Day 8
Day 9	 Oasis	Day 9

Names: .... Class: ....

Dividing Shapes

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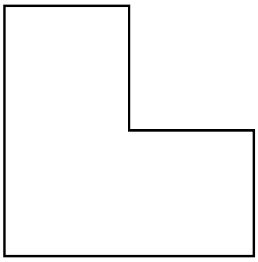
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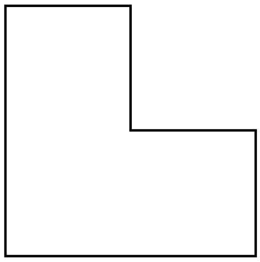
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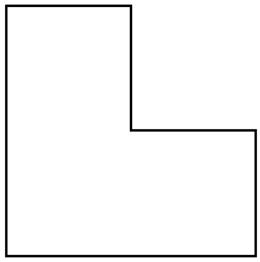
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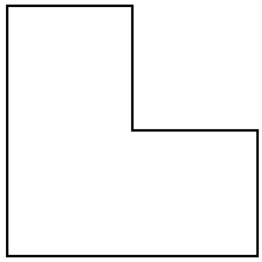
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Dividing Shapes

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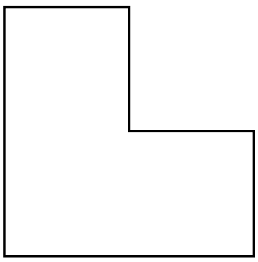
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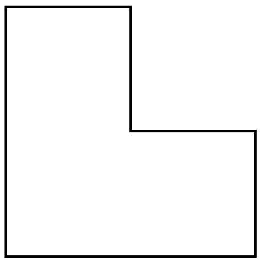
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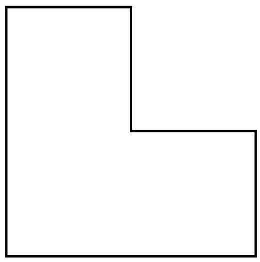
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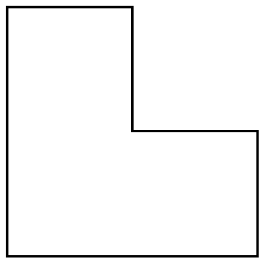
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Names: .... Class: ....

Dividing Shapes

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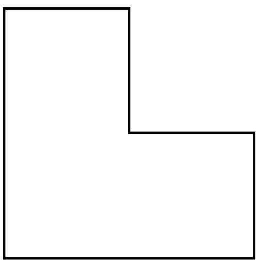
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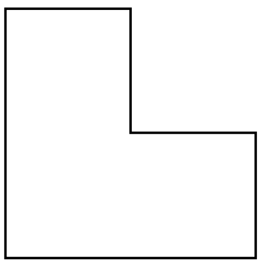
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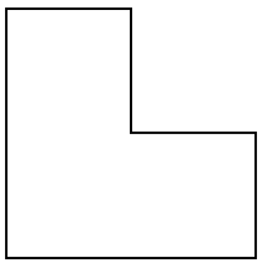
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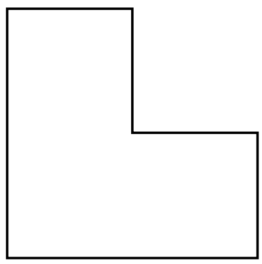
4 identical shapes

Names: .... Class: ....

Racetrack Results

Record your time in seconds for each track. You don't have to do them all today.

Racetrack One

	First Try	Second Try	Third Try
Jockey A			
Jockey B			

Racetrack Two

	First Try	Second Try	Third Try
Jockey A			
Jockey B			

Racetrack Three

	First Try	Second Try	Third Try
Jockey A			
Jockey B			