maths with attitude

2nd Edition

Pattern & Algebra Years 7 & 8

Charles Lovitt
Doug Williams

Mathematics Task Centre & Maths300
helping to create happy healthy cheerful productive inspiring classrooms

maths with attitude

2nd Edition

Pattern & Algebra

Years 7 & 8

In this kit:

- *Hands-on problem solving tasks*
- *Detailed curriculum planning*

Access from Maths300:

- *Extensive lesson plans*
- *Software*

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Maths With Attitude includes reference to hands-on problem solving tasks and Maths300 material.

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- To support teachers, schools and systems wanting to create:

  happy, healthy, cheerful, productive, inspiring classrooms

Our Attitude

- to learning:

  learning is a personal journey stimulated by achievable challenge
- to learners:

  stimulated students are creative and love to learn
- to pedagogy:

  the art of choosing teaching strategies to involve and interest all students
- to mathematics:

  mathematics is concrete, visual and makes sense
- to learning mathematics:

  all students can learn to work like a mathematician
- to teachers:

  the teacher is the most important resource in education
- to professional development:

  teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

Our Objective in Detail

What do we mean by creating:

happy, healthy, cheerful, productive, inspiring classrooms

Happy…

means the elimination of the unnecessary fear of failure that hangs over so many students in their mathematics studies. Learning experiences *can* be structured so that all students see there is something in it for them and hence make a commitment to the learning. In so many ‘threatening’ situations, students see the impending failure and withhold their participation.

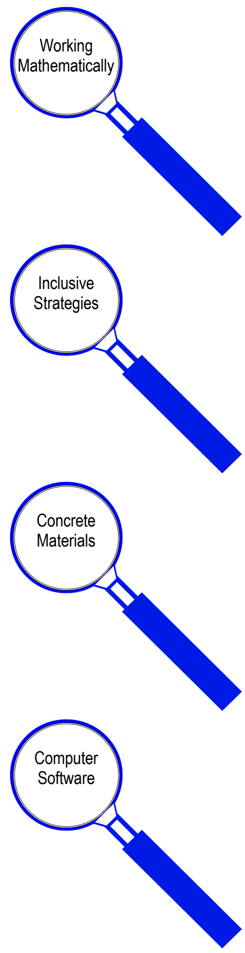
A phrase which describes the structure allowing all students to perceive something in it for them is *multiple entry points and multiple exit points*. That is, students can enter at a variety of levels, make progress and exit the problem having visibly achieved.

Healthy…

means *educationally healthy*. The learning environment should be a reflection of all that our community knows about how students learn. This translates into a rich array of teaching strategies that could and should be evident within the learning experience.

If we scrutinise the *exploration* through any lens, it should confirm to us that it is well structured or alert us to missed opportunities. For example, peering through a pedagogy lens we should see such features as:

- a story shell to embed the situation in a meaningful context
- significant active use of concrete materials
- a problem solving challenge which provides ownership for students
- small group work
- a strong visual component
- access to supportive software



Cheerful…

because we want 'happy' in the title twice!

Productive…

is the clear acknowledgment that students are working towards recognisable outcomes. They should know what these are and have guidelines to show they have either reached them or made progress. Teachers are accountable to these outcomes as well as to the quality of the learning environment.

Inspiring…

is about creating experiences that are uplifting or exalting; that actually *turn students on*. Experiences that make students feel great about themselves and empowered to act in meaningful ways.

Pattern & Algebra Resources

To help you create

happy, healthy, cheerful, productive, inspiring classrooms

this kit contains

- 20 hands-on problem solving tasks from Mathematics Centre

and a Teachers' Manual which integrates the use of the tasks with

- 16 detailed lesson plans from Maths300

The kit offers **8 weeks** of Scope & Sequence planning in Pattern and Algebra for *each* of Year 7 and Year 8. This is detailed in *Part 2: Planning Curriculum* which begins on Page 12. You are invited to map these weeks into your Year Planner. Together, the four kits available for these levels provide 25 weeks of core curriculum in Working Mathematically (working like a mathematician).

Note: Membership of Maths300 is assumed.
The kit will be useful without it, but it will be much more useful with it.

Tasks

- 4 Arm Shapes
- Addition Totals
- Algebra Through Geometry
- Crossing The River 1
- Eric The Sheep
- Garden Beds
- Lining Up
- Making Monuments
- Match Triangles
- Mirror Patterns 2
- Painted Rods
- Pointy Fences
- Shape Algebra
- Smooth Edge Tiles
- Snail Trail
- Sphinx
- Square Numbers
- The Mushroom Hunt
- Time For Tiling
- Unseen Triangles

Part 2 of this manual introduces each task. The latest information can be found at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm

Maths300 Lessons

- 4 Arm Shapes
- Addition Totals
- Algebra Walk
- Back Tracking
- Colour Spots on a Number Line
- Crossing The River
- Game of 31
- Garden Beds
- Heads & Legs
- Lining Up
- Painted Rods
- Snail Trail
- Sphinx
- Trial, Record & Improve
- Unseen Triangles
- What's My Rule?

Lessons with Software

- Game of 31
- Garden Beds
- Heads & Legs
- What's My Rule?

Part 2 of this manual introduces each lesson. Full details can be found at:

- http://www.maths300.com

Working Like A Mathematician

Our attitude is:

all students can learn to work like a mathematician

What does a mathematician's work actually involve? Mathematicians have provided their answer on Page 8. In particular we are indebted to Dr. Derek Holton for the clarity of his contribution to this description.

Perhaps the most important aspect of Working Mathematically is the recognition that *knowledge is created by a community and becomes part of the fabric of that community*. Recognising, and engaging in, the process by which that knowledge is generated can help students to see themselves as able to work like a mathematician. Hence Working Mathematically is the framework of **Maths With Attitude.**

Skills, Strategies & Working Mathematically

A Working Mathematically curriculum places learning mathematical skills and problem solving strategies in their true context. Skills and strategies are the tools mathematicians employ in their struggle to solve problems. Lessons on skills or lessons on strategies are not an end in themselves.

- **Our skill toolbox** can be added to in the same way as the mechanic or carpenter adds tools to their toolbox. Equally, the addition of the tools is not for the sake of collecting them, but rather for the purpose of getting on with a job. A mathematician's job is to attempt to solve problems, not to collect tools that might one day help solve a problem.
- **Our strategy toolbox** has been provided through the collective wisdom of mathematicians from the past. All mathematical problems (and indeed life problems) that have ever been solved have been solved by the application of this concise set of strategies.

About Tasks

Our attitude is:

mathematics is concrete, visual and makes sense

Tasks are from Mathematics Task Centre. They are an invitation to two students to work like a mathematician (see Page 8).

The Task Centre concept began in Australia in the late 1970s as a collection of rich tasks housed in a special room, which came to be called a Task Centre. Since that time hundreds of Australian teachers, and, more recently, teachers from other countries, have adapted and modified the concept to work in their schools. For example, the special purpose room is no longer seen as an essential component, although many schools continue to opt for this facility.

A brief history of Task Centre development, considerable support for using tasks, for example Task Cameos, and a catalogue of all currently available tasks can be found at:

- http://www.mathematicscentre.com/taskcentre

Key principles are:
- A good task is the tip of an iceberg
- Each task has three lives
- Tasks involve students in the Working Mathematically process

The Task Centre Room or the Classroom?

There are good reasons for using the tasks in a special room which the students visit regularly. There are also different good reasons for keeping the tasks in classrooms. Either system can work well if staff are committed to a core curriculum built around learning to work like a mathematician.
- A task centre room creates a focus and presence for mathematics in the school. Tasks are often housed in clear plastic 'cake storer' type boxes. Display space can be more easily managed. The visual impact can be vibrant and purposeful.
- However, tasks can be more readily integrated into the curriculum if teachers have them at their finger tips in the classrooms. In this case tasks are often housed in press-seal plastic bags which take up less space and are more readily moved from classroom to classroom.

Tip of an Iceberg

The initial problem on the card can usually be solved in 10 to 20 minutes. The investigation iceberg which lies beneath may take many lessons (even a lifetime!). Tasks are designed so that the original problem reveals just the 'tip of the iceberg'. Task Cameos and Maths300 lessons help to dig deeper into the iceberg.

We are constantly surprised by the creative steps teachers and students take that lead us further into a task. No task is ever 'finished'.

> *Most tasks have many levels of entry and exit and therefore offer an on-going invitation to revisit them, and, importantly, multiple levels of success for students.*

Three Lives of a Task

This phrase, coined by a teacher, captures the full potential and flexibility of the tasks. Teachers say they like using them in three distinct ways:
1. As on the card, which is designed for two students.
2. As a whole class lesson involving all students, as supported by outlines in the Task Cameos and in detail through the Maths300 site.
3. Extended by an Investigation Guide (project), examples of which are included in both Task Cameos and Maths300.

The first life involves just the 'tip of the iceberg' of each task, but nonetheless provides a worthwhile problem solving challenge - one which 'demands' concrete materials in its solution. This is the invitation to work like a mathematician. Most students will experience some level of success and accomplishment in a short time.

The second life involves adapting the materials to involve the whole class in the investigation, in the first instance to model the work of a mathematician, but also to develop key outcomes or specific content knowledge. This involves choosing teaching craft to interest the students in the problem and then absorb them in it.

The third life challenges students to explore the 'rest of the iceberg' independently. Investigation Guides are used to probe aspects and extensions of the task and can be introduced into either the first or second life. Typically this involves providing suggestions for the direction the investigation might take. Students submit the 'story' of their work for 'portfolio assessment'. Typically a major criteria for assessment is application of the Working Mathematically process.

About Maths300

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

Maths300 is a subscription based web site. It is an attempt to collect and publish the 300 most 'interesting' maths lessons (K - 12).

- *Lessons have been successfully trialed in a range of classrooms.*
- *About one third of the lessons are supported by specially written software.*
- *Lessons are also supported by investigation sheets (with answers) and game boards where relevant.*
- *A 'living' Classroom Contributions section in each lesson includes the latest information from schools.*
- *The search engine allows teachers to find lessons by pedagogical feature, curriculum strand, content and year level.*
- *Lesson plans can be printed directly from the site.*
- *Each lesson supports teaches to model the Working Mathematically process.*

Modern internet facilities and computers allow teachers easy access to these lesson plans. Lesson plans need to be researched, reflected upon in the light of your own students and activated by collecting and organising materials as necessary.

Maths300 Software

Our attitude is:

> *stimulated students are creative and love to learn*

Pedagogically sound software is one feature likely to encourage enthusiastic learning and for that reason it has been included as an element in about one third of Maths300 lesson plans. The software is used to develop an investigation beyond its introduction and early exploration which is likely to include other pedagogical techniques such as concrete materials, physical involvement, estimation or mathematical conversation. The software is not the lesson plan. It is a feature of the lesson plan used at the teacher's discretion.

For school-wide use, the software needs to be downloaded from the site and installed in the school's network image. You will need to consult your IT Manager about these arrangements. It can also be downloaded to stand alone machines covered by the site licence, in particular a teacher's own laptop, from where it can be used with the whole class through a data projector.

Note:

- *Maths300 lessons and software may only be used by Maths300 members.*

Working Mathematically

First give me an interesting problem.

When mathematicians become interested in a problem they:

- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

Questions which help mathematicians learn more are:

- Can I check this another way?
- What happens if ...?
- How many solutions are there?
- How will I know when I have found them all?

When mathematicians have a problem they:

- Read & understand the problem.
- Plan a strategy to start the problem.
- Carry out their plan.
- Check the result.

A mathematician's strategy toolbox includes:

- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- Break a problem into smaller parts
- ...

If one way doesn't work, I just start again another way.

Professional Development Purpose

Our attitude is:

the teacher is the most important resource in education

> *We had our first study group on Monday. The session will be repeated again on Thursday. I had 15 teachers attend. We looked at the task Farmyard Friends (Task 129 from the Mathematics Task Centre). We extended it out like the questions from the companion Maths300 lesson suggested, and talked for quite a while about the concept of a factorial. This is exactly the type of dialog that I feel is essential for our elementary teachers to support the development of their math background. So anytime we can use the tasks to extend the teacher's math knowledge we are ahead of the game.*
> District Math Coordinator, Denver, Colorado

Research suggests that professional development most likely to succeed:

- is requested by the teachers
- takes place as close to the teacher's own working environment as possible
- takes place over an extended period of time
- provides opportunities for reflection and feedback
- enables participants to feel a substantial degree of ownership
- involves conscious commitment by the teacher
- involves groups of teachers rather than individuals from a school
- increases the participant's mathematical knowledge in some way
- uses the services of a consultant and/or critical friend

Maths With Attitude has been designed with these principles in mind. All the materials have been tried, tested and modified by teachers from a wide range of classrooms. We hope the resources will enable teacher groups to lead themselves further along the professional development road, and support systems to improve the learning outcomes for students K - 12.

With the support of Maths300 ETuTE, professional development can be a regular component of in-house professional development. See:

- http://www.mathematicscentre.com/taskcentre/resource.htm#etute

For external assistance with professional development, contact:

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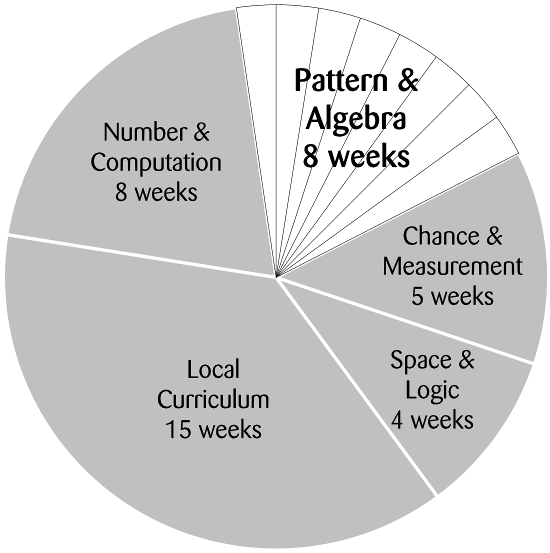
Part 2: Planning Curriculum

Curriculum Planners

Our attitude is:

learning is a personal journey stimulated by achievable challenge



Curriculum Planners:

- show one way these resources can be integrated into your weekly planning
- provide a starting point for those new to these materials
- offer a flexible structure for those more experienced

You are invited to map Planner weeks into your school year planner as the core of the curriculum.

Planners:

- detail each week lesson by lesson
- offer structures for using tasks and lessons
- are sequenced from lesson to lesson, week to week and year to year to 'grow' learning

Teachers and schools will map the material in their own way, but all will be making use of extensively trialed materials and pedagogy.

Using Resources

- Your kit contains 20 hands-on problem solving tasks and reference to relevant Maths300 lessons.
- Tasks are introduced in this manual and supported by the Task Cameos at:
  http://www.mathematicscentre.com/taskcentre/iceberg.htm
- Maths300 lessons are introduced in this manual and supported by detailed lesson plans at:
  http://www.maths300.com

In your preparation, please note:

- Planners assume 4 lessons per week of about 1 hour each.
- Planners are *not* prescribing a continuous block of work.
- Weeks can be interspersed with other learning; perhaps a **Maths With Attitude** week from a different strand.
- Weeks can sometimes be interchanged within the planner.
- Lessons can sometimes be interchanged within weeks.
- The four **Maths With Attitude** kits available at each year level offer 25 weeks of a Working Mathematically core curriculum.

A Way to Begin

- Glance over the Planner for your class. Skim through the comments for each task and lesson as it is named. This will provide an overview of the kit.
- Task Comments begin after the Planners. Lesson Comments begin after Task Comments. The index will also lead you to any task or lesson comments.
- Select your preferred starting week - usually Week 1.
- Now plan in detail by researching the comments and web support. Enjoy!

Research, Reflect, Activate

Curriculum Planner
Pattern & Algebra: Year 7

	Session 1	Session 2	Session 3	Session 4
Weeks 1 - 3	**Replacement Unit A:** Called a Replacement Unit because it replaces the traditional way of teaching a strand. The structure (see Page 16) begins with a week using all 20 tasks. The second week is a four session investigation of **Match Triangles** to model how a mathematician works (see Page 17). In Week 3, Investigation Guides (Appendix 1) support students as they dig deeper into one of the tasks on their own.			
Week 4	**Whole Class Investigations:** *Lining Up & Colour Spots on a Number Line* both start at a point accessible to all students and move to sophisticated algebra in a way that students seem to find comfortable. *Lining Up* focuses on equivalent algebraic expressions and *Colour Spots* provides opportunity for generalisation, substitution, and solving equations and bridges into linear graphing, gradients and equations to straight lines.			
Week 5	**Whole Class Investigations & Group Work:** Both investigations - *Game of 31* and *What's My Rule?* - begin in a game context. The algebra involved can be extended as the teacher sees appropriate. Lots of mental arithmetic practice in each lesson. Both lessons include software with a 'beat the computer' aspect that seems to appeal and is rich enough to revisit.			
Week 6	**Concept & Skill Development:** *Trial, Record & Improve* and *Back Tracking* offer two approaches to first creating and then solving equations. *Trial...* uses numeric methods and is calculator dependent in the more difficult cases. *Back Tracking* pivots on the strategy of working backwards.			
Week 7	**Whole Class Investigation:** *Crossing The River* and *4 Arm Shapes* are tasks in the Replacement Unit above. Some students would have investigated them in depth, but there is always more to learn about a problem. The lessons serve as a revision of Working Mathematically and a confirmation of the value of working in community.			
Week 8	**Whole Class Investigation:** *Painted Rods* works best with access to Cuisenaire Rods or the like. However, many students will soon be able to visualise the physical situation. There are several extensions in the Lesson Plan that could provide up to four sessions of involvement.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Curriculum Planner
Pattern & Algebra: Year 8

	Session 1	Session 2	Session 3	Session 4
Week 1	**Whole Class Investigation:** *Unseen Triangles* can be approached in an almost identical way to *Match Triangles* (see Page 17). As a whole class investigation it revises Year 7 and establishes the groundwork for visualising algebra that is called on in Garden Beds in the next unit.			
Weeks 2 - 4	**Replacement Unit B:** Called a Replacement Unit because it replaces the traditional way of teaching a strand. The structure (see Page 16) begins with a week using all 20 tasks. The second week is a four session investigation of **Garden Beds** to model how a mathematician works. In Week 3, Investigation Guides (Appendix 1) support students as they dig deeper into one of the tasks on their own. Software from Garden Beds can be used through the following weeks.			
Week 5	**Whole Class Investigation:** *Addition Totals* is an easy to manage investigation that involves considerable arithmetic practice as it works towards a generalisation. The twist in this problem is that the result has to allow for the slightly different behaviour of odd and even numbers. In essence the result is a step function. Teachers might link this to *Eric The Sheep* (Lesson 17, Maths300) which also involves a step function.			
Week 6	**Whole Class Investigation:** *Snail Trail* begins outside with physical involvement. There are many aspects of the initial problem that can be varied. Some students may have studied the task in the Replacement Unit, but there is always more that can be learned about a problem.			
Week 7	**Whole Class Investigation:** *Sphinx* is an easily accessible task that opens the door to quadratic algebra. There are many aspects to the problem and although the generalisation may be discovered in the first lesson, using it to interpolate results for Size 3, 5, 6, 7 etc. sphinxes and then trying to make them confirms the algebra while challenging spatial perception.			
Week 8	**Whole Class Investigation:** *Algebra Walk* uses physical involvement to refresh the linear algebra that permeates this kit and reinforce and extend the quadratic algebra that has begun to develop with *Sphinx*. It provides both a summary of the past two years and a springboard for the next two.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Planning Notes

Enhancing Maths With Attitude

Resources to support learning to work like a mathematician are extensive and growing. There are more tasks and lessons available than have been included in this Pattern & Algebra kit. You could use the following to enhance this kit.

Additional Tasks

- Task 59, 13 Away

  This is a game situation that has an underlying strategy waiting to be discovered. It is easy to state and easy to start, which is one feature that makes it suitable for younger as well as older students. The starting point is a pile of 13 counters. The person who takes the last one loses. Players take turns to remove either 1, 2 or 3 counters on any move. The unwritten challenge is to find a way to always win.
- Task 72, Farmyard

  A spatial challenge with aspects of area. A farmer wants to divide a field shaped like an L into smaller areas which are all the same size and shape. This task relates to Task 237, Trisquares and to Task 115, Dividing Shapes which is in Space & Logic, Years 7 & 8.
- Task 139, Squound

  A square and a circle intersect to form a 'Squound'. The total number of counters is known, as is the total in the square and circle. But how many in the Squound? Students must realise that the number in the intersection can't be counted twice and are led to this by an Investigation Guide. Then comes the big question which leads to surprising algebra. For a given number of counters, how many Squound questions can be asked?

More information about these tasks may be available in the Task Cameo Library:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Keep in touch with new developments which enhance **Maths With Attitude** at:

- http://www.mathematicscentre.com/taskcentre/enhance.htm

Additional Materials

As stated, our attitude is that mathematics is concrete, visual and makes sense. We assume that all classrooms will have easy access to many materials beyond what we supply. For this unit you will need:

- Newspaper and masking tape
- Square tiles
- Cuisenaire Rods
- Packs of playing cards
- Sphinx pieces
- Popsticks
- Unifix or MultiLink or the like

Apart from the first listed, all are available from Mathematics Centre Resources:

- http://www.mathematicscentre.com/taskcentre/resource.htm

Special Comments Year 7

- The first additional material you will need is popsticks. Check whether they are already available in the school.
- Look ahead to Planner Week 8. You will need Cuisenaire Rods. Cuisenaire Rods were once common in schools, but this may no longer be the case, so you may have to look around. An alternative is for students to build rods with Unifix or MultiLink.

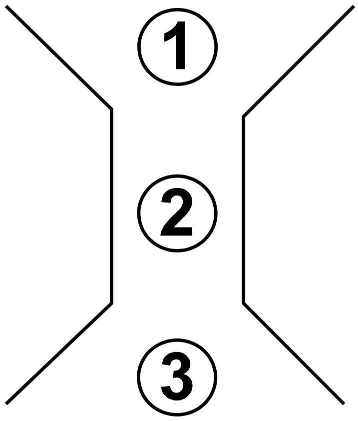
Special Comments Year 7 & 8

- The Replacement Unit plan may be a new unit structure for you and the students so look ahead and plan carefully. Trial teachers suggest the planning is more effective and efficient when carried out as a team.

Special Comments Year 8

- Look ahead to Planner Week 7. You will need time to prepare or order Sphinx Shapes.
- Look ahead to Planner Week 8. You will need to find an appropriate outside or gymnasium space.

Replacement Unit



Week Zero - Planning

Staff familiarise themselves with the material and jointly plan the unit. This is not a model that can be 'planned on the way to class'.

> *Getting together turned out to be great professional development for our group.*

Week 1 - Introduction

Students explore the 20 tasks listed on a printed menu:

- students explore the tip of the task, as on the card
- students move from task to task following teacher questioning that suggests there is more to the task than the tip
- in discussion with students, teachers gather informal assessment information that guides lesson planning for the following week.

> *We gave the kids an 'encouragement talk' first about joining us in an experiment in ways of learning maths and then gave out the tasks. The response was intelligent and there was quite a buzz in the room.*

A Menu of tasks is provided in Appendix 2: Recording Sheets.

Week 2 - Formalisation

> *It was good for both us and the students that the lessons in this week were a bit more traditional. However, they weren't text book based. We used whole class lessons based on the tasks they had been exploring and taught the Working Mathematically process, content and report writing.*

Assessment was via standard teacher-designed tests, quizzes and homework.

For Replacement Unit A, intended for Year 7, the lesson Match Triangles is included below. As described it provides at least four lessons consistent with this teacher's comment. Teaching these four sessions also prepares the students for the type of question included in the Investigation Guides for Week 3. For Replacement Unit B, Year 8, it is suggested teachers use Maths300 Lesson 16, *Garden Beds*.

Week 3 - Investigations

We were most delighted with Week 3. Each student chose one task from the menu and carried out an in-depth investigation into the iceberg guided by an investigation sheet. They had to publish a report of their investigation and we were quite surprised at the outcomes. It was clear that the first two weeks had lifted the image of mathematics from 'boring repetition' to a higher level of intellectual activity.

Over Years 7 & 8 students will have two weeks of opportunity to use these tasks. However, they will only investigate two tasks in depth led by an Investigation Guide - one in Year 7 and one in Year 8. See Appendix 1 for Investigation Guides.

Student Publishing

It is inappropriate to simply expect students to publish a report of their investigation. We have to devote lesson time to teaching how to keep a journal while investigating and how to plan and present a report. The Recording & Publishing section of Mathematics Task Centre includes two different approaches to scaffolding this process with the class. Both include sample student work and suggest that a report can be presented in forms other than pencil and paper, for example PowerPoint. The links are titled 'Learning to Write a Maths Report' and 'Learning to Write a Maths Report 2' and can be found at:

- http://www.mathematicscentre.com/taskcentre/record.htm

Match Triangles

Session 1: Introducing the Problem

Equipment

- One or two newspapers.
- One roll of masking tape.
- Packet of 1000 pop sticks (or tongue depressors or alternative - check the science department).
- Sufficient floor or hall space to lay out the pattern and have the students stand around to discuss it.

Preparation

- As students enter, or in advance if possible, arrange for monitors to hand out one sheet of newspaper and a strip of masking tape about 10cm long to each student. The masking tape can be hung from the edge of the table.

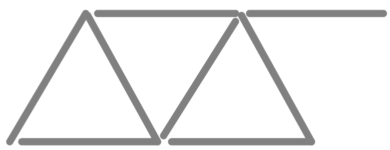
Plan

> *Today we are going to investigate this task together (hold up Match Triangles). Some of you looked at it last week. What we learn from this will get you ready for next week when you will go back to the tasks and explore one of them in much more depth.*

Ask the students to roll up their paper into a tube by starting from the fold line of the sheet. They should be able to get their thumb into the ends of the tube. Secure the two ends and the middle with a piece of tape.

> *Now I want you to bring your roll over here and stand around this space.*

Take the rolls from the first few students one at a time and begin to build the Match Triangles pattern:



> *Now follow on around the line and keep making this chain of triangles with your rolls.*

When everyone has placed their roll, count together the number of triangles that have been formed. Introduce the challenge for the day.

> *So with this many rolls we have made ... triangles. My first challenge for you today is to work out the number of rolls we would need to make a chain of 100 triangles.*

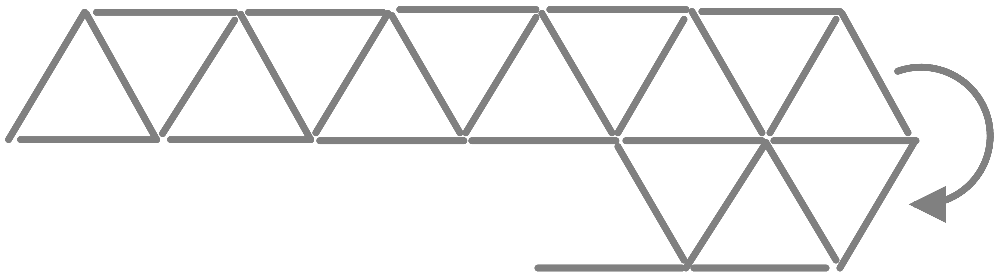
Go straight on to explain that you are going to give each pair a bundle of popsticks so they can experiment on their table, then send the students back to their seats to explore. Leave the newspaper rolls on the floor - if you have a double period this will be particularly convenient to begin Session 2. If not, near the end of the lesson, you can organise monitors to toss them into a box.

As the students are working, look for opportunities to comment and compliment on anything that reflects the Working Mathematically process (see Page 8). It could be that students are looking for patterns, or they see the structure in two ways (*Can I check this another way?*), or someone uses a particular strategy or... As with all **Maths With Attitude** tasks and lessons, this lesson is a vehicle for helping students learn to work like a mathematician. The answer to this challenge is actually quite easy to derive (at least for teachers!) and if the answer was the primary objective, the activity would be used up in half a period.

You will also find students who start to build other patterns with the sticks. Congratulate them on being mathematicians because they are creating a new problem to investigate, but ask them to stick to the current problem for now:

> *...so we can see just how much we can learn from it.*

Note: Sometimes the dimensions of the student's table forces them to make the pattern 'turn back':



It can sometimes take quite a bit of explaining before students realise that this is not the same problem because an additional stick has been needed to 'turn the corner' and in some places the same stick is being used for two purposes.

As groups indicate that they 'know the answer', visit each one and ask for an explanation. Try to take the focus off the answer by complimenting the thinking offered. Encourage the students to check their own answer by finding another way to 'see' and explain the problem.

Towards the end of the lesson gain consensus on the answer to the 100 challenge. Ask students to record the answer and also a diagram or notes to remind them how they worked it out.

> *Next lesson you will be asked to explain to someone else how to work out the number of rolls for any number of triangles.*

Session 2: Explaining and Justifying

Equipment

- Newspaper rolls from previous lesson.
- Packet of 1000 pop sticks.
- Sufficient floor or hall space to lay out the pattern and have the students stand around to discuss it.

Preparation

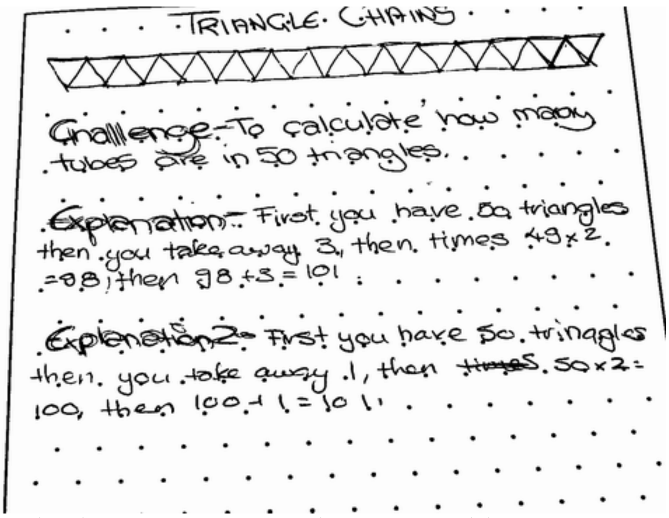
- As quickly as possible organise the rebuilding of the triangle chain. Many classrooms use this design as strengthening struts through the ceiling, so if yours is one of those lucky ones draw the students' attention to this application of the pattern.

Plan

You did great thinking last lesson about how this pattern was made. Today we focus on explaining to someone else how they can work out the number of rolls needed for any number of triangles.

Ask for a volunteer group to start off the discussion. Make sure they use/move the actual rolls to illustrate their explanation.

Keep the discussion rolling until students have exhausted all the ways they can 'see' the problem. Ask them to return to their seats. On the board sketch the layout you want for their journal notes (heading, date etc.) and ask them to record in notes and diagrams the two ways of solving the problem that they understand best.


TRIANGLE CHAINS

Challenge - To calculate how many tubes are in 50 triangles.

Explanation - First you have 50 triangles then you take away 3, then times 49 x 2 = 98, then 98 + 3 = 101

Explanation 2 - First you have 50 triangles then you take away 1, then ~~times~~ 50 x 2 = 100, then 100 + 1 = 101.

Look for students who begin to convert their natural language explanations into symbolic algebraic expressions. Encourage this without making it a requirement. The generalisation - which is the algebraic process involved - is in being able to explain for any case, not in *having to* explain in someone else's symbolic language.

The *Garden Beds* lesson suggested for Week 2 of Replacement Unit B focuses much more on this symbolic representation and sets up equivalent algebraic expressions in a concrete context.

Now I am going to test you to see if you really can work out the number of rolls for any number of triangles.

Sketch up a set of exercises like the following (which have been taken from the Investigation Guide for this task):

Substituting

Triangles	Rolls
19	
20	
35	
128	
319	
1000	

Copy & complete these tables

Solving - Working Backwards

Triangles	Rolls
	25
	39
	57
	71
	513
	1009

Students record answers in their journal. As necessary, encourage discussion between individuals, groups and as a class. Round off by asking:

> *So if someone told you any number of rolls explain how they could work out the number of triangles that could be made.*

Ask for these explanations to be demonstrated with the rolls in the same way as you encouraged at the beginning of this lesson. A full explanation would have to include the fact that it can't be 'any number' because some numbers of rolls (eg: 1, 2, 4, 6, 8...) don't make complete triangles.

> *Next lesson we will look more closely at the pairs of numbers we have made in these tables on the board.*

Session 3: Linking to Linear Algebra

Equipment

- Graph Paper (see Appendix 2: Recording Sheets).
- Overhead projector with a transparency of the same graph paper. Alternatively, a graph grid prepared on the whiteboard (or electronic whiteboard) or even a grid prepared on a Flip Chart or piece of poster paper.
- Computer that is able to run the Excel program and data show projector, or graphics calculators that achieve the outcomes described below.

Preparation

- Ask a monitor to hand out graph paper as the students enter and sketch a table on the board with the headings Triangles/Rolls. There is no need to make any entries in the table.

Plan

> *Last session when we were exploring Match Triangles we filled in a table like this one. These tables make pairs of numbers across the rows. In our case the pairs are (**T**riangles, **R**olls) pairs.*

Write the ordered pair on the board in words and in symbols (**T**, **R**).

I am going to come around to ten people and tell them a number. It will be your number of triangles. You have to work out how many rolls will be needed to make that many triangles.

Use numbers up to 20. Encourage checking in two ways and discussing with their partner if necessary. Ask students to write their ordered pair on a slip of paper when they are sure.

It will be a few minutes before we use your number pair so write it down now to remember it.

On the overhead demonstrate how to draw up a graph of Triangles against Rolls and choose one other number to show how to plot these pairs. Ask all students to copy your graph.

Next the students with number pairs will come out to the front one at a time and put a dot on the graph for their pair. As they do, they will say their pair out loud and you copy it onto your graph. At the end I want you to tell me two ways of knowing whether their pairs are correct.

Keep the students' slips of paper as they come out and copy their ordered pairs across the board in the order the students come up, eg: (3, 7) (8, 17) (1, 3) etc.

When the pairs are entered on the graph, ask for two ways of knowing that they are correct. One could be that 'we checked the calculation in each pair', but one should be that all the dots seem to be in a straight line.

Make the point that whenever there is a number pattern, there will also be a picture pattern of some sort. Point out too that in this case there are two picture patterns to go with the number pattern:

...this graph and the pattern of newspaper rolls we made on the floor.

Explain that since the pattern came from the pairs, it is reasonable to expect that we can find something the same about each pair. Ask the students to examine the pairs and look for

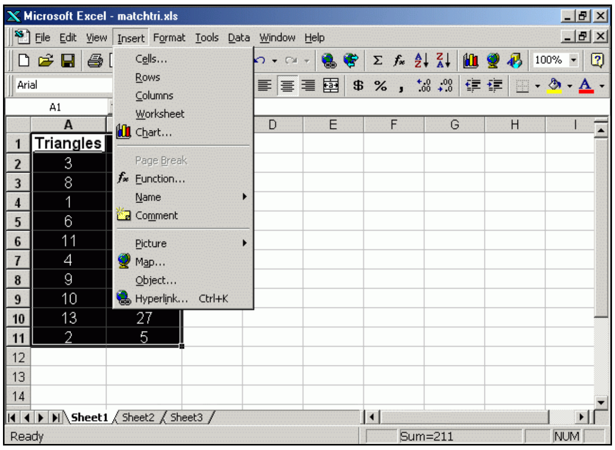
...something the same that is true in all the number pairs.

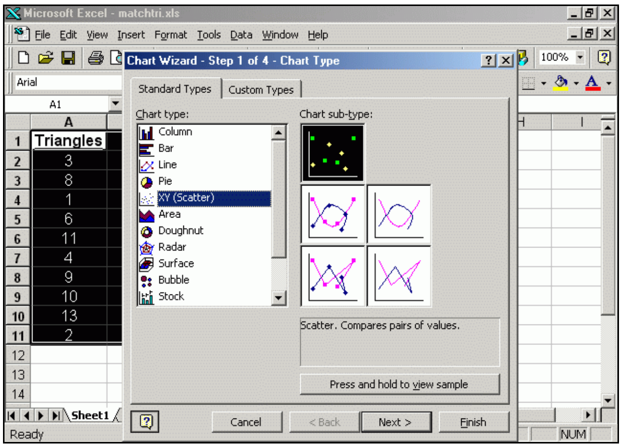
You will get answers like:

- two times the first plus one = the second
- second one take away twice the first one = 1

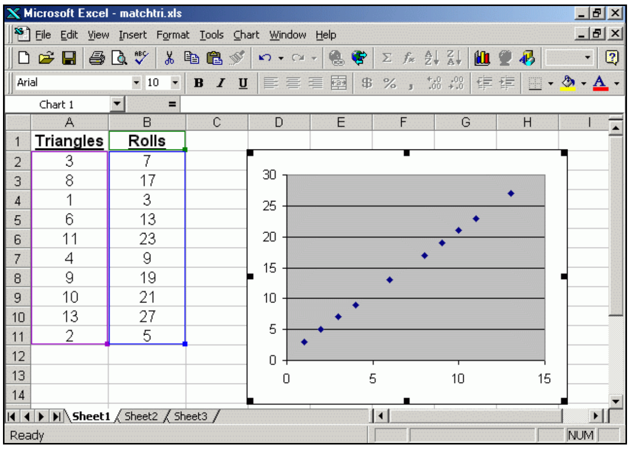
Record these in words and symbols and ask how they relate to the ways the students were seeing the triangles made with rolls.

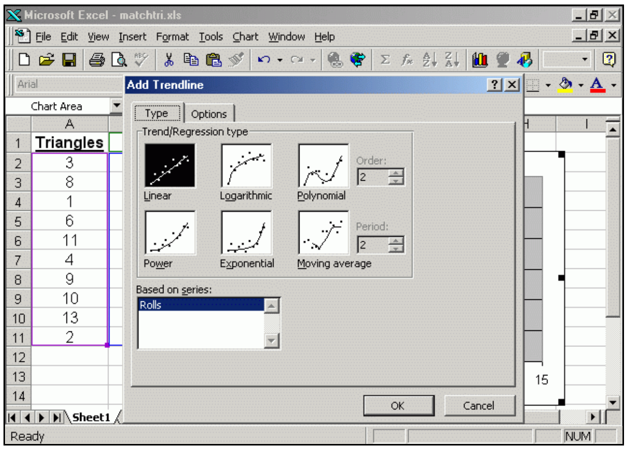
Demonstrate how these pairs can be entered into a table in a spreadsheet. Select the table and use Insert/Chart/XY(Scatter) to add a graph.





Select the chart, then choose Chart/Add Trendline. Leave the first sub-type selected (Linear) and on the Options Tab select 'Display equation on chart'.





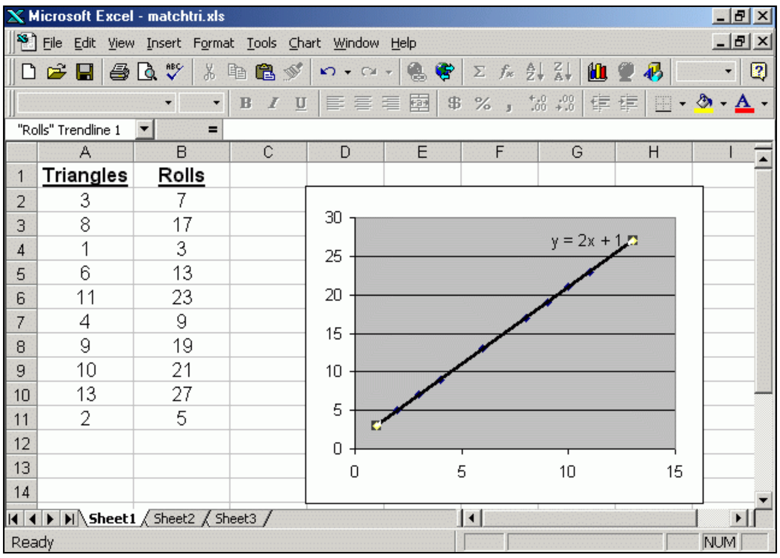
Ask how this equation relates to what was just discovered to be true about all number pairs.

(Note: This aspect of the lesson sequence may be better as a lesson in a computer lab (a) if you have the facilities and (b) if you can access five lessons in the week. Perhaps work with the computer teacher on this.)

The use of this tool demonstrates that the ordered pairs do lie on the line governed by the equation: $y = 2x + 1$ or $y - 2x = 1$.

Again confirm the links between:

- the original visual/construction pattern



- the oral and written generalisations
- the number pairs and the rules that link them
- the graph of the pairs and the equation of the line

> *The Excel trendline has joined the dots on the graph. Do you think it is right to do that?*

Explore the implication that, for example, there could be 3¼ triangles using 7½ rolls. How do the students feel about this in the light of the actual problem? Reinforce the idea that mathematics must make sense in the context of the problem that generates it. There is also opportunity here to introduce the ideas of discrete and continuous variables and domain and range.

Session 4: Publishing

Equipment

- Whiteboard or overhead projector.
- Paper for drafting a report.

Preparation

- Place the lesson in the context of a mathematician who has explored a problem in depth wanting to publish her/his work for all to learn.

Plan

- See Student Publishing Page 17 for reference to lessons plans which support students to produce quality mathematics reports. The references include examples of student work.
- Consider the possibility of publishing in forms other than written text, eg: PowerPoint, web page, wall display, cartoon strip, video…
- Open the door to other problems by reminding students that in Session 1 some of them started to develop other patterns worthy of investigation.
- Highlight that the investigative process just journeyed as a class will be repeated by each pair in Week 3 and show an Investigation Guide.

Task Comments

- For each task, an Investigation Guide (see Appendix 1) provides additional support. These are designed to lead the students through Week 3 of each Replacement Unit, but they will also assist you in learning more about the iceberg of the problem. In addition, many of the tasks have an alternative Investigation Sheet linked to their companion Maths300 lesson.
- Tasks, lessons and unit plans prepare students for the more traditional skill practice lessons, which we invite you to weave into your curriculum. Teachers who have used practical, hands-on investigations as the focus of their curriculum, rather than focussing on the drill and practice diet of traditional mathematics, report success in referring to skill practice lessons as Toolbox Lessons. This links to the idea of a mathematician dipping into a toolbox to find and use skills to solve problems.

4 Arm Shapes

A straightforward introduction to the idea of building a spatial pattern and discovering that a number pattern results. Some will 'see' the generalisation almost immediately. None-the-less encourage recording in the table from the viewpoint of rehearsing the skills of a mathematician. Also encourage taking a second look for a way to solve the problem by asking the mathematician's question, *Can you check it another way?*

It is very important that the students attempt, orally and in written form, to make links between the visual pattern and the numbers used in the generalisation. *Why is it four times? Where does that plus one come from?*

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Addition Totals

This task appears to be as straightforward as **4 Arm Shapes**, but it is a little more challenging because the generalisation has to account for a slight twist related to whether the chosen number is odd or even. The task becomes a class investigation if you are following the planner and by that week, some students may have used it already. However, teachers find it reinforces the purposefulness of using tasks if they begin such a lesson by selecting the task 'out of the box', holding it up and saying something like: *Some of you have already explored this task with your partner. Today we will see what more we can learn from it when we work together to explore it like mathematicians.*

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Algebra Through Geometry

This task is a more sophisticated partner to **Shape Algebra**. The traditional x and y of algebra are given immediate physical presence in the task and the intriguing foam shapes allow students to see and touch algebraic procedures related to operations on like and unlike terms. The context invites students to apply visual and tactile intelligences, and even the fact that it includes algebraic fraction operations related to halves and quarters is not a barrier to most. *The Recording Sheet which could assist students with this task can be found in Appendix 2.*

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Crossing The River 1

No mathematics as such is required to work out this task. It can be thought through by asking what the choices are for the first move and exploring the steps that follow until a successful sequence is discovered. A visual/kinaesthetic pattern develops when the successful path is chosen. Students come to realise that it takes 4 trips to shift each adult across the river. It will help to consolidate that pattern if the students are asked to record their solution. The task can be extended by asking: *What if we change the number of adults or children?*.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Eric The Sheep

Most students set this problem up with materials and easily find the one answer required on the card. However the iceberg of the task involves multiples of 3, visual representation and algebraic generalisation.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Garden Beds

The task suggests making a table to reveal a number pattern. This is one of the strategies from the mathematician's toolbox. The pattern can be used to answer the extension of the problem in Question 2, but it is also important to encourage students to see and express one or more visual ways of building the tiles around the garden bed. There is a double line on the card before the Challenge. Many task centre teachers use double lines like this as a signal to students that they are expected to communicate with the teacher at this stage. This procedure provides opportunity for teachers to ask questions which bring out the students' visual generalisation of the problem, and support them to continue. There are several ways of 'seeing' the pattern other than the 'double and add six' approach that tends to develop from the table.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Lining Up

If you know your position from each end of a line, can you work out how many people in the line? This is a familiar context because children are often put in lines. There are multiple visualisations. Encouraging students to find these 'ways of seeing' reinforces that the mathematician's question *Can I check it another way?* is not a hollow one. They also provide several equivalent algebraic expressions. Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Making Monuments

This is something like **4 Arm Shapes**, but in three dimensions. Encourage the students to visualise the counting in at least two ways, then they can check their own work. The task is structured to highlight aspects of the Working Mathematically process - collecting and organising data, seeking and seeing a pattern, making and testing hypotheses, recording the steps of the discovery. From the algebraic point of view, when a generalisation is found, each term in it links to the visual pattern of the construction. In this way, algebra makes sense.

Match Triangles

Again a task structured to lead students through the Working Mathematically process. The context is different, but the process used to investigate the problem is the same. Again, algebra makes sense because the visual/tactile model suggests the generalisation and, conversely, the elements of the generalisation relate to the model. Can the students see the pattern in more than one way? If so, how do the words they use to describe it vary? What do these different words imply about the symbolic generalisation that can be developed from each description? *The Recording Sheet which could assist students with this task can be found in Appendix 2.*

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Mirror Patterns 2

This intriguing task makes links between geometry, number, pattern and algebra. It takes the students past the fun of creating mirror patterns by requiring them to measure the angle between the mirrors. Recording the measured angles in the table, as suggested, leads to the hypothesis that the angle $A = 360 \div n$, where n is the number of sides of the polygon.

The first part of the task explores regular polygons whose side numbers are a factor of 360. The task then suggests working backwards and asks about angles like 100° that would imply a shape with 3.6 sides. How is this to be interpreted?

Painted Rods

Another algebra in context situation. This time there is a three dimensional aspect to the problem. Being able to hold and manipulate the six given rods, encourages a brain picture that can extend to the rod of length 100, and indeed to a visualised rod of any length. The emphasis is on being able to explain what to do regardless of rod length. This task has a companion Maths300 lesson with more detail.

Pointy Fences

This task is similar to **Garden Beds**. The plants have become pyramids and each pyramid has four triangular faces, but the tiles surround the pyramids in the same way as they surround the plants in **Garden Beds**. It is not necessary to try **Garden Beds** before **Pointy Fences**, however, the cameo for *Garden Beds* will also support teachers with many ideas for this task. *The Recording Sheet which could assist students with this task can be found in Appendix 2.*

Shape Algebra

Shape Algebra blows away the traditional opening line of an algebra lesson, ie: *Let x stand for any number*. In this task *x* is a shape! Well actually it is the number that stands for the area of the shape, but the exciting feature of the task is that the algebra becomes geometry. The task focuses on creating and manipulating algebraic terms, but all stages of the symbol manipulation are grounded in the concrete material. Students are often heard making statements like: *Shape D must be 4x - y because there were four x pieces there and a y bit has been cut out of it*.

You might like to turn the lesson into a whole class investigation by scanning or tracing the pieces and printing a copy for each pair. Students can carefully cut out their own sets. What other shapes can they make and record by combining the shapes in the set? What is the algebraic representation of each shape? Can the students transfer their experience to text book work on algebraic expressions and like and unlike terms? *The Recording Sheet which could assist students with this task can be found in Appendix 2.*

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Smooth Edge Tiles

The thinking in this task involves realising that:

- The L-shape tiles are always in the corner, so there will always be four of them.
- Further, because the L-shape tiles take up the corners there will be two less single smooth edge tiles on each length and width.
- The tiles with zero smooth edges will always form a rectangle in the middle that is 2 less than each of the length and the width of the patio.

So, for an M by N patio:

- 2 smooth edges: 4
- 1 smooth edge: 2(M - 2) + 2(N - 2)
- 0 smooth edges: (M - 2) x (N - 2)

However, the development of this symbolic notation is not the objective of the task. The objective is to learn to:

- 'see' how the problem is constructed
- 'see' the construction in alternative ways if possible
- describe the general principles of construction to someone else in natural language

Then, when symbolic notation is used, to be able to give meaning to the symbols and operations in terms of the physical problem.

To extend the task, consider asking backwards questions like:

- *If Janine had only 14 single smooth edge tiles and all the other tiles she needed, what size patios could she build?*

Snail Trail

Any proposed solution the students think of when they first read the card can be readily checked with the equipment. This is important because it is often the case in this problem that first responses are inaccurate. The double line drawn across the card after Question 1 is to remind students to check with the teacher after this first step. Simply asking them to show you their solution with the wood is enough to confirm or deny their proposal.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Sphinx

Not only is the shape used in this task imagined to look like the profile of Egypt's Sphinx, but the depth of the iceberg of this task is as extensive as the mystery that is imagined to surround that national monument. In this unit the aspect emphasised is the algebra developing from the growth pattern in the task. Four sphinxes make a sphinx. So four of the new size sphinx will make the next sphinx and so on. It is also true that one can imagine each of the original sphinxes being made from four smaller sphinxes and each of these from four smaller ones and so on. Within this conceptualisation is the opportunity to explore an introduction to Limits, as used in calculus. *The Recording Sheet which could assist students with this task can be found in Appendix 2.*

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Square Numbers

The patterns in this task help students discover two pieces of information that will continue to be useful throughout their mathematical studies. Firstly, that square numbers actually represent 'real' squares and secondly that a square number is the sum of consecutive odd numbers. The task also links to the concept of area as counting tessellating units (usually squares). *The Recording Sheet which could assist students with this task can be found in Appendix 2.* Cut it into strips to supply each pair of students.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

The Mushroom Hunt

Perhaps the first challenge in this task is to be clear about how the problem works. Two conditions have to be satisfied simultaneously. The total number of mushrooms must be sixty-three *and* combining and re-combining baskets must produce every number from 1 through to 63. The example makes this clear. The total of the baskets *is* 63. Some required totals can be made, including those shown. But how, for example, can the number 1 be made using the illustrated baskets? It can't be, and this realisation begins the search for the solution. There has to be a basket with just one mushroom in it.

Note that mushrooms are not shifted from basket to basket to make totals. Each calculation is based on *If we combined these baskets, what would the total be?*. The pattern that develops in the solution is based on powers of 2, or as the students may perceive it, continuous doubling. The relevant arrangement is 1, 2, 4, 8, 16, 32. The task also offers a way to introduce the Binary Number system. If a 1 means *use one of these baskets* and a 0 means *use none of these baskets* then the table shows the binary equivalent of each decimal number listed down the side.

	Basket Containing					
	32	16	8	4	2	1
1						1
2					1	0
3					1	1
4				1	0	0
5				1	0	1
...						
14			1	1	1	0
...						
38	1	0	0	1	1	0
...						

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Time For Tiling

As with **Smooth Edge Tiles**, the objective of this task is to:

- 'see' how the problem is constructed
- 'see' the construction in alternative ways if possible
- describe the general principles of construction to someone else in natural language

Then, when symbolic notation is used, to be able to give meaning to the symbols and operations in terms of the physical problem.

Odd Squares 1

The construction could be seen as the number of dark tiles being twice the side length less 1 because the diagonals are the same number of tiles as the side and counting both diagonals counts the centre tile twice. Once the dark tiles are known the light tiles can be found by subtraction from the number of tiles in the square.

Odd Squares 2

The light tiles are in four clusters. The longest line of the cluster is two less than the side of the square. Each successive row of the cluster is two less than the previous row because there are two dark tiles in each row - one from each diagonal. This pattern continues down to a one tile row. Adding this sequence for one cluster and multiplying by four gives the total of white tiles. Subtraction from the total number of tiles in the square gives the total of dark tiles.

No doubt there are also other ways of seeing this construction. The more the merrier. One of the mathematician's questions is *Can I check this another way?*.

Once students learn that the search for other ways is valued, they have less need to ask *Am I right Miss?*. Teachers meet this question with *Can YOU check it another way?* and thereby encourage students to be more responsible for their own learning.

Even Squares 1

Focussing on the dark tiles we might see a central square of four with four shortened diagonals leading out to the corners. Each of these short pieces is one less than one half of the diagonal length, which, of course, is the same length as the side of the square. Having calculated the dark tiles in this way, the light tiles can be found by subtraction.

Even Squares 2

The light tiles are in the same four clusters as above and the successive row lengths are shortened by two each time for the same reason as above. The difference from the odd case is that the sequence counts down to a row of two tiles, rather than a row of one.

Again, there will be other ways to view these constructions.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Unseen Triangles

Build a 'mountain range' with matches (or popsticks) and work out the number of matches for any length. This task is a more complex version of **Match Triangles** and can be approached in equal depth. For a start, there is more than one way to 'see' the generalisation. This task has a companion Maths300 lesson with more detail. *The Recording Sheet which could assist students with this task can be found in Appendix 2.*

Lesson Comments

- These comments introduce you to each Maths300 lesson. The complete plan is easily accessed through the lesson library available to members at:
  http://www.maths300.com
  where they are listed alphabetically by lesson name.

4 Arm Shapes

The lesson uses square tiles; a concrete material readily available in most schools. The lesson could be presented with a whole class introduction by gathering the students at a central table and using large size cardboard tile cut-outs. Students would then move to their groups and use the square tiles to continue the investigation. Alternatively teachers might like to begin in groups and supply an investigation sheet to guide students into the iceberg of the problem.

As with *Match Triangles*, if the primary objective of the lesson is seen as obtaining the oral or symbolic form of the generalisation which describes the pattern, then the activity is likely to be seen as 'finished' very quickly. Alternatively the lesson offers a strong visual/tactile dimension that certainly makes the rule 'easy' for many, but in so doing allows the emphasis to move to the Working Mathematically process. For example:

- Most will see the structure of the problem as four arms with one special tile in the middle. This leads to the generalisation of four times one arm length plus one, as in the notes. But mathematicians ask the question: Can I check this another way?. Asking that question in this circumstance may lead to seeing the structure as one horizontal line crossed by a vertical line of the same length. In other words the total is twice the length of the horizontal line. But this counts the central tile twice, so one tile has to be taken away. The verbalisation, and indeed the symbolic representation, of each of these generalisations is different, but each can be directly related to the concrete situation. The pattern building, and its complementary algebra, make sense.
- When mathematicians have solved a problem, they ask what else they can learn from it. The lesson offers several What if...? questions that could be followed through. These include alternative shapes to build with tiles.
- Since the problem is readily solvable by most students, the mathematics is unlikely to 'get in the way' if the lesson is used to model how to publish a mathematics report. See Student Publishing Page 17.

Addition Totals

The rule that describes the pattern in this task is a little more sophisticated than *4 Arm Shapes*, *if* symbolic representation is seen as the objective. The generalisation requires differentiation between what to do with odd and even number totals. The Lesson Plan clearly flags the steps in the Working Mathematically process as they are used, so as well as helping students work towards a symbolic generalisation, the overall objective of learning to work like a mathematician is being achieved.

Algebra Walk

As the title suggests this lesson is dependent on whole body involvement. The students become points on a co-ordinate plane but their movements are governed by given rules. In this way the equations connected with lines, and especially those related to quadratics, become real because the students *are* the graph. The lesson does require a little planning ahead to find the appropriate space to step out the functions, have a length of rope available, and even to consider the weather forecast. A major benefit of the lesson is the visual imagery which remains with the students long after the activity is over.

- *This lesson saved me at least two sessions of teaching because of the powerful imagery.*

Back Tracking

After only one lesson students are able to use a pictorial method, combined with the problem solving strategies of working backwards and breaking a problem into smaller parts, to solve equations so complex that the text book examples seem like child's play. Some experience with algebraic symbols and facility with conventions for order of operations is required, but beyond that the lesson is open to students with a wide range of abilities. The lesson is supported by an Investigation Sheet which is much 'tougher' than the material offered in most texts. However, the experience of many teachers is that students 'love' the work. A great lesson to use before beginning the textbook chapter on equations, and plenty of room in the week to insert any relevant local material.

All the evidence is that the lesson 'succeeds' - so the question becomes *Why does it succeed?*, when so many text lessons on this topic fail. The answer has much to do with the effort that goes into attaching meaning to the equations rather than into instrumental techniques for their solution.

Colour Spots on a Number Line

Students generally like colouring in and this lesson captures that interest to assist the objective of learning to work like a mathematician. The colouring is as quick as making a small circle with a felt tip pen. The spots are added to the line according to a tactile rule and the spots begin to make a visual pattern. The students soon learn that where there is a visual pattern there will be a number pattern and vice versa.

The lesson becomes open-ended because the number line has no numerals marked. It *does* display equal divisions, but it is the students who choose the starting number and the 'jump' represented by each division. The outcome is that every student is able to explore patterns of their own choice and can compare and contrast with those being explored by their classmates. Links are easily made to symbolic representation of the pattern rules and to linear graphing.

As suggested in the Lesson Plan, there is a strong link to the *What's My Rule?* lesson. Further, the number line has been provided with ten divisions per section of the line. This is not sacrosanct. *What happens if ...?* we design number lines with 5, or 15, divisions in each section?

Crossing The River

Eight adults and two children have to cross a river in a small boat that restricts the number of people who can travel simultaneously. How can everyone get across? Teachers strongly recommend beginning by acting out and it has been used this way up to at least Year 9. This is followed with concrete materials and recording in words and pictures. The Classroom Contributions section of the lesson includes some examples of Year 7 recording.

Interestingly the rule that describes the total number of crossings in the problem is the same - in symbols - as the usual rule that describes *4 Arm Shapes*. This is a rather neat demonstration that algebra in context adds meaning that cannot be accessed through naked symbolic representation. It also opens the door to giving student groups a symbolic representation as a starting point (eg: $P = 5A + 2$) and asking them to find a context that makes sense of the rule.

However that is a corollary to the main lesson. Once the original problem is solved, the question *What happens if...?* takes over. Changing the number of adults or the number of children or both leads to investigation of a family of problems. As with other lessons of this type, there is also potential for broadening the lesson beyond the focus on generalisation, substitution and solving equations into graphical algebra.

Game of 31

It is easy to start this simple card game and the playing rules are straightforward enough to allow the mind to begin thinking about strategy. The game begins as simple arithmetic, but can lead as deeply as you wish into the world of Algebra.

As the game continues the basic arithmetic practice gives way to consideration of the logical strategies needed to win. These expose patterns which eventually make the game as much about patterns and algebra as about number skills.

It is also extremely flexible, in that it can be as simple or complex as desired. Consequently it has been successfully used from Year 2 to Year 10. The companion software allows students to test their emerging theories and understanding as they try to beat the computer.

Garden Beds

This Lesson Plan (and that for *Game of 31*) has been structured using teaching techniques developed by English as a Second Language (ESL) teachers. The devices within it that build mathematical conversation and recording/publishing of the investigation provide support for all students and can be abstracted for use with any lesson.

Garden Beds is a very rich context from which many mathematical concepts can be explored. The story of building a set of tiles around a garden captures students' interest. The mathematics of counting, area and perimeter, pattern and algebra are all evident. Within algebra the lesson covers several ideas concurrently, namely, the concepts of variable and function, substitution, solving equations, equivalence, and domain and range.

Garden Beds is a concrete and active lesson which suits many levels of ability. It provides opportunity for algebra to 'sneak up' on the students because the structure of the lesson encourages constructing and drawing, which leads to number patterns which the students naturally (and usually quite quickly) generalise. However, it is most often the case that within any class there is more than one way of seeing the generalisation. All are valid and the algebra makes sense because it can be related to the physical context. The investigation expands to seek:

- *...all the different ways we can predict the number of tiles for any garden.*

These rules can be expressed first in natural language, then meaningfully converted to symbols.

There is also ample opportunity to link the lesson with graphing and many ways to extend the lesson by investigating other ways of arranging the plants.

The software is possibly best used after all this has been explored. It offers just one view of the way the structure is built, so to use it too early may stifle the variety of responses the rest of the lesson is after. However, the software is extremely valuable in providing mental arithmetic challenges in the context of 'racing the computer'; something most students are very willing to try.

Heads & Legs

Two children go to a farm. There are some roosters in the cow paddock. Sarah counts the heads they see. Sam counts the legs they see. When they run back to the farmhouse to tell Mum what they found, she is able to use the two pieces of information to tell them how many animals there were altogether.

This is a classic problem that can be introduced in a concrete way using the animal cards provided. It offers significant mental arithmetic opportunity with the two and four times tables embedded in a problem solving context.

There are several strategies Mum could have used to solve this problem. The students have ample opportunity to explore them. The emphasis is on finding the answer in more than one way, then explaining the solution to someone else. Both of these aspects are essential elements of the Working Mathematically process.

The software continues the challenge by offering an extensive range of problems presented either visually or symbolically. Almost all students will be able to find a satisfactory challenge at their own level.

A similarly structured problem has Sarah and Sam standing on one side of a garden fence and a group of children riding trikes and bikes passing on the other. Only the number of helmeted heads can be seen above the top paling of the fence and only the number of wheels can be counted below the bottom paling. Again Mum is able to work out the number of bike and trike riders. How?

Lining Up

Lining up is part of most student's lives, so the context seems to generate a brain picture fairly readily. Most students quickly come up with a way to analyse the initial problem. It is also usually the case that more than one way of viewing the problem is described. This opens the door to highlighting the Working Mathematically process. The focus of the lesson shifts from the answer to working as a community of mathematicians. Equivalent algebraic expressions are found and explored. There is also plenty of opportunity to rehearse skills related to substituting into, and solving, equations.

Painted Rods

If this lesson is to be available to as wide a range of learning style preferences as possible, then it is dependent upon access to Cuisenaire Rods or equivalent. It may also help if you create your own larger size demonstration set as suggested in the Lesson Plan.

The story shell surrounding the lesson is that a person decides to paint the outside surface of a series of rods of various lengths.

- *How can you predict the number of unit squares of area that will be painted for any length of rod?*

Students use Cuisenaire rods to model simple cases, collect data from these and generalise the results into a rule. The first challenge is to work out the painted area of a rod 100 units long.

A central aspect of the lesson is explaining and justifying the rule so that others can see how it is done. The collection of data points created as the problem is solved (ie: Size 1 rod = 6 units, Size 2 rod = 10 units, Size 3 rod = 14 units ... so ordered pairs [1, 6], [2, 10], [3, 14]...) provides opportunity for graphing and therefore reinforcing with the students that whenever there is a visual pattern there will be an underlying number pattern and vice versa.

Snail Trail

A snail determined to climb out of a well sets out at a steady speed, but needs to rest after a given time. During the rest period it slips back a given amount. The challenge is to decide when it will escape. This is a well known problem that has appeared in many guises in many countries. For example, the snail is often a frog. The puzzle can help students develop logical skills, however a world of algebra opens up by exploring the effect of changing the variables involved.

The lesson is perfect for a kinaesthetic start by taking the students outside and quickly marking some chalk lines on the asphalt or marking positions on the grass with twigs. Once begun it can be continued inside with the game board supplied in the lesson.

It is also a great lesson for conversation and 'argument'. Students often get different answers and then argue among themselves in order to justify both their logic and calculations.

Sphinx

Perhaps the surprise in this lesson is that an essentially simple jigsaw puzzle has such extensive algebraic connections. However, you will need to have many sphinx pieces available. They can be prepared from the master in the lesson, or obtained from Mathematics Centre.

The algebraic pattern in this lesson is based around $y = x^2$ and therefore offers an introduction to quadratic algebra and quadratic graphs. There are also questions related to continuous and discrete functions, domain and range, interpolation and limits and asymptotes. The Classroom Contributions remind us that the perimeter of the growing sphinxes is another function worthy of study.

Beyond the algebra connections there are even more surprises when it is realised that the growth pattern of building each new sphinx from four of the previous size is also laying the groundwork for similarity and therefore trigonometry. As one Year 8 student said: *No matter how big the sphinx gets, the angles all stay the same*.

Trial, Record & Improve

Many students seem to find solving equations like $6 + x = 11$ to be a waste of time because *It's obvious what the answer is*. Yet, in the same class others may find the solution difficult especially if they don't have automatic number bond knowledge. This is the reality of the seven year spread of ability which research indicates faces Year 7 teachers.

Consequently teachers tend to balk at including equations like $0{\cdot}35 - x = {}^{-}1{\cdot}24$. However, this lesson sets up a situation in which students are happy to tackle such equations.

The keys to success are:

- being involved in the creation of equations before being asked to solve them
- free, self-directed access to calculators
- accepting the trial, record and improve technique as a valid and much used approach of professional mathematicians

and, perhaps most importantly, recognising that the text book driven recipe model for solving equations is less important than the ability to solve them and explain how to do it.

The lesson has a direct connection with Back Tracking and, like Back Tracking, can help build an *I can do Maths* attitude among students.

Unseen Triangles

To lift this lesson from its life as a task for two students (as included in the kit), you only need a packet of icy-pole (popsicle) sticks or something equivalent. Matches are less satisfactory in the whole class context because they are 'too fiddly'. Also the pop-sticks encourage working on the floor, which some students prefer.

This is a wonderful lesson that captures and extends algebra ideas in the same way as the *Match Triangles* lesson. One of the important aspects of the lesson is that there are several ways to see and describe the physical pattern. As a result, the teacher can reinforce the idea that there is more than one way to tackle a problem and that is what allows mathematicians to anser 'Yes' when they ask *Can I check this another way?*. This also leads, like the *Garden Beds* lesson and others, to equivalent algebraic expressions and graphing algebraic functions.

What's My Rule?

This is a favourite 'number machine' type of lesson. Often text books provide exercises with a similar basis and these can be used for practice. However, in the lesson presentation, the mathematics seems to have more life. The lesson links very well with *Colour Spots* because, if you plumb into the depths of the *Colour Spots* iceberg, it provides a clue for solving any of the (linear) *What's My Rule?*

challenges. This link is clearly made in the *Colour Spots* lesson plan. However it is not the intention that the students be told this clue. Rather that they are provided with opportunity to 'work it out for themselves'.

One of the features of this lesson is the natural way you can emphasise the need for recording and therefore highlight that aspect of a mathematician's work. This idea is backed up by the software because it is based around a table for recording the trials. The recording sheet and the computer reinforce each other in this regard. The recording sheet itself then becomes a feature that can help students learn to bridge from describing the rule in natural language to doing so in symbolic form.

Part 3: Value Adding

The Poster Problem Clinic

Maths With Attitude kits offer several models for building a Working Mathematically curriculum around tasks. Each kit uses a different model, so across the range of 16 kits, teachers' professional learning continues and students experience variety. The Poster Problem Clinic is an additional model. It can be used to lead students into working with tasks, or it can be used in a briefer form as a opening component of each task session.

> *I was apprehensive about using tasks when it seemed such a different way of working. I felt my children had little or no experience of problem solving and I wanted to prepare them to think more deeply. The Clinic proved a perfect way in.*

Careful thought needs to be given to management in such lessons. One approach to getting the class started on the tasks and giving it a sense of direction and purpose is to start with a whole class problem. Usually this is displayed on a poster that all can see, perhaps in a Maths Corner. Another approach is to print a copy for each person. A Poster Problem Clinic fosters class discussion and thought about problem solving strategies.

Starting the lesson this way also means that just prior to liberating the students into the task session, they are all together to allow the teacher to make any short, general observations about classroom organisation, or to celebrate any problem solving ideas that have arisen.

One teacher describes the session like this:

> *I like starting with a class problem - for just a few minutes - it focuses the class attention, and often allows me to introduce a particular strategy that is new or needs emphasis.*

It only takes a short time to introduce a poster and get some initial ideas going. The class discussion develops a way of thinking. It allows class members to hear, and learn from their peers, about problem solving strategies that work for them.

> *If we don't collectively solve the problem in 5 minutes, I will leave the problem 'hanging' and it gives a purpose to the class review session at the end.*
> *Sometimes I require everyone to work out and write down their solution to the whole class problem. The staggered finishing time for this allows me to get organised and help students get started on tasks without being besieged.*
> *I try to never interrupt the task session, but all pupils know we have a five minute review session at the end to allow them to comment on such things as an activity they particularly liked. We often close then with an agreed answer to our whole class problem.*

A Clinic in Action

The aims of the regular clinic are:

- to provide children with the opportunity to learn a variety of strategies
- to familiarise children with a process for solving problems.

The following example illustrates a structure which many teachers have found successful when running a clinic.

Preparation

For each session teachers need:

- a Strategy Board as below
- a How To Solve A Problem chart as below
- to choose a suitable problem and prepare it as a poster
- to organise children into groups of two or three.

The Strategy Board can be prepared in advance as a reference for the children, or may be developed *with* the children as they explore problem solving and suggest their own versions of the strategies.

The problem can be chosen from

- a book
- the task collection
- prepared collections such as Professor Morris Puzzles which can be viewed at: http://www.mathematicscentre.com/taskcentre/resource.htm#profmorr

The example which follows is from the task collection. The teacher copied it onto a large sheet of paper and asked some children to illustrate it. *The teacher also changed the number of sheep to sixty* to make the poster a little different from the one in the task collection.

The Strategy Board and the How To Solve A Problem chart can be used in any maths activity and are frequently referred to in Maths300 lessons.

The Clinic

The poster used for this example session is:

> Eric the Sheep is lining up to be shorn before the hot summer ahead. There are sixty [60] sheep in front of him. Eric can't be bothered waiting in the queue properly, so he decides to sneak towards the front.
>
> Every time one [1] sheep is taken to be shorn, Eric <u>then</u> sneaks past two [2] sheep. How many sheep will be shorn <u>before</u> Eric?

This Poster Problem Clinic approach is also extensively explored in Maths300 Lesson 14, *The Farmer's Puzzle*.

Strategy Board

DO I KNOW A SIMILAR PROBLEM?	ACT IT OUT
GUESS, CHECK AND IMPROVE	DRAW A PICTURE OR GRAPH
TRY A SIMPLER PROBLEM	MAKE A MODEL
WRITE AN EQUATION	LOOK FOR A PATTERN
MAKE A LIST OR TABLE	TRY ALL POSSIBILITIES
WORK BACKWARDS	SEEK AN EXCEPTION
BREAK INTO SMALLER PARTS	...

How To Solve A Problem

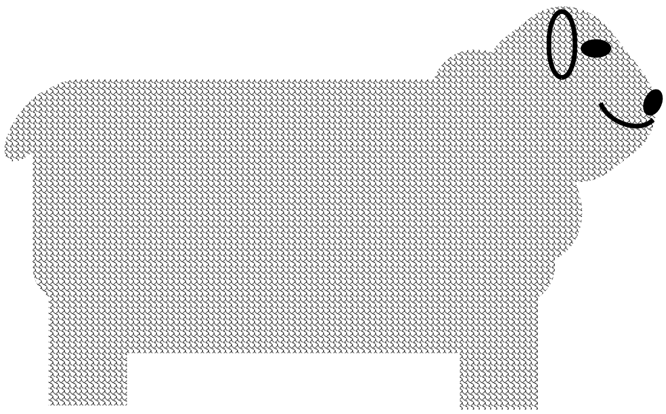
SEE & UNDERSTAND	Do I understand what the problem is asking? Discuss
PLANNING	Select a strategy from the board. Plan how you intend solving the problem.
DOING IT	Try out your idea.
CHECK IT	Did it work out? If so reflect on the activity. If not, go back to step one.

Step 1

- Tell the children that we are at Stage 1 of our four stage plan ... **See & Understand** ... Point to it! Read the problem with the class. Discuss the problem and clarify any misunderstandings.
- If children do not clearly understand what the problem is asking, they will not cope with the next stage. A good way of finding out if a child understands a problem is for her/him to retell it.
- Allow time for questions - approximately 3 to 5 minutes.

Step 2

- Tell the children that we are at Stage 2 of our four stage plan ... **Planning**. In their groups children select one or more strategies from the Strategy Board and discuss/organise how to go about solving the problem.
- Without guidance, children will often skip this step and go straight to Doing It. It is vital to emphasise that this stage is simply planning, not solving, the problem.
- After about 3 minutes, ask the children to share their plans.



Plan 1

Well we're drawing a picture and sort of making a model.

Can you give me more information please Brigid?

We're putting 60 crosses on our paper for sheep and the pen top will be Eric. Then Claire will circle one from that end, and I will pass two crosses with my pen top.

Plan 2

Our strategy is Guess and Check.

That's good Nick, but how are you going to check your guess?

Oh, we're making a model.

Go on ...

> *John's getting MAB smalls to be sheep and I'm getting a domino to be Eric and the chalk box to be the shed for shearing.*

Plan 3

> *We are doing it for 3 sheep then 4 sheep then 5 sheep and so on. Later we will look at 60.*

Great so you are going to try a simpler problem, make a table and look for a pattern.

This sharing of strategies is invaluable as it provides children who would normally feel lost in this type of activity with an opportunity to listen to their peers and make sense out of strategy selection. Note that such children are not given the answer. Rather they are assisted with understanding the power of selecting and applying strategies.

Step 3

- Tell the children that we are at Stage 3 of our four stage plan ... **Doing It**. Children collect what they need and carry out their plan.

Step 4

- Tell the children that we are at Stage 4 of our four stage plan ... **Check It**. Come together as a class for groups to share their findings. Again emphasis is on strategies.

> *We used the drawing strategy, but we changed while we were doing it because we saw a pattern.*

So Jake, you used the Look For A Pattern strategy. What was it?

> *We found that when Eric passed 10 sheep, 5 had been shorn, so 20 sheep meant 10 had been shorn ... and that means when Eric passes 40 sheep, 20 were shorn and that makes the 60 altogether.*

Great Jake. How would you work out the answer for 59 sheep or 62 sheep?

Sharing time is also a good opportunity to add in a strategy which no one may have used. For example:

> *Maybe we could've used the Number Sentence strategy, ie: 1 sheep goes to be shorn and Eric passes two sheep. That's 3 sheep, so perhaps, 60 divided into groups of 3, or 60 ÷ 3 gives the answer.*

Round off the lesson by referring to the Working Mathematically chart. There will be many opportunities to compliment the students on working like a mathematician.

Curriculum Planning Stories

Our attitude is:

teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

In more than a decade of using tasks and many years of using the detailed whole class lessons of Maths300, teachers have developed several models for integrating tasks and whole class lessons. Some of those stories are retold here. Others can be found at:

- http://www.mathematicscentre.com/taskcentre/plans.htm

Story 1: Threading

Educational research caused me a dilemma. It tells us that students construct their own learning and that this process takes time. My understanding of the history of mathematics told me that certain concepts, such as place value and fractions, took thousands of years for mathematicians to understand. The dilemma was being faced with a textbook that expected students to 'get it' in a concentrated one, two or three week block of work and then usually not revisit the topic again until the next academic year.

A Working Mathematically curriculum reflects the need to provide time to learn in a supportive, non-threatening environment and...

When I was involved in a Calculating Changes PD program I realised that:

- *choosing rich and revisitable activities, which are familiar in structure but fresh in challenge each time they are used, and*
- *threading them through the curriculum over weeks for a small amount of time in each of several lessons per week*

resulted in deeper learning, especially when partnered with purposeful discussion and recording.

Calculating Changes:

- http://www.mathematicscentre.com/calchange

Story 2: Your turn

Some teachers are making extensive use of a partnership between the whole class lessons of Maths300 and small group work with the tasks. Setting aside a lesson for using the tasks in the way they were originally designed now seems to have more meaning, as indicated by this teacher's story:

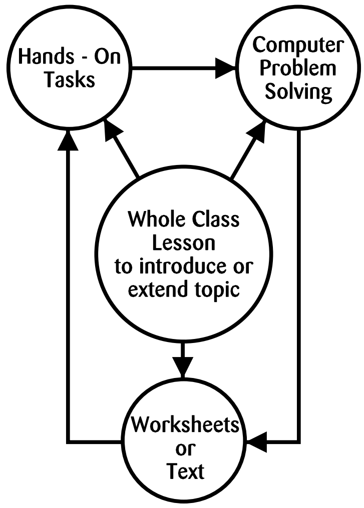
When I was thinking about helping students learn to work like a mathematician, my mind drifted to my daughter learning to drive. She *needed me to model how to do it and then she needed lots of opportunity to try it for herself.*

That's when the idea clicked of using the Maths300 lessons as a model and the tasks as a chance for the students to have their turn to be a mathematician.

The Maths300 lessons illustrate how other teachers have modelled the process, so I felt I could do it too. Now the process is always on display on the wall or pasted inside the student's journal.

A session just using the tasks had seemed a bit like play time before this. Now I see it as an integral part of learning to work mathematically.

Story 3: Mixed Media



It was our staff discussion on Gardner's theory of Multiple Intelligences that led us into creating mixed media units. That and the access you have provided to tasks and Maths300 software.

We felt challenged to integrate these resources into our syllabus. There was really no excuse for a text book diet that favours the formal learners. We now often use four different modes of learning in the work station structure shown. It can be easily managed by one teacher, but it is better when we plan and execute it together.

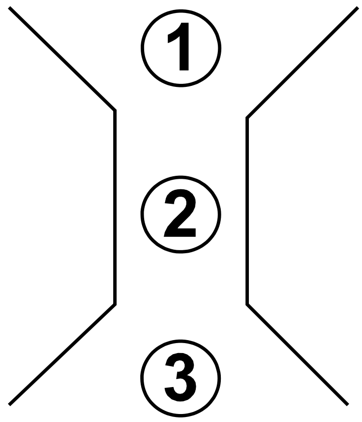
Story 4: Replacement Unit

We started meeting with the secondary school maths teachers to try to make transition between systems easier for the students. After considerable discussion we contracted a consultant who suggested that school might look too much the same across the transition when the students were hoping for something new. On the other hand our experience suggested that there needed to be some consistency in the way teachers worked.

We decided to 'bite the bullet' and try a hands-on problem solving unit in one strand. We selected two menus of twenty hands-on tasks, one for the primary and one for the secondary, that became the core of the unit. We deliberately overlapped some tasks that we knew were very rich and added some new ones for the high school.

Class lessons and investigation sheets were used to extend the tasks, within a three week model.

It is important to note that although these teachers structured a 3 week unit for the students, they strongly advised an additional *Week Zero* for staff preparation. The units came to be called Replacement Units.



Week Zero - Planning

Staff familiarise themselves with the material and jointly plan the unit. This is not a model that can be 'planned on the way to class'.

> *Getting together turned out to be great professional development for our group.*

Week 1 - Introduction

Students explore the 20 tasks listed on a printed menu:
- students explore the tip of the task, as on the card
- students move from task to task following teacher questioning that suggests there is more to the task than the tip
- in discussion with students, teachers gather informal assessment information that guides lesson planning for the following week.

> *We gave the kids an 'encouragement talk' first about joining us in an experiment in ways of learning maths and then gave out the tasks. The response was intelligent and there was quite a buzz in the room.*

Week 2 - Formalisation

> *It was good for both us and the students that the lessons in this week were a bit more traditional. However, they weren't text book based. We used whole class lessons based on the tasks they had been exploring and taught the Working Mathematically process, content and report writing.*
>
> *Assessment was via standard teacher-designed tests, quizzes and homework.*

Week 3 - Investigations

> *We were most delighted with Week 3. Each student chose one task from the menu and carried out an in-depth investigation into the iceberg guided by an investigation sheet. They had to publish a report of their investigation and we were quite surprised at the outcomes. It was clear that the first two weeks had lifted the image of mathematics from 'boring repetition' to a higher level of intellectual activity.*

Story 5: Curriculum shift

> *I think our school was like many others. The syllabus pattern was 10 units of three weeks each through the year. We had drifted into that through a text book driven curriculum and we knew the students weren't responding.*

Our consultant suggested that there was sameness about the intellectual demands of this approach which gave the impression that maths was the pursuit of skills. We agreed to select two deeper investigations to add to each unit. It took some time and considerable commitment, but we know that we have now made a curriculum shift. We are more satisfied and so are the students.

The principles guiding this shift were:

- Agree

  The 20 particular investigations for the year are agreed to by all teachers. If, for example, Cube Nets *is decided as one of these, then all the teachers are committed to present this within its unit.*

- Publish

  The investigations are written into the published syllabus. Students and parents are made aware of their existence and expect them to occur.

- Commit

  Once agreed, teachers are required to present the chosen investigations. They are not a negotiable 'extra'.

- Value

  The investigations each illustrate an explicit form of the Working Mathematically process. This is promoted to students, constantly referenced and valued.

- Assess

  The process provides students with scaffolding for their written reports and is also known by them as the criteria for assessment. (See next page.)

- Report

  The assessment component features within the school reporting structure.

A Final Comment

Including investigations has become policy.

Why? Because to not do so is to offer a diminished learning experience.

The investigative process ranks equally with skill development and needs to be planned for, delivered, assessed and reported.

Perhaps most of all we are grateful to our consultant because he was prepared to begin where we were. We never felt as if we had to throw out the baby and the bath water.

Assessment

Our attitude is:

stimulated students are creative and love to learn

Regardless of the way you use your **Maths With Attitude** resource, a variety of procedures can be employed to assess this learning.

Where these assessment procedures are applied to task sessions and involve written responses from students, teachers will need to be careful that the writing does not become too onerous. Students who get bogged down in doing the writing may lose interest in doing the tasks.

In addition to the ideas below, useful references are:

- http://www.mathematicscentre.com/taskcentre/assess.htm
- http://www.mathematicscentre.com/taskcentre/report.htm

The first offers several methods of assessment with examples and the second is a detailed lesson plan to support students to prepare a Maths Report.

Journal Writing

Journal writing is a way of determining whether the task or lesson has been understood by the student. The pupil can comment on such things as:

- What I learned in this task.
- What strategies I/we tried (refer to the Strategy Board).
- What went wrong.
- How I/we fixed it.
- Jottings - ie: any special thoughts or observations

Some teachers may prefer to have the page folded vertically, so that children's reflective thoughts can be recorded adjacent to critical working.

Assessment Form

An assessment form uses questions to help students reflect upon specific issues related to a specific task.

Anecdotal Records

Some teachers keep ongoing records about how students are tackling the tasks. These include jottings on whether students were showing initiative, whether they were working co-operatively, whether they could explain ideas clearly, whether they showed perseverance.

Checklists

A simple approach is to create a checklist based on the Working Mathematically process. Teachers might fill it in following questioning of individuals, or the students may fill it in and add comments appropriately.

Pupil Self-Reflection

Many theorists value and promote metacognition, the notion that learning is more permanent if pupils deliberately and consciously analyse their own learning. The deliberate teaching strategy of oral questioning and the way pupils record their work is an attempt to manifest this philosophy in action. The alternative is the tempting 'butterfly' approach which is to madly do as many activities as possible, mostly superficially, in the mistaken belief that quantity equates to quality.

> *I had to work quite hard to overcome previously entrenched habits of just getting the answer, any answer, and moving on to the next task.*

Thinking about *what* was learned *how* it was learned consolidates and adds to the learning.

When it follows an extensive whole class investigation, a reflection lesson such as this helps to shift entrenched approaches to mathematics learning. It is also an important component of the assessment process. On the one hand it gives you a lot of real data to assist your assessment. On the other it prepares the students for any formal assessment which you may choose to round off a unit.

Introduction

Ask students to recall what was done during the unit or lesson by asking a few individuals to say what *they* did, eg:

> *What did you do or learn that was new?*
> *What can you now do/understand that is new?*
> *What do you know now that you didn't know 1 (2, 3, ...) lesson ago?*

Continuing Discussion

Get a few ideas from the first students you ask, then:

- organise 5 -10 minute buzz groups of three or four students to chat together with one person to act as a recorder. These groups address the same questions as above.
- have a reporting session, with the recorder from each group telling the class about the group's ideas.

Student comments could be recorded on the board, perhaps in three groups.

Ideas & Facts	Maths Skills	Process (learning) Skills

If you need more questions to probe deeper and encourage more thought about process, try the following:

> *What new things did you do that were part of* how *you learned?*
> *Who uses this kind of knowledge and skill in their work?*

Student Recording

Hand out the REFLECTION sheet (next page) and ask students to write their own reflection about what they did, based on the ideas shared by the class. Collect these for interest and, possibly, assessment information.

REFLECTION

me looking at me learning

NAME: CLASS:

Working With Parents

Balancing Problem Solving with Basic Skill Practice

Many schools find that parents respond well to an evening where they have an opportunity to work with the tasks and perhaps work a task together as a 'whole class'. Resourced by the materials in this kit, teachers often feel quite confident to run these practical sessions. Comments from parents like:

> *I wish I had learnt maths like this.*

are very supportive. Letting students 'host' the evening is an additional benefit to the home/school relationship.

The 4½ Minute Talk

Charles Lovitt has considerable experience working with parents and has developed a crisp, parent-friendly talk which he shares below. Many others have used it verbatim with great success.

Why the Four and a Half Minute Talk?

When talking with parents about Problem Solving or the meaning of the term Working Mathematically, I have often found myself in the position, after having promoted inquiry based or investigative learning, of the parents saying:

> *Well - that's all very well - BUT ...*

at which stage they often express their concern for basic (meaning arithmetic) skill development.

The weakness of my previous attempts has been that I have been unable to reassure parents that problem solving does not mean sacrificing our belief in the virtues of such basic skill development.

One of the unfortunate perceptions about problem solving is that if a student is engaged in it, then somehow they are not doing, or it may be at the expense of, important skill based work.

This Four and a Half Minute Talk to parents is an attempt to express my belief that basic skill practice and problem solving development can be closely intertwined and not seen as in some way mutually exclusive.

(I'm still somewhat uncomfortable using the expression 'basic skills' in the above way as I am certain that some thinking, reasoning, strategy and communication skills are also 'basic'.)

Another aspect of the following 'talk' is that, as teachers put more emphasis on including investigative problem solving into their courses, a question arises about the source of suitable tasks.

This talk argues that we can learn to create them for ourselves by 'tweaking' the closed tasks that heavily populate out existing text exercises, and hence not be dependent on external suppliers. (Even better if students begin to create such opportunities for themselves.)

The Talk

In preparation, write the following graphic on the board:

CLOSED	OPEN	EXTENDED INVESTIGATION
		How many solutions exist? How do you know you have found them all?

I would like to show you what teachers are beginning to do to achieve some of the thinking and reasoning and communication skills we hope students will develop. I would like to show you three examples.

Example One: *6 + 5 = ?*

I write this question under the 'closed' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
$\begin{array}{r} 6 \\ \underline{+\ 5} \\ \_\_ \end{array}$		How many solutions exist? How do you know you have found them all?

And I ask:

What is the answer to this question?

I then explain that:

We often ask students many closed questions such as 6 + 5 = ?

The only response the students can tell us is "The answer is 11." ... and as a reward for getting it correct we ask another twenty questions just like it.

What some teachers are doing is trying to *tweak* the question and ask it a different way, for example:

I have two counting numbers that add to 11. What might the numbers be?

[Counting numbers = positive whole numbers including zero]

I write this under the 'open' label on the diagram:

CLOSED	**OPEN**	**EXTENDED INVESTIGATION**
6 + 5 ___	? + ? 11	How many solutions exist? How do you know you have found them all?

What is the answer to the question now?

At this stage it becomes apparent there are several solutions:

The question is now a bit more open than it was before, allowing students to tell you things like 8 + 3, or 10 + 1, or 11 + 0 etc.

Let's see what happens if the teacher 'tweaks' it even further with the investigative challenge *or* extended investigation question:

How many solutions are there altogether?

and more importantly, and with greater emphasis on the second question:

How could you convince someone else that you have found them all?

Now the original question is definitely different - it still involves the skills of addition but now also involves thinking, reasoning and problem solving skills, strategy development and particularly communication skills.

Young students will soon tell you the answer is 'six different ones', but they must also confront the communication and reasoning challenge of convincing you that there are only six and no more.

Example Two: *Finding Averages*

Again, as I go through this example, I write it into the diagram on the board in the relevant sections.

The CLOSED question is: *11, 12, 13 - find the average*

Tweaking this makes it an OPEN question and it becomes:

I have three counting numbers whose average is 12. What might the numbers be?

Students will often say:

10, 12, 14 … or 9, 12, 15 … or even 12, 12, 12

After realising there are many answers, you can tweak it some more and turn it into an EXTENDED INVESTIGATION:

How many solutions exist? … AND …

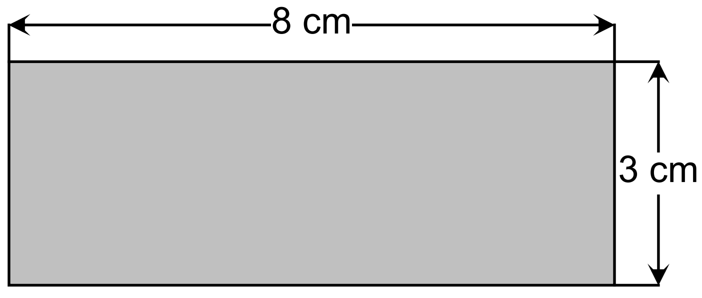
How do you know you have found them all?

Now the question is of a quite different nature. It still involves the arithmetic skill, but has something else as well - and that something else is the thinking, reasoning and communication skills necessary to find all of the combinations and convince someone else that you have done so.

By the time a student announces, with confidence, there are 127 different ways (which there are) that student will have engaged in all of these aspects, ie: the skill of calculating averages, (and some combination number theory) as well as significant strategy and reasoning experiences.

Example Three: *Finding the Area of a Rectangle*

A typical CLOSED question is:



Find the area. Find the perimeter.

The OPEN question is:

A rectangle has 24 squares inside:

What might its length and width be?

What might its perimeter be?

The EXTENDED INVESTIGATION version is:

Given they are whole number lengths, how many different rectangles are there? … AND …

How do you know you have found them all?

In summary, mathematics teachers are trying to convert *some* (not all) of the many closed questions that populate our courses and 'push' them towards the investigation direction. In doing so, we keep the skills we obviously value, but also activate the thinking, reasoning and justification skills we hope students will also develop.

This sequence of three examples hopefully shows two major features:

- That skills and problem solving can 'live alongside each other' and be developed concurrently.

- That the process of creating open-ended investigations can be done by anyone - just go to any source of closed questions and try 'tweaking' them as above. If it only worked for one question per page it would still provide a very large supply of investigations.

In terms of the effect of the talk on parents, I have usually found them to be reassured that we are not compromising important skill development (and nor do we want to). The only debate then becomes whether the additional skills of thinking, reasoning and communication are also desirable.

I've also been told that parents appreciate it because of the essential simplicity of the examples - no complicated theoretical jargon.

A Working Mathematically Curriculum

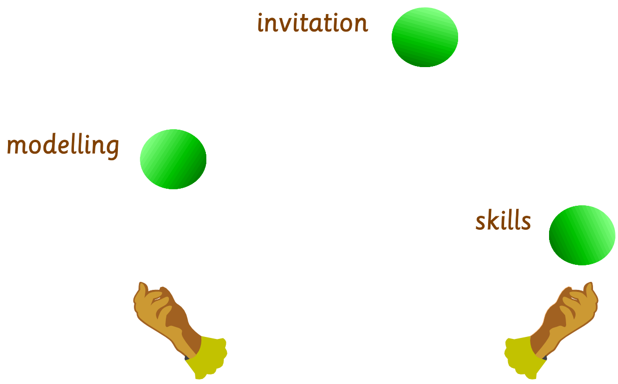
An Investigative Approach to Learning

The aim of a Working Mathematically curriculum is to help students learn to work like a mathematician. This process is detailed earlier (Page 8) in a one page document which becomes central to such a curriculum.

The change of emphasis brings a change of direction which *implies and requires* a balance between:

- the process of being a mathematician, and
- the development of skills needed to be a *successful* mathematician.

This journey is not two paths. It is one path made of two interwoven threads in the same way as DNA, the building block of life, is one compound made of two interwoven coils. To achieve a Working Mathematically curriculum teachers need to balance three components.



The task component of **Maths With Attitude** offers each pair of students an invitation to work like a mathematician.

The Maths300 component of **Maths With Attitude** assists teachers to model working like a mathematician.

Content skills are developed in context. They *are* important, but it is the application of skills within the process of Working Mathematically that has developed, and is developing, the human community's mathematical knowledge.

A focus for the Working Mathematically teacher is to help students develop mathematical skills in the context of problem posing and solving.

> *We are all 'born' with the same size mathematical toolbox, in the same way as I can own the same size toolbox as my motor mechanic. However, my motor mechanic has many more tools in her box than I and she has had more experience than I using them in context. Someone has helped her learn to use those tools while crawling under a car.*

Afzal Ahmed, Professor of Mathematics at Chichester, UK, once quipped:

> *If teachers of mathematics had to teach soccer, they would start off with a lesson on kicking the ball, follow it with lessons on trapping the ball and end with a lesson on heading the ball. At no time would they play a game of football.*

Such is not the case when teaching a Working Mathematically curriculum.

Elements of a Working Mathematically Curriculum

Working Mathematically is a K - 12 experience offering a balanced curriculum structured around the components below.

Hands-on Problem Solving Play

Mathematicians don't know the answer to a problem when they start it. If they did, it wouldn't be a problem. They have to play around with it. Each task invites students to play with mathematics 'like a mathematician'.

Skill Development

A mathematician needs skills to solve problems. Many teachers find it makes sense to students to place skill practice in the context of *Toolbox Lessons* which *help us better use the Working Mathematically Process* (Page 8).

Focus on Process

This is what mathematicians do; engage in the problem solving process.

Strategy Development

Mathematicians also make use of a strategy toolbox. These strategies are embedded in Maths300 lessons, but may also have a separate focus. Poster Problem Clinics are a useful way to approach this component.

Concept Development

A few major concepts in mathematics took centuries for the human race to develop and apply. Examples are place value, fractions and probability. In the past students have been expected to understand such concepts after having 'done' them for a two week slot. Typically they were not visited again until the next year. A Working Mathematically curriculum identifies these concepts and regularly 'threads' them through the curriculum.

Planning to Work Mathematically

The class, school or system that shifts towards a Working Mathematically curriculum will no longer use a curriculum document that looks like a list of content skills. The document would be clear in:

- choosing genuine problems to initiate investigation
- choosing a range of best practice teaching strategies to interest a wider range of students
- practising skills for the purpose of problem solving

Some teachers have found the planning template on the next page assists them to keep this framework at the forefront of their planning. It can be used to plan single lessons, or units built of several lessons. There are examples from schools in the Curriculum & Planning section of Maths300 and a Word document version of the template.

Unit Planning Page

Class

Topic

Pedagogy	Problem Solving	Skills
	In this topic how will I engage my students in the Working Mathematically process?	
How do I create an environment where students know what they are doing and why they have accepted the challenge?		Does the challenge identify skills to practise? Are there other skills to practise in preparation for future problem solving?

Notes

As a general guide:

- Find a problem(s) to solve related to the topic.
- Choose the best teaching craft likely to engage the learners.
- Where possible link skill practice to the problem solving process.

More on Professional Development

For many teachers there will be new ideas within **Maths With Attitude**, such as unit structures, views of how students learn, teaching strategies, classroom organisation, assessment techniques and use of concrete materials. It is anticipated (and expected) that as teachers explore the material in their classrooms they will meet, experiment with and reflect upon these ideas with a view to long term implications for the school program and for their own personal teaching.

Being explored 'on-the-job' so to speak, in the teacher's own classroom, makes the professional development more meaningful and practical for the teacher. This is also a practical and economic alternative for a local authority.

Strategic Use by Systems

From Years 3 - 10, **Maths With Attitude** is designed as a professional development vehicle by schools or clusters or systems because it carries a variety of sound educational messages. They might choose **Maths With Attitude** because:

- It can be used to highlight how investigative approaches to mathematics can be built into balanced unit plans without compromising skill development and without being relegated to the margins of a syllabus as something to be done only after 'the real' content has been covered.
- It can be used to focus on how a balance of concept, skill and application work can all be achieved within the one manageable unit structure.
- It can be used to show how a variety of assessment practices can be used concurrently to build a picture of student progress.
- It can be used to focus on transition between primary and secondary school by moving towards harmony and consistency of approach.
- It can be used to raise and continue debate about the pedagogy (art of teaching) that supports deeper mathematical learning for a wider range of students.

Teachers in Years K - 2 are similarly encouraged in professional growth through **Working Mathematically with Infants**, which derives from Calculating Changes, a network of teachers enhancing children's number skills from Years K - 6.

In supporting its teachers by supplying these resources in conjunction with targeted professional development over time, a system can fuel and encourage classroom-based debate on improving outcomes. There is evidence that by exploring alternative teaching strategies and encouraging curriculum shift towards Working Mathematically, learners improve and teachers are more satisfied. For more detail visit Research & Stories at:

- http://www.mathematicscentre.com/taskcentre/do.htm

We would be happy to discuss professional development with system leaders.

Web Reference

The starting point for all aspects of learning to work like a mathematician, including Calculating Changes, and the teaching craft which encourages it is:

- http://www.mathematicscentre.com/mathematicscentre

Appendix 1: Investigation Guides

4 Arm Shapes - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising

a. One arm has 50 tiles. Explain how to find the total of tiles.
b. Explain in a different way if you can.
c. If someone told you **any number** of tiles in one arm, explain how you would find the total of tiles.
d. Write an equation that shows how **T** (**T**otal of tiles) is found from **A** (number in one **A**rm).

2. Substituting

One Arm	Total of Tiles
10	
25	
49	
101	
3.25	
$5\frac{3}{4}$	

Copy & complete these tables

3. Solving - Working Backwards

One Arm	Total of Tiles
	25
	13
	97
	1004
	75
	58

Explain as much as you can about how to find one Arm if someone tells you any total number of tiles.

4. Making Pairs

a. Choose any five numbers up to 20 for one arm.
For each number find the total of tiles and make five number pairs like this: (A, T)
b. Choose any five numbers up to 50 for the total of tiles.
For each number find one arm and make five more number pairs like this: (A, T)
c. If you do the same calculation in each pair the answer is always 1.
Explain the calculation.
d. Does this calculation still work if there are $10\frac{1}{2}$ or 6.7 tiles in one arm?

5. Graphing Pairs

a. Show your ten pairs from Question 4 on a graph and explain what you see.
b. Make one more dot that you think belongs on your graph. Call it **My Dot**.
Draw the 4 Arm Shape picture that goes with My Dot.

6. Graphing in Excel

a. Use Excel to record your ten pairs in a table.
b. Select the table and use it to insert a chart.
Use the XY (Scatter) graph with the first sub-type.
c. Select the chart and choose Chart/Add Trendline. On the Options Tab select Display equation on chart. Explain how this equation links to the calculation in Question 4.

7. What happens if...?

Use tiles to create your own picture pattern like 4 Arm Shapes and investigate it.

Algebra Through Geometry - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Extra Information 1

Some of the pieces have a small quadrant. The area of this small quadrant is $\frac{1}{4}y$.

a. Use your knowledge of area of a circle to explain this.

b. Can you explain it another way?

2. Evaluating: Whole Numbers

a. Use the recording sheet to trace the combined perimeter of each of these combinations.
A + B, D + H

b. Record the area of each combined shape.

3a. Like & Unlike Terms

Shapes	Area
A + D	
A + E	
B + H	
B + D	

Copy & complete these tables

b.

Shapes	Area
	$5x$
	$3x + 2y$
	$4x + y$
	$4x + y$

4. Evaluating: Fractions

a. Use the recording sheet to trace the combined perimeter of each of these combinations.
B + F, C + H

b. Record the area of each combined shape.

5a. Like & Unlike Terms

Shapes	Area
A + C	
A + F	
B + G	
D + C	

Copy & complete these tables

b.

Shapes	Area
	$5x - \frac{1}{4}y$
	$6x - 1\frac{1}{4}y$
	$4x + \frac{3}{4}y$
	$5x - \frac{1}{4}y$

6. Exploring Addition

a. Put any three shapes together to make a new shape.
Trace its perimeter. Work out its area.

6. Exploring Subtraction

a. Put H on top of D to make the best fit. Trace what you make.
How big is the piece sticking out on D?

b. Put B on top of D to make the best fit. Trace what you make.
What is the total size of the pieces sticking out on D?

c. Put F on top of G to make the best fit. Trace what you make.
How big is the piece sticking out on F?
How big is the piece sticking out on G?

7. Extra Information 2

A straight side of y is the same length as one side of x. Their areas are related: $y = 0.785x$.
Use what you know about area of a circle to explain this connection.

Addition Totals - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising

a. The total is 50. Explain how to find all the whole number pairs that add to 50.
b. Explain in a different way if you can.
c. The total is 51. Explain how to find all the whole number pairs that add to 51.
d. Explain in a different way if you can.
e. If someone told you **any total**, explain how you would find all the whole number pairs there are.
f. Write an equation that shows how **P** (whole number **P**airs) is found for any even number total **E**.
g. Write an equation that shows how **P** (whole number **P**airs) is found for any odd number total **D**. (Use **D** for odd instead of O so it doesn't look like zero.)

2. Substituting

Total	Pairs
19	
20	
21	
128	
319	
1000	

Copy & complete these tables

3. Solving - Working Backwards

Total(s)	Pairs
	10
	23
	97
	58
	75
	1000

Explain as much as you can about how to find the number of pairs if someone tells you any total.

4. Making A Table

Use the numbers 1 to 12 as the totals and make an organised table showing the number of pairs that can be made for each total.

5. Graphing Pairs

a. Make a graph from your table in Question 4 and explain what you see.
b. Make one more dot that you think belongs on your graph. Call it **My Dot**. Explain the information My Dot gives about totals and their whole number pairs.

6. Graphing in Excel

a. Use the information in your table to make two tables in Excel - one for **Even Totals** and one for **Odd Totals**.
b. Select the **Even** table and use it to insert a chart. Use the XY (Scatter) graph with the first sub-type.
c. Select the chart and choose Chart/Add Trendline. On the Options Tab select Display equation on chart. Explain how this equation links to Question 1f.
d. Repeat for the **Odd** table and link to Question 1g.

7. What happens if...?

a. The total is 4. Work out all the whole number **triples** that add to 4.
b. Investigate finding whole number triples for any total.

Crossing The River 1 - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising

a. There are 2 children and 50 Adults on one side.
Explain how to find the number of trips on the river.

b. Explain in a different way if you can.

c. If there are 2 children and someone told you **any number of Adults**, explain how you would find the number of trips on the river.

d. Write an equation that shows how **T** (number of **T**rips) is worked out from **A** (number of **A**dults).

2. Substituting

Adults (2 chn.)	Trips
19	
20	
35	
128	
319	
1000	

Copy & complete these tables

3. Solving - Working Backwards

Adults(2 chn.)	Trips
	37
	65
	21
	476
	75
	1000

Explain as much as you can about how to find the number of adults if you know there are only two children and someone tells you any number for the trips.

4. Making Pairs

a. Choose any five numbers up to 20 for the adults. (Still 2 children.)
For each number find the trips and make five number pairs like this: (A, T)

b. Choose any five numbers up to 50 that will work for trips. (Still 2 children.)
For each number find the adults and make five more number pairs like this: (A, T)

c. If you do the same calculation in each pair the answer is always 1.
Explain the calculation.

5. Graphing Pairs

a. Show your ten pairs from Question 4 on a graph and explain what you see.

b. Make one more dot that you think belongs on your graph. Call it **My Dot**.
Explain the information My Dot gives about Adults and Trips.

6. Graphing in Excel

a. Use Excel to record your ten pairs in a table.

b. Select the table and use it to insert a chart.
Use the XY (Scatter) graph with the first sub-type.

c. Select the chart and choose Chart/Add Trendline. On the Options Tab select Display equation on chart. Explain how this equation links to the calculation in Question 4.

7. What happens if...?

a. There are 8 adults and 3 children to cross the river. Find out how many trips.

b. Investigate the number of trips if you are given any number of adults and any number of children.

Eric The Sheep - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising

a. There are 100 sheep in front of Eric.
Explain how to find the number of sheep shorn before Eric reaches the shearer.

b. Explain in a different way if you can.

c. If someone told you **any number of sheep in front of Eric**, explain how you would find the number of sheep shorn before Eric reaches the shearer.

2. Substituting

Number in front	Sheep Shorn
19	
20	
35	
40	
319	
1000	

Copy & complete these tables

3. Solving - Working Backwards

Number in front	Sheep Shorn
	11
	22
	33
	44
	55
	100

Explain as much as you can about how to work out the number of sheep in front if someone tells you any number of sheep shorn.

4. Making A Table

Use the numbers 1 to 15 as the number in front of Eric and make an organised table showing the number of sheep shorn before Eric is at the shearer.

5. Graphing Pairs

a. Make a graph from your table in Question 4 and explain what you see.

b. Make one more dot that you think belongs on your graph. Call it **My Dot**.
Explain the information My Dot gives about the number in front of Eric to start and the number shorn before Eric is at the shearer.

6. Graphing in Excel

a. Use Excel to record your table.

b. Select the table and use it to insert a chart.
Use the XY (Scatter) graph with the first sub-type.

c. Explain the pattern in the graph.

7. What happens if...?

a. Investigate what happens if Eric sneaks past 3 sheep each time the shearer takes one from the front of the line.

b. If someone tells you any number of sheep Eric sneaks past, explain how to find the number of sheep shorn before Eric is at the shearer.

c. Investigate what happens if Eric sneaks past 2 sheep AND there are 3 shearers at the other end who all take their sheep at the same time.

d. Investigate what happens if Eric sneaks past any number of sheep AND there are any number of shearers at the other end who all take their sheep at the same time.

Garden Beds - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising

a. The garden bed has 50 plants in one row. Explain how to find the number of tiles.
b. Explain in a different way if you can.
c. If someone told you **any number** of plants in one row, explain how you would find the number of tiles.
d. Write an equation that shows how **T** (number of **T**iles) is found from **P** (number of plants).

2. Substituting

Plants	Tiles
19	
20	
35	
128	
319	
1000	

Copy & complete these tables

3. Solving - Working Backwards

Plants	Tiles
	28
	37
	65
	512
	75
	1000

Explain as much as you can about how to find the number of plants if someone tells you any number for the tiles.

4. Making Pairs

a. Choose any five numbers up to 20 for the number of plants.
For each number find the number of tiles and make five number pairs like this: (P, T)
b. Choose any five numbers up to 50 that work for the number of tiles. For each tile number find the number of plants and make five more number pairs like this: (P, T)
c. If you do the same calculation in each pair the answer is always 6.
Explain the calculation.

5. Graphing Pairs

a. Show your ten pairs from Question 4 on a graph and explain what you see.
b. Make one more dot that you think belongs on your graph. Call it **My Dot**.
Draw the Garden Bed picture that goes with My Dot.

6. Graphing in Excel

a. Use Excel to record your ten pairs in a table.
b. Select the table and use it to insert a chart.
Use the XY (Scatter) graph with the first sub-type.
c. Select the chart and choose Chart/Add Trendline. On the Options Tab select Display equation on chart. Explain how this equation links to the calculation in Question 4.

7. What happens if...?

Investigate the number of tiles to surround gardens shaped your way. You could have:

- plants in many equal rows
- plants in squares only
- plants in L shapes

or any other pattern that you want to investigate.

Lining Up - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising

a. *You are 50th from each end. Explain how to find the number of people in the line.*
b. *Explain in a different way if you can.*
c. *If someone told you* **any position** *from each end, explain how to find the number in the line.*
d. *Write an equation that shows how* **N** *(***N***umber of people) is found from* **P** *(***P***osition in the line).*

2. Substituting

Position	Number in Line
19	
20	
35	
128	
319	
1000	

Copy & complete these tables

NOTE: You are the same place from both ends

3. Solving - Working Backwards

Position	Number in Line
	28
	37
	65
	512
	75
	1000

Explain as much as you can about how to find the position if someone tells you any number of people in the line.

4. Making Pairs

a. *Choose any five numbers up to 20 for the position from each end. For each position find the number in the line and make 5 number pairs like this: (P, N).*
b. *Choose any five numbers up to 50 for the number in the line. For each number find the position from each end and make 5 more number pairs like this: (P, N).*
c. *If you do the same calculation in each pair the answer is always 1. Explain the calculation.*

5. Graphing Pairs

a. *Show your ten pairs from Question 4 on a graph and explain what you see.*
b. *Make one more dot that you think belongs on your graph. Call it* **My Dot***. Draw the Lining Up picture that goes with My Dot.*

6. Graphing in Excel

a. *Use Excel to record your ten pairs in a table.*
b. *Select the table and use it to insert a chart. Use the XY (Scatter) graph with the first sub-type.*
c. *Select the chart and choose Chart/Add Trendline. On the Options Tab select Display equation on chart. Explain how this equation links to the calculation in Question 4.*

7. What happens if...?

Investigate how to find the number of people in the line if you are NOT in the same position from each end.

Making Monuments - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising

a. *Explain how to find the total number of tiles needed to build a Size 50 monument.*
b. *Explain in a different way if you can.*
c. *If someone told you* **any size monument** *explain how you would find the total of tiles.*
d. *Write an equation that shows how* **T** *(***T***otal tiles) is found from* **S** *(***S***ize of monument).*

2. Substituting

Size	Total Tiles
19	
20	
35	
128	
319	
1000	

Copy & complete these tables

3. Solving - Working Backwards

Size	Total Tiles
	25
	41
	57
	72
	512
	1005

Explain as much as you can about how to find the size of the monument if someone tells you any total number of tiles.

4. Making Pairs

a. *Choose any five numbers up to 20 for the size of the monument.*
For each number find the total of tiles and make five number pairs like this: (S, T)
b. *Choose any five numbers up to 100 for the total of tiles.*
For each number find the size and make five more number pairs like this: (S, T)
c. *If you do the same calculation in each pair the answer is always 0.*
Explain the calculation.

5. Graphing Pairs

a. *Show your ten pairs from Question 4 on a graph and explain what you see.*
b. *Make one more dot that you think belongs on your graph. Call it* **My Dot**.
Explain the information My Dot gives about the size of the monument and the tiles.

6. Graphing in Excel

a. *Use Excel to record your ten pairs in a table.*
b. *Select the table and use it to insert a chart.*
Use the XY (Scatter) graph with the first sub-type.
c. *Select the chart and choose Chart/Add Trendline. On the Options Tab select Display equation on chart. Explain how this equation links to the calculation in Question 4.*

7. What happens if...?

Create your own monument and paths pattern and investigate how the number of tiles is related to the size of your monument.

eg: suppose the monuments were always cubes and there were paths leading up to each block in each side of the cube.

Match Triangles - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising

a. The chain has 50 triangles. Explain how to find the number of matches.
b. Explain in a different way if you can.
c. If someone told you **any number** of triangles in the chain, explain how you would find the number of matches.
d. Write an equation that shows how **M** (number of **M**atches) is found from **T** (number of **T**riangles).

2. Substituting

Triangles	Matches
19	
20	
35	
128	
319	
1000	

Copy & complete these tables

3. Solving - Working Backwards

Triangles	Matches
	25
	39
	57
	71
	513
	1009

Explain as much as you can about how to find the number of triangles if someone tells you any number of matches.

4. Making Pairs

a. Choose any five numbers up to 20 for the number of triangles.
For each number find the total of matches and make five number pairs like this: (T, M)
b. Choose any five numbers up to 100 for the total of matches.
For each total find the number of triangles and make five more number pairs like this: (T, M)
c. If you do the same calculation in each pair the answer is always 1.
Explain the calculation.

5. Graphing Pairs

a. Show your ten pairs from Question 4 on a graph and explain what you see.
b. Make one more dot that you think belongs on your graph. Call it **My Dot**.
Draw the Match Triangle picture that goes with My Dot.

6. Graphing in Excel

a. Use Excel to record your ten pairs in a table.
b. Select the table and use it to insert a chart.
Use the XY (Scatter) graph with the first sub-type.
c. Select the chart and choose Chart/Add Trendline. On the Options Tab select Display equation on chart. Explain how this equation links to the calculation in Question 4.

7. What happens if...?

Create your own pattern with matches and investigate it.
eg: you might make fence sections with two rails between each post.
How many matches would you need to make 10 sections of fence?
If someone told you any...

Mirror Patterns 2 - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising

a. Explain how the numbers 3, 4, 5, 6, 8, 9, 10, 12, 15, 18 are connected to 360°.
b. Each of the numbers in (a) has a partner. Write out all the partner numbers.
c. If someone told you to make a shape that had **any of the numbers in (a) or (b) as sides,** explain how you would find the angle between the mirrors.
d. Write an equation that shows how **A** (**A**ngle between mirrors) is found from **N** (**N**umber of sides).

2. Substituting

Sides	Angle
3	
4	
8	
9	
15	
20	

Copy & complete these tables

3. Solving - Working Backwards

Sides	Angle
	4°
	20°
	30°
	36°
	60°
	72°

Some of the shapes in Questions 2 and 3 can't actually be made with the mirrors you have. Write down which ones and explain as much as you can about why they can't be made.

4. Making Pairs

a. Use all the numbers in 1(a) to make pairs like this: (N, A)
c. If you do the same calculation in each pair the answer is always 360. Explain the calculation. How is it connected to the equation in 1(d)?

5. Graphing Pairs

a. Show your ten pairs from Question 4 on a graph and explain what you see.
b. If you made pairs from the numbers in 1(b) explain where they would be on the graph.
c. Make one more dot that you think belongs on your graph. Call it **My Dot**. Work out as accurately as you can the sides and the angle for My Dot.
d. Set the mirrors at this angle and try to explain the number of sides for My Dot

6. Graphing in Excel

a. Use Excel to record your ten pairs in a table.
b. Select the table and use it to insert a chart. Use the XY (Scatter) graph with the first sub-type.
c. What is the rule (equation) for all the points on this graph?

7. What happens if...?

a. The product of each pair in your Excel table is 360. Choose any other number as the product and make ten pairs. Investigate their graph.
b. The product is 4.5. Work out ten pairs that equal this product - a calculator might help. Graph the pairs.
c. Suppose someone tells you any number and says it is the product of a pair of numbers. Explain as much as you can about the graph of pairs that multiply to make their product.

Painted Rods - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising
a. The rod is 100 units long. Explain how to find how many squares of area are painted.
b. Explain in a different way if you can.
c. If someone told you **any length of rod**, explain how to find the squares of area.
d. Write an equation that shows how **S** (**S**quares of area) is found from **L** (**L**ength of rod).

2. Substituting

Length of Rod	Squares of Area
10	
25	
49	
101	
3.25	
5¾	

Copy & complete these tables

3. Solving - Working Backwards

Length of Rod	Squares of Area
	26
	14
	98
	1003
	60
	75

Explain as much as you can about how to find the length of the rod if someone tells you any number of squares of painted area.

4. Making Pairs
a. Choose any five numbers up to 20 for the length of the rod.
For each number find the squares of area and make five number pairs like this: (L, S)
b. Choose any five numbers up to 80 for the squares of area.
For each area find the length of the rod and make five more number pairs like this: (L, S)
c. If you do the same calculation in each pair the answer is always 2.
Explain the calculation.

5. Graphing Pairs
a. Show your ten pairs from Question 4 on a graph and explain what you see.
b. Make one more dot that you think belongs on your graph. Call it **My Dot**.
Explain the information My Dot gives about the size of the monument and the tiles.

6. Graphing in Excel
a. Use Excel to record your ten pairs in a table.
b. Select the table and use it to insert a chart.
Use the XY (Scatter) graph with the first sub-type.
c. Select the chart and choose Chart/Add Trendline. On the Options Tab select Display equation on chart. Explain how this equation links to the calculation in Question 4.

7. What happens if...?
a. Investigate how to find the painted area if two rods are placed side by side and then painted.
b. Investigate how to find the painted area if any number of rods are placed side by side and then painted.
c. Investigate how to find the painted area if a rod is surrounded left/right/top/bottom by rods the same length and then painted.

Pointy Fences - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising

a. Imagine the 100 Fence. Explain how to find the number of shapes (squares and triangles) it takes to make it.
b. Explain in a different way if you can.
c. If someone told you **any fence number**, explain how to find the number of shapes.
d. Write an equation that shows how **S** (number of **S**hapes) is found from **F** (**F**ence number).

2. Substituting

Fence Number	Shapes
19	
20	
35	
128	
319	
1000	

Copy & complete these tables

3. Solving - Working Backwards

Fence Number	Shapes
	24
	42
	60
	96
	498
	1008

Explain as much as you can about how to find the fence number if someone tells you any number of shapes.

4. Making Pairs

a. Choose any five numbers up to 20 for fence number.
For each number find how many shapes and make five number pairs like this: (F, S)
b. Choose any five numbers up to 120 for the number of shapes.
For each shape number find the fence number and make five more number pairs like this: (F, S)
c. If you do the same calculation in each pair the answer is always 6.
Explain the calculation.

5. Graphing Pairs

a. Show your ten pairs from Question 4 on a graph and explain what you see.
b. Make one more dot that you think belongs on your graph. Call it **My Dot**.
Explain the information My Dot gives about the fence number and the shapes.

6. Graphing in Excel

a. Use Excel to record your ten pairs in a table.
b. Select the table and use it to insert a chart.
Use the XY (Scatter) graph with the first sub-type.
c. Select the chart and choose Chart/Add Trendline. On the Options Tab select Display equation on chart. Explain how this equation links to the calculation in Question 4.

7. What happens if...?

Use the squares and triangles to create your own structures that grow in a pattern. Investigate the pattern.

eg: you might build 'connected houses' that start as one cube with a pyramid roof.

(You may need more shapes from your teacher to make your special design.)

Shape Algebra - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Shapes & Symbols
a. A + B = B + G
Explain in two ways why this is true. The recording sheet will help.
b. A + C = C + G
Explain in two ways why this is true. The recording sheet will help.
c. A + E = E + G
Explain in two ways why this is true. The recording sheet will help.
d. B + F = C + D
Explain in two ways why this is true. The recording sheet will help.

2a. Like & Unlike Terms

Shapes	Area
A + D	
A + F	
A + G	
B + C	

Copy & complete these tables

b.

Shapes	Area
	6x
	3x + y
	5x + y
	5x - y

3. Exploring Addition
a. Put two shapes together to make a new shape that only has right angle corners.
Trace its perimeter. Work out its area.
b. Put three shapes together to make a new shape that only has right angle corners.
Trace its perimeter. Work out its area.
c. Put E, F & G together to make a new shape.
Trace its perimeter. Work out its area.
d. Put D, F & G together to make a new shape.
Trace its perimeter. Work out its area.
e. Put any other three shapes together to make a new shape.
Trace its perimeter. Work out its area.

6. Exploring Subtraction
a. Put C on top of B to make the best fit. Trace what you make.
How big is the piece sticking out on B?
Show in another way that B - C = x
b. Put C on top of G to make the best fit. Trace what you make.
How big is the piece sticking out on G?
Show in another way that G - C = x + y
c. Put C on top of A to make the best fit. Trace what you make.
How big is the piece sticking out on A?
Show in another way that C - A = x + y
d. Put F on top of D to make the best fit. Trace what you make.
How big is the piece sticking out on D?
Show in another way that D - F = x
e. Put F on top of A so you can see 2y sticking our on A.
Turn the combined pieces over. What is the area of the little piece that is sticking out on F?
Find out the area of A - F in at least two ways.

Smooth Edge Tiles - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising

Suppose Janine has a contract to build patios in a housing estate. The patios must be 5 tiles wide, but the owners may choose their own length.

a. How many smooth edge tiles will she need if the patio is 5 by 50?
b. If someone told you **any length**, explain how to find the number of smooth edge tiles.
c. Explain in a different way if you can. (The patios are still five tiles wide.)
d. Write an equation that shows how **S** (**S**mooth edge tiles) is found from **L** (**L**ength of patio) if the width of the patio is always 5.

2. Substituting

Length	Smooth Tiles
19	
20	
35	
128	
319	
1000	

Copy & complete these tables

3. Solving - Working Backwards

Length	Smooth Tiles
	28
	37
	65
	512
	75
	1000

Explain as much as you can about how to find the length of the patio if someone tells you any number for the smooth tiles.

4. Making Pairs

a. Choose any five numbers up to 20 for length of the patio.
For each length find how many smooth tiles and make five number pairs like this: (L, S)
b. Choose any five numbers up to 50 for the number of smooth edge tiles.
For each number find the length of the patio and make five more number pairs like this: (L, S)
c. If you do the same calculation in each pair the answer is always 6.
Explain the calculation.

5. Graphing Pairs

a. Show your ten pairs from Question 4 on a graph and explain what you see.
b. Make one more dot that you think belongs on your graph. Call it **My Dot**.
Draw the Smooth Edge Tiles picture that goes with My Dot.

6. Graphing in Excel

a. Use Excel to record your ten pairs in a table.
b. Select the table and use it to insert a chart.
Use the XY (Scatter) graph with the first sub-type.
c. Select the chart and choose Chart/Add Trendline. On the Options Tab select Display equation on chart. Explain how this equation links to the calculation in Question 4.

7. What happens if...?

Investigate what happens to the graph if the patios are all 6 wide, or 7 wide or ... 10 wide.

Snail Trail - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

This investigation explores what happens when parts of the problem are changed.
Mathematicians often learn by asking, What happens if ...?
In this problem they could ask What happens if ...

we change the time of climbing or resting?
we change the height of the well?
we change the speed of the climb?
we change the rate of slippage?
we change the snail to climbing down instead of climbing up?
we change ...

1. A New Problem: changing the time

A snail is at the bottom of an 11 metre well. It begins to climb the wall. In half an hour it climbs 5 metres, then it rests for half an hour, during which it slips back 3 metres.

a. Identify how this problem is different from the one the class worked on.
b. In which hour will the snail climb out of the well if this pattern of climbing and resting continues? Explain how you work this out.

2. A New Problem: changing the height

Suppose everything else was the same as Question 1 but the height of the well was different.

a. Work out the hour in which the snail would climb out of the well for each of these heights: 8, 15, 20, 35, 80, 100
(Hint: A mathematician would try to organise data. Perhaps it would help to work out well heights starting from 1, 2, 3, ...)
b. Find a rule so that as soon as someone tells you the height of the well, you can explain how to find the time. Try to write your rule in Algebra.

3. A New Problem - exact times

Go back to Question 2a and work out the exact time the snail reaches the top of the well, instead of just the hour in which it gets out.

4. A New Problem - changing the crawl distance

Suppose the distance the snail crawls up in half an hour changes? Investigate what happens if:

a. It climbs 6, or 7, or 8, ... or U metres up each half an hour.
Try to write an Algebra rule to explain how to work these out.

5. A New Problem - changing the slip distance

Suppose the distance the snail slips down in half an hour changes? Investigate what happens if:

a. It slips 4, or 5, or 6, ... or D metres down each half hour.

6. Other Investigations

Design your own snail trail investigation, for example:

- Work out exact times in all the rules, not just in which hour.
- What if the snail is crawling down instead of crawling up the wall?

Sphinx - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

You will need some more Sphinx Shapes to carry out this investigation. You may have to cut them from the master provide, or your teacher may have some plastic ones.

1. Generalising

a. The task card asks you to put 4 Size 1 Sphinxes together to make a larger Sphinx. Tape this larger Sphinx so you can pick it up. This is the Size 2 Sphinx. Draw the Size 1 and Size 2 Sphinxes near the top of your Sphinx Paper.
b. Make 4 Size 2 Sphinxes and put them together to make one Size 4 Sphinx.
c. Draw the result on Sphinx Paper and explain why this is called the Size 4 Sphinx. (Hint: Look at the Size 1 Sphinxes inside it.)
d. Predict how many Size 1 sphinx pieces it would take to make the Size 8 Sphinx. Write your prediction then check by making it with sphinx pieces, or by making four drawings and cutting them out.
e. If someone told you **any size Sphinx**, explain how to find the number of Size 1 Sphinxes needed to make it.
f. Explain in a different way if you can.
g. Write an equation that shows how **P** (size 1 **P**ieces) is found from **S** (**S**ize of Sphinx).

2. Substituting

Size of Sphinx	Pieces of Size 1
1	
2	
4	
8	
16	
32	

Copy & complete these tables

3. Solving - Working Backwards

Size of Sphinx	Pieces of Size 1
	4096
	65,536
	9
	25
	36
	49

Wait a minute! Can you really make a Sphinx with 9 pieces? Try. Draw your result.

4. Graphing Pairs

a. Graph the pairs in Question 2 and explain what you see.
b. Show where the dot would be for the Size 3 Sphinx. How many pieces does this dot tell you to use?
c. Show where the dot would be for the Size 11 Sphinx. How many pieces does this dot tell you to use?

5. Graphing in Excel

a. Use Excel to record the pairs in Question 2. Add some more pairs to the table.
b. Select the table and use it to insert a chart. Use the XY (Scatter) graph with the first sub-type.
c. Select the chart and choose Chart/Add Trendline. On the Type Tab select Power. On the Options Tab select Display equation on chart. How does this Excel equation relate to your equation in Question 1.

6. What happens if...?

Excel joins the points of the graph. Explain what you think about this.

Square Numbers - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising

The task shows you how to make square numbers by adding odd numbers starting from 1. But do you really need to know all the odd numbers in the list?

a. Suppose someone tells you that the last odd number in the list is 11.
What is the square number? (Hint: It's NOT 121)

b. If someone told you **any odd number as the last**, explain how to find the square.

c. Explain in a different way if you can.

d. Write an equation that shows how **S** (**S**quare number) is found from **L** (**L**ast odd).

2. Substituting

Last Odd	Square Number
19	
15	
35	
47	
319	
1001	

Copy & complete these tables

3. Solving - Working Backwards

Last Odd	Square Number
	36
	64
	121
	81
	169
	10,000

Explain as much as you can about how to find the last odd number in the list if someone tells you any square number.

4. Making Pairs

a. Choose any five numbers up to 21 for the last odd number in the list.
For each odd number find the square number and make 5 number pairs like this: (L, S)

b. Choose any five square numbers up to 144. For each square number find the last odd number in the list and make five more number pairs like this: (L, S)

c. In each pair it is true that $S = [{}^{(L-1)}/_{2} + 1]^2$
Draw a picture to explain this equation.

d. Can you write this equation another way?

5. Graphing Pairs

a. Show your ten pairs from Question 4 on a graph and explain what you see.

b. Make one more dot that you think belongs on your graph. Call it **My Dot**.
Explain the information My Dot gives about the last odd number in the list and the square number.

6. Graphing in Excel

a. Use Excel to record your ten pairs in a table.

b. Select the table and use it to insert a chart.
Use the XY (Scatter) graph with the first sub-type.

c. Select the chart and choose Chart/Add Trendline. On the Type Tab select Polynomial.
On the Options Tab select Display equation on chart.
How does this Excel equation relate to equations in Questions 1, 4c and 4d

7. What happens if...?

Excel joins the points of the graph. Explain what you think about this.

The Mushroom Hunt - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising - Fullest Basket

a. *Suppose the Big Bad Wolf also went out mushrooming with the group. If the basket pattern continued, how many mushrooms would you expect in her basket?*
b. *If someone told you* **any number of people** *who go mushrooming, explain how to find the number of mushrooms in the fullest basket if the doubling pattern continues.*
c. *Explain in a different way if you can.*
d. *Write an equation that shows how* **M** *(***M***ushrooms in fullest basket) is found from* **P** *(***P***eople who go mushrooming).*

2. Substituting

People	Fullest Basket
1	
2	
4	
7	
17	

Copy & complete these tables

3. Solving - Working Backwards

People	Fullest Basket
	4
	32
	256
	2048
	65,536

Explain as much as you can about how to find the number of people if someone tells you any doubling number for the fullest basket.

4. Generalising - Total of Mushrooms

a. *When the Big Bad Wolf goes out mushrooming with the group, what total of mushrooms would you expect if the pattern continued?*
b. *If someone told you* **any number of people** *who go mushrooming, explain how to find the total number of mushrooms in all their baskets.*
c. *Explain in a different way if you can.*
d. *Write an equation that shows how* **T** *(***T***otal of mushrooms) is found from* **P** *(***P***eople who go mushrooming).*

5. Substituting

People	Total
1	
2	
4	
7	
17	

Copy & complete these tables

6. Solving - Working Backwards

People	Total
	7
	127
	1023
	8191
	131,071

Explain as much as you can about how to find the number of people if someone tells you the total number of mushrooms in all the doubling pattern baskets.

7. Graphing Pairs

a. *Use formulas in Excel to make a table with three columns:*
People from 1 - 10 ... Fullest Basket ... Total Mushrooms
b. *Select the table and insert and XY (Scatter) chart with the first sub type.*
c. *Explain as much as you can about the information in the graph.*

Time For Tiling - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising

a. The square is 50 x 50. Explain how to find the number of dark tiles.
b. Explain in a different way if you can.
c. The square is 51 x 51. Explain how to find the number of dark tiles.
d. Explain in a different way if you can.
e. If someone told you **any square**, explain how you would find the number of dark tiles.
f. Write an equation that shows how **D** (**D**ark tiles) is found for any even square **ES**.
g. Write an equation that shows how **D** (**D**ark tiles) is found for any odd square **OS**.

2. Substituting

Side of Square	Dark Tiles
19	
20	
21	
128	
319	
1000	

Copy & complete these tables

3. Solving - Working Backwards

Side of Square	Dark Tiles
	12
	25
	97
	60
	100
	1001

Explain as much as you can about how to find the side of the square if someone tells you any total.

4. Making A Table

Use the numbers 1 to 12 as the sides of the squares and make an organised table showing the dark tiles for each one.

5. Graphing Pairs

a. Make a graph from your table in Question 4 and explain what you see.
b. Make one more dot that you think belongs on your graph. Call it **My Dot**.
Explain the information My Dot gives about squares and dark tiles.

6. Graphing in Excel

a. Use the information in your table to make two tables in Excel - one for **Even Squares** and one for **Odd Squares**.
b. Select the **Even** table and use it to insert a chart.
Use the XY (Scatter) graph with the first sub-type.
c. Select the chart and choose Chart/Add Trendline. On the Options Tab select Display equation on chart. Explain how this equation links to Question 1f.
d. Repeat for the **Odd** table and link to Question 1g.

7. What happens if...?

a. Investigate how to find the Light Tiles if you know the side of the square.
b. Design your own dark and light tile pattern and investigate it.

Unseen Triangles - Investigation Guide

Do the task first. Use this Guide to find out more. Prepare a report.

1. Generalising
a. The pattern has 50 peaks. Explain how to find the number of matches.
b. Explain in a different way if you can.
c. If someone told you **any number** of peaks in the pattern, explain how you would find the number of matches.
d. Write an equation that shows how **M** (number of **M**atches) is found from **P** (number of **P**eaks).

2. Substituting

Peaks	Matches
19	
20	
35	
128	
319	
1000	

Copy & complete these tables

3. Solving - Working Backwards

Peaks	Matches
	26
	38
	62
	174
	13
	1009

Explain as much as you can about how to find the number of peaks if someone tells you any number of matches.

4. Making Pairs
a. Choose any five numbers up to 20 for the number of peaks.
For each number find the total of matches and make five number pairs like this: (P, M)
b. Choose any five numbers up to 100 for the total of matches.
For each total find the number of triangles and make five more number pairs like this: (P, M)
c. If you do the same calculation in each pair the answer is always 2.
Explain the calculation.

5. Graphing Pairs
a. Show your ten pairs from Question 4 on a graph and explain what you see.
b. Make one more dot that you think belongs on your graph. Call it **My Dot**.
Draw the Unseen Triangles picture that goes with My Dot.

6. Graphing in Excel
a. Use Excel to record your ten pairs in a table.
b. Select the table and use it to insert a chart.
Use the XY (Scatter) graph with the first sub-type.
c. Select the chart and choose Chart/Add Trendline. On the Options Tab select Display equation on chart. Explain how this equation links to the calculation in Question 4.

7. What happens if...?
Create your own pattern with matches and investigate it.
eg: You might make equilateral triangles. The first picture on the card is a Size 2 equilateral triangle and there are 4 single equilateral triangles inside it.
How many matches would you need to make any Size equilateral triangle?

Appendix 2: Recording Sheets

Pattern & Algebra Task Menu

- ☐ 4 Arm Shapes
- ☐ Addition Totals
- ☐ Algebra Through Geometry
- ☐ Crossing The River 1
- ☐ Eric The Sheep
- ☐ Garden Beds
- ☐ Lining Up
- ☐ Making Monuments
- ☐ Match Triangles
- ☐ Mirror Patterns 2
- ☐ Painted Rods
- ☐ Pointy Fences
- ☐ Shape Algebra
- ☐ Smooth Edge Tiles
- ☐ Snail Trail
- ☐ Sphinx
- ☐ Square Numbers
- ☐ The Mushroom Hunt
- ☐ Time For Tiling
- ☐ Unseen Triangles

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- ☐ Unseen Triangles

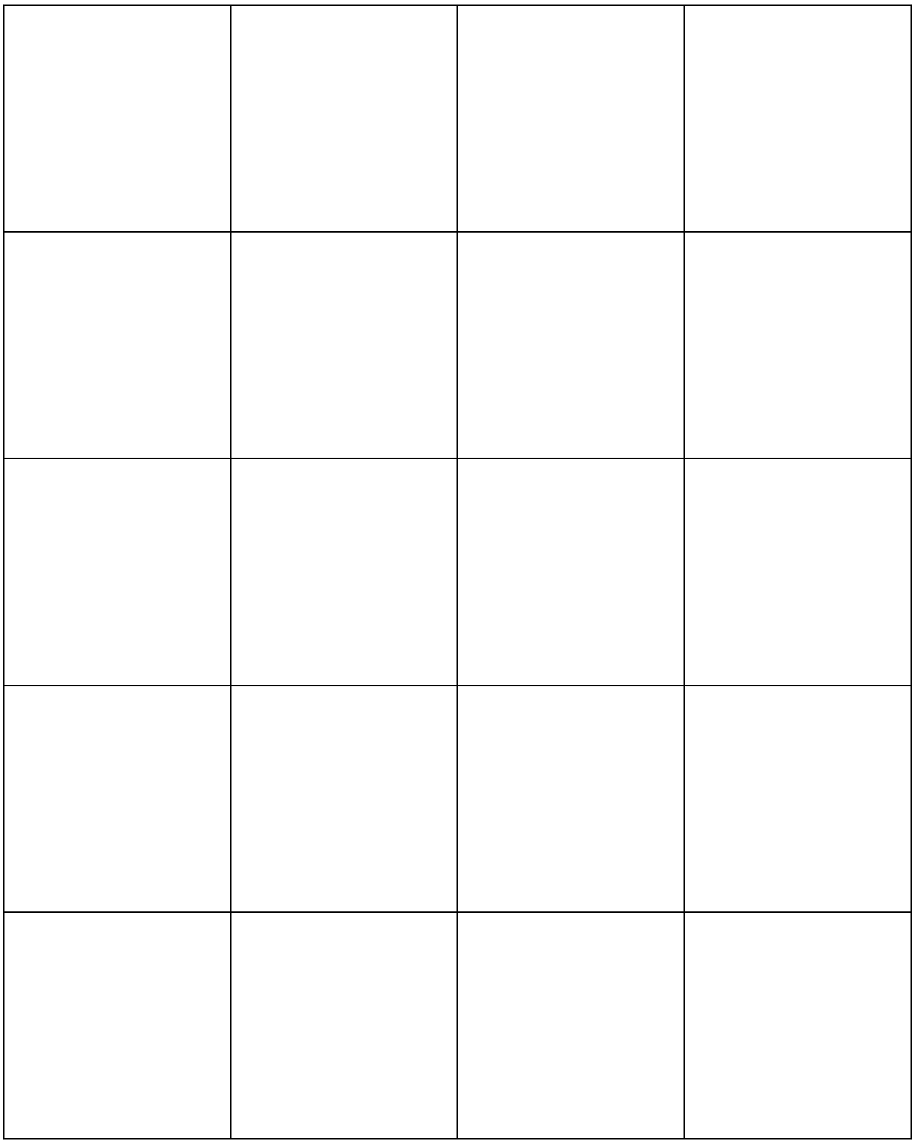
Graph Paper

Reproducible Page

Names: .... Class: ....

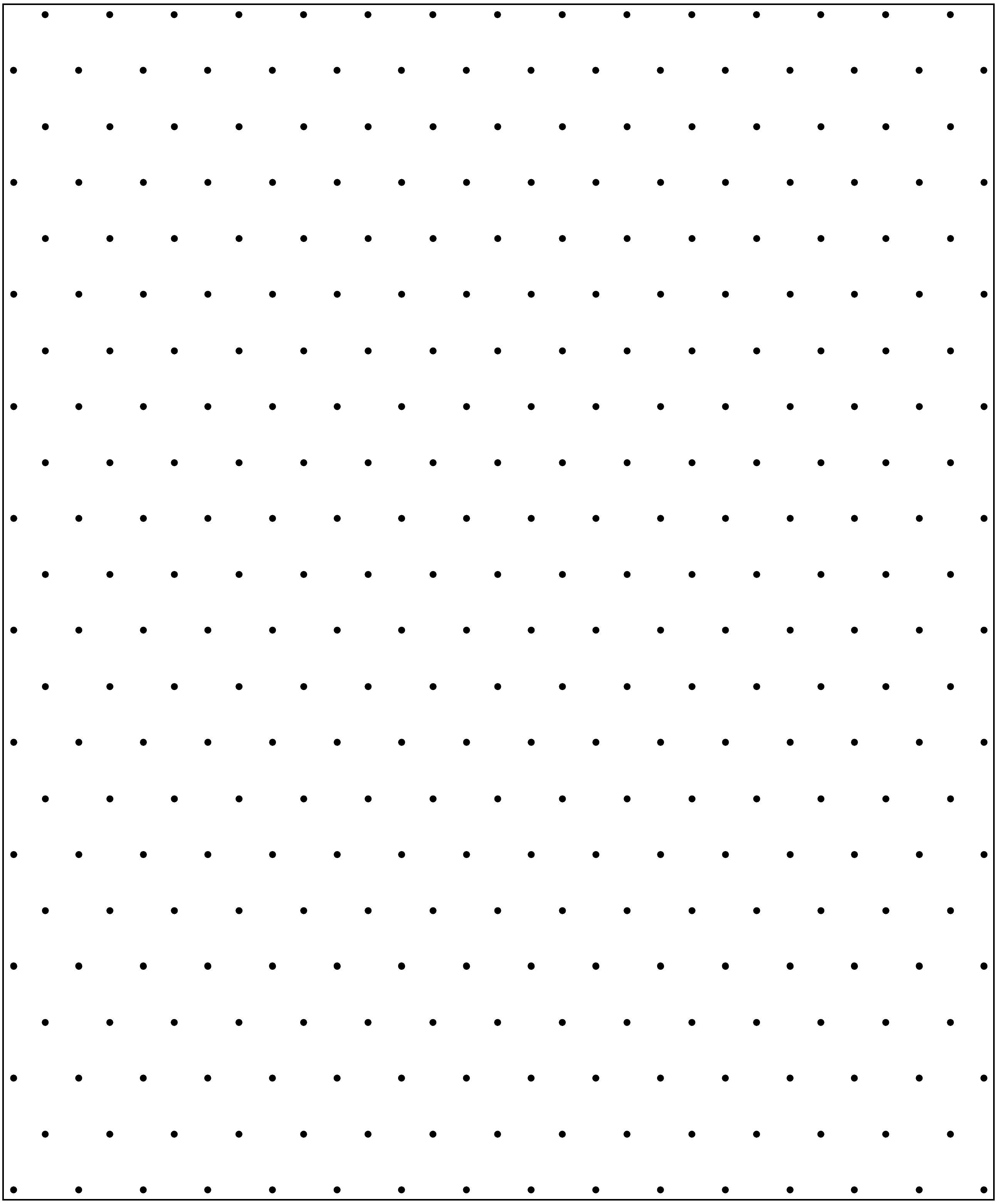
Algebra Through Geometry

Each square is the same size as the shape named x.
The sheet can help you work out the area of combinations of shapes.
You might trace the shapes and then remove them to work out the area.



Names: .... Class: ....

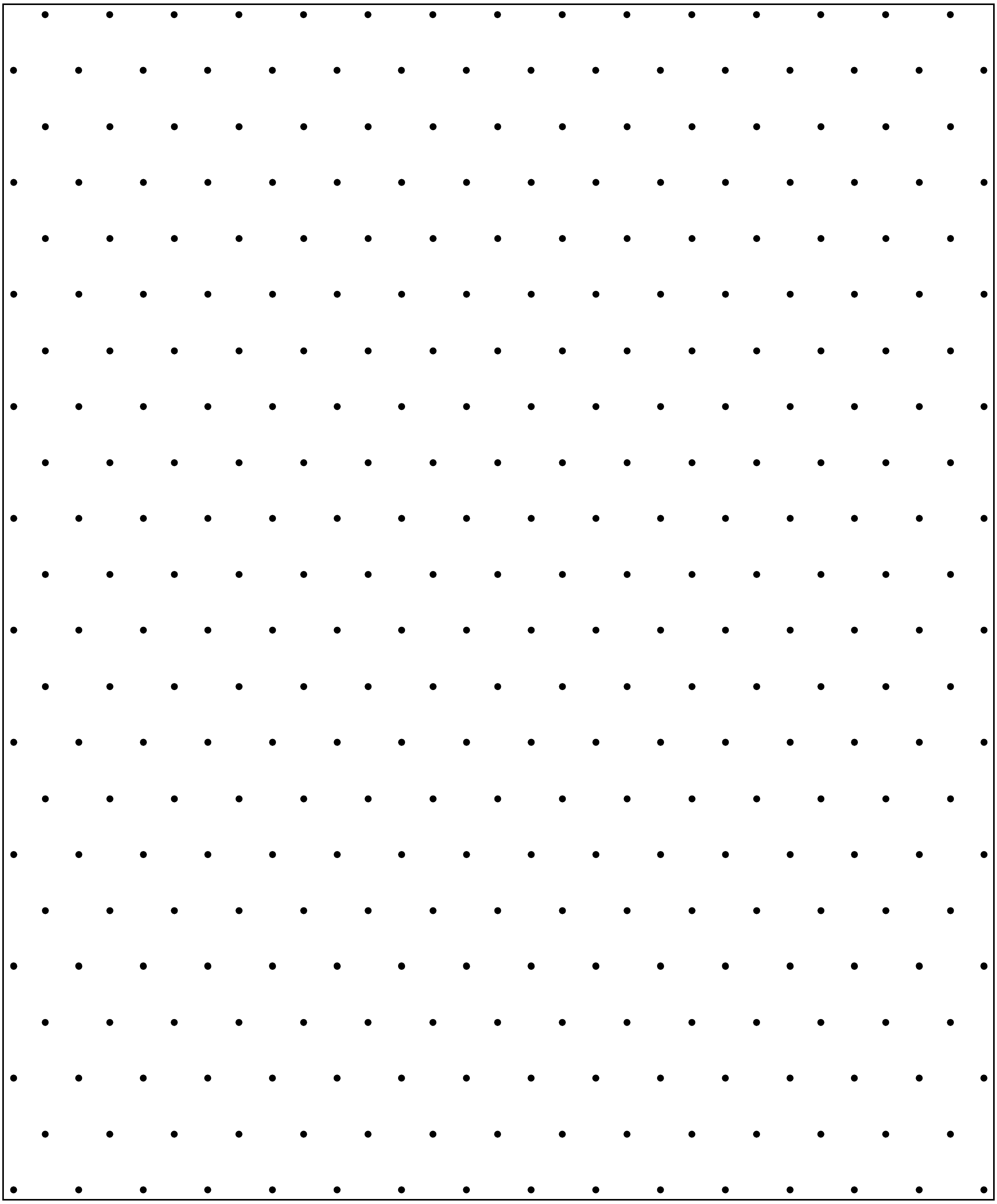
Match Triangles



Names: .... Class: ....

Pointy Fences

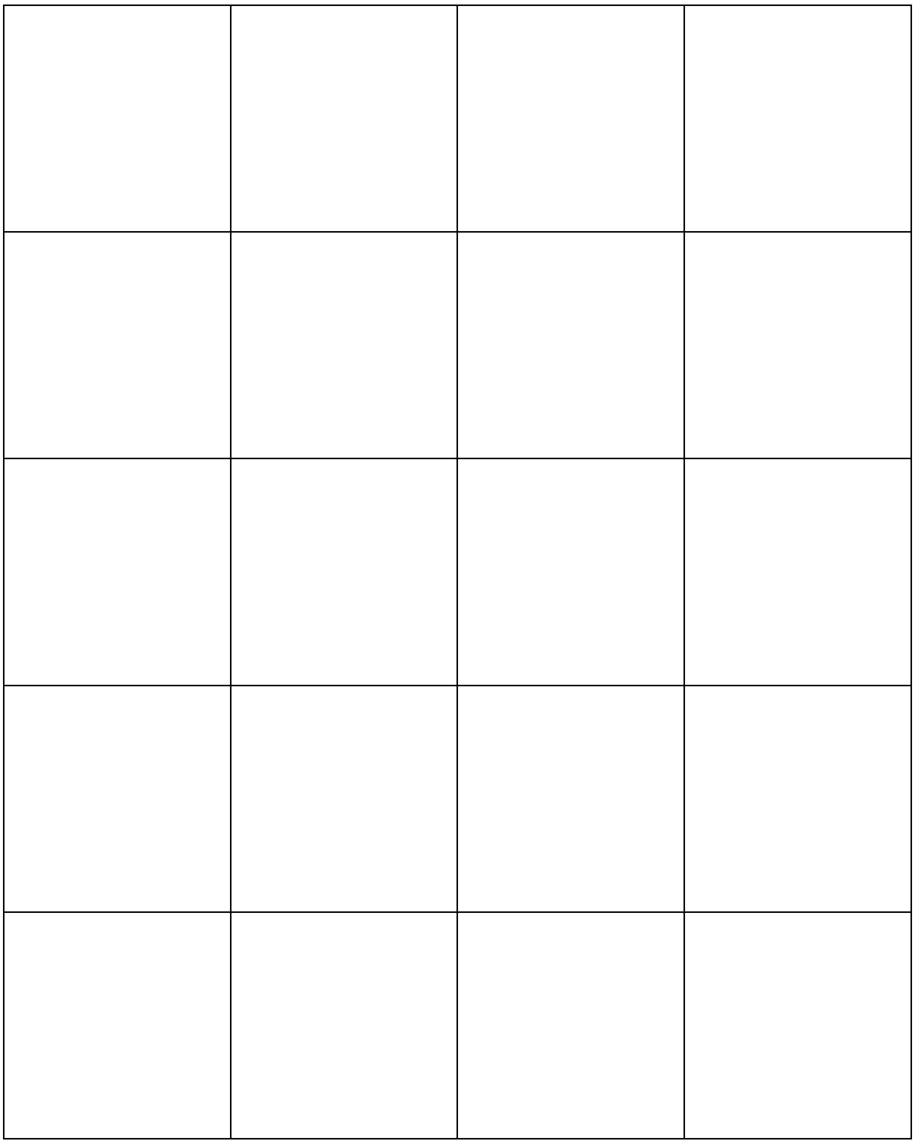
Reproducible Page



Names: .... Class: ....

Shape Algebra

Each square is the same size as the shape named x.
The sheet can help you work out the area of combinations of shapes.
You might trace the shapes and then remove them to work out the area.



Names: .... Class: ....

Sphinx Paper

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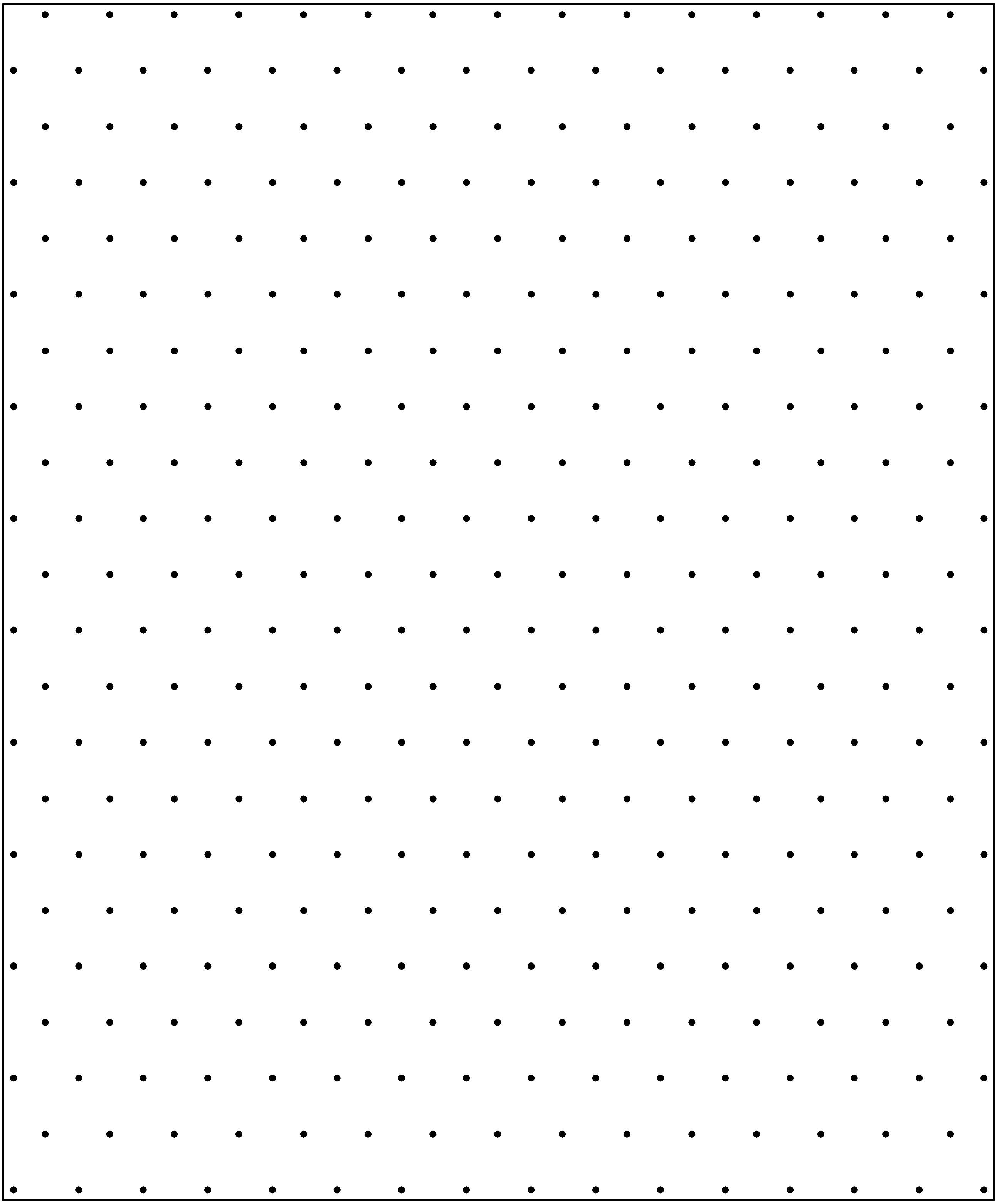

Names: .... Class: ....

Square Numbers

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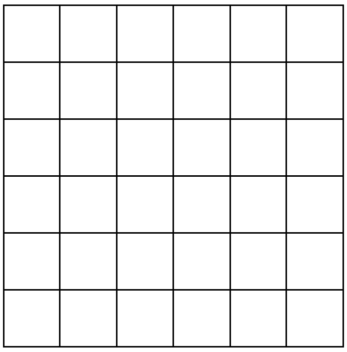
Names: .... Class: ....

1. Colour the grid to show how the squares from areas 1 to 25 fit inside this area 36 square.



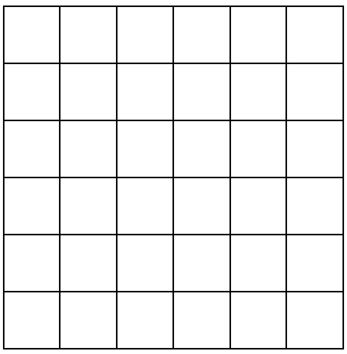
2. 36 = ...
3. The 20th square number would be: ....
   Explain your reasoning here:

Square Numbers

Reproducible Page © Mathematics Task Centre

Names: .... Class: ....

1. Colour the grid to show how the squares from areas 1 to 25 fit inside this area 36 square.



2. 36 = ...
3. The 20th square number would be: ....
   Explain your reasoning here:

Unseen Triangles

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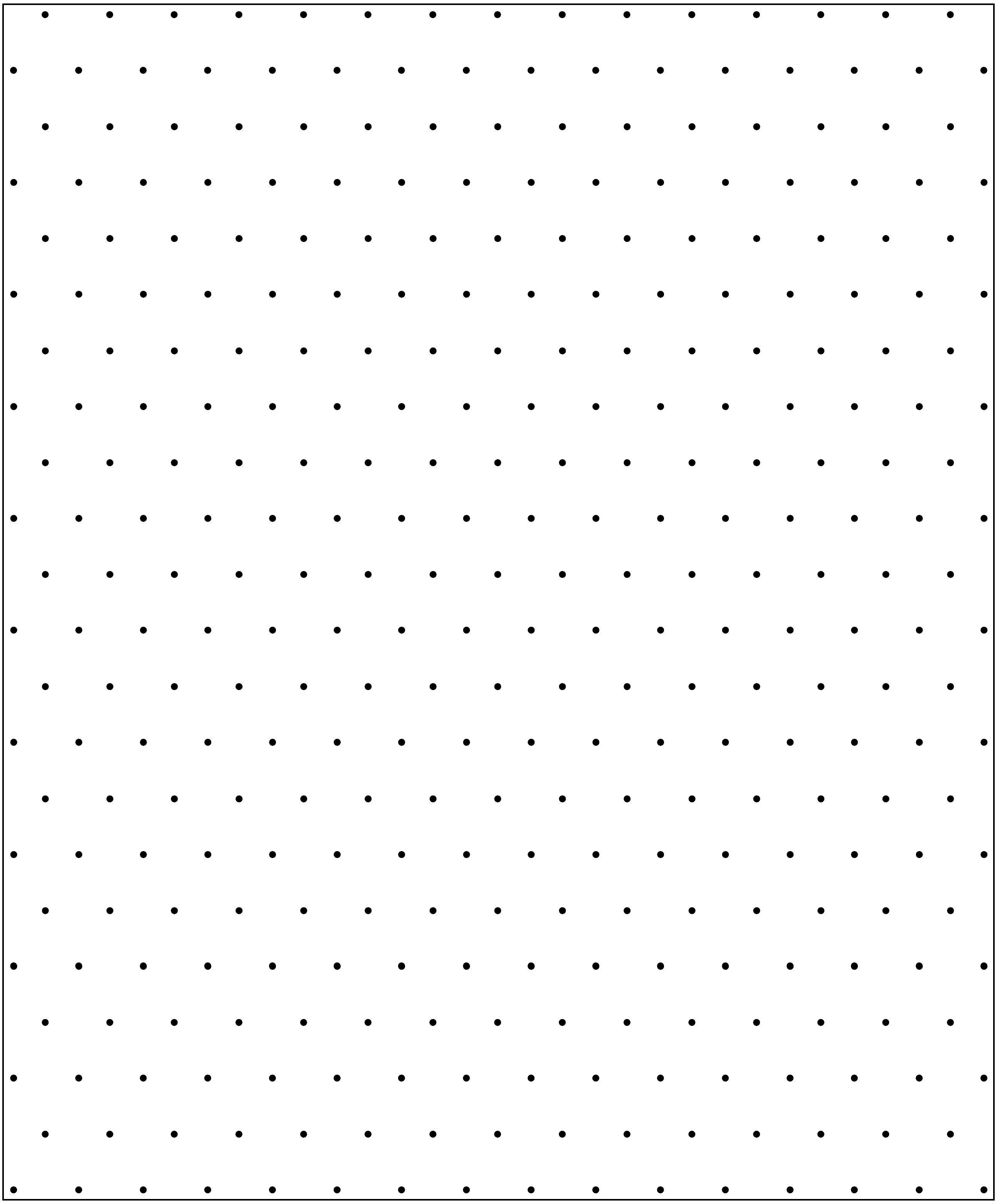

Names: .... Class: ....