maths with attitude

2nd Edition

Space & Logic Years 5 & 6

Charles Lovitt
Doug Williams

Mathematics Task Centre & Maths300
helping to create happy healthy cheerful productive inspiring classrooms

Space & Logic

Years 5 & 6

In this kit:

- *Hands-on problem solving tasks*
- *Detailed curriculum planning*

Access from Maths300:

- *Extensive lesson plans*
- *Software*

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Maths With Attitude includes reference to hands-on problem solving tasks and Maths300 material.

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TABLE OF CONTENTS

Part 1: Preparing To Teach

Our Objective

- To support teachers, schools and systems wanting to create:

    happy, healthy, cheerful, productive, inspiring classrooms

Our Attitude

- to learning:

    learning is a personal journey stimulated by achievable challenge
- to learners:

    stimulated students are creative and love to learn
- to pedagogy:

    the art of choosing teaching strategies to involve and interest all students
- to mathematics:

    mathematics is concrete, visual and makes sense
- to learning mathematics:

    all students can learn to work like a mathematician
- to teachers:

    the teacher is the most important resource in education
- to professional development:

    teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

Our Objective in Detail

What do we mean by creating:

happy, healthy, cheerful, productive, inspiring classrooms

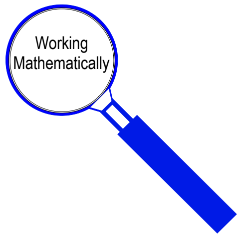
Happy…

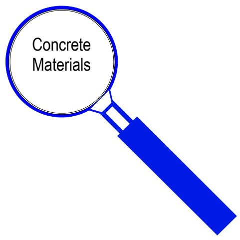
means the elimination of the unnecessary fear of failure that hangs over so many students in their mathematics studies. Learning experiences *can* be structured so that all students see there is something in it for them and hence make a commitment to the learning. In so many 'threatening' situations, students see the impending failure and withhold their participation.

A phrase which describes the structure allowing all students to perceive something in it for them is *multiple entry points and multiple exit points*. That is, students can enter at a variety of levels, make progress and exit the problem having visibly achieved.

Healthy…

means *educationally healthy*. The learning environment should be a reflection of all that our community knows about how students learn. This translates into a rich array of teaching strategies that could and should be evident within the learning experience.





If we scrutinise the *exploration* through any lens, it should confirm to us that it is well structured or alert us to missed opportunities. For example, peering through a pedagogy lens we should see such features as:

- a story shell to embed the situation in a meaningful context
- significant active use of concrete materials
- a problem solving challenge which provides ownership for students
- small group work
- a strong visual component
- access to supportive software

Cheerful…

because we want 'happy' in the title twice!

Productive…

is the clear acknowledgment that students are working towards recognisable outcomes. They should know what these are and have guidelines to show they have either reached them or made progress. Teachers are accountable to these outcomes as well as to the quality of the learning environment.

Inspiring…

is about creating experiences that are uplifting or exalting; that actually *turn students on*. Experiences that make students feel great about themselves and empowered to act in meaningful ways.

Space & Logic Resources

To help you create

happy, healthy, cheerful, productive, inspiring classrooms

this kit contains

- 20 hands-on problem solving tasks from Mathematics Centre

and a Teachers' Manual which integrates the use of the tasks with

- 8 detailed lesson plans from Maths300

The kit offers **5 weeks** of Scope & Sequence planning in Space and Logic for *each* of Year 5 and Year 6. This is detailed in *Part 2: Planning Curriculum* which begins on Page 12. You are invited to map these weeks into your Year Planner. Together, the four kits available for these levels provide 25 weeks of core curriculum in Working Mathematically (working like a mathematician).

Note: Membership of Maths300 is assumed.
The kit will be useful without it, but it will be much more useful with it.

Tasks

- Crossing The Desert
- Crossing The River 2
- Cube Nets
- Diamonds & Rectangles
- Eight Queens
- Football Ladder
- Four Cube Houses
- Koala Carts
- Making Triangles
- Number Discs
- Octaflex
- Pattern Cube
- Pentominoes
- Reflections
- Sliding Tiles
- Soma Cube 2
- The Farmer's Puzzles
- Tricube Constructions A
- Which View?
- Who Owns The Monkey?

Part 2 of this manual introduces each task. The latest information can be found at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm

Maths300 Lessons

- Cube Nets
- Football Ladder
- Four Cube Houses
- Land of ET
- Nim
- Red To Blue
- Sphinx
- Where Do We Sit?

Lessons with Software

- There are no lessons with software support in this **Maths With Attitude** kit.

Part 2 of this manual introduces each lesson. Full details can be found at:

- http://www.maths300.com

Working Like A Mathematician

Our attitude is:

all students can learn to work like a mathematician

What does a mathematician's work actually involve? Mathematicians have provided their answer on Page 8. In particular we are indebted to Dr. Derek Holton for the clarity of his contribution to this description.

Perhaps the most important aspect of Working Mathematically is the recognition that *knowledge is created by a community and becomes part of the fabric of that community*. Recognising, and engaging in, the process by which that knowledge is generated can help students to see themselves as able to work like a mathematician. Hence Working Mathematically is the framework of **Maths With Attitude.**

Skills, Strategies & Working Mathematically

A Working Mathematically curriculum places learning mathematical skills and problem solving strategies in their true context. Skills and strategies are the tools mathematicians employ in their struggle to solve problems. Lessons on skills or lessons on strategies are not an end in themselves.

- **Our skill toolbox** can be added to in the same way as the mechanic or carpenter adds tools to their toolbox. Equally, the addition of the tools is not for the sake of collecting them, but rather for the purpose of getting on with a job. A mathematician's job is to attempt to solve problems, not to collect tools that might one day help solve a problem.
- **Our strategy toolbox** has been provided through the collective wisdom of mathematicians from the past. All mathematical problems (and indeed life problems) that have ever been solved have been solved by the application of this concise set of strategies.

About Tasks

Our attitude is:

mathematics is concrete, visual and makes sense

Tasks are from Mathematics Task Centre. They are an invitation to two students to work like a mathematician (see Page 8).

The Task Centre concept began in Australia in the late 1970s as a collection of rich tasks housed in a special room, which came to be called a Task Centre. Since that time hundreds of Australian teachers, and, more recently, teachers from other countries, have adapted and modified the concept to work in their schools. For example, the special purpose room is no longer seen as an essential component, although many schools continue to opt for this facility.

A brief history of Task Centre development, considerable support for using tasks, for example Task Cameos, and a catalogue of all currently available tasks can be found at:

- http://www.mathematicscentre.com/taskcentre

Key principles are:

- A good task is the tip of an iceberg
- Each task has three lives
- Tasks involve students in the Working Mathematically process

The Task Centre Room or the Classroom?

There are good reasons for using the tasks in a special room which the students visit regularly. There are also different good reasons for keeping the tasks in classrooms. Either system can work well if staff are committed to a core curriculum built around learning to work like a mathematician.

- A task centre room creates a focus and presence for mathematics in the school. Tasks are often housed in clear plastic 'cake storer' type boxes. Display space can be more easily managed. The visual impact can be vibrant and purposeful.
- However, tasks can be more readily integrated into the curriculum if teachers have them at their finger tips in the classrooms. In this case tasks are often housed in press-seal plastic bags which take up less space and are more readily moved from classroom to classroom.

Tip of an Iceberg

The initial problem on the card can usually be solved in 10 to 20 minutes. The investigation iceberg which lies beneath may take many lessons (even a lifetime!). Tasks are designed so that the original problem reveals just the 'tip of the iceberg'. Task Cameos and Maths300 lessons help to dig deeper into the iceberg.

We are constantly surprised by the creative steps teachers and students take that lead us further into a task. No task is ever 'finished'.

> *Most tasks have many levels of entry and exit and therefore offer an on-going invitation to revisit them, and, importantly, multiple levels of success for students.*

Three Lives of a Task

This phrase, coined by a teacher, captures the full potential and flexibility of the tasks. Teachers say they like using them in three distinct ways:

1. As on the card, which is designed for two students.
2. As a whole class lesson involving all students, as supported by outlines in the Task Cameos and in detail through the Maths300 site.
3. Extended by an Investigation Guide (project), examples of which are included in both Task Cameos and Maths300.

The first life involves just the 'tip of the iceberg' of each task, but nonetheless provides a worthwhile problem solving challenge - one which 'demands' concrete materials in its solution. This is the invitation to work like a mathematician. Most students will experience some level of success and accomplishment in a short time.

The second life involves adapting the materials to involve the whole class in the investigation, in the first instance to model the work of a mathematician, but also to develop key outcomes or specific content knowledge. This involves choosing teaching craft to interest the students in the problem and then absorb them in it.

The third life challenges students to explore the 'rest of the iceberg' independently. Investigation Guides are used to probe aspects and extensions of the task and can be introduced into either the first or second life. Typically this involves providing suggestions for the direction the investigation might take. Students submit the 'story' of their work for 'portfolio assessment'. Typically a major criteria for assessment is application of the Working Mathematically process.

About Maths300

Our attitude is:

teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

Maths300 is a subscription based web site. It is an attempt to collect and publish the 300 most 'interesting' maths lessons (K - 12).

- *Lessons have been successfully trialed in a range of classrooms.*
- *About one third of the lessons are supported by specially written software.*
- *Lessons are also supported by investigation sheets (with answers) and game boards where relevant.*
- *A 'living' Classroom Contributions section in each lesson includes the latest information from schools.*
- *The search engine allows teachers to find lessons by pedagogical feature, curriculum strand, content and year level.*
- *Lesson plans can be printed directly from the site.*
- *Each lesson supports teaches to model the Working Mathematically process.*

Modern internet facilities and computers allow teachers easy access to these lesson plans. Lesson plans need to be researched, reflected upon in the light of your own students and activated by collecting and organising materials as necessary.

Maths300 Software

Our attitude is:

stimulated students are creative and love to learn

Pedagogically sound software is one feature likely to encourage enthusiastic learning and for that reason it has been included as an element in about one third of Maths300 lesson plans. The software is used to develop an investigation beyond its introduction and early exploration which is likely to include other pedagogical techniques such as concrete materials, physical involvement, estimation or mathematical conversation. The software is not the lesson plan. It is a feature of the lesson plan used at the teacher's discretion.

For school-wide use, the software needs to be downloaded from the site and installed in the school's network image. You will need to consult your IT Manager about these arrangements. It can also be downloaded to stand alone machines covered by the site licence, in particular a teacher's own laptop, from where it can be used with the whole class through a data projector.

Note:

- *Maths300 lessons and software may only be used by Maths300 members.*

Working Mathematically

First give me an interesting problem.

When mathematicians become interested in a problem they:

- ♦ Play with the problem to collect & organise data about it.
- ♦ Discuss & record notes and diagrams.
- ♦ Seek & see patterns or connections in the organised data.
- ♦ Make & test hypotheses based on the patterns or connections.
- ♦ Look in their strategy toolbox for problem solving strategies which could help.
- ♦ Look in their skill toolbox for mathematical skills which could help.
- ♦ Check their answer and think about what else they can learn from it.
- ♦ Publish their results.

Questions which help mathematicians learn more are:

- ♦ Can I check this another way?
- ♦ What happens if ...?
- ♦ How many solutions are there?
- ♦ How will I know when I have found them all?

When mathematicians have a problem they:

- ♦ Read & understand the problem.
- ♦ Plan a strategy to start the problem.
- ♦ Carry out their plan.
- ♦ Check the result.

A mathematician's strategy toolbox includes:

- ♦ Do I know a similar problem?
- ♦ Guess, check and improve
- ♦ Try a simpler problem
- ♦ Write an equation
- ♦ Make a list or table
- ♦ Work backwards
- ♦ Act it out
- ♦ Draw a picture or graph
- ♦ Make a model
- ♦ Look for a pattern
- ♦ Try all possibilities
- ♦ Seek an exception
- ♦ Break a problem into smaller parts
- ♦ ...

If one way doesn't work, I just start again another way.

Professional Development Purpose

Our attitude is:

the teacher is the most important resource in education

> *We had our first study group on Monday. The session will be repeated again on Thursday. I had 15 teachers attend. We looked at the task Farmyard Friends (Task 129 from the Mathematics Task Centre). We extended it out like the questions from the companion Maths300 lesson suggested, and talked for quite a while about the concept of a factorial. This is exactly the type of dialog that I feel is essential for our elementary teachers to support the development of their math background. So anytime we can use the tasks to extend the teacher's math knowledge we are ahead of the game.*
> District Math Coordinator, Denver, Colorado

Research suggests that professional development most likely to succeed:

- is requested by the teachers
- takes place as close to the teacher's own working environment as possible
- takes place over an extended period of time
- provides opportunities for reflection and feedback
- enables participants to feel a substantial degree of ownership
- involves conscious commitment by the teacher
- involves groups of teachers rather than individuals from a school
- increases the participant's mathematical knowledge in some way
- uses the services of a consultant and/or critical friend

Maths With Attitude has been designed with these principles in mind. All the materials have been tried, tested and modified by teachers from a wide range of classrooms. We hope the resources will enable teacher groups to lead themselves further along the professional development road, and support systems to improve the learning outcomes for students K - 12.

With the support of Maths300 ETuTE, professional development can be a regular component of in-house professional development. See:

- http://www.mathematicscentre.com/taskcentre/resource.htm#etute

For external assistance with professional development, contact:

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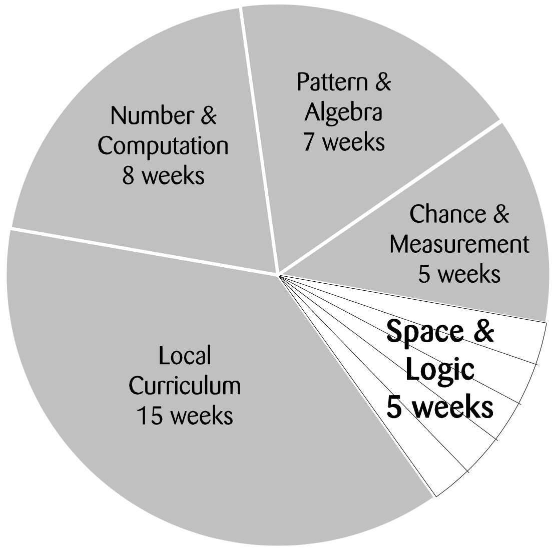
Part 2: Planning Curriculum

Curriculum Planners

Our attitude is:

learning is a personal journey stimulated by achievable challenge



Curriculum Planners:

- show one way these resources can be integrated into your weekly planning
- provide a starting point for those new to these materials
- offer a flexible structure for those more experienced

You are invited to map Planner weeks into your school year planner as the core of the curriculum.

Planners:

- detail each week lesson by lesson
- offer structures for using tasks and lessons
- are sequenced from lesson to lesson, week to week and year to year to 'grow' learning

Teachers and schools will map the material in their own way, but all will be making use of extensively trialed materials and pedagogy.

Using Resources

- Your kit contains 20 hands-on problem solving tasks and reference to relevant Maths300 lessons.
- Tasks are introduced in this manual and supported by the Task Cameos at:
  http://www.mathematicscentre.com/taskcentre/iceberg.htm
- Maths300 lessons are introduced in this manual and supported by detailed lesson plans at:
  http://www.maths300.com

In your preparation, please note:

- Planners assume 4 lessons per week of about 1 hour each.
- Planners are *not* prescribing a continuous block of work.
- Weeks can be interspersed with other learning; perhaps a **Maths With Attitude** week from a different strand.
- Weeks can sometimes be interchanged within the planner.
- Lessons can sometimes be interchanged within weeks.
- The four **Maths With Attitude** kits available at each year level offer 25 weeks of a Working Mathematically core curriculum.

A Way to Begin

- Glance over the Planner for your class. Skim through the comments for each task and lesson as it is named. This will provide an overview of the kit.
- Task Comments begin after the Planners. Lesson Comments begin after Task Comments. The index will also lead you to any task or lesson comments.
- Select your preferred starting week - usually Week 1.
- Now plan in detail by researching the comments and web support. Enjoy!

Research, Reflect, Activate

Curriculum Planner
Space & Logic: Year 5

	Session 1	Session 2	Session 3	Session 4
Week 1	**Small Group Problem Solving:** The kit includes 20 hands-on problem solving tasks. In this week, pairs of students are encouraged to select 1 or 2 and investigate in depth to show how they are learning to work like a mathematician (p. 8). An expectation can be the preparation of an investigation report. This report can be included in the student's assessment portfolio. See this link for a lesson plan on reporting: http://www.mathematicscentre.com/taskcentre/record.htm			
Week 2	**Whole Class Investigation:** *Where Do We Sit?* may be most relevant at the beginning of the year because students want to know where they will sit in the new room. The lesson includes 'clever counting' of arrangements and asks the mathematician's questions: *How many ways are there? How will I know when I have found them all?*			
Week 3	**Whole Class Investigation:** *Red To Blue* is an opportunity to model how a mathematician works. The problem requires no specific background; only a willingness to think through the choices at each step. It begins with an easy to state challenge that may be presented kinaesthetically. Once the initial problem is solved there are several extensions.			
Week 4	**Whole Class Investigation:** Four sphinxes make a sphinx; therefore four of this new size sphinx also makes a sphinx. The spatial connection grows into a study of pattern and tessellation. There are many parts to the iceberg, but for this year the focus is on 'chasing' spatial aspects. *Sphinx* appears to be a simple puzzle - it is only a jigsaw-style problem with four pieces - yet even students who have previously experienced Sphinx as a task in the Space & Logic Years 3 & 4 kit may find it a challenging.			
Week 5	**Whole Class Investigation:** Although presented in the context of the Australian Football League (AFL) teams, *Football Ladder* is easily adapted to other codes. Indeed, once explored as presented, one extension is for the students to create a similar puzzle for their chosen code. The task is based in language and logic and offers several levels of entry by using the alternative clue sets provided. There is plenty of opportunity for group work and mathematical conversation.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Curriculum Planner

Space & Logic: Year 6

	Session 1	Session 2	Session 3	Session 4
Week 1	**Small Group Problem Solving:** The kit includes 20 hands-on problem solving tasks. In this week, pairs of students are encouraged to select 1 or 2 and investigate in depth to show how they are learning to work like a mathematician (p. 8). An expectation can be the preparation of an investigation report. This report can be included in the student's assessment portfolio. See this link for a lesson plan on reporting: http://www.mathematicscentre.com/taskcentre/record.htm			
Week 2	**Whole Class Investigation:** *Cube Nets* depends on the equipment *3d Geoshape*. The task version of the problem contains sufficient equipment for just two students. However as an investigation it models the Working Mathematically process so well that schools obtain the 'right' equipment. (Visit http://www.mathematicscentre.com/taskcentre/resource.htm). All the data in this investigation is spatial/visual. So the lesson illustrates that Working Mathematically is more than working with numbers.			
Week 3	**Whole Class Investigation:** *Four Cube Houses* involves an organised search in 3-dimensions for all the 'houses' that can be made with four cubic modules. The investigation may extend into costing the construction of various designs, and all that would be associated with building a housing estate from them and marketing its features.			
Week 4	**Whole Class Investigation:** *Nim* may be known to the students but it is unlikely that they have explored its logic. The investigation is easy to introduce and its game context is motivating. The strength of the lesson is the opportunity to discuss and share strategies. The lesson may be guided by the investigation sheet provided.			
Week 5	**Whole Class Investigation:** *Land of ET* begins as a language and logic puzzle embedded in a fantasy story shell about a mythical land where the King and Queen make rules for the language. The structure of the language leads to surprisingly few unique words. One extension of the lesson is exploring symmetries of the equilateral triangle, which turn out to behave 'just like the ET words do'.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Planning Notes

Enhancing Maths With Attitude

Resources to support learning to work like a mathematician are extensive and growing. There are more tasks and lessons available than have been included in this Space & Logic kit. You could use the following to enhance this kit.

Additional Tasks

- Task 21, Tactical
  A logic game with clear rules and a spatial dimension. The aim is to force your opponent to take the last counter.
- Task 74, Button Sort
  A logical challenge that uses the attributes of buttons to sort and classify them. The task informally introduces significant ideas in mathematical logic such as the connectives 'and', 'or' and 'not' and introduces diagrams similar to flow charts as a way of sorting and making decisions.
- Task 96, Networks
  A game with laminated cardboard tiles which encourages thinking ahead in a spatial situation that is complicated by a growing network of lines. A player can either win by placing the piece which reaches the Finish square, or by placing a piece that forces their opponent to lose.
- Task 153, Knight Protectors
  A famous chess puzzle that results in a beautiful rotationally symmetric solution. The challenge is to place the minimum number of knights on a chess board so that every square is either occupied or attacked.
- Task 164, Symmetric Tiles
  Using two designs of laminated cardboard tiles, the task encourages exploration of symmetry, where the line of symmetry will most likely be seen as diagonal. One shape is the key to finding many solutions, and even when this is realised, the challenge is not easy to resolve.
- Task 185, Coloured Cubes
  A classic puzzle requiring a great deal of thought to solve without hint. Each of the four cubes have their faces coloured in a different way using four different colours. The challenge is to place the cubes in a line so that all 4 colours show along the surface of the 'square-section rod' created.

More information about these tasks may be available in the Task Cameo Library:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Additional Lessons

- Lesson 123, Mirror Bounce
  How were artists deep in the Egyptian pyramids able to find sufficient light to create the elaborate paintings on the walls of the burial chambers? The question generates interest in the properties of light, in particular in the intensity of light and its reflection. Students use mirrors to 'bounce' light around the classroom in a group activity with the challenge of 'hitting' a stated target.

Keep in touch with new developments which enhance **Maths With Attitude** at:

- http://www.mathematicscentre.com/taskcentre/enhance.htm

Additional Materials

As stated, our attitude is that mathematics is concrete, visual and makes sense. We assume that all classrooms will have easy access to many materials beyond what we supply. For this unit you will need:

- Wooden cubes or linking cubes like Multilink that join in three dimensions
- Counters
- Two-sided counters, which can be made by adding sticker dots to one side of a single-colour counter.

In Year 5 you might like to use colourful plastic Sphinx pieces rather than printing, laminating and cutting your own from the master supplied with the Maths300 lesson. Sphinx pieces are a Mathematics Centre resource:

- http://www.mathematicscentre.com/taskcentre/resource.htm

In Year 6 you *will* need special equipment called 3d Geoshapes (square pieces only) for the Cube Nets lesson. You can also find 3d Geoshapes through Mathematics Centre Resources.

Special Comments Year 5

- *Red to Blue*, Planner Week 3 may be enhanced by counters with a different colour on each side, or Poly Plug (see below). However, the concrete material can just as easily be ripped up pieces of paper with R written on one side and B written on the other.
- *Sphinx*, Planner Week 4. You will need to prepare your Sphinx shapes. A template is provided in the lesson, but it will take some time to print on card, laminate and cut out about 100 shapes.
- *Football Ladder*, Planner Week 5. You will need to prepare the cards provided with the lesson. If you print in colour on card, then laminate and guillotine, you will have a reusable set for future years.

Special Comments Year 6

- *Cube Nets*, Planner Week 2. You will need to ensure the school has a supply of 3d Geoshape squares.
- *Land of ET*, Planner Week 5, requires two colours of counter, but if you have Poly Plug the yellow/blue plugs exactly model the Y and B used as letters of the words in the language of ET.

Find Poly Plug information at:

- http://www.mathematicscentre.com/taskcentre/polyplug.htm

Task Comments

- Tasks, lessons and unit plans prepare students for the more traditional skill practice lessons, which we invite you to weave into your curriculum. Teachers who have used practical, hands-on investigations as the focus of their curriculum, rather than focussing on the drill and practice diet of traditional mathematics, report success in referring to skill practice lessons as Toolbox Lessons. This links to the idea of a mathematician dipping into a toolbox to find and use skills to solve problems.

Crossing The Desert

This task is sure to generate discussion. There are interpretations of the problem which are important to explore. Perhaps the most relevant reason for encouraging the discussion is that a mathematician needs to understand, in fact in many cases clarify and refine, the problem before attempting a solution. Usually this process involves 'talking it through' with colleagues. In this sense Crossing The Desert encourages justification of answers against agreed criteria, rather than production of *the* solution, or sequence of solutions.

Discussion will identify some or all of the following aspects of the problem:

- Travelling to the oasis requires 8 days food each way per person and one more food portion for Day 9.
- That is 17 food portions per person, so 34 for two people if they both reach the oasis and return. Therefore the 24 food portions allowed are not enough for both travellers to reach the oasis and return.
- One person will need 17 food portions (the Messenger) so there are 7 others to 'play with'.
- We could assume that each person eats at noon.
- We could assume that the message is delivered 'in seconds' and the Messenger then turns for home. They don't spend a day resting at the oasis and therefore only need one portion of food on Day 9.
- In a similar way, we could also assume that on the day one person (the Companion) turns around, they also only need one portion of food.

Since the card suggests burying food, students usually try that option in conjunction with the trial and improve strategy. However burying alone doesn't reveal a solution; although burying on Day 5 comes close and an interpretation based on this potential solution is made below.

The key is to realise that the Companion needs food for their own return journey, could bury some for the Messenger to use on the way back and could *give* some to the Messenger to make their total back up to the maximum of 12.

This understanding leads to the following solutions:

	Day	Give	Bury
Solution 1	3	3	4
Solution 2	4	4	1
Solution 3	4	3	2
Solution 4	4	2	3
Solution 5	4	1	4

Notes

- Solution 1 gets the messenger home with food to spare.
- It could be reasoned that more solutions are possible if you allow progressive burial and/or giving. For example in Solution 5, the four buried portions could be buried 1 on each day, or 2 on Day 2 and 2 on Day 4 and so on. Similarly the giving of 1 could occur on any of the four days. Students can make the decision whether these are seen as merely variations, or as new solutions. It really doesn't matter which. What matters is the reasoned discussion and agreement should the issue arise.
- Equally the students could debate the merits of this potential Bury Only solution. If you accept that in real life the food you eat on Day N provides the energy to travel on Day (N + 1), then they could travel together to Day 5. The Companion would need only 3 pieces of food for the return journey because the third of these would provide the energy to travel home on the last day (Day 1 on the board). The Companion's remaining 4 food portions would be buried. The Messenger would continue the journey, and be able to return to Day 5 on the strength of the food eaten the day before. The four buried portions would be sufficient (consistent with this interpretation) to complete the return journey. Again the real issue is not whether this is a solution, but rather whether it is a solution in the context of this interpretation.
- Newcomb Secondary College assisted with the design of the reproducible board provided at the end of this manual.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Crossing The River 2

The clue in bold on the card is very important. *At no stage* means neither in the boat, nor on either bank. In the boat isn't an issue since the only possibilities are AA, AC, AC_r or C_rC, where C_r means the rowing child. So there are two questions to consider at each move:

- What does this move do to the adult/child mix on the bank I am leaving?
- What does this move do to the adult/child mix on the bank where I arrive?

The problem can begin successfully in one of two ways, but both lead to the three children being on the required bank by themselves after three moves. As students explore the possibilities from here, each of the children will actually return to the original bank, before finally arriving safely on the required bank in the last moves of the problem. In total there will be 13 trips across the river.

Once the problem is solved (it's not easy but it does resolve to the strategy of try every possible case), focus attention on recording/explaining/publishing the solution.

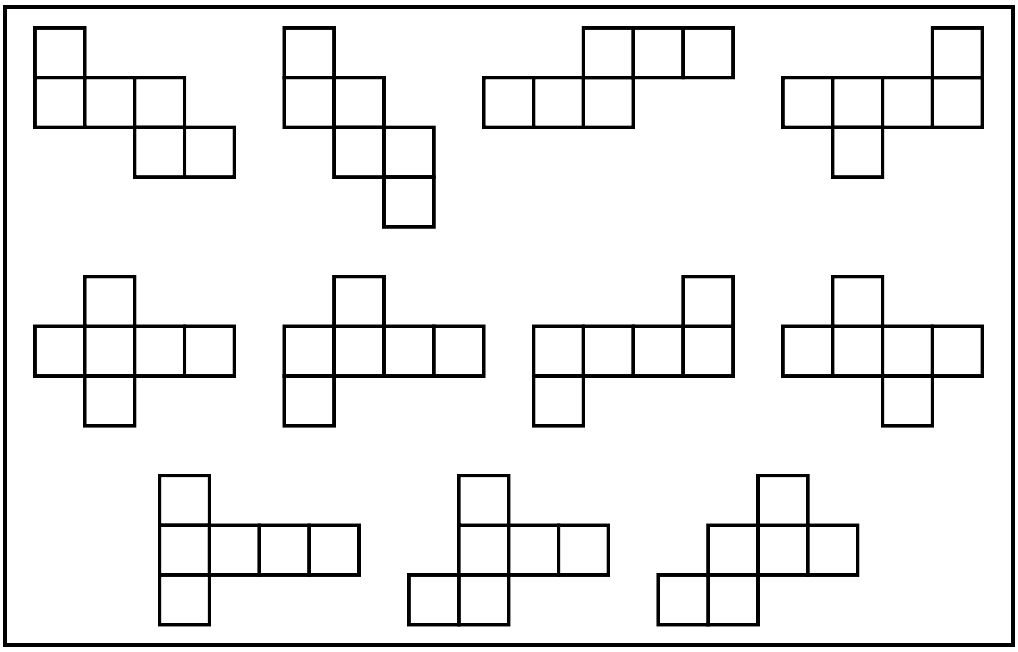
- What happens if all the children can row?
- Search River Crossing problems on the web and try other variations. These problems have a long history. Variations have been traced back at least as far as the 8th century. This thought could be the basis of a project.

The solution is not recorded here because you might like to tackle the task yourself as you plan. However it can be found in the Cameo for Task 106 at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm

Cube Nets

Students use the *3d Geoshapes* squares to make a cube, and then unfold them to form all 11 possible nets. They need to keep careful record of the solutions they have found and check that the solutions they have found are all genuinely different - that is, that they cannot be matched using rotations, reflections or flips. The solutions are given below.



Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Diamonds & Rectangles

This task requires sophisticated thinking. As usual it is the struggle of finding the answer for yourself that is more valuable than the answer. The corollary of this is that there is little value in the task to someone who is told the answer by another. The task does not have to be solved today. It may need to be revisited several times before it is solved.

Engaging in the struggle is also important for readers of this manual, so *read no further* unless you have already solved the problem.

The crucial steps in the solution are:
- Choose from the Mixed bag first. Whatever you choose tells you what is in that bag, because the wrong labelling implies it is not a mixture. Therefore the only possibility is a bag of what you bring out.
- There are two bags remaining. One is incorrectly labelled with the name of the shape you just discovered. Forget this bag for a minute and consider the other one.
- This other bag can't be what it is labelled; and it can't be what you just discovered in the Mixed bag. So it must be the only remaining choice.
- Now you know what two bags must be so you also know the third.

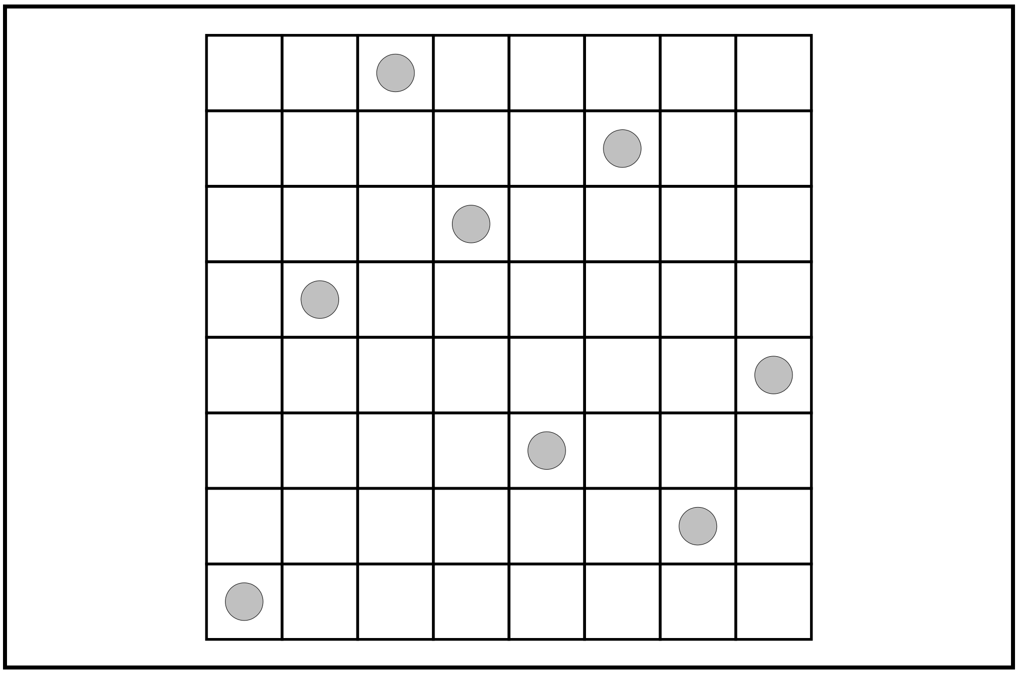
Try the steps a few times for yourself. They will become clear.

Wrongly labelling *all three* bags leads to a unique solution. Would there be a unique solution if two bags were incorrectly labelled? If not, can the students explain the dilemmas that result? Can the students explain the consequences of being asked to explore one incorrectly labelled bag?

Find more information about this task in the Task Cameo Library at:
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Eight Queens

There are twelve solutions to this puzzle, and, because of rotational symmetry, each can be represented four ways. One solution is:



Turning the page through four quarter turns shows the other ways this solution could appear.

Finding just one solution can be challenging and will engage the students in strategy considerations such as:

- Avoid the centre because it can be 'seen' from too many directions.
- Go for the corners because they can only be 'seen' from three directions.
- Start with a full row then shuffle.

You might start an **8 Queens** display corner and display solutions as they are found. Allowing students to claim a solution by naming it with their initials can increase the interest in searching for solutions.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Football Ladder

It doesn't matter whether students have an interest or not in this code of football. The puzzle is based in the language and logic of the clues and is not dependent on knowledge of the code. The solution (from the top) is:

> Hawthorn, Fremantle, Essendon, Carlton, Brisbane, Sydney, Adelaide, Melbourne, Collingwood, Geelong, Richmond, Kangaroos, West Coast, St. Kilda, Port Adelaide, Bulldogs.

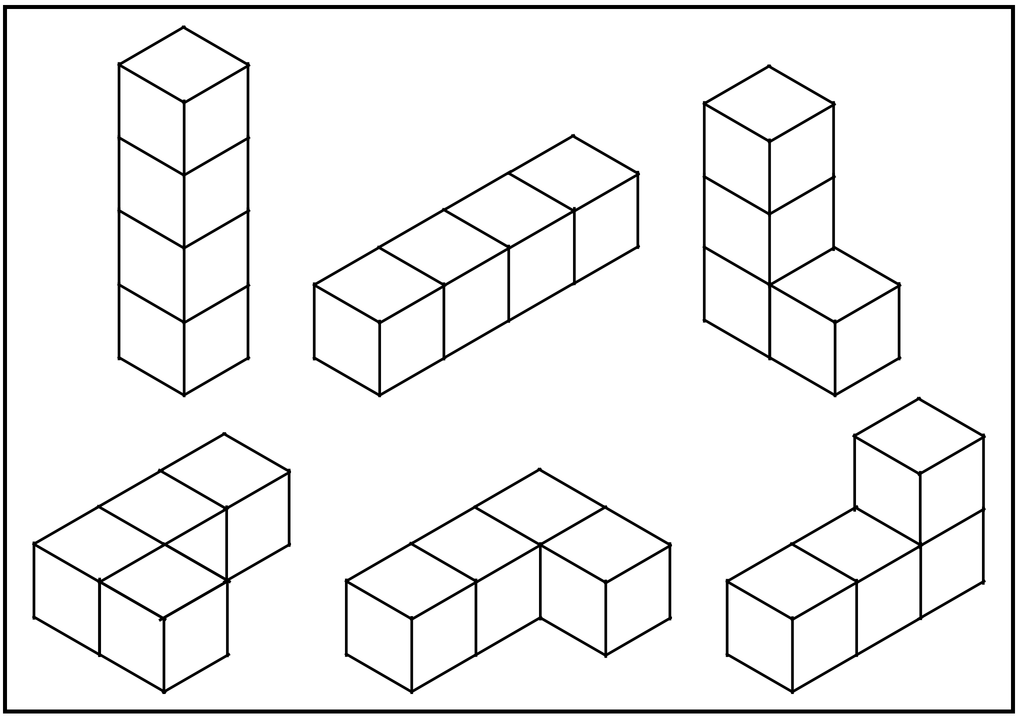
Once solved, an obvious extension is to create and trial a similar puzzle based on the student's chosen sport.

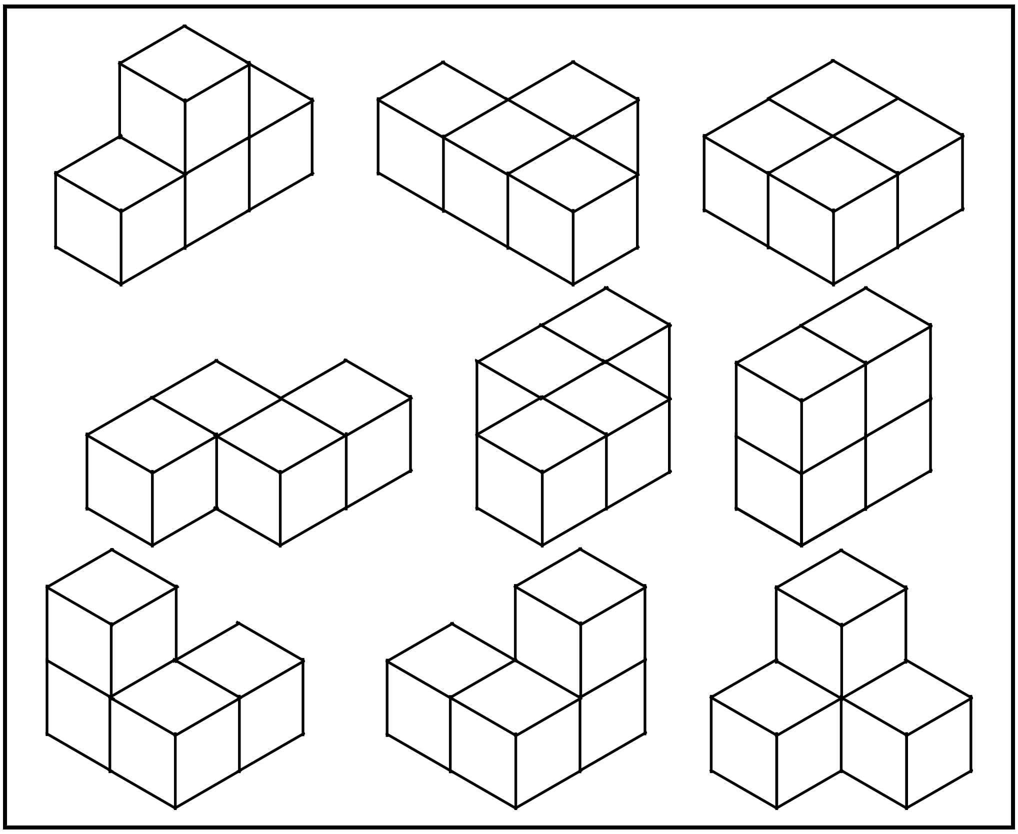
Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Four Cube Houses

In this task students have to find and draw isometrically all the possible houses made with four cubes. Faces must match completely. Houses are said to be the same if they may be rotated (on the block of land) to look alike. The isometric paper used as the recording sheet can be found at the end of this manual.





There are fifteen houses in all:

- 2 with four cubes in a row horizontally
- 6 with three in a row horizontally
- 7 with two in a row horizontally

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Koala Carts

Koala Carts is a concrete way to explore sorting and classifying. When a road is labelled with a particular card, all the koalas with that attribute must travel on the labelled fork (at least until the next forward junction). Also, all the ones that do not have that characteristic must travel the other fork. So the koalas that collect at the top road in Question 1 are blue - it doesn't matter what size they are. In Question 2, with the label card on the other fork, the koalas on the top road are green, red and yellow, ie: NOT blue.

In Question 3, blue koalas travel the full length of the top road, but at the next fork they are joined by any small koalas that have been sent up from the bottom road. Therefore, the koalas that collect at the end of the top row are either blue OR small.

How could road segments and cards be arranged so that the koalas collected at the top road were blue AND small? How many such koalas are there in the set?

As the students move on with the task and begin to create their own roads and sorting conditions, encourage them to use only the logical conjunctives AND, OR and NOT to uniquely describe the koalas collected at the end of the top road.

If you have other materials with a range of attributes, eg: Attribute (or Logic) Blocks, you can make extra cards and use the same road pieces to extend the complexity of the investigation.

Another variation is to consider delivering objects other than koalas and changing the condition cards appropriately. For example the cart might be used to deliver quadrilaterals with the condition cards changed to statements like:

- opposite sides parallel
- opposite sides equal
- opposite sides right angles

Another example might be to deliver numbers in the cart with conditions like:

- divisible by three
- greater than 100
- between 1 and 2

Making Triangles

In this task the only thing that matters is the colours in the triangle. The order of the colours doesn't matter. For example R-**B**-Y is the same as Y-**B**-R. In this description, the bold colour is thought of as the side placed horizontally in front of you on the table as a base.

This is another task that uses criteria to sort and classify. Perhaps the students will use the criteria of the types of triangles as suggested by the card. Perhaps they will focus on the number of each colour and come back to the triangle types later.

Some students begin quite randomly and it is heartening to see them reconsider this approach and try a more systematic strategy. Mathematical conversation is a key feature of the task and one of the high points for teachers is when students realise that some triangles belong to more than one category. For example all equilateral triangles are also isosceles.

The students are asked to draw each triangle, but the purpose of this is to reinforce that a mathematician records notes and diagrams as they investigate a problem. So, if they choose to use a code rather than lines and colours that is fine.

One way to approach an organised search is to start with the smallest rod, Red, and make all the possible 3-side, 2-side and 1-side red triangles, then to repeat this approach for Yellow, Green and Blue, being careful to avoid repeats.

Red as Base

- 3 sides: R-**R**-R
- 2 sides: R-**R**-Y ... R-**R**-G ... R-**R**-B
- 1 side: Y-**R**-G ... Y-**R**-B ... G-**R**-B (other 2-side situations occur below)

Yellow as Base

- 3 sides: Y-**Y**-Y
- 2 sides: Y-**Y**-R ... Y-**Y**-G ... Y-**Y**-B
- 1 side: G-**Y**-B (other possibilities have been made)

Green as Base

- 3 sides: G-**G**-G
- 2 sides: G-**G**-R ... G-**G**-Y ... G-**G**-B
- 1 side: All possibilities have been made.

Blue as Base
- 3 sides: B-**B**-B
- 2 sides: B-**B**-R ... B-**B**-Y ... B-**B**-G
- 1 side: All possibilities have been made.

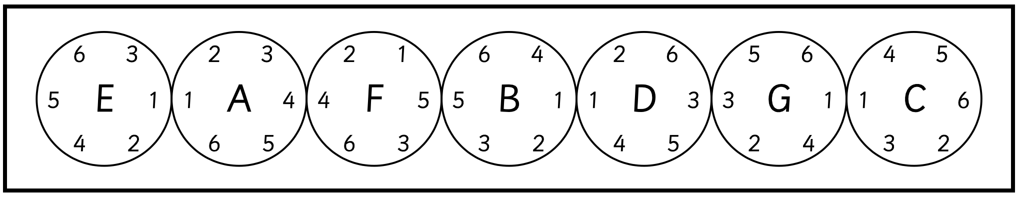
There are 20 different triangles in all. Four are Equilateral, 16 are Isosceles (which includes the 4 equilateral) and 4 are scalene. One of the scalene ones (G-**R**-B) is also Right Angled.

Extensions to the task could include:
- Exploring the possibilities if using a fifth colour.
- Exploring the possibilities of making quadrilaterals.
- Investigating how you know without making the shape whether three sticks will make a triangle.

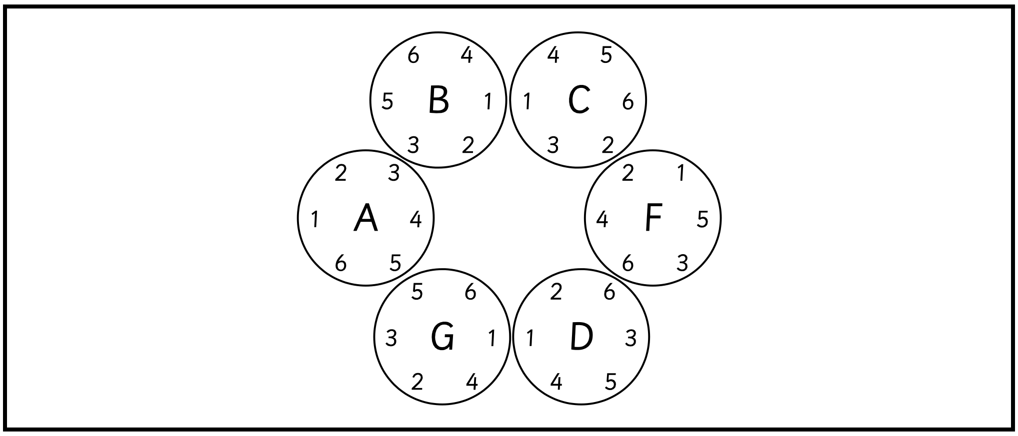
Find more information about this task in the Task Cameo Library at:
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Number Discs

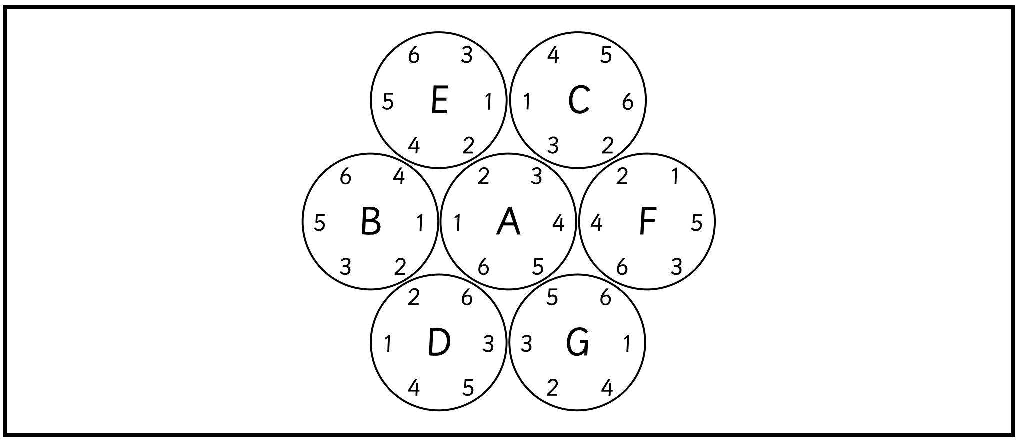
One solution for Question 1 is:



One solution for Question 2 is:



One solution for Question 3 is:



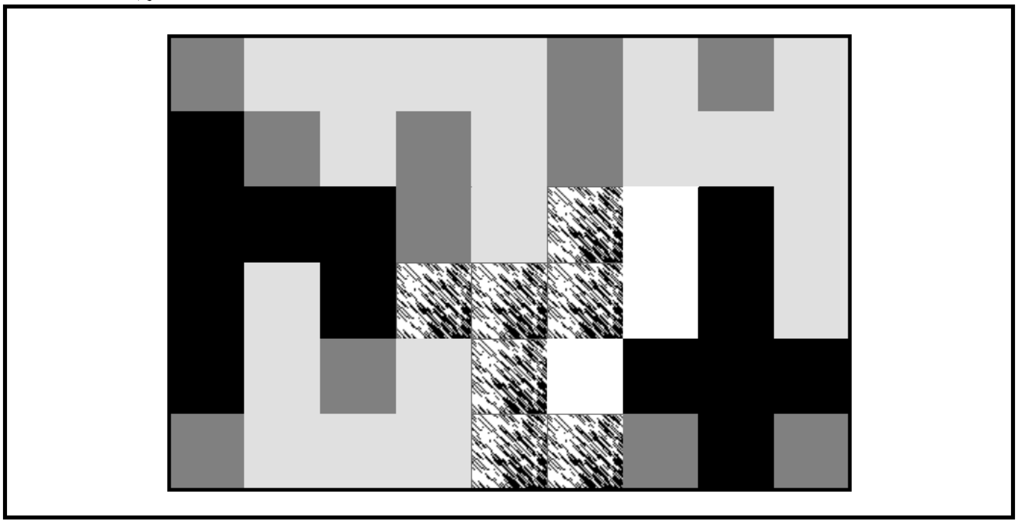
However, if A is taken from the centre the remaining discs are a second solution for Question 2. Are there other solutions for Question 2? Perhaps any solutions the students find can be displayed in the Maths Corner.

Octaflex

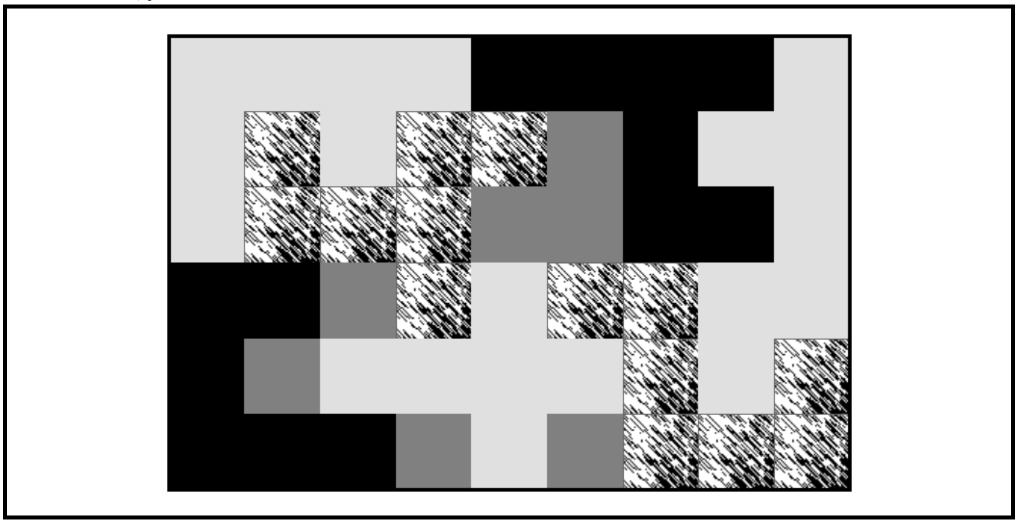
A mathematician frequently visualises a solution to a problem before being able to solve it. **Octaflex** is a puzzle that contributes to the development of student's spatial perception, and therefore to their developing ability to work like a mathematician. The first challenge on the card is reasonably straightforward. The second is harder and may take several attempts over time to solve.

Can the students make up similar puzzles for their classmates?

Solution: Question 1



Solution: Question 3



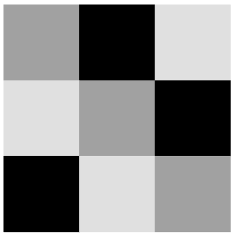
Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Pattern Cube

Pattern Cube grows out of a task called **Latin Squares**, which is two dimensional. It is included in the *Years 5 & 6 Pattern & Algebra* kit. A Latin Square is a square made from the same number of colours as the number of units in the base *and* has no colour the same in any row or column. For example:

3x3 Latin Square



To create the Pattern Cube, begin with a Latin Square in the base. Notice that each row is formed from the previous one by a cyclic movement of colours. The top row is:

Red, Black, Yellow

The next row is formed by taking the Yellow from the right end, putting it on left end and 'pushing' the other two blocks to the right to make:

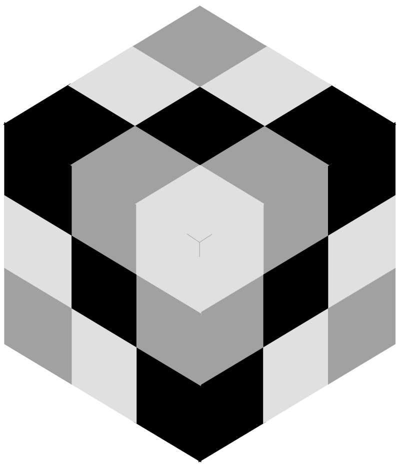
Yellow, Red, Black

Repeating this process with the Black from the right end makes the bottom row:

Black, Yellow Red

To make the cube, build a Latin Square upward from each row. The three Latin Square 'walls' you make will become a pattern cube.

One solution is:



There are variations on this solution depending on how the cube is rotated in 2 or 3 dimensions, however, these are essentially the same solution. To highlight this, students could be asked to sit opposite each other with a completed cube between them. They then draw and shade the cube on the isometric paper supplied at the end of the manual. They are looking at the same solution, but their diagrams will appear to be different.

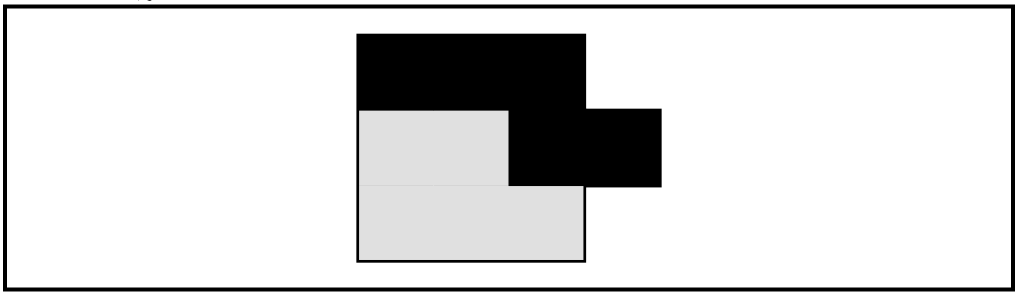
So, are there other unique solutions for **Pattern Cube**? Perhaps the students could begin to answer this, as a mathematician might, by breaking the problem into smaller parts and asking if there are other unique Latin Squares that could be used in the base.

An additional investigation is to start with a 4x4 Latin Square and try to make a 4x4 pattern cube from it.

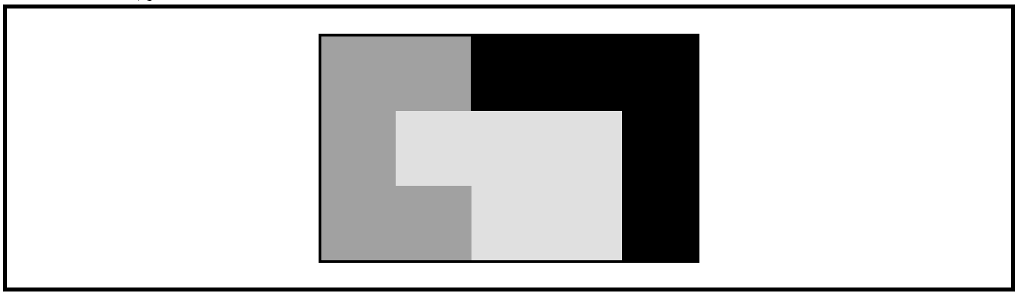
Pentominoes

This is a task with a long history as a mathematical recreation. The three challenges on the card are only a few of the seemingly endless number that have been created over centuries. Even the additional puzzles suggested as extensions below still only represent the tip of the iceberg.

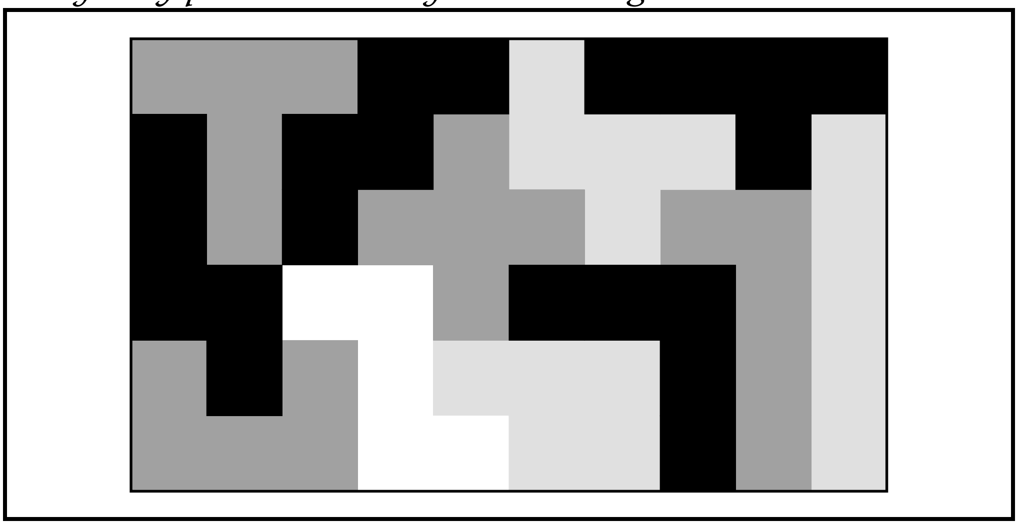
Solution: Question 1



Solution: Question 2



One of many possible solutions for the Challenge:



There is no need for the Challenge to be solved at the first attempt. A mathematician takes as much time as necessary to solve a problem. Confirm for the students that it is okay to put the problem away and try it again another time.

Additional challenges:

- Question 3 can become a game. Players take turns to select from the twelve pieces until they have six each. Then they take turns to place the pieces on the loungeroom floor one at a time. The first player unable to place a piece loses.
- Each of the twelve pentominoes is 5 square units. Their total area is 60 square units. The loungeroom floor is 60 square units as a 10 x 6 board. Other factors of 60 are 4 x 15 and 5 x 12. Students could make these boards and try to place all the pieces on them.
- Can the pieces be placed on an 8 x 8 board so that there is either a 2 x 2 gap in the centre or an uncovered square in each corner?
- One piece is a + shape. Nine of the other pieces can be put together to make a + which has each length three times longer than the single piece +.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Reflections

The objective of the task is to provide further experience with the properties of reflection. In particular the students are likely to develop, or confirm, the idea that each point and its reflection are an equal distance from the mirror.

Extensions include:

- Asking the students to make the initial of their name and its reflection.
- Shifting the task to graph paper to encourage further experiment with reflection - perhaps with two mirror lines.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Sliding Tiles

This task is a simplified version of a puzzle that has been very popular over the years, namely 15 tiles that can slide within a 4x4 frame. The user is challenged to slide the tiles into certain configurations and is also given 'impossible' challenges. Using a 3 x 2 frame with 5 tiles, this simplified version explores the mathematics behind the challenge and shows why some arrangements are 'impossible'.

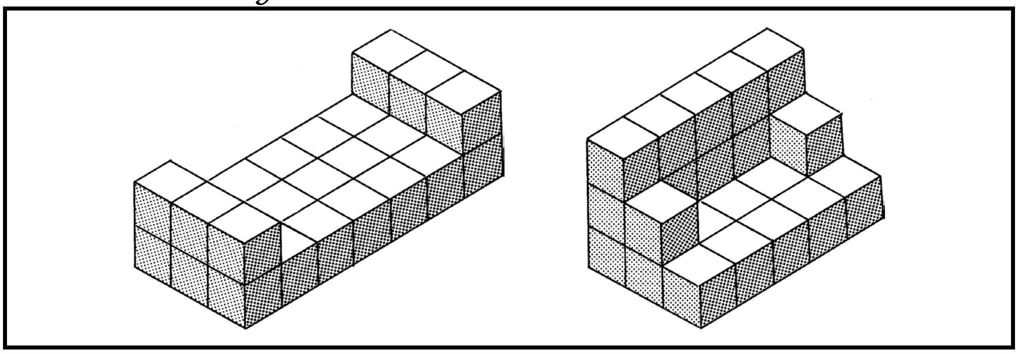
First, reduce the 6 piece puzzle (5 tiles and a blank) to 4 (by covering up two positions) and only using the tiles numbered 1 to 3. Students can discover there are only 24 configurations (or arrangements). Once the tiles are randomly placed in any positions to start, only 12 of the combinations can be made - the other 12 are 'impossible'. Indeed the combinations turn into two sets of 12. The 12 of the 'opposite' set are like reflections of each combination of the first 12.

Similarly with the 6 pieces of the task (5 & blank). It is a worthwhile challenge to discover that there are 720 (6!) arrangements of the tiles. These form into two sets of 360. Each arrangement within the 360 is like a 'rotation' of the others, and the 360 'impossible' arrangements are reflections of the first set of 360. So, if the tiles are placed down at random then there is a 50:50 chance that they can be moved into any given arrangement.

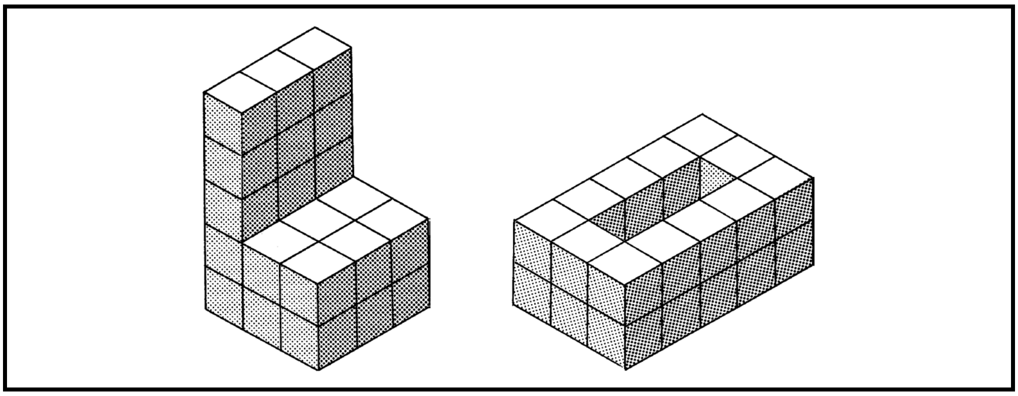
Soma Cube 2

Students are usually attracted by the challenge of making the cube from the pieces, and may then get quite competitive about the speed with which they can put it together. In the process the activity is supporting the development of the student's spatial sense, a very useful attribute for a mathematician. However, one of the reasons that the original puzzle has become so popular in the community at various times since its invention is the myriad of different shapes that can be made by connecting the pieces in various ways. Some of these shapes are quite 'geometric', for example rectangles and cuboids, and others are quite 'symmetric', eg:

The Bed and the Sofa



The Chair and the Bath



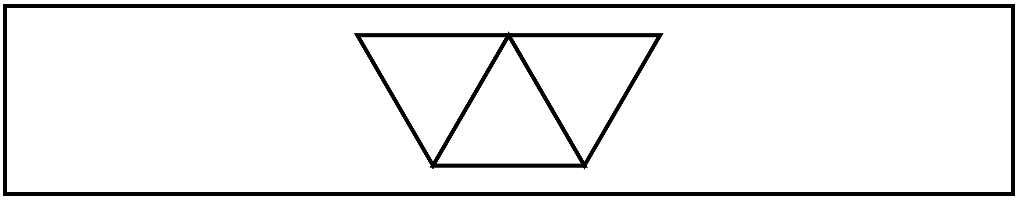
There is considerable spatial and numerical reasoning involved in trying to make a given shape. There are many clues about lengths and volumes given in the pictures which leads to selection of the possible pieces to fit in a position.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

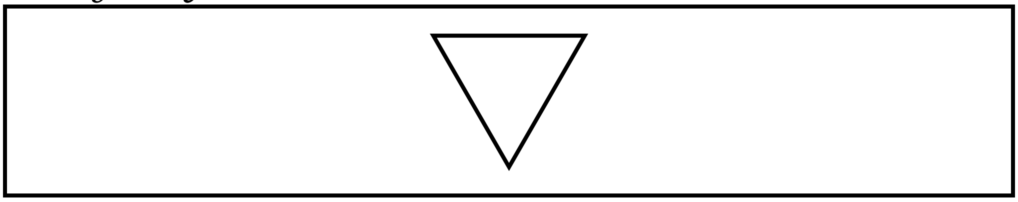
The Farmer's Puzzles

Students will soon discover that making square pens won't work. The geometric property that gets in the way is that the diagonal of square is longer than its side, so the sticks can't be placed end to end within the square to make triangular pens. However, triangular pens turns out to be the solution when they are arranged like this:

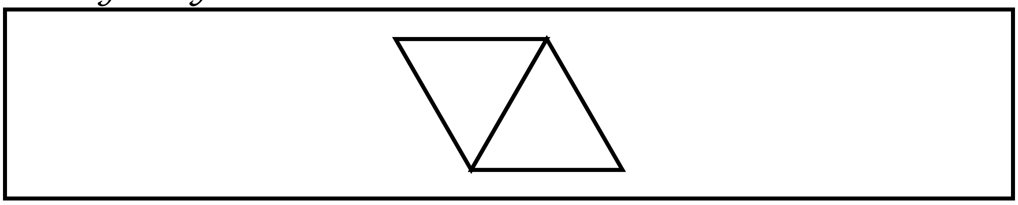


The solution suggests a way of making pens that leads to a pattern. Generalising the pattern leads to algebra.

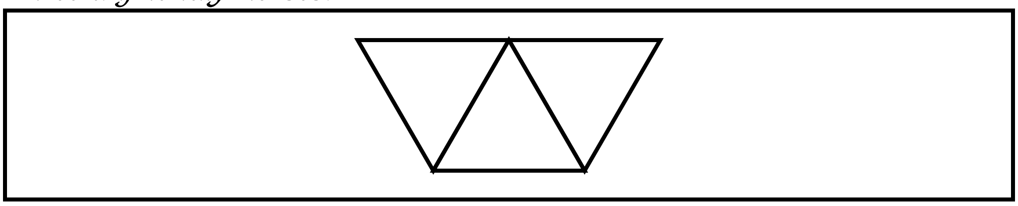
One unfriendly horse:



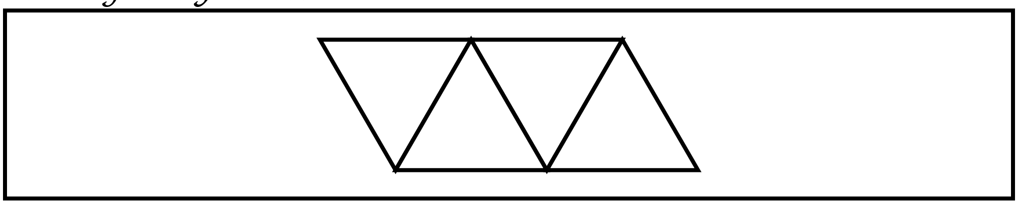
Two unfriendly horses:



Three unfriendly horses:



Four unfriendly horses:

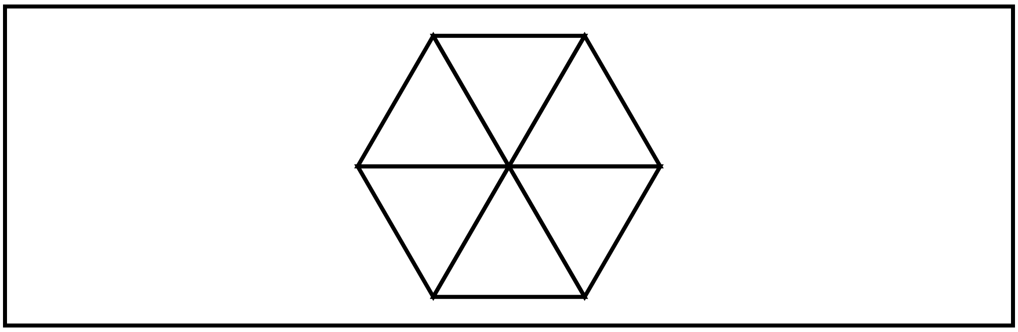


- If I told you any number of unfriendly horses could you tell me how many lengths of fence?
- If I told you how many lengths of fence, could you tell me how many unfriendly horses I could pen?
- The horses and fence pieces are pairs of numbers that belong together because of the structure of the problem, ie: (1, 3), (2, 5), (3, 7), (4, 9)… If these pairs were plotted on a graph, what would you expect to see?
- Would it make sense (in the context of the problem) to join the dots?

If you wish to follow this path there are other questions that could be asked to extend the algebra that develops from this spatial task.

Question 1 of the task also suggests the solution to Question 2.

Two of the trapeziums make a hexagon like this:



All lengths are equal and the pigs can wallow in equal areas.

Extensions suggested by this solution include:

- Many classrooms have trapezoidal tables. The designers chose these because they can be rearranged in many ways. Encourage students to explore (with actual tables or on triangle paper) all the arrangements that can be made with 2, 3, 4, 5, 6 trapezoidal tables.
- The solution is made from six equilateral triangles. This is one example of a hexiamond. Hexiamonds are made from six equilateral triangles arranged so sides (not vertices) match. How many other hexiamonds can the students find?

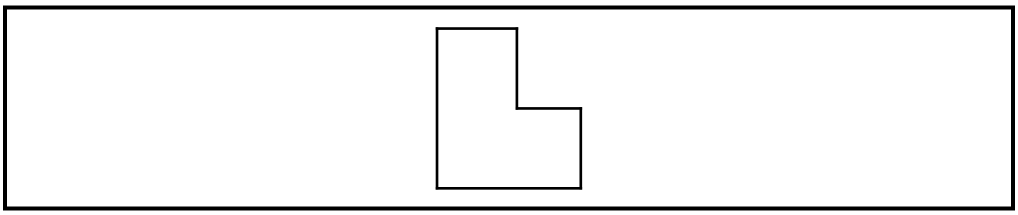
Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Tricube Constructions A

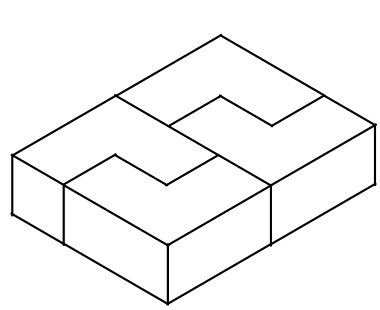
Plans are given for buildings made from tricubes. Students have to construct the buildings and draw them on isometric paper. After this they use the tricubes to create their own puzzles for each other drawing the plans and isometric views, and challenging a friend to recreate the building using only the plan view.

Answers:
Note that the comment on the card that drawings (d) and (e) are two layers high means that they are two layers high in some (which might include all) parts. This is quite justified use of language since if a building looked like this if you were standing on the street:

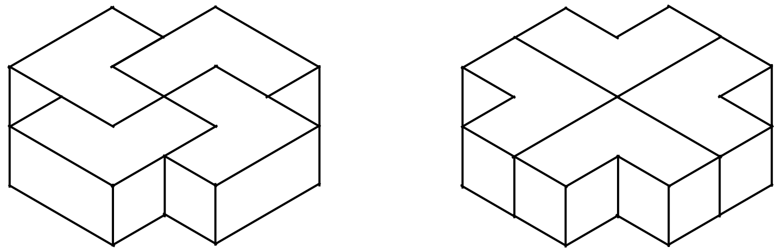


then you would say the building was two storeys high.

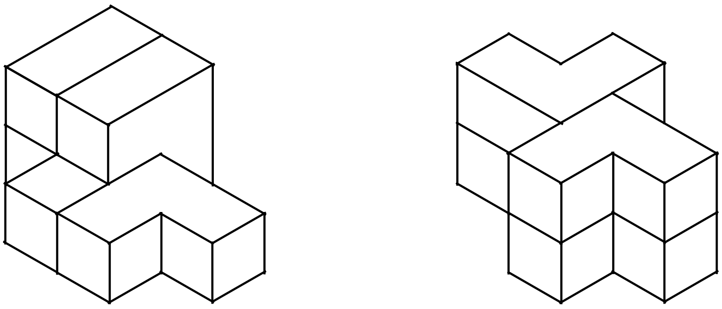
Solutions A & B



Two solutions for C



Solutions D & E



Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Which View?

Students are given top, front and left side views of a solid shape, and are asked to construct the solid using wooden cubes. They also build their own solid from cubes, and draw the three views of it. The answers below assume that there is at least one cube in each position. Even then there are four possible answers for Question 2.

Question 1

3	1	1
2	1	1
1	1	1

Question 2

1	1	2	1
1	1	2	2
1	1	1	3

1	1	1	2
1	1	2	1
1	1	2	3

1	1	2	2
1	1	2	1
1	1	1	3

1	1	2	2
1	1	1	2
1	1	1	3

Note: If the activities involving 2D representations of 3D objects has excited your students, you might consider using Mixed Media Unit 1: *Points of View*. For more information, find Points of View in the Mathematics Centre Resources page:

- http://www.mathematicscentre.com/taskcentre/resource.htm

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Who Owns The Monkey?

The Malaysians do - but the challenge is in convincing yourself that this is the only conclusion the clues allow. The challenge is not easy and it is important that the students have an opportunity to struggle with it. It is not necessary that the problem is solved in one sitting.

Once it is solved, the next question is *How did the designer come up with the problem?*. The most likely answer is that (s)he came up with the story line, set out the cards and then invented the clues. This realisation suggests the possibility of the students making up a similar problem of their own. Story shells might be:

- Who Owns The Footy Shorts?
- Who's Hobby Is Camping?
- Who Gets The Dessert?

Designing the puzzle, inventing the clues and validating that users find it interesting and challenging is a major project that can simultaneously address mathematics, language and technology outcomes.

Lesson Comments

- These comments introduce you to each Maths300 lesson. The complete plan is easily accessed through the lesson library available to members at:
  http://www.maths300.com
  where they are listed alphabetically by lesson name.

Cube Nets

Almost all teachers have asked students to construct a cube from its net; usually with all students using the same T-shaped net. That is a great lesson for the measurement toolbox, but this *Cube Nets* lesson investigates *all* the different nets of a cube and illustrates so much more of the Working Mathematically process.

However, the lesson *is* equipment specific. It is one case where only this equipment makes the lesson possible. So, if you don't have access to plenty of 3d Geoshape squares then you can't use this lesson.

With the manipulative available, the data becomes visual - an opportunity for students to see that data doesn't necessarily mean numbers - and the students see ways of classifying the various nets. Using this mathematician's skill of sorting and classifying, the students are able to attack the deeper questions:

- *How many nets of a cube are there?*
- *How do we know when we have found them all?*

At many stages in the lesson, the teacher is able to draw a parallel with the way a professional mathematician works. This puts the teacher in the position of being able to congratulate and celebrate a student's efforts as being consistent with the way a mathematician works. Students learn that they *can* work mathematically. There are several extensions to the problem, each of which is a comprehensive investigation in itself.

Football Ladder

The context of the well known Australian Football League (AFL) provides the setting for this language and logic puzzle. A major attraction is the provision of concrete materials (cards) which turns the hands-on task from the kit into a whole class investigation accessible to, and popular with, almost all students. You will need to prepare these cards in advance.

Content outcomes are numerical order, awareness of a range of problem solving strategies and correct use of spatial language. Another feature is a kinaesthetic option where the class members 'act out' the ladder positions. The third puzzle also offers arithmetic skill practice related to the way points are assigned for a win, loss or draw. For some students, the clue set provided for Years 1 - 3 may be more appropriate as a beginning.

Although the particular sport may not be relevant to your classroom culture, after using the Lesson Plan as it is, the students can be challenged to prepare similar problems based on local sporting codes. Alternatively, the plan provides you with the model to prepare a relevant puzzle of your own.

Four Cube Houses

Architectural teams (students working in groups) are challenged to design as many houses as possible using four cubic modules. The resulting designs have to be drawn up, costed and justified as part of a housing development. Embedded in the story shell, this lesson is a lot of fun. However finding the complete set of houses such that:

- the modules all touch face to face
- there are no 'cantilevered' structures
- designs cannot be transformed into each other by a simple rotation around a vertical axis

is quite a spatial challenge.

For some students it may be appropriate to solve the simpler problem of 2-cube or 3-cube houses.

Costing the designs adds further interest and results in the lesson not only achieving objectives in the Space and Measurement Strands, but also in the Number strand.

The Classroom Contributions in this lesson suggest a wealth of extension work. One class has donated the web site it created to promote their new housing estate made of four cube houses. In another class the challenge was to use sufficient cubes, and the costings in the 'architectural brief', to create a million dollar house.

The lesson works well as a follow up to *Cube Nets*. Teachers may want to lead from *Cube Nets* into creating cardboard cubes from nets, then using those for *Four Cube Houses*. This will create an extensive space/geometry unit.

Land of ET

In the Land of ET the words were too long and they had too many letters in the alphabet. So the King and Queen decreed henceforth there would be only two letters - B and Y.

Then they were upset that some of the words seemed to be getting even longer, eg: BBBBBYYBYBYBYBBYY, so they passed some rules for shortening the words.

- Rule 1: BBB next to each other can be added or removed.
- Rule 2: YY next to each other can be added or removed.
- Rule 3: BYB can be replaced by Y or vice versa.

Problem: After shortening, how many words are left in the language of ET?

On the surface this lesson looks quite 'different'. On one level it appears as a fantasy language puzzle, however the structure underlying the puzzle goes to the heart of number theory. Teachers who have Poly Plug as a resource in their room will be able to build concrete models of the ET words. (Visit the Poly Plug link on the Mathematics Centre site.)

ET might be thought of as 'extra-terrestrial' in terms of the fantasy - however it is actually named for the identity between the structure of the puzzle and the symmetry properties of an Equilateral Triangle (ET). So this is a logic puzzle with a link to transformations of the ET when rotated and flipped. At an even deeper level the table of combined transformations reminds us of charts showing operations on numbers.

Nim

This is one version of a famous two person strategy game. Counters are arranged in 4 rows with 1, 3, 5 & 7 counters in each row. Players take turns to remove one or more counters from one row. The player who has to remove the last counter loses the game. So, on any turn, any number of counters may be taken (including the whole row), but they may only be taken *from one row*.

The purpose of the task is to encourage problem solving strategies:

- looking ahead
- reasoning ('what-if')
- working backwards

It is almost essential for students to use counters; at least at first. They somehow make the problem 'real'. The lesson may be guided by the investigation sheet supplied and there are a number of extensions suggested.

Red To Blue

Four students stand facing the class. The class instructs all except one (ie: any three at a time) to turn around 180 degrees. Using this rule, is it possible to make all four turn their backs to the class? A challenge that is simple to express, easy to begin and rich with possibilities for a wide range of abilities and ages.

The kinaesthetic whole class introduction is followed by checking the logic with concrete materials at a personal/small group level. This sets the problem up for later exploration of other cases such as *What if there were five or six or seven or ... students?*. However, before that, the problem is an excellent springboard for examining the number of different strategies a mathematician could use to try to solve this problem. The thrust of the lesson at this stage is therefore to highlight the value of the Strategy Toolbox.

Exploring the *What if...?* cases leads to a generalisation about all cases that have the rule of turning all except one each time, ie: the turning (n - 1) case.

Of course this answer only opens the door to another series of questions, ie: *What happens if the rule is turn (n - 2) or (n - 3) or ...?*.

Teachers will take this lesson as far as is appropriate for this age and the students' interest. There is no need to do it 'all' now. A mathematician might take many days, weeks or even years to solve a problem.

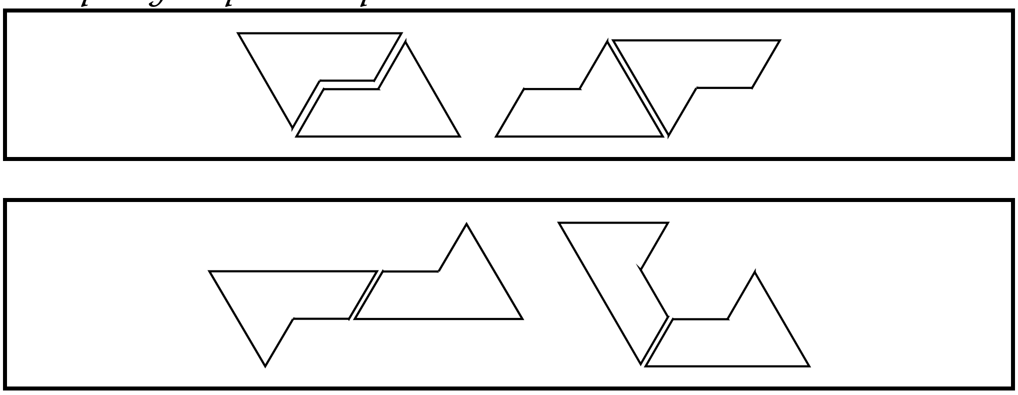
Sphinx

This lesson begins with an apparently simple four piece jigsaw puzzle. The same shape is used four times to make a larger version of that shape. The shape is called a Sphinx - probably because of its similarity to the profile of the ancient Egyptian monument. There are almost endless directions that this shape has been taken by teachers. The lesson as written shows how the Sphinx continues to grow spatially and links that growth with a number pattern. At this level children find plenty of interest in the spatial aspects of the growth. The investigation related to the numerical form of the growth will be returned to in later years.

From the spatial point of view, teachers have used *Sphinx* in these ways:

- Build the Sphinx growth to the largest size that time, card and student interest will allow.
- Lead this development into an exploration of tiling patterns, also called tessellations, which involves covering a flat surface with a repeated pattern of the same shape. This may even extend to covering a surface with a repeated pattern of two shapes, such as octagons partnered with squares. One practical application of this idea is in decorating bathroom floors. Another is in the intricate designs displayed in many Moslem mosques.
- Use a tessellation of Sphinxes to ask about the angle measures of the Sphinxes joining at any point, given the angle sum around a point is 360°.
- Consider the not-Sphinxes which the students create as they work towards a solution of the initial puzzle as equally valid sources of interest. One web link listed in the extensions at the end of the Lesson Plan leads to a lesson that uses this idea to explore perimeter and area.
- Use multiple Sphinx shapes to make and record shapes in the same way as Tangrams are used. This involves a good deal of mathematical language related to shape and exploration of the properties of shapes. Sphinx Paper is provided at the end of this manual to assist with recording these creations. The first recording challenge is to make the smallest possible drawing of one Sphinx by joining the dots. This sets the scale for future drawings and saves paper.
- Noticing that the pattern in the Lesson Plan lists Sphinxes of Size 1, 2, 4, 8... and asking about the possibility of creating Size 3 or 5 or 6 and so on.
- Using just two Sphinxes to make new shapes and discussing mathematics suggested by these shapes, in particular reflective and rotational symmetry. Another approach to using two Sphinxes to make new shapes is to ask: *How many 2-Sphinx shapes are there and how will we know when we have found them all?*.

Examples of 2-Sphinx shapes



Where Do We Sit?

As written, the Lesson Plan reads as if for an infant class, but teachers will have no problem adjusting the story shell to work for this age group. For example simply consider three students starting the year in a new class and sitting in the three front seats. Perhaps they even plan to sit in a different arrangement each day to confuse the new teacher.

- *How many days can they sit in a different arrangement as seen from the teacher's desk?*

The lesson involves clever counting to answer the mathematician's questions:

- *How many solutions are there?*
- *How will we know when we have found them all?*

The clever counting is explained in the lesson and in the Content Epilogue link. Formally this is the mathematics of selections and arrangements, which is also called permutations and combinations. At this level, some classes will add the Tree Diagram tool discussed in the lesson to their skill toolbox.

There are many extensions listed in the Lesson Plan and they also remind us of another question mathematician's ask to help them learn more, ie: *What if…?*.

Part 3:
Value
Adding

The Poster Problem Clinic

Maths With Attitude kits offer several models for building a Working Mathematically curriculum around tasks. Each kit uses a different model, so across the range of 16 kits, teachers' professional learning continues and students experience variety. The Poster Problem Clinic is an additional model. It can be used to lead students into working with tasks, or it can be used in a briefer form as a opening component of each task session.

> *I was apprehensive about using tasks when it seemed such a different way of working. I felt my children had little or no experience of problem solving and I wanted to prepare them to think more deeply. The Clinic proved a perfect way in.*

Careful thought needs to be given to management in such lessons. One approach to getting the class started on the tasks and giving it a sense of direction and purpose is to start with a whole class problem. Usually this is displayed on a poster that all can see, perhaps in a Maths Corner. Another approach is to print a copy for each person. A Poster Problem Clinic fosters class discussion and thought about problem solving strategies.

Starting the lesson this way also means that just prior to liberating the students into the task session, they are all together to allow the teacher to make any short, general observations about classroom organisation, or to celebrate any problem solving ideas that have arisen.

One teacher describes the session like this:

> *I like starting with a class problem - for just a few minutes - it focuses the class attention, and often allows me to introduce a particular strategy that is new or needs emphasis.*

It only takes a short time to introduce a poster and get some initial ideas going. The class discussion develops a way of thinking. It allows class members to hear, and learn from their peers, about problem solving strategies that work for them.

> *If we don't collectively solve the problem in 5 minutes, I will leave the problem 'hanging' and it gives a purpose to the class review session at the end.*
> *Sometimes I require everyone to work out and write down their solution to the whole class problem. The staggered finishing time for this allows me to get organised and help students get started on tasks without being besieged.*
> *I try to never interrupt the task session, but all pupils know we have a five minute review session at the end to allow them to comment on such things as an activity they particularly liked. We often close then with an agreed answer to our whole class problem.*

A Clinic in Action

The aims of the regular clinic are:

- to provide children with the opportunity to learn a variety of strategies
- to familiarise children with a process for solving problems.

The following example illustrates a structure which many teachers have found successful when running a clinic.

Preparation

For each session teachers need:

- a Strategy Board as below
- a How To Solve A Problem chart as below
- to choose a suitable problem and prepare it as a poster
- to organise children into groups of two or three.

The Strategy Board can be prepared in advance as a reference for the children, or may be developed *with* the children as they explore problem solving and suggest their own versions of the strategies.

The problem can be chosen from

- a book
- the task collection
- prepared collections such as Professor Morris Puzzles which can be viewed at: http://www.mathematicscentre.com/taskcentre/resource.htm#profmorr

The example which follows is from the task collection. The teacher copied it onto a large sheet of paper and asked some children to illustrate it. *The teacher also changed the number of sheep to sixty* to make the poster a little different from the one in the task collection.

The Strategy Board and the How To Solve A Problem chart can be used in any maths activity and are frequently referred to in Maths300 lessons.

The Clinic

The poster used for this example session is:

> Eric the Sheep is lining up to be shorn before the hot summer ahead. There are sixty [60] sheep in front of him. Eric can't be bothered waiting in the queue properly, so he decides to sneak towards the front.
>
> Every time one [1] sheep is taken to be shorn, Eric <u>then</u> sneaks past two [2] sheep. How many sheep will be shorn <u>before</u> Eric?

This Poster Problem Clinic approach is also extensively explored in Maths300 Lesson 14, *The Farmer's Puzzle*.

Strategy Board

DO I KNOW A SIMILAR PROBLEM?	ACT IT OUT
GUESS, CHECK AND IMPROVE	DRAW A PICTURE OR GRAPH
TRY A SIMPLER PROBLEM	MAKE A MODEL
WRITE AN EQUATION	LOOK FOR A PATTERN
MAKE A LIST OR TABLE	TRY ALL POSSIBILITIES
WORK BACKWARDS	SEEK AN EXCEPTION
BREAK INTO SMALLER PARTS	...

How To Solve A Problem

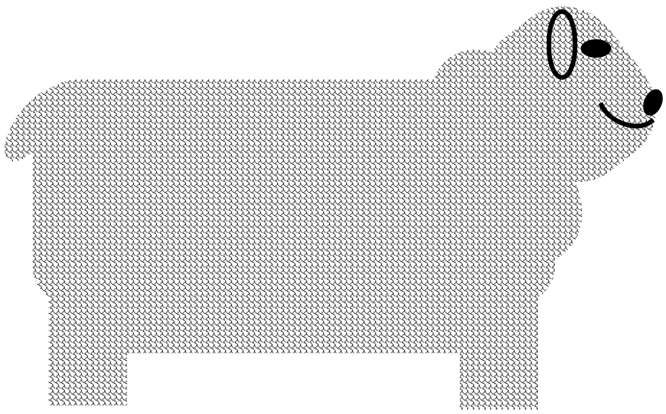
SEE & UNDERSTAND	Do I understand what the problem is asking? Discuss
PLANNING	Select a strategy from the board. Plan how you intend solving the problem.
DOING IT	Try out your idea.
CHECK IT	Did it work out? If so reflect on the activity. If not, go back to step one.

Step 1

- Tell the children that we are at Stage 1 of our four stage plan ... **See & Understand** ... Point to it! Read the problem with the class. Discuss the problem and clarify any misunderstandings.
- If children do not clearly understand what the problem is asking, they will not cope with the next stage. A good way of finding out if a child understands a problem is for her/him to retell it.
- Allow time for questions - approximately 3 to 5 minutes.

Step 2

- Tell the children that we are at Stage 2 of our four stage plan ... **Planning**. In their groups children select one or more strategies from the Strategy Board and discuss/organise how to go about solving the problem.
- Without guidance, children will often skip this step and go straight to Doing It. It is vital to emphasise that this stage is simply planning, not solving, the problem.
- After about 3 minutes, ask the children to share their plans.



Plan 1

Well we're drawing a picture and sort of making a model.

Can you give me more information please Brigid?

We're putting 60 crosses on our paper for sheep and the pen top will be Eric. Then Claire will circle one from that end, and I will pass two crosses with my pen top.

Plan 2

Our strategy is Guess and Check.

That's good Nick, but how are you going to check your guess?

Oh, we're making a model.

Go on ...

> *John's getting MAB smalls to be sheep and I'm getting a domino to be Eric and the chalk box to be the shed for shearing.*

Plan 3

> *We are doing it for 3 sheep then 4 sheep then 5 sheep and so on. Later we will look at 60.*

Great so you are going to try a simpler problem, make a table and look for a pattern.

This sharing of strategies is invaluable as it provides children who would normally feel lost in this type of activity with an opportunity to listen to their peers and make sense out of strategy selection. Note that such children are not given the answer. Rather they are assisted with understanding the power of selecting and applying strategies.

Step 3

- Tell the children that we are at Stage 3 of our four stage plan ... **Doing It**. Children collect what they need and carry out their plan.

Step 4

- Tell the children that we are at Stage 4 of our four stage plan ... **Check It**. Come together as a class for groups to share their findings. Again emphasis is on strategies.

> *We used the drawing strategy, but we changed while we were doing it because we saw a pattern.*

So Jake, you used the Look For A Pattern strategy. What was it?

> *We found that when Eric passed 10 sheep, 5 had been shorn, so 20 sheep meant 10 had been shorn ... and that means when Eric passes 40 sheep, 20 were shorn and that makes the 60 altogether.*

Great Jake. How would you work out the answer for 59 sheep or 62 sheep?

Sharing time is also a good opportunity to add in a strategy which no one may have used. For example:

> *Maybe we could've used the Number Sentence strategy, ie: 1 sheep goes to be shorn and Eric passes two sheep. That's 3 sheep, so perhaps, 60 divided into groups of 3, or 60 ÷ 3 gives the answer.*

Round off the lesson by referring to the Working Mathematically chart. There will be many opportunities to compliment the students on working like a mathematician.

Curriculum Planning Stories

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

In more than a decade of using tasks and many years of using the detailed whole class lessons of Maths300, teachers have developed several models for integrating tasks and whole class lessons. Some of those stories are retold here. Others can be found at:

- http://www.mathematicscentre.com/taskcentre/plans.htm

Story 1: Threading

> *Educational research caused me a dilemma. It tells us that students construct their own learning and that this process takes time. My understanding of the history of mathematics told me that certain concepts, such as place value and fractions, took thousands of years for mathematicians to understand. The dilemma was being faced with a textbook that expected students to 'get it' in a concentrated one, two or three week block of work and then usually not revisit the topic again until the next academic year.*

A Working Mathematically curriculum reflects the need to provide time to learn in a supportive, non-threatening environment and...

> *When I was involved in a Calculating Changes PD program I realised that:*
>
> - *choosing rich and revisitable activities, which are familiar in structure but fresh in challenge each time they are used, and*
> - *threading them through the curriculum over weeks for a small amount of time in each of several lessons per week*
>
> *resulted in deeper learning, especially when partnered with purposeful discussion and recording.*

Calculating Changes:

- http://www.mathematicscentre.com/calchange

Story 2: Your turn

Some teachers are making extensive use of a partnership between the whole class lessons of Maths300 and small group work with the tasks. Setting aside a lesson for using the tasks in the way they were originally designed now seems to have more meaning, as indicated by this teacher's story:

> *When I was thinking about helping students learn to work like a mathematician, my mind drifted to my daughter learning to drive. She*

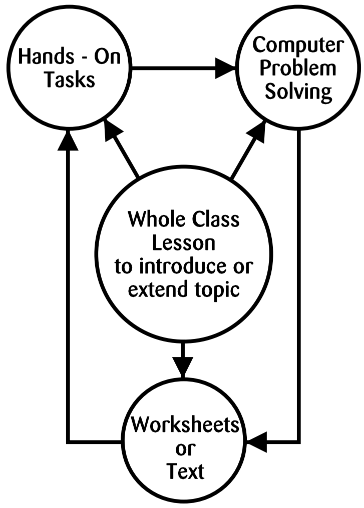
needed me to model how to do it and then she needed lots of opportunity to try it for herself.

That's when the idea clicked of using the Maths300 lessons as a model and the tasks as a chance for the students to have their turn to be a mathematician.

The Maths300 lessons illustrate how other teachers have modelled the process, so I felt I could do it too. Now the process is always on display on the wall or pasted inside the student's journal.

A session just using the tasks had seemed a bit like play time before this. Now I see it as an integral part of learning to work mathematically.

Story 3: Mixed Media



It was our staff discussion on Gardner's theory of Multiple Intelligences that led us into creating mixed media units. That and the access you have provided to tasks and Maths300 software.

We felt challenged to integrate these resources into our syllabus. There was really no excuse for a text book diet that favours the formal learners. We now often use four different modes of learning in the work station structure shown. It can be easily managed by one teacher, but it is better when we plan and execute it together.

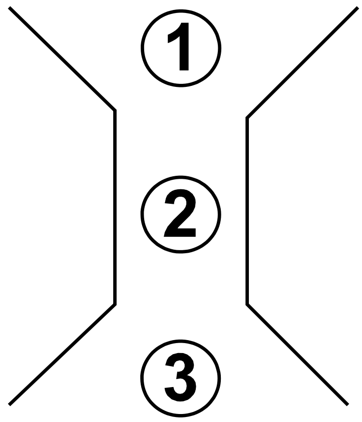
Story 4: Replacement Unit

We started meeting with the secondary school maths teachers to try to make transition between systems easier for the students. After considerable discussion we contracted a consultant who suggested that school might look too much the same across the transition when the students were hoping for something new. On the other hand our experience suggested that there needed to be some consistency in the way teachers worked.

We decided to 'bite the bullet' and try a hands-on problem solving unit in one strand. We selected two menus of twenty hands-on tasks, one for the primary and one for the secondary, that became the core of the unit. We deliberately overlapped some tasks that we knew were very rich and added some new ones for the high school.

Class lessons and investigation sheets were used to extend the tasks, within a three week model.

It is important to note that although these teachers structured a 3 week unit for the students, they strongly advised an additional *Week Zero* for staff preparation. The units came to be called Replacement Units.



Week Zero - Planning

Staff familiarise themselves with the material and jointly plan the unit. This is not a model that can be 'planned on the way to class'.

> *Getting together turned out to be great professional development for our group.*

Week 1 - Introduction

Students explore the 20 tasks listed on a printed menu:
- students explore the tip of the task, as on the card
- students move from task to task following teacher questioning that suggests there is more to the task than the tip
- in discussion with students, teachers gather informal assessment information that guides lesson planning for the following week.

> *We gave the kids an 'encouragement talk' first about joining us in an experiment in ways of learning maths and then gave out the tasks. The response was intelligent and there was quite a buzz in the room.*

Week 2 - Formalisation

> *It was good for both us and the students that the lessons in this week were a bit more traditional. However, they weren't text book based. We used whole class lessons based on the tasks they had been exploring and taught the Working Mathematically process, content and report writing.*
>
> *Assessment was via standard teacher-designed tests, quizzes and homework.*

Week 3 - Investigations

> *We were most delighted with Week 3. Each student chose one task from the menu and carried out an in-depth investigation into the iceberg guided by an investigation sheet. They had to publish a report of their investigation and we were quite surprised at the outcomes. It was clear that the first two weeks had lifted the image of mathematics from 'boring repetition' to a higher level of intellectual activity.*

Story 5: Curriculum shift

> *I think our school was like many others. The syllabus pattern was 10 units of three weeks each through the year. We had drifted into that through a text book driven curriculum and we knew the students weren't responding.*

Our consultant suggested that there was sameness about the intellectual demands of this approach which gave the impression that maths was the pursuit of skills. We agreed to select two deeper investigations to add to each unit. It took some time and considerable commitment, but we know that we have now made a curriculum shift. We are more satisfied and so are the students.

The principles guiding this shift were:

- Agree

  The 20 particular investigations for the year are agreed to by all teachers. If, for example, Cube Nets *is decided as one of these, then all the teachers are committed to present this within its unit.*

- Publish

  The investigations are written into the published syllabus. Students and parents are made aware of their existence and expect them to occur.

- Commit

  Once agreed, teachers are required to present the chosen investigations. They are not a negotiable 'extra'.

- Value

  The investigations each illustrate an explicit form of the Working Mathematically process. This is promoted to students, constantly referenced and valued.

- Assess

  The process provides students with scaffolding for their written reports and is also known by them as the criteria for assessment. (See next page.)

- Report

  The assessment component features within the school reporting structure.

A Final Comment

Including investigations has become policy.

Why? Because to not do so is to offer a diminished learning experience.

The investigative process ranks equally with skill development and needs to be planned for, delivered, assessed and reported.

Perhaps most of all we are grateful to our consultant because he was prepared to begin where we were. We never felt as if we had to throw out the baby and the bath water.

Assessment

Our attitude is:

stimulated students are creative and love to learn

Regardless of the way you use your **Maths With Attitude** resource, a variety of procedures can be employed to assess this learning.

Where these assessment procedures are applied to task sessions and involve written responses from students, teachers will need to be careful that the writing does not become too onerous. Students who get bogged down in doing the writing may lose interest in doing the tasks.

In addition to the ideas below, useful references are:

- http://www.mathematicscentre.com/taskcentre/assess.htm
- http://www.mathematicscentre.com/taskcentre/report.htm

The first offers several methods of assessment with examples and the second is a detailed lesson plan to support students to prepare a Maths Report.

Journal Writing

Journal writing is a way of determining whether the task or lesson has been understood by the student. The pupil can comment on such things as:

- What I learned in this task.
- What strategies I/we tried (refer to the Strategy Board).
- What went wrong.
- How I/we fixed it.
- Jottings - ie: any special thoughts or observations

Some teachers may prefer to have the page folded vertically, so that children's reflective thoughts can be recorded adjacent to critical working.

Assessment Form

An assessment form uses questions to help students reflect upon specific issues related to a specific task.

Anecdotal Records

Some teachers keep ongoing records about how students are tackling the tasks. These include jottings on whether students were showing initiative, whether they were working co-operatively, whether they could explain ideas clearly, whether they showed perseverance.

Checklists

A simple approach is to create a checklist based on the Working Mathematically process. Teachers might fill it in following questioning of individuals, or the students may fill it in and add comments appropriately.

Pupil Self-Reflection

Many theorists value and promote metacognition, the notion that learning is more permanent if pupils deliberately and consciously analyse their own learning. The deliberate teaching strategy of oral questioning and the way pupils record their work is an attempt to manifest this philosophy in action. The alternative is the tempting 'butterfly' approach which is to madly do as many activities as possible, mostly superficially, in the mistaken belief that quantity equates to quality.

> *I had to work quite hard to overcome previously entrenched habits of just getting the answer, any answer, and moving on to the next task.*

Thinking about *what* was learned *how* it was learned consolidates and adds to the learning.

When it follows an extensive whole class investigation, a reflection lesson such as this helps to shift entrenched approaches to mathematics learning. It is also an important component of the assessment process. On the one hand it gives you a lot of real data to assist your assessment. On the other it prepares the students for any formal assessment which you may choose to round off a unit.

Introduction

Ask students to recall what was done during the unit or lesson by asking a few individuals to say what *they* did, eg:

> *What did you do or learn that was new?*
> *What can you now do/understand that is new?*
> *What do you know now that you didn't know 1 (2, 3, ...) lesson ago?*

Continuing Discussion

Get a few ideas from the first students you ask, then:
- organise 5 -10 minute buzz groups of three or four students to chat together with one person to act as a recorder. These groups address the same questions as above.
- have a reporting session, with the recorder from each group telling the class about the group's ideas.

Student comments could be recorded on the board, perhaps in three groups.

Ideas & Facts	Maths Skills	Process (learning) Skills

If you need more questions to probe deeper and encourage more thought about process, try the following:

> *What new things did you do that were part of* how *you learned?*
> *Who uses this kind of knowledge and skill in their work?*

Student Recording

Hand out the REFLECTION sheet (next page) and ask students to write their own reflection about what they did, based on the ideas shared by the class. Collect these for interest and, possibly, assessment information.

REFLECTION

me looking at me learning

NAME: CLASS:

Working With Parents

Balancing Problem Solving with Basic Skill Practice

Many schools find that parents respond well to an evening where they have an opportunity to work with the tasks and perhaps work a task together as a 'whole class'. Resourced by the materials in this kit, teachers often feel quite confident to run these practical sessions. Comments from parents like:

> *I wish I had learnt maths like this.*

are very supportive. Letting students 'host' the evening is an additional benefit to the home/school relationship.

The 4½ Minute Talk

Charles Lovitt has considerable experience working with parents and has developed a crisp, parent-friendly talk which he shares below. Many others have used it verbatim with great success.

Why the Four and a Half Minute Talk?

When talking with parents about Problem Solving or the meaning of the term Working Mathematically, I have often found myself in the position, after having promoted inquiry based or investigative learning, of the parents saying:

> *Well - that's all very well - BUT ...*

at which stage they often express their concern for basic (meaning arithmetic) skill development.

The weakness of my previous attempts has been that I have been unable to reassure parents that problem solving does not mean sacrificing our belief in the virtues of such basic skill development.

One of the unfortunate perceptions about problem solving is that if a student is engaged in it, then somehow they are not doing, or it may be at the expense of, important skill based work.

This Four and a Half Minute Talk to parents is an attempt to express my belief that basic skill practice and problem solving development can be closely intertwined and not seen as in some way mutually exclusive.

(I'm still somewhat uncomfortable using the expression 'basic skills' in the above way as I am certain that some thinking, reasoning, strategy and communication skills are also 'basic'.)

Another aspect of the following 'talk' is that, as teachers put more emphasis on including investigative problem solving into their courses, a question arises about the source of suitable tasks.

This talk argues that we can learn to create them for ourselves by 'tweaking' the closed tasks that heavily populate out existing text exercises, and hence not be dependent on external suppliers. (Even better if students begin to create such opportunities for themselves.)

The Talk

In preparation, write the following graphic on the board:

CLOSED	OPEN	EXTENDED INVESTIGATION
		How many solutions exist? How do you know you have found them all?

I would like to show you what teachers are beginning to do to achieve some of the thinking and reasoning and communication skills we hope students will develop. I would like to show you three examples.

Example One: *6 + 5 = ?*

I write this question under the 'closed' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
$\begin{array}{r} 6 \\ \underline{+\ 5} \\ \underline{} \end{array}$		How many solutions exist? How do you know you have found them all?

And I ask:

What is the answer to this question?

I then explain that:

We often ask students many closed questions such as 6 + 5 = ?

The only response the students can tell us is "The answer is 11." … and as a reward for getting it correct we ask another twenty questions just like it.

What some teachers are doing is trying to *tweak* the question and ask it a different way, for example:

I have two counting numbers that add to 11. What might the numbers be?

[Counting numbers = positive whole numbers including zero]

I write this under the 'open' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 ___	? + ? 11	How many solutions exist? How do you know you have found them all?

What is the answer to the question now?

At this stage it becomes apparent there are several solutions:

The question is now a bit more open than it was before, allowing students to tell you things like 8 + 3, or 10 + 1, or 11 + 0 etc.

Let's see what happens if the teacher 'tweaks' it even further with the investigative challenge *or* extended investigation question:

How many solutions are there altogether?

and more importantly, and with greater emphasis on the second question:

How could you convince someone else that you have found them all?

Now the original question is definitely different - it still involves the skills of addition but now also involves thinking, reasoning and problem solving skills, strategy development and particularly communication skills.

Young students will soon tell you the answer is 'six different ones', but they must also confront the communication and reasoning challenge of convincing you that there are only six and no more.

Example Two: *Finding Averages*

Again, as I go through this example, I write it into the diagram on the board in the relevant sections.

The CLOSED question is: *11, 12, 13 - find the average*

Tweaking this makes it an OPEN question and it becomes:

I have three counting numbers whose average is 12. What might the numbers be?

Students will often say:

10, 12, 14 … or 9, 12, 15 … or even 12, 12, 12

After realising there are many answers, you can tweak it some more and turn it into an EXTENDED INVESTIGATION:

How many solutions exist? … AND …

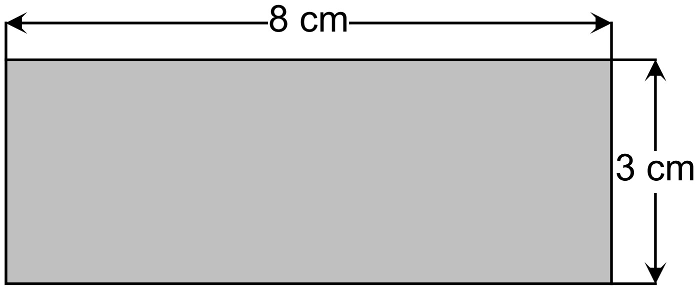
How do you know you have found them all?

Now the question is of a quite different nature. It still involves the arithmetic skill, but has something else as well - and that something else is the thinking, reasoning and communication skills necessary to find all of the combinations and convince someone else that you have done so.

By the time a student announces, with confidence, there are 127 different ways (which there are) that student will have engaged in all of these aspects, ie: the skill of calculating averages, (and some combination number theory) as well as significant strategy and reasoning experiences.

Example Three: *Finding the Area of a Rectangle*

A typical CLOSED question is:



Find the area. Find the perimeter.

The OPEN question is:

A rectangle has 24 squares inside:

What might its length and width be?

What might its perimeter be?

The EXTENDED INVESTIGATION version is:

Given they are whole number lengths, how many different rectangles are there? … AND …

How do you know you have found them all?

In summary, mathematics teachers are trying to convert *some* (not all) of the many closed questions that populate our courses and 'push' them towards the investigation direction. In doing so, we keep the skills we obviously value, but also activate the thinking, reasoning and justification skills we hope students will also develop.

This sequence of three examples hopefully shows two major features:

- That skills and problem solving can 'live alongside each other' and be developed concurrently.

- That the process of creating open-ended investigations can be done by anyone - just go to any source of closed questions and try 'tweaking' them as above. If it only worked for one question per page it would still provide a very large supply of investigations.

In terms of the effect of the talk on parents, I have usually found them to be reassured that we are not compromising important skill development (and nor do we want to). The only debate then becomes whether the additional skills of thinking, reasoning and communication are also desirable.

I've also been told that parents appreciate it because of the essential simplicity of the examples - no complicated theoretical jargon.

A Working Mathematically Curriculum

An Investigative Approach to Learning

The aim of a Working Mathematically curriculum is to help students learn to work like a mathematician. This process is detailed earlier (Page 8) in a one page document which becomes central to such a curriculum.

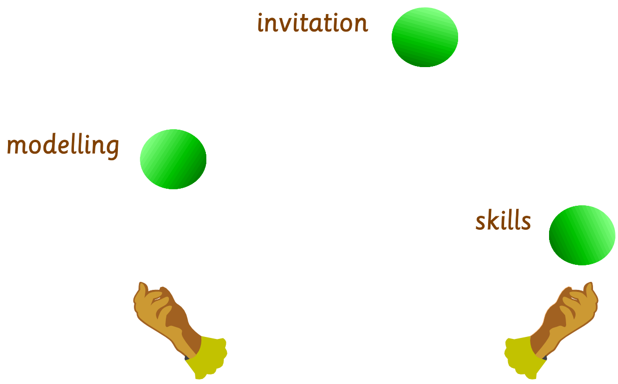
The change of emphasis brings a change of direction which *implies and requires* a balance between:

- the process of being a mathematician, and
- the development of skills needed to be a *successful* mathematician.

This journey is not two paths. It is one path made of two interwoven threads in the same way as DNA, the building block of life, is one compound made of two interwoven coils. To achieve a Working Mathematically curriculum teachers need to balance three components.

The task component of **Maths With Attitude** offers each pair of students an invitation to work like a mathematician.

The Maths300 component of **Maths With Attitude** assists teachers to model working like a mathematician.



Content skills are developed in context. They *are* important, but it is the application of skills within the process of Working Mathematically that has developed, and is developing, the human community's mathematical knowledge.

A focus for the Working Mathematically teacher is to help students develop mathematical skills in the context of problem posing and solving.

> *We are all 'born' with the same size mathematical toolbox, in the same way as I can own the same size toolbox as my motor mechanic. However, my motor mechanic has many more tools in her box than I and she has had more experience than I using them in context. Someone has helped her learn to use those tools while crawling under a car.*

Afzal Ahmed, Professor of Mathematics at Chichester, UK, once quipped:

> *If teachers of mathematics had to teach soccer, they would start off with a lesson on kicking the ball, follow it with lessons on trapping the ball and end with a lesson on heading the ball. At no time would they play a game of football.*

Such is not the case when teaching a Working Mathematically curriculum.

Elements of a Working Mathematically Curriculum

Working Mathematically is a K - 12 experience offering a balanced curriculum structured around the components below.

Hands-on Problem Solving Play

Mathematicians don't know the answer to a problem when they start it. If they did, it wouldn't be a problem. They have to play around with it. Each task invites students to play with mathematics 'like a mathematician'.

Skill Development

A mathematician needs skills to solve problems. Many teachers find it makes sense to students to place skill practice in the context of *Toolbox Lessons* which *help us better use the Working Mathematically Process* (Page 8).

Focus on Process

This is what mathematicians do; engage in the problem solving process.

Strategy Development

Mathematicians also make use of a strategy toolbox. These strategies are embedded in Maths300 lessons, but may also have a separate focus. Poster Problem Clinics are a useful way to approach this component.

Concept Development

A few major concepts in mathematics took centuries for the human race to develop and apply. Examples are place value, fractions and probability. In the past students have been expected to understand such concepts after having 'done' them for a two week slot. Typically they were not visited again until the next year. A Working Mathematically curriculum identifies these concepts and regularly 'threads' them through the curriculum.

Planning to Work Mathematically

The class, school or system that shifts towards a Working Mathematically curriculum will no longer use a curriculum document that looks like a list of content skills. The document would be clear in:

- choosing genuine problems to initiate investigation
- choosing a range of best practice teaching strategies to interest a wider range of students
- practising skills for the purpose of problem solving

Some teachers have found the planning template on the next page assists them to keep this framework at the forefront of their planning. It can be used to plan single lessons, or units built of several lessons. There are examples from schools in the Curriculum & Planning section of Maths300 and a Word document version of the template.

Unit Planning Page

Class

Topic

Pedagogy	Problem Solving	Skills
	In this topic how will I engage my students in the Working Mathematically process?	
How do I create an environment where students know what they are doing and why they have accepted the challenge?		Does the challenge identify skills to practise? Are there other skills to practise in preparation for future problem solving?

Notes

As a general guide:

- Find a problem(s) to solve related to the topic.
- Choose the best teaching craft likely to engage the learners.
- Where possible link skill practice to the problem solving process.

More on Professional Development

For many teachers there will be new ideas within **Maths With Attitude**, such as unit structures, views of how students learn, teaching strategies, classroom organisation, assessment techniques and use of concrete materials. It is anticipated (and expected) that as teachers explore the material in their classrooms they will meet, experiment with and reflect upon these ideas with a view to long term implications for the school program and for their own personal teaching.

Being explored 'on-the-job' so to speak, in the teacher's own classroom, makes the professional development more meaningful and practical for the teacher. This is also a practical and economic alternative for a local authority.

Strategic Use by Systems

From Years 3 - 10, **Maths With Attitude** is designed as a professional development vehicle by schools or clusters or systems because it carries a variety of sound educational messages. They might choose **Maths With Attitude** because:

- It can be used to highlight how investigative approaches to mathematics can be built into balanced unit plans without compromising skill development and without being relegated to the margins of a syllabus as something to be done only after 'the real' content has been covered.
- It can be used to focus on how a balance of concept, skill and application work can all be achieved within the one manageable unit structure.
- It can be used to show how a variety of assessment practices can be used concurrently to build a picture of student progress.
- It can be used to focus on transition between primary and secondary school by moving towards harmony and consistency of approach.
- It can be used to raise and continue debate about the pedagogy (art of teaching) that supports deeper mathematical learning for a wider range of students.

Teachers in Years K - 2 are similarly encouraged in professional growth through **Working Mathematically with Infants**, which derives from Calculating Changes, a network of teachers enhancing children's number skills from Years K - 6.

In supporting its teachers by supplying these resources in conjunction with targeted professional development over time, a system can fuel and encourage classroom-based debate on improving outcomes. There is evidence that by exploring alternative teaching strategies and encouraging curriculum shift towards Working Mathematically, learners improve and teachers are more satisfied. For more detail visit Research & Stories at:

- http://www.mathematicscentre.com/taskcentre/do.htm

We would be happy to discuss professional development with system leaders.

Web Reference

The starting point for all aspects of learning to work like a mathematician, including Calculating Changes, and the teaching craft which encourages it is:

- http://www.mathematicscentre.com/mathematicscentre

Appendix: Recording Sheets

Crossing The Desert

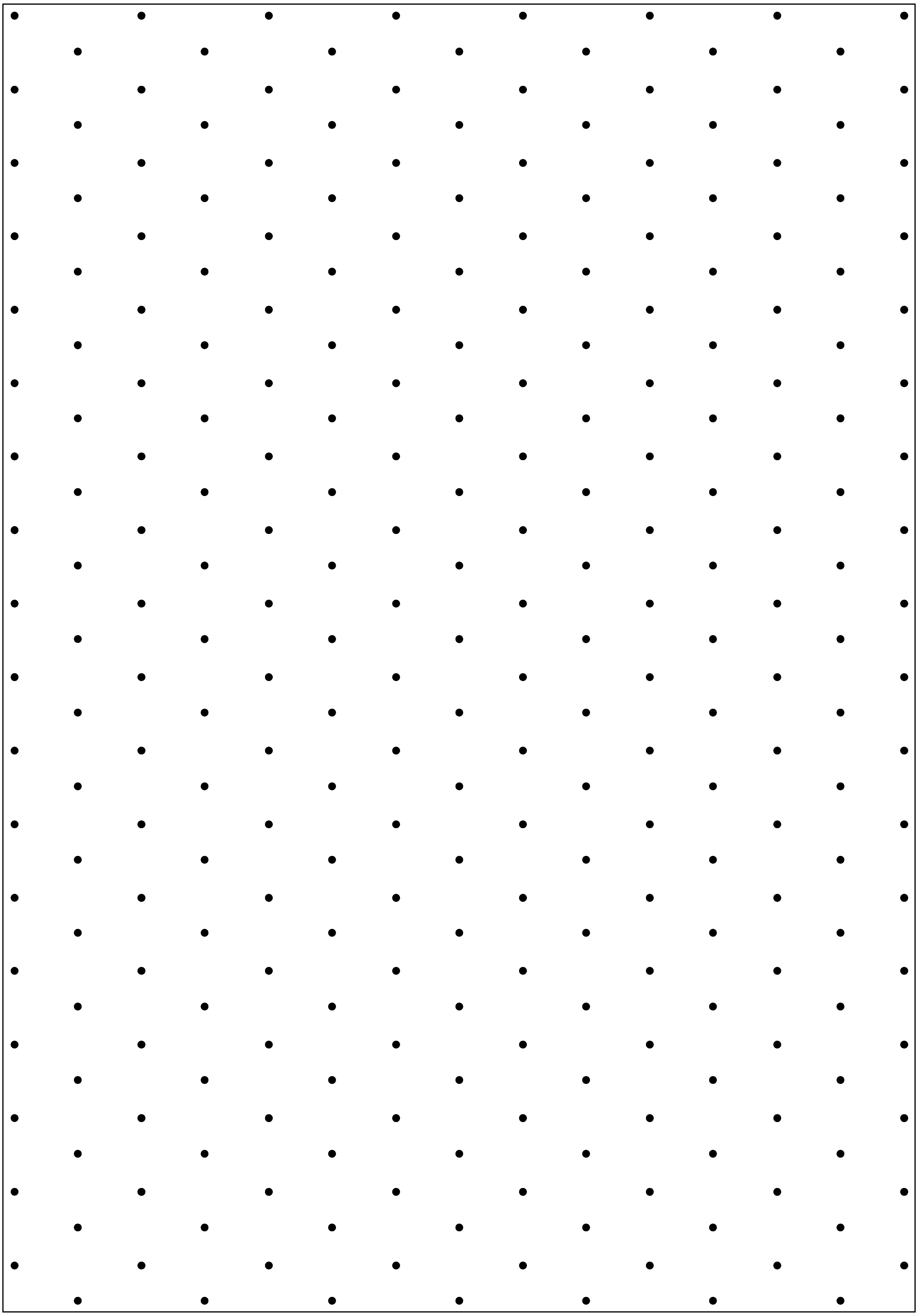
Reproducible Page

	Desert View Caravan Park & Kiosk	
Day 1	Start 	Day 1
Day 2		Day 2
Day 3		Day 3
Day 4		Day 4
Day 5		Day 5
Day 6		Day 6
Day 7		Day 7
Day 8		Day 8
Day 9	 Oasis	Day 9

Names: .... Class: ....

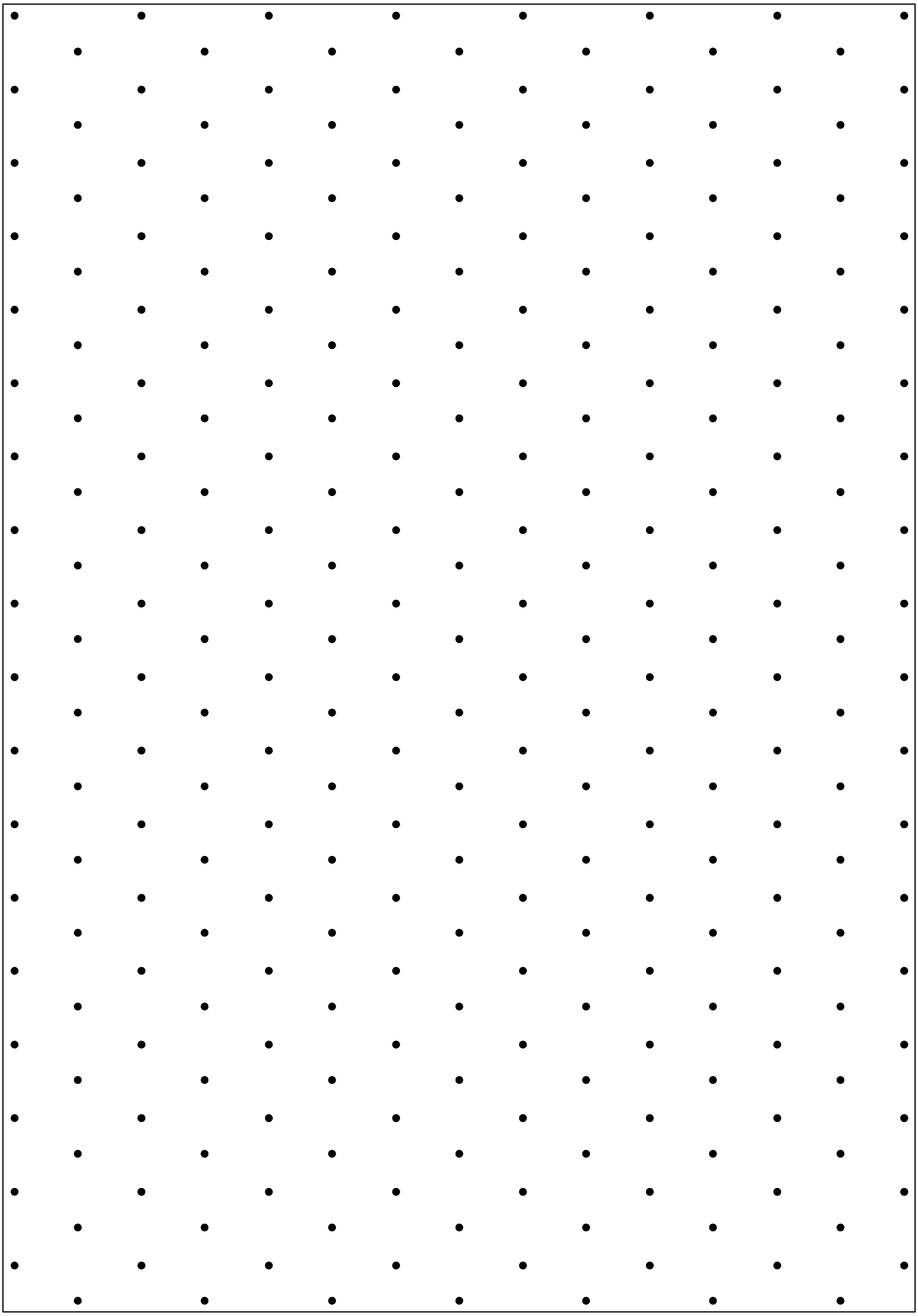
Four Cube Houses Recording Sheet

Reproducible Page



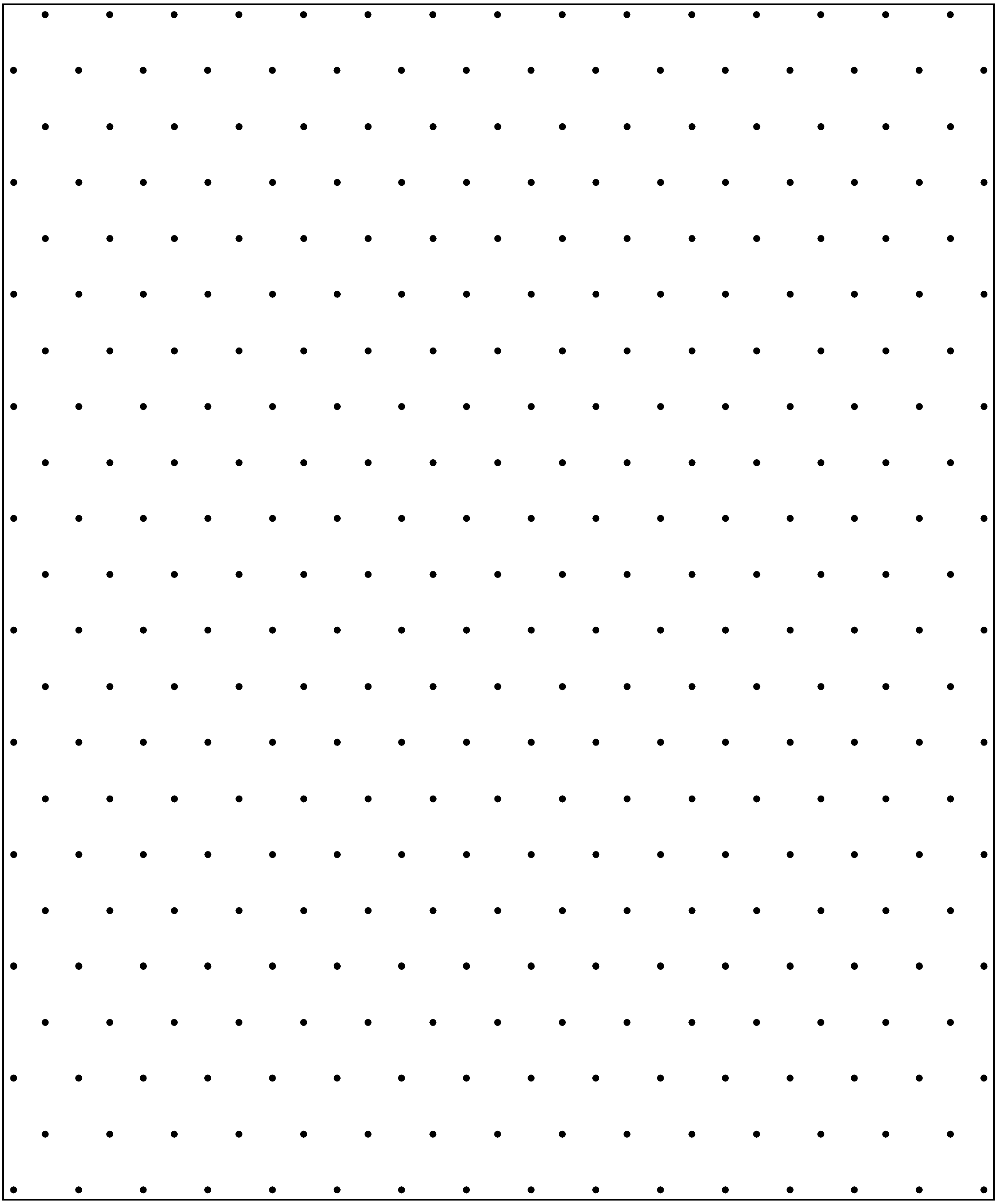
Pattern Cube Recording Sheet

Reproducible Page



Sphinx Paper

Reproducible Page



Names: .... Class: ....

Tricube Constructions A Recording Sheet

Reproducible Page