Chance & Measurement Years 3 & 4

Charles Lovitt
Doug Williams

Mathematics Task Centre & Maths300
helping to create happy healthy cheerful productive inspiring classrooms

Chance & Measurement

Years 3 & 4

In this kit:

- *Hands-on problem solving tasks*
- *Detailed curriculum planning*

Access from Maths300:

- *Extensive lesson plans*
- *Software*

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The **Maths With Attitude** series has been developed by The Task Centre Collective and is published by Black Douglas Professional Education Services.

Prepared for free distribution February 2025.

Design and Desktop Publishing:
Black Douglas Professional Education Services

ISBN: 0-958-59795-2 ... 978-0-958-59795-1
First published 2002
2nd Edition 2012
2016: Designed for web delivery as a computer-based file

Maths With Attitude includes reference to hands-on problem solving tasks and Maths300 material.

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- To support teachers, schools and systems wanting to create:

  happy, healthy, cheerful, productive, inspiring classrooms

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- to learners:

  stimulated students are creative and love to learn
- to pedagogy:

  the art of choosing teaching strategies to involve and interest all students
- to mathematics:

  mathematics is concrete, visual and makes sense
- to learning mathematics:

  all students can learn to work like a mathematician
- to teachers:

  the teacher is the most important resource in education
- to professional development:

  teachers improve their teaching by re-enacting stories from the classrooms of their colleagues

Our Objective in Detail

What do we mean by creating:

happy, healthy, cheerful, productive, inspiring classrooms

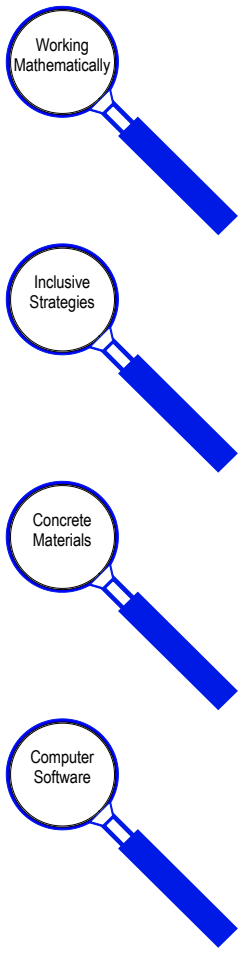
Happy…

means the elimination of the unnecessary fear of failure that hangs over so many students in their mathematics studies. Learning experiences *can* be structured so that all students see there is something in it for them and hence make a commitment to the learning. In so many 'threatening' situations, students see the impending failure and withhold their participation.

A phrase which describes the structure allowing all students to perceive something in it for them is *multiple entry points and multiple exit points*. That is, students can enter at a variety of levels, make progress and exit the problem having visibly achieved.

Healthy…

means *educationally healthy*. The learning environment should be a reflection of all that our community knows about how students learn. This translates into a rich array of teaching strategies that could and should be evident within the learning experience.



If we scrutinise the *exploration* through any lens, it should confirm to us that it is well structured or alert us to missed opportunities. For example, peering through a pedagogy lens we should see such features as:

- a story shell to embed the situation in a meaningful context
- significant active use of concrete materials
- a problem solving challenge which provides ownership for students
- small group work
- a strong visual component
- access to supportive software

Cheerful…

because we want 'happy' in the title twice!

Productive…

is the clear acknowledgment that students are working towards recognisable outcomes. They should know what these are and have guidelines to show they have either reached them or made progress. Teachers are accountable to these outcomes as well as to the quality of the learning environment.

Inspiring…

is about creating experiences that are uplifting or exalting; that actually *turn students on*. Experiences that make students feel great about themselves and empowered to act in meaningful ways.

Chance & Measurement Resources

To help you create

happy, healthy, cheerful, productive, inspiring classrooms

this kit contains

- 20 hands-on problem solving tasks from Mathematics Centre

and a Teachers' Manual which integrates the use of the tasks with

- 6 detailed lesson plans from Maths300

The kit offers **5 weeks** of Scope & Sequence planning in Chance & Measurement for *each* of Year 3 and Year 4. This is detailed in *Part 2: Planning Curriculum* which begins on Page 12. You are invited to map these weeks into your Year Planner.
Together, the four kits available for these levels provide 25 weeks of core curriculum in Working Mathematically (working like a mathematician).

Note: Membership of Maths300 is assumed.
The kit will be useful without it, but it will be much more useful with it.

Tasks

- 12 Counters
- Choosing Beads
- Counter Escape
- Decimals With A Tape
- Dice Differences
- Duelling Dice
- First Down The Mountain
- Fried Rice
- Greedy Pig
- Have A Hexagon
- Pack The Box
- See-Saw
- Same Or Different
- The Frog Pond
- Travelling Australia
- Triangle Perimeters
- Tube Toss
- Walk The Plank
- What's In The Bag?
- Win At The Fair

Part 2 of this manual introduces each task. The latest information can be found at:

- http://mathematicscentre.com/taskcentre/iceberg.htm

Maths300 Lessons

- Dice Differences
- Estimation Walks
- Finger Knitting Good
- Potato Olympics
- Problem Dice
- Win At The Fair

Lessons with Software

- Dice Differences
- Problem Dice
- Win At The Fair

Part 2 of this manual introduces each lesson. Full details can be found at:

- http://www.maths300.com

Working Like A Mathematician

Our attitude is:

all students can learn to work like a mathematician

What does a mathematician's work actually involve? Mathematicians have provided their answer on Page 8. In particular we are indebted to Dr. Derek Holton for the clarity of his contribution to this description.

Perhaps the most important aspect of Working Mathematically is the recognition that *knowledge is created by a community and becomes part of the fabric of that community*. Recognising, and engaging in, the process by which that knowledge is generated can help students to see themselves as able to work like a mathematician. Hence Working Mathematically is the framework of **Maths With Attitude**.

Skills, Strategies & Working Mathematically

A Working Mathematically curriculum places learning mathematical skills and problem solving strategies in their true context. Skills and strategies are the tools mathematicians employ in their struggle to solve problems. Lessons on skills or lessons on strategies are not an end in themselves.

- **Our skill toolbox** can be added to in the same way as the mechanic or carpenter adds tools to their toolbox. Equally, the addition of the tools is not for the sake of collecting them, but rather for the purpose of getting on with a job. A mathematician's job is to attempt to solve problems, not to collect tools that might one day help solve a problem.
- **Our strategy toolbox** has been provided through the collective wisdom of mathematicians from the past. All mathematical problems (and indeed life problems) that have ever been solved have been solved by the application of this concise set of strategies.

About Tasks

Our attitude is:

mathematics is concrete, visual and makes sense

Tasks are from Mathematics Task Centre. They are an invitation to two students to work like a mathematician (see Page 8).

The Task Centre concept began in Australia in the late 1970s as a collection of rich tasks housed in a special room, which came to be called a Task Centre. Since that time hundreds of Australian teachers, and, more recently, teachers from other countries, have adapted and modified the concept to work in their schools. For example, the special purpose room is no longer seen as an essential component, although many schools continue to opt for this facility.

A brief history of Task Centre development, considerable support for using tasks, for example Task Cameos, and a catalogue of all currently available tasks can be found at:

- http://www.mathematicscentre.com/taskcentre

Key principles are:

- A good task is the tip of an iceberg
- Each task has three lives
- Tasks involve students in the Working Mathematically process

The Task Centre Room or the Classroom?

There are good reasons for using the tasks in a special room which the students visit regularly. There are also different good reasons for keeping the tasks in classrooms. Either system can work well if staff are committed to a core curriculum built around learning to work like a mathematician.

- A task centre room creates a focus and presence for mathematics in the school. Tasks are often housed in clear plastic 'cake storer' type boxes. Display space can be more easily managed. The visual impact can be vibrant and purposeful.
- However, tasks can be more readily integrated into the curriculum if teachers have them at their finger tips in the classrooms. In this case tasks are often housed in press-seal plastic bags which take up less space and are more readily moved from classroom to classroom.

Tip of an Iceberg

The initial problem on the card can usually be solved in 10 to 20 minutes. The investigation iceberg which lies beneath may take many lessons (even a lifetime!). Tasks are designed so that the original problem reveals just the 'tip of the iceberg'. Task Cameos and Maths300 lessons help to dig deeper into the iceberg.

We are constantly surprised by the creative steps teachers and students take that lead us further into a task. No task is ever 'finished'.

> *Most tasks have many levels of entry and exit and therefore offer an on-going invitation to revisit them, and, importantly, multiple levels of success for students.*

Three Lives of a Task

This phrase, coined by a teacher, captures the full potential and flexibility of the tasks. Teachers say they like using them in three distinct ways:

1. As on the card, which is designed for two students.
2. As a whole class lesson involving all students, as supported by outlines in the Task Cameos and in detail through the Maths300 site.
3. Extended by an Investigation Guide (project), examples of which are included in both Task Cameos and Maths300.

The first life involves just the 'tip of the iceberg' of each task, but nonetheless provides a worthwhile problem solving challenge - one which 'demands' concrete materials in its solution. This is the invitation to work like a mathematician. Most students will experience some level of success and accomplishment in a short time.

The second life involves adapting the materials to involve the whole class in the investigation, in the first instance to model the work of a mathematician, but also to develop key outcomes or specific content knowledge. This involves choosing teaching craft to interest the students in the problem and then absorb them in it.

The third life challenges students to explore the 'rest of the iceberg' independently. Investigation Guides are used to probe aspects and extensions of the task and can be introduced into either the first or second life. Typically this involves providing suggestions for the direction the investigation might take. Students submit the 'story' of their work for 'portfolio assessment'. Typically a major criteria for assessment is application of the Working Mathematically process.

About Maths300

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

Maths300 is a subscription based web site. It is an attempt to collect and publish the 300 most 'interesting' maths lessons (K - 12).

- *Lessons have been successfully trialed in a range of classrooms.*
- *About one third of the lessons are supported by specially written software.*
- *Lessons are also supported by investigation sheets (with answers) and game boards where relevant.*
- *A 'living' Classroom Contributions section in each lesson includes the latest information from schools.*
- *The search engine allows teachers to find lessons by pedagogical feature, curriculum strand, content and year level.*
- *Lesson plans can be printed directly from the site.*
- *Each lesson supports teaches to model the Working Mathematically process.*

Modern internet facilities and computers allow teachers easy access to these lesson plans. Lesson plans need to be researched, reflected upon in the light of your own students and activated by collecting and organising materials as necessary.

Maths300 Software

Our attitude is:

> *stimulated students are creative and love to learn*

Pedagogically sound software is one feature likely to encourage enthusiastic learning and for that reason it has been included as an element in about one third of Maths300 lesson plans. The software is used to develop an investigation beyond its introduction and early exploration which is likely to include other pedagogical techniques such as concrete materials, physical involvement, estimation or mathematical conversation. The software is not the lesson plan. It is a feature of the lesson plan used at the teacher's discretion.

For school-wide use, the software needs to be downloaded from the site and installed in the school's network image. You will need to consult your IT Manager about these arrangements. It can also be downloaded to stand alone machines covered by the site licence, in particular a teacher's own laptop, from where it can be used with the whole class through a data projector.

Note:

- *Maths300 lessons and software may only be used by Maths300 members.*

Working Mathematically

First give me an interesting problem.

When mathematicians become interested in a problem they:

- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

Questions which help mathematicians learn more are:

- Can I check this another way?
- What happens if ...?
- How many solutions are there?
- How will I know when I have found them all?

When mathematicians have a problem they:

- Read & understand the problem.
- Plan a strategy to start the problem.
- Carry out their plan.
- Check the result.

A mathematician's strategy toolbox includes:

- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- Break a problem into smaller parts
- ...

If one way doesn't work, I just start again another way.

Professional Development Purpose

Our attitude is:

the teacher is the most important resource in education

> *We had our first study group on Monday. The session will be repeated again on Thursday. I had 15 teachers attend. We looked at the task Farmyard Friends (Task 129 from the Mathematics Task Centre). We extended it out like the questions from the companion Maths300 lesson suggested, and talked for quite a while about the concept of a factorial. This is exactly the type of dialog that I feel is essential for our elementary teachers to support the development of their math background. So anytime we can use the tasks to extend the teacher's math knowledge we are ahead of the game.*
> District Math Coordinator, Denver, Colorado

Research suggests that professional development most likely to succeed:

- is requested by the teachers
- takes place as close to the teacher's own working environment as possible
- takes place over an extended period of time
- provides opportunities for reflection and feedback
- enables participants to feel a substantial degree of ownership
- involves conscious commitment by the teacher
- involves groups of teachers rather than individuals from a school
- increases the participant's mathematical knowledge in some way
- uses the services of a consultant and/or critical friend

Maths With Attitude has been designed with these principles in mind. All the materials have been tried, tested and modified by teachers from a wide range of classrooms. We hope the resources will enable teacher groups to lead themselves further along the professional development road, and support systems to improve the learning outcomes for students K - 12.

With the support of Maths300 ETuTE, professional development can be a regular component of in-house professional development. See:

- http://www.mathematicscentre.com/taskcentre/resource.htm#etute

For external assistance with professional development, contact:

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Part 2: Planning Curriculum

Curriculum Planners

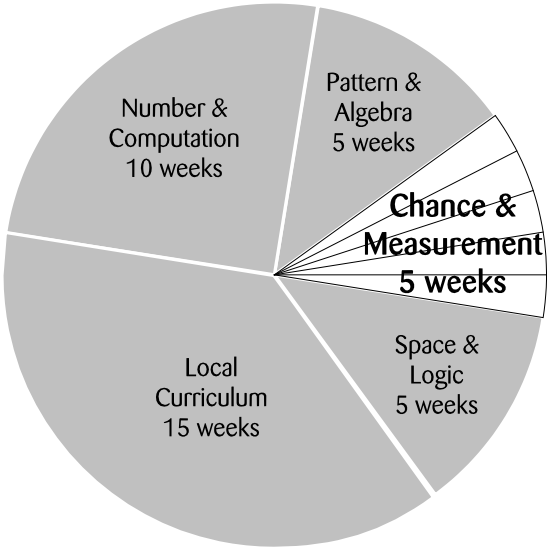
Our attitude is:

learning is a personal journey stimulated by achievable challenge

Curriculum Planners:

- show one way these resources can be integrated into your weekly planning
- provide a starting point for those new to these materials
- offer a flexible structure for those more experienced

You are invited to map Planner weeks into your school year planner as the core of the curriculum.



Planners:

- detail each week lesson by lesson
- offer structures for using tasks and lessons
- are sequenced from lesson to lesson, week to week and year to year to 'grow' learning

Teachers and schools will map the material in their own way, but all will be making use of extensively trialed materials and pedagogy.

Using Resources

- Your kit contains 20 hands-on problem solving tasks and reference to relevant Maths300 lessons.
- Tasks are introduced in this manual and supported by the Task Cameos at:
  http://www.mathematicscentre.com/taskcentre/iceberg.htm
- Maths300 lessons are introduced in this manual and supported by detailed lesson plans at:
  http://www.maths300.com

In your preparation, please note:

- Planners assume 4 lessons per week of about 1 hour each.
- Planners are *not* prescribing a continuous block of work.
- Weeks can be interspersed with other learning; perhaps a **Maths With Attitude** week from a different strand.
- Weeks can sometimes be interchanged within the planner.
- Lessons can sometimes be interchanged within weeks.
- The four **Maths With Attitude** kits available at each year level offer 25 weeks of a Working Mathematically core curriculum.

A Way to Begin

- Glance over the Planner for your class. Skim through the comments for each task and lesson as it is named. This will provide an overview of the kit.
- Task Comments begin after the Planners. Lesson Comments begin after Task Comments. The index will also lead you to any task or lesson comments.
- Select your preferred starting week - usually Week 1.
- Now plan in detail by researching the comments and web support. Enjoy!

Research, Reflect, Activate

Curriculum Planner

Chance & Measurement: Year 3

	Session 1	Session 2	Session 3	Session 4
Week 1	*Estimation Walks*: The lesson focuses on metres and parts of a metre, but is readily adaptable to other systems. At this level, rather than use decimals to record the closeness of estimates to actual measure (as in the Lesson Plan), it is more appropriate to use metres and centimetres. It will take one full session to establish the lesson. In the other three sessions the lesson is continued, but only forms part of the session. During these sessions one group at a time goes outside to estimate and walk. Managing this is made easier if the remainder of the work for the week is a combination of text material and tasks related to measurement and data. Relevant tasks are **Decimals With A Tape**, **Dice Differences**, **See-Saw**, **Travelling Australia** and **Win At The Fair**.			
Week 2	*Problem Dice & Dice Differences*: The concept of difference between two measurements was introduced in context in the previous week and this week the concept is applied to the difference between the numbers on two dice. Both lessons use this arithmetic idea, so you can choose to use either or both. In each case it will take a full session to establish the lesson. The software can be introduced in a second session and exploring the natural variability in chance events, which is another component of both lessons, can be continued through the week with tasks such as **Greedy Pig**, **Have A Hexagon**, **The Frog Pond**, **Tube Toss** and **Walk The Plank**.			
Weeks 3 - 5	*Finger Knitting Good*: This lesson is incredibly rich and perhaps, like many colleagues previously, you may soon feel that 3 weeks is not really enough time to follow the students' interest and capitalise on the many content areas that develop. The Lesson Plan includes work on: ♦ estimating and measuring length with informal and standard measures ♦ difference between estimated and actual length knitted ♦ experiencing the passage of time ♦ ordering lengths ♦ measuring body parts ♦ collecting and displaying data and much more. The ten tasks in bold type above can be reused as a work station during these weeks. The other tasks in the kit are used in Year 4.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Curriculum Planner

Chance & Measurement: Year 4

	Session 1	Session 2	Session 3	Session 4
Week 1	*Estimation Walks*: If previously used by the Year 3 teacher, the lesson is extended by asking for estimates and measures in decimal form. Year 4 teachers may also like to do some data collection and representation using a spreadsheet. As for Year 3, managing this lesson is easier if the remainder of the work for the week is a combination of text material and tasks related to measurement and data. Using the following tasks will complement the linear measurement and introduce other measures which will also be part of the *Potato Olympics* lesson. Relevant tasks are **Decimals With A Tape**, **Fried Rice**, **Pack The Box**, **See-Saw**, **Travelling Australia** and **Triangle Perimeters**, some of which may have been used in Year 3.			
Week 2	*Win At The Fair*: This lesson is the focus of a Chance & Data week. Use the lesson as the opening session for the week and set up three work stations for the other three sessions. One station will be *Win At The Fair* software. One will be the 14 Chance tasks in the kit (all the tasks not named for Week 1). One will be text work of your choice. It might also be valuable to consult with the Year 3 teacher to see if either *Problem Dice* or *Dice Differences* could have a place in the planning of this week.			
Weeks 3 - 5	*Potato Olympics*: Like *Finger Knitting Good* in Year 3 many have found the biggest challenge for the teacher in this lesson is deciding when to stop. The Lesson Plan includes work on: ♦ measurement of length, mass, time and displacement of water ♦ collection & display of data ♦ analysis of data using statistics like range, mode, median, mean ♦ displaying data in Stem & Leaf Plots, Histograms, Bar Graphs & Pie Charts ♦ the concept of percentage and percentage calculations ♦ use of a spreadsheet and much more. One of the strongest elements of the lesson is its cross-curriculum potential. With appropriate planning you will find this one theme can achieve objectives across many key learning areas during these three weeks.			

- Weeks can be interchanged.
- An activity named in **bold** refers to a hands-on task.
- An activity named in *italic* refers to a lesson from Maths300.
- Text book style Toolbox Lessons can be interwoven or set for homework.

Planning Notes

Enhancing Maths With Attitude

Resources to support learning to work like a mathematician are extensive and growing. There are more tasks and lessons available than have been included in this Chance & Measurement kit. You could use the following to enhance this kit.

Additional Tasks

- Task 22, Time Together

  A task to help students explore the passing of time. The focus is on those moments in a 12 hour cycle when the hands are 'on top of each other'. The task encourages manipulation of the clock hands, estimation and, for the more mathematically mature, precise calculation. The task also offers experience with the meaning of clockwise and opportunity for informal learning related to counting, angles and fractions.

- Task 25, In Between Time

  As time passes, both the minute hand and the hour hand move. This task first uses a thirty minute difference (embodying the idea of half of the hourly minute hand journey) to establish that students understand: (a) Time difference can be clockwise or anti-clockwise, and (b) If the minute hand moves half an hour, then the hour hand has moved half of its journey towards the next (or previous) hour. From this basis the students are challenged to set a position for one of the hands and explore where the other hand must, or in some cases could, be.

- Task 237, Trisquares

  Students explore the creation of new shapes from this simple shape made of three squares joined as an L. This leads to the problem of finding all the new shapes that can be made with just 2 Trisquares and to an investigation of area and perimeter. Each of these aspects of the investigation have a number/computation component.

More information about these tasks may be available in the Task Cameo Library:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Additional Lessons

- Lesson 171, Pick's Rule

  This easy to run lesson follows the classic Working Mathematically steps of exploring a context leading to data, noticing patterns in the data, which lead to generalisations, theories and conjectures, making and testing predictions from the theory and ultimately the process of proof. Students draw polygons on grid paper and count: the number of border dots (on the edge of the shape), the number of dots inside the shape, the area (number of squares) within the shape. There are underlying patterns and rules to discover that are true for all such shapes.

Keep in touch with new developments which enhance **Maths With Attitude** at:

- http://www.mathematicscentre.com/taskcentre/enhance.htm

Additional Materials

As stated, our attitude is that mathematics is concrete, visual and makes sense. We assume that all classrooms will have easy access to many materials beyond what we supply. For this unit you will need:

- Balls of knitting wool collected over time
- Measuring instruments of various sorts, but especially a tape 30 - 50m long
- Calculators

Special Comments Year 3

- *Finger Knitting Good*, Planner Week 3. You will need a great deal of knitting wool. Parents are usually able to supply all you need, but best to begin collection early.
- By Week 5 you will also be asking "What will we do with all this finger knitting?" The students may well provide answers, but it would be useful to dig into any art resources you have to look for creative solutions to this problem. One possibility is to build a giant spider web in the class. Another is to make use of wallpaper paste and Bulky News paper. Each thread is dipped in its turn into the wallpaper paste. As it is drawn out, gently squeeze off excess paste. The thread will now stick to the absorbent art paper to form interesting lines, curves and shapes. Leave it to dry for a week. When dry use felt-tip markers to begin a repeated pattern of 'contour lines' shaped by the lie of the threads. Again using the felt-tip markers, creatively add repeated decoration marks between the contours.

Special Comments Year 4

- *Win At The Fair*, Planner Week 2. The focus of this week is chance & probability. There are several ways to organise this. The Planner is based on the Mixed Media Model with Win At The Fair software as one work station. Find details of this model in the Curriculum Planning section on Page 34. However, teachers have created several models for structuring investigations into the classroom. You might read through the whole section on Curriculum Planning Stories and choose a different model.
- *Potato Olympics*, Planner Week 3. You are going to need a potato for each student. Clean, dry washed potatoes are necessary so start checking the shops or arranging for a parent to do some potato cleaning. The lesson actually begins with an art and language focus, so read it through well before launching in.

Chance & Data Tasks

- 12 Counters
- Choosing Beads
- Counter Escape
- Dice Differences
- Duelling Dice
- First Down Mountain
- Greedy Pig
- Have A Hexagon
- Same Or Different
- The Frog Pond
- Tube Toss
- Walk The Plank
- What's In The Bag?
- Win At The Fair

Measurement Tasks

- Decimals With A Tape
- Fried Rice
- Pack The Box
- See-Saw
- Travelling Australia
- Triangle Perimeters

Task Comments

- Tasks, lessons and unit plans prepare students for the more traditional skill practice lessons, which we invite you to weave into your curriculum. Teachers who have used practical, hands-on investigations as the focus of their curriculum, rather than focussing on the drill and practice diet of traditional mathematics, report success in referring to skill practice lessons as Toolbox Lessons. This links to the idea of a mathematician dipping into a toolbox to find and use skills to solve problems.

12 Counters

Students place 12 counters into 12 numbered rectangles in any way they wish, including using more than one counter in each rectangle. Counters are removed according to the sum of two dice.

At first students will probably place their counters at random. That is good because as they play the game, they work like a mathematician collecting and organising data (albeit internally) until they realise that the possible sums of two dice are not all equally likely. In fact it is not possible at all to remove a counter from Rectangle 1. The game then becomes a problem solving challenge as they search for the 'best' way to place their counters to ensure a win. It may help students to see the 36 possibilities for the sum of two dice if the dice are different colours.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Choosing Beads

For the most part at this level, the task is about the experience of sampling a mixed 'population' without replacement. Three colours of beads (A, B, C) are provided in a given ratio (15:12:4) and the initial challenge is:

> *What is the smallest number of beads you must select to be certain of having 3 the same colour?*

With eyes closed, students choose a bead, check its colour, choose again, check the colours and so on until three are the same colour.

Clearly choosing 3 beads is not enough. The three *might* be the same but we can't be *certain* that they will be. The card suggests an empirical beginning to the task and many students may need to do more experiments than the five that are suggested. As they do so, it may begin to occur to them that the chances of getting each colour is not what matters in this problem. Rather, what matters is the number already the same after each selection. This may lead to reasoning something like:

Selection 1:

- Problem not solved - select again.

Selection 2:

- Problem not solved - select again.

Selection 3:

Problem may be solved, but we can't be certain that it is.

The possibilities are:
- 3 the same - which solves the problem
- 2 the same
- all different

Each of these last two possibilities has to be followed separately at the next step.

Selection 4a (starting with 2 the same out of 3):
- 3 the same and 1 different - which solves the problem
- 2 pairs of 2 the same
- 2 the same and 2 different others

Selection 4b (starting with all different)
- 2 the same and 2 different others

Again there are two possibilities to follow through at the next step.

Selection 5a (starting with 2 pairs of 2 the same):
- 3 the same and 1 pair different - which solves the problem
- 2 pairs of 2 the same and 1 different other

Selection 5b (starting with 2 the same and 2 different others)
- 3 the same and 2 different others - which solves the problem
- 2 pairs of 2 the same and 1 different other

Now there is only one possibility to check at the next step.

Selection 6 (starting with 2 pairs of 2 the same and 1 different other):
- 3 the same, 1 pair the same and 1 different other - which solves the problem
- 3 pairs of 2 the same

Again there is only one possibility to check at the next step.

Selection 7 (starting with 3 pairs of 2 the same)
- 3 the same and 2 pairs of different others - which solves the problem.

So the smallest number of beads that would have to be selected to be certain of three the same is 7. Did any of the students' trials take more than 7 beads?

The problem card also opens another direction for investigation by identifying the variables in the original problem as:
- the number of colours in the container
- the number that have to be the same colour

Counter Escape

This task is similar to **Dice Differences** but the probabilities are governed by a single dice, rather than the difference between two dice. Three squares, six counters and permission to place counters into the squares in any arrangement. But where can they be placed so that all are removed in the least number of moves, given the dice roll rules governing removal from each square?

The task is easy to start, but to tackle the challenge requires recognition and discussion (at an intuitive level) of the chances involved. As they play, students will develop and test hypotheses just like a mathematician. It is likely that one question that will arise is 'How much data is enough to test an hypothesis?'.

Find more information about this task in the Task Cameo Library at:
- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Decimals With A Tape

The task takes the arithmetic of decimals out of the text book and into the sort of practical context that students may have already experienced in helping an adult with carpentry or sewing. It may be that the task is a bit advanced for some, but on the other hand, the measurement context may stimulate them to ask questions about the way decimals operate - a point of need learning opportunity. The calculator and the tape help to make the experience non-threatening and the question *Can you check it another way?* places the experience firmly in the domain of working like a mathematician.

Even if the students only use the tape to estimate, measure and record 'stuff', the task provides a significant learning experience.

Dice Differences

Easy to learn and play. What is the best strategy for placing six counters in six cells numbered 0 - 5, given that:

- the counters are to be removed in the least number of rolls of two dice
- the difference between the dice determines the cell from which one counter will be removed on each roll

The outcomes of the experiment are not equally likely. There are, for example, far more chances of scoring a difference of 1 than there are of scoring a difference of 5. The task invites the students to design an experiment, keep data and decide on statistics which help them make decisions about which placement strategies work better.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Duelling Dice

This is a fascinating task that will remind students of a playground game sometimes called 'Scissors, Rock & Paper'. Of course students can play it for fun and simply experience the unequal chances. However to answer the challenge on the card will require working like a mathematician. Once all the possibilities have been explored - and it is a challenge in itself to decide what these are - the somewhat unexpected result is that no dice is 'best'. Whichever dice is chosen there is always one that can beat it.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

First Down The Mountain

This task can keep players engaged in serious discussion at all levels from Infant to University. Encourage students to guess before each trial which player will win. Exploring their reasoning will provide insight into their current understanding. Younger students are more likely to reason that 2 and 12 are closest to the Win square and therefore are more likely to win. That reasoning is counter-intuitive for many older students - let's include teachers there too, since your authors reasoned incorrectly when first exploring the task. Our initial reasoning was that the steps are arranged so all players have an equal chance of reaching the square one before the Win, so 7 is most likely to win because it is most likely to be rolled. The

fascinating thing is that the infants are correct in their hypothesis, even if their reasoning is incomplete.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Fried Rice

This is really a logic problem involving measurement of mass rather than a task depending directly on measurement. The materials are provided as an aid to thinking. One key restriction in the problem is that Mr. Wang has *only* a 50g and a 30g container. There is nothing else to transfer rice to except the big tub of rice.

Students need to realise that the difference between the 50g and 30g containers is 20g, which could be a clue to finding 40g. So filling the 50g and pouring it into the 30g would leave 20g in the 50g container. The 30g container is now emptied back into the rice bucket and the 20g is poured out of the 50g container into the 30g container.

Now the 50g container is filled again and as much as possible (10g in fact) is poured into the 30g container. almost magically, this leaves 40g in the 50g container as required. Encourage the students to try recording those steps.

Ah! but there is another way to get the 40g and the card suggests that there might also be other measures of rice that Mr. Wang could create with these two containers.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Greedy Pig

Roll a dice to receive the number of points shown by the dice. Keep rolling and adding on points to create a progressive total until you lose ... or you decide to quit. How do you lose? Simple if you roll a 2 your score immediately becomes zero and your round is over.

- But the first two rolls are free, so you get those points even if a two is involved.
- Therefore, one strategy for always getting points is to quit after the first two rolls.
- But is it the strategy that will gain most points if a full game is five rounds?

The game might be played for the addition practice. However, its major aim is the investigation of the best playing strategy and that will require collecting and comparing data. Students should be encouraged to justify their choice of winning strategy. There is more information in the companion Maths300 lesson.

Have A Hexagon

The best hexagon to choose is actually the centre one which has the sections 1 - 6. This result is counter-intuitive for most students (and teachers). The arithmetic behind the task is the products which can be made by multiplying the numbers on two standard dice. Gathering sufficient data by playing the game several times reveals that, although the other hexagons initially look more attractive, several of the numbers on them can only be made in a limited number of ways. The chance of

filling either of these hexagons first is actually less than the chance of filling the centre one. Having gathered evidence to support that hypothesis, the what if question becomes: *What if we change the arrangement of the numbers on the hexagon? Could we make a fair playing board?*.

Exploring this investigative question leads to considering that the chances of winning a hexagon may be influenced not only by the number of ways a product can be formed, but also by the number of products on a particular hexagon which can only be formed in one way from the dice.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Pack The Box

This spatial puzzle can be extremely frustrating, but there is no need to rush to a solution. If mathematicians are facing a real problem, they don't immediately know the answer. Many students will enjoy returning to the task time and again. The clue to its solution lies in counting cubes, which is itself the basis of measuring volume. The box can take 27 cubes. The blocks only total 24 cubes. So perhaps the problem can best be solved by deciding how to arrange the three empty spaces. Hmm, three empty spaces and three layers of blocks. Maybe there is one empty space in each layer?

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Same Or Different

Using only two colours, students put a given number of blocks in a bag. They take turns to withdraw a block. Will the two blocks drawn be the same colour or different colours? The investigation is about discovering which combinations of the two colours produces a fair game, that is, one in which it is equally likely for the drawn blocks to be either the same colour or different colours.

The task requires the students to carry out experiments and collect sufficient data to decide which combinations of blocks are fair. One appeal of the game is that the results are quite 'unexpected and counter-intuitive'. Many begin by thinking that if the two colours in the bag are the same number (eg: 3 red and 3 blue), the same/different result will be fair. Not true.

This is a very easy game to set up and provides a wealth of probability experiences at this level. Equally the same task has an iceberg that can take you to Year 12 level or beyond.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

See-Saw

This is a very practical experience with balancing around a fulcrum. Finding a way to balance two objects on either side of the ruler requires consideration of both weight and distance. At one level the task can be dealt with intuitively - lighter things go further out, heavier things go closer in. At another, the objects could be

weighed and distances accurately measured. Can a pattern be found to link the measurements?

At one remote Aboriginal school, the playground had a real see-saw. After this task was used in class the teacher found students of all ages gathered around it balancing younger students against older ones.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

The Frog Pond

This task combines basic arithmetic practice with intuitive probability. When the students begin to reason that both players have an equal chance of winning, the door is open to some *What if...* questions:

- *What if...* one player started with 5 frogs and the other started with 4?
- *What if...* the number of frogs was the same but one player had to miss a turn if they rolled 1 OUT (or IN)?
- *What if...* you were asked to design your own game like this? How would you make it interesting and challenging?

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Travelling Australia

The yellow card in this task is a continuous 24 hour clock, but it is delivered flat packed. So, to prepare this task the yellow card has to be trimmed, curved into a circle and taped.

Although in essence the task is about measurement of time, it has the potential to cross many subject boundaries. Some teachers make it an essential part of an integrated unit in geography, language, history culture, and so on which can focus learning for several weeks. Mathematically the task also suggests many follow up investigations with questions like:

- How many ways are there to fly from A to B?
- How far can a jet travel in three hours?
- How far is it between...?
- If you travelled for the same time in a jet across USA, Asia, Europe (or any other continent or country) starting from ... where would you finish?
- Do the times given allow for time zone changes?
- How do time zones change around the world?

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Triangle Perimeters

Within a broad problem solving situation, this task is an exploration of perimeter at a concrete level that is consistent with the concept of counting equal units. This experience is held within the larger problem of fixing the area of a shape (ie: fixing the number of triangle tiles) and investigating how perimeter varies. Remember that if the students are Working Mathematically they would record their drawings and notes as they went.

Remember, perimeter and area are not about formulas. Formulas are a shortcut for counting. Perimeter is about counting the number of length units that fit around the boundary of a shape and area is about counting units of a tessellated shape which cover the flat space inside the boundary.

Tube Toss

There are actually three investigations on this card and it may be sufficient for the students to focus on the first one, which is the chances of a 3 landing face up. Just applying the Working Mathematically process to this one requires considerable research (perhaps 100 trials) and considerable discussion of ideas related to the psychology of gambling.

In this case the 3 has a one in two chance of occurring because the relevant outcome of the experiment is either a 3 up or not. The other discs don't matter. So, if it costs $1 per game to play and you paid out $2 for a win, you would expect no profit or loss.

It is important in the development of understanding about chance that the students do carry out the 100 trials to compare the 1 in 2 theory with the short term variation that appears in the experimental approach. For example in one set of 55 trials we carried out, 3 UP scored 25 and 3 DOWN scored 30.

- Would the scores have become more equal after 100 trials?
- Is it more instructive to keep a running record of the changing *percentage* of each possibility compared to the total number of experiments?

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Walk The Plank

This task is related to **The Frog Pond** but takes the arithmetic of integers further. The students will not be aware of it, but a conceptual groundwork is being laid for the representation of addition and subtraction of positive and negative numbers on a number line. (There are two ways to face - equivalent to positive and negative - and two operations - walking forwards and walking backwards which are equivalent to addition and subtraction.) However, in the context of this **Maths With Attitude** kit, the focus is on the question:

- *What happens if we change the length of the plank?*

In fact:

- *Which length of plank makes the best game?*

requiring students to both gather data for the average number of rolls to complete a given plank length and personal/psychological judgements about what makes 'the best game'.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

What's In The Bag?

The mathematics in this task is quite sophisticated, but it is appropriate that students of this age begin to consider the challenge. It is the same type of reasoning mathematicians have to use when they take surveys of comparatively

few people and try to decide the opinion of the whole population from that limited information.

The task is fun to do and can be put in terms of a detective being given three clues to find out "what's in the bag". Discussion is the primary objective at this level.

Any approach the students choose to solving the challenge is likely to reflect a growing understanding of proportion. It is also relevant to draw parallels with real life events mentioned above, such as election predictions.

- Do those who predict a certain voting pattern actually ask every member of the population?
- If not, how many do they have to ask to be reasonably certain of their predictions?
- What does 'reasonably certain' mean? Can it be quantified?

Win At The Fair

Here is another fairground game situation like **Tube Toss**. This time though the designer of the game hasn't considered the chances involved. It becomes clear (usually before 100 trials) that the designer is paying out more than is being received. The challenge then is to 'fix' the game to either break even, or make a reasonable profit. There are extensive notes in the companion Maths300 lesson, which also includes software that allows students to experiment with 'fixes'. *The recording sheet you may need for this task is at the end of this manual.* It allows the class to keep a running results sheet as each pair uses the task and thereby builds a greater collection of data on which to make judgements.

Find more information about this task in the Task Cameo Library at:

- http://www.mathematicscentre.com/taskcentre/iceberg.htm#cameos

Lesson Comments

- These comments introduce you to each Maths300 lesson. The complete plan is easily accessed through the lesson library available to members at:
  http://www.maths300.com
  where they are listed alphabetically by lesson name.

Dice Differences

The fantasy story context of a 'prisoner release scheme' seems to add both a structure and motivation to the lesson. It also suggests acting out the problem as a means of introduction and this seems to work well with young children. When the difference between two dice are calculated, there are 6 possible outcomes, from 0 to 5. The non-equal probabilities of each of these provides the underlying theory.

The investigation is easy to get started, but finding and proving the 'best' arrangement of prisoners is challenging. Hence the lesson can be effectively used at many levels and will be returned to in later years. The lesson is a strong backup for the skill of finding the difference between two numbers and at the same time allows students to 'play' with probabilities. The idea of average is introduced in context as a way of comparing sets of data. The computer simulation allows data to be gathered quickly to either confirm or disprove hypotheses.

Estimation Walks

The lesson is built around each pair of children walking from a starting point to a spot they estimate is a given distance away. The estimate is checked with a tape. The lesson produces data about how close the estimates are to the measured distances. The lesson plan explores ways of representing the data and introduces the statistic of average (or mean).

Year 3
Teachers may find it appropriate to use measurements in units and sub-units, eg: 4m 35cm

Year 4
At this level it may be more appropriate to write measurements like 4m 35cm in decimal form, eg: 4.35m. Differences between estimates and measures are then in decimal form, thus encouraging decimal arithmetic in context.

Finger Knitting Good

Almost all children find finger knitting fascinating. Once introduced (and it may help to do that with a couple of parent helpers around) they will merrily churn out metres of the stuff. In this lesson we view the strands as home made maths material. Finger knitting can become a daily pursuit which complements activities like reading to the class - children knit while the teacher reads. The literature chosen can also complement the knitting theme. The focus of the lesson is measurement, but there is also extensive scope for statistics, visual representation and even a little graphical algebra.

Potato Olympics

Who would believe that students could build a personal relationship with a potato? Well in this lesson they do! As a result of developing their potato into a character the students prepare it for entry into specially designed Potato Olympic events. They have to measure their potato in various ways and produce measurements that will allow the efforts of their potato athletes to be compared.

All this measurement produces data which needs to be displayed, analysed and reported.

The integrated cross-curriculum opportunities for language, art, science and health lessons are almost endless. And you don't even have to wait for an Olympic year to use this lesson. The Spudville Athletic Carnival is an annual event.

> *I just had to share with you the fun and excitement of 'Potato Olympics' at Mt. Dandenong Primary school over the past few weeks. ... The students have had weeks of training, event organising, personal bests, as well as measuring, estimating, sorting and comparing whilst discovering 'Where is the maths?' in potatoes. The students have had a ball, with our playgrounds and corridors filled with the excitement of events.*
>
> *It also just happened that the same two classes where scheduled to present their class Friday Assemblies, where students from all areas of the school showcase the learning from their level to the whole school. One class presented their learning in*

the form of an Opening Ceremony with a parade of participating potatoes and their flags from each state of Australia - (there were) speeches and interviews with coaches about their potatoes. The other class concluded the Potato Olympics yesterday with a closing ceremony with all our award winning potatoes on show. A slide show of the events and medal presentations captured the fun and excitement experienced by all the students.

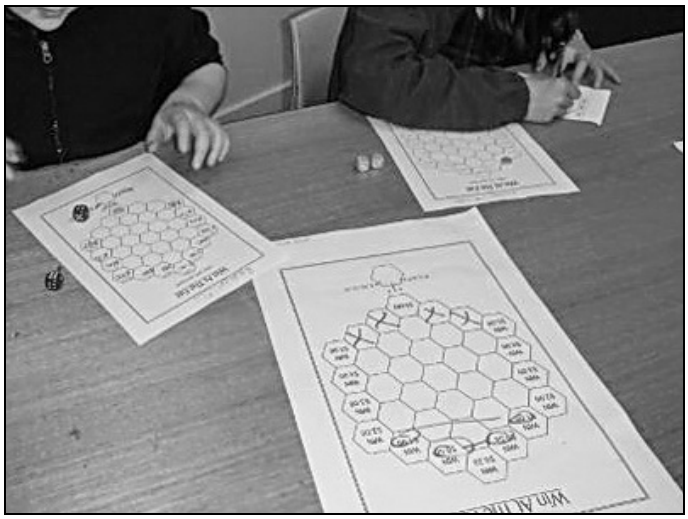
Sue Gibson

Problem Dice

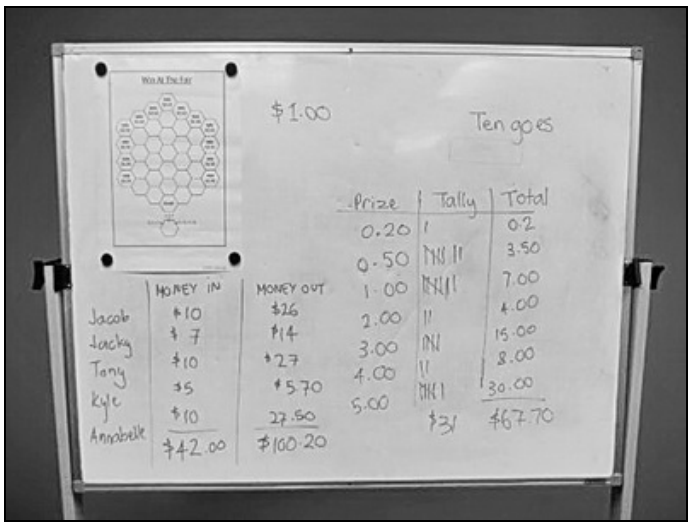
Some teachers consider this lesson to be more appropriate to younger students than *Dice Differences*. *Problem Dice* focuses on the concept of a fair game and it offers opportunity to explore the students' possible misconceptions of fairness. Pairs take turns to roll two dice and find the difference. The rules of the opening game are Player A wins a point if the difference is 0, 1, 2 and Player B wins if the difference is 3, 4, 5. Many students expect this to be a fair situation because each player 'owns' three numbers. Having the class play several games suggests otherwise and leads to the class investigating further to find a fair game.

Win At The Fair

The lesson begins as a 'fun' game on an unusual hexagonal board. Rolling two dice and summing them determines which direction (up, left, right) the marker can move. Eventually it will reach a boundary cell and win a prize, which may be more, less or the same as the amount they 'paid' to play.



Once sufficient data has been collected it becomes obvious that the 'owner' of the game is losing money. The students 'help out' the owner by changing the board or the rules to find a game that makes a fair profit.



Software allows students to run many experiments of the original game and supports them in seeking an alternative board design to support the 'owner'.

Part 3: Value Adding

The Poster Problem Clinic

Maths With Attitude kits offer several models for building a Working Mathematically curriculum around tasks. Each kit uses a different model, so across the range of 16 kits, teachers' professional learning continues and students experience variety. The Poster Problem Clinic is an additional model. It can be used to lead students into working with tasks, or it can be used in a briefer form as a opening component of each task session.

> *I was apprehensive about using tasks when it seemed such a different way of working. I felt my children had little or no experience of problem solving and I wanted to prepare them to think more deeply. The Clinic proved a perfect way in.*

Careful thought needs to be given to management in such lessons. One approach to getting the class started on the tasks and giving it a sense of direction and purpose is to start with a whole class problem. Usually this is displayed on a poster that all can see, perhaps in a Maths Corner. Another approach is to print a copy for each person. A Poster Problem Clinic fosters class discussion and thought about problem solving strategies.

Starting the lesson this way also means that just prior to liberating the students into the task session, they are all together to allow the teacher to make any short, general observations about classroom organisation, or to celebrate any problem solving ideas that have arisen.

One teacher describes the session like this:

> *I like starting with a class problem - for just a few minutes - it focuses the class attention, and often allows me to introduce a particular strategy that is new or needs emphasis.*

It only takes a short time to introduce a poster and get some initial ideas going. The class discussion develops a way of thinking. It allows class members to hear, and learn from their peers, about problem solving strategies that work for them.

> *If we don't collectively solve the problem in 5 minutes, I will leave the problem 'hanging' and it gives a purpose to the class review session at the end.*
> *Sometimes I require everyone to work out and write down their solution to the whole class problem. The staggered finishing time for this allows me to get organised and help students get started on tasks without being besieged.*
> *I try to never interrupt the task session, but all pupils know we have a five minute review session at the end to allow them to comment on such things as an activity they particularly liked. We often close then with an agreed answer to our whole class problem.*

A Clinic in Action

The aims of the regular clinic are:

- to provide children with the opportunity to learn a variety of strategies
- to familiarise children with a process for solving problems.

The following example illustrates a structure which many teachers have found successful when running a clinic.

Preparation

For each session teachers need:

- a Strategy Board as below
- a How To Solve A Problem chart as below
- to choose a suitable problem and prepare it as a poster
- to organise children into groups of two or three.

The Strategy Board can be prepared in advance as a reference for the children, or may be developed *with* the children as they explore problem solving and suggest their own versions of the strategies.

The problem can be chosen from

- a book
- the task collection
- prepared collections such as Professor Morris Puzzles which can be viewed at: http://www.mathematicscentre.com/taskcentre/resource.htm#profmorr

The example which follows is from the task collection. The teacher copied it onto a large sheet of paper and asked some children to illustrate it. *The teacher also changed the number of sheep to sixty* to make the poster a little different from the one in the task collection.

The Strategy Board and the How To Solve A Problem chart can be used in any maths activity and are frequently referred to in Maths300 lessons.

The Clinic

The poster used for this example session is:

> Eric the Sheep is lining up to be shorn before the hot summer ahead. There are sixty [60] sheep in front of him. Eric can't be bothered waiting in the queue properly, so he decides to sneak towards the front.
>
> Every time one [1] sheep is taken to be shorn, Eric then sneaks past two [2] sheep. How many sheep will be shorn before Eric?

This Poster Problem Clinic approach is also extensively explored in Maths300 Lesson 14, *The Farmer's Puzzle*.

Strategy Board

DO I KNOW A SIMILAR PROBLEM?	ACT IT OUT
GUESS, CHECK AND IMPROVE	DRAW A PICTURE OR GRAPH
TRY A SIMPLER PROBLEM	MAKE A MODEL
WRITE AN EQUATION	LOOK FOR A PATTERN
MAKE A LIST OR TABLE	TRY ALL POSSIBILITIES
WORK BACKWARDS	SEEK AN EXCEPTION
BREAK INTO SMALLER PARTS	...

How To Solve A Problem

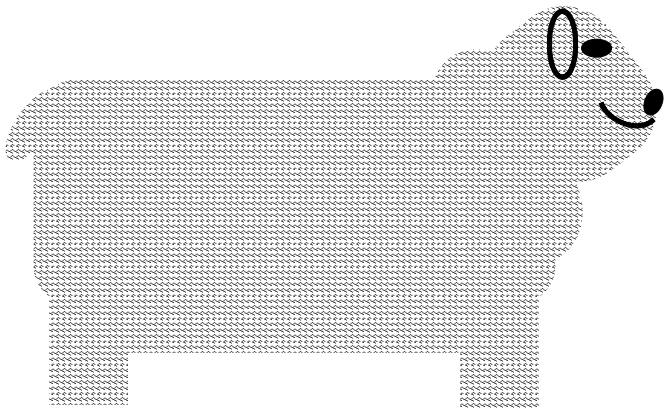
SEE & UNDERSTAND	Do I understand what the problem is asking? Discuss
PLANNING	Select a strategy from the board. Plan how you intend solving the problem.
DOING IT	Try out your idea.
CHECK IT	Did it work out? If so reflect on the activity. If not, go back to step one.

Step 1

- Tell the children that we are at Stage 1 of our four stage plan ... **See & Understand** ... Point to it! Read the problem with the class. Discuss the problem and clarify any misunderstandings.
- If children do not clearly understand what the problem is asking, they will not cope with the next stage. A good way of finding out if a child understands a problem is for her/him to retell it.
- Allow time for questions - approximately 3 to 5 minutes.

Step 2

- Tell the children that we are at Stage 2 of our four stage plan ... **Planning**. In their groups children select one or more strategies from the Strategy Board and discuss/organise how to go about solving the problem.
- Without guidance, children will often skip this step and go straight to Doing It. It is vital to emphasise that this stage is simply planning, not solving, the problem.
- After about 3 minutes, ask the children to share their plans.



Plan 1

Well we're drawing a picture and sort of making a model.

Can you give me more information please Brigid?

We're putting 60 crosses on our paper for sheep and the pen top will be Eric. Then Claire will circle one from that end, and I will pass two crosses with my pen top.

Plan 2

Our strategy is Guess and Check.

That's good Nick, but how are you going to check your guess?

Oh, we're making a model.

Go on ...

> *John's getting MAB smalls to be sheep and I'm getting a domino to be Eric and the chalk box to be the shed for shearing.*

Plan 3

> *We are doing it for 3 sheep then 4 sheep then 5 sheep and so on. Later we will look at 60.*

Great so you are going to try a simpler problem, make a table and look for a pattern.

This sharing of strategies is invaluable as it provides children who would normally feel lost in this type of activity with an opportunity to listen to their peers and make sense out of strategy selection. Note that such children are not given the answer. Rather they are assisted with understanding the power of selecting and applying strategies.

Step 3

- Tell the children that we are at Stage 3 of our four stage plan ... **Doing It**. Children collect what they need and carry out their plan.

Step 4

- Tell the children that we are at Stage 4 of our four stage plan ... **Check It**. Come together as a class for groups to share their findings. Again emphasis is on strategies.

> *We used the drawing strategy, but we changed while we were doing it because we saw a pattern.*

So Jake, you used the Look For A Pattern strategy. What was it?

> *We found that when Eric passed 10 sheep, 5 had been shorn, so 20 sheep meant 10 had been shorn ... and that means when Eric passes 40 sheep, 20 were shorn and that makes the 60 altogether.*

Great Jake. How would you work out the answer for 59 sheep or 62 sheep?

Sharing time is also a good opportunity to add in a strategy which no one may have used. For example:

> *Maybe we could've used the Number Sentence strategy, ie: 1 sheep goes to be shorn and Eric passes two sheep. That's 3 sheep, so perhaps, 60 divided into groups of 3, or 60 ÷ 3 gives the answer.*

Round off the lesson by referring to the Working Mathematically chart. There will be many opportunities to compliment the students on working like a mathematician.

Curriculum Planning Stories

Our attitude is:

> *teachers improve their teaching by re-enacting stories from the classrooms of their colleagues*

In more than a decade of using tasks and many years of using the detailed whole class lessons of Maths300, teachers have developed several models for integrating tasks and whole class lessons. Some of those stories are retold here. Others can be found at:

- http://www.mathematicscentre.com/taskcentre/plans.htm

Story 1: Threading

> *Educational research caused me a dilemma. It tells us that students construct their own learning and that this process takes time. My understanding of the history of mathematics told me that certain concepts, such as place value and fractions, took thousands of years for mathematicians to understand. The dilemma was being faced with a textbook that expected students to 'get it' in a concentrated one, two or three week block of work and then usually not revisit the topic again until the next academic year.*

A Working Mathematically curriculum reflects the need to provide time to learn in a supportive, non-threatening environment and…

> *When I was involved in a Calculating Changes PD program I realised that:*
>
> - *choosing rich and revisitable activities, which are familiar in structure but fresh in challenge each time they are used, and*
> - *threading them through the curriculum over weeks for a small amount of time in each of several lessons per week*
>
> *resulted in deeper learning, especially when partnered with purposeful discussion and recording.*

Calculating Changes:

- http://www.mathematicscentre.com/calchange

Story 2: Your turn

Some teachers are making extensive use of a partnership between the whole class lessons of Maths300 and small group work with the tasks. Setting aside a lesson for using the tasks in the way they were originally designed now seems to have more meaning, as indicated by this teacher's story:

> *When I was thinking about helping students learn to work like a mathematician, my mind drifted to my daughter learning to drive. She*

needed me to model how to do it and then she needed lots of opportunity to try it for herself.

That's when the idea clicked of using the Maths300 lessons as a model and the tasks as a chance for the students to have their turn to be a mathematician.

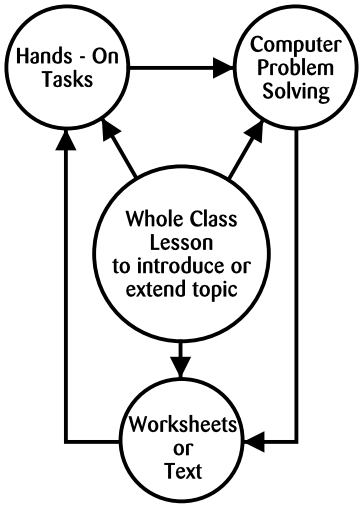
The Maths300 lessons illustrate how other teachers have modelled the process, so I felt I could do it too. Now the process is always on display on the wall or pasted inside the student's journal.

A session just using the tasks had seemed a bit like play time before this. Now I see it as an integral part of learning to work mathematically.

Story 3: Mixed Media

It was our staff discussion on Gardner's theory of Multiple Intelligences that led us into creating mixed media units. That and the access you have provided to tasks and Maths300 software.

We felt challenged to integrate these resources into our syllabus. There was really no excuse for a text book diet that favours the formal learners. We now often use four different modes of learning in the work station structure shown. It can be easily managed by one teacher, but it is better when we plan and execute it together.



Story 4: Replacement Unit

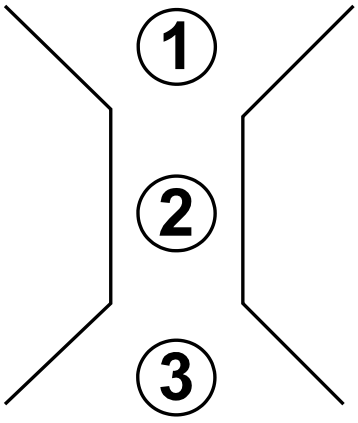
We started meeting with the secondary school maths teachers to try to make transition between systems easier for the students. After considerable discussion we contracted a consultant who suggested that school might look too much the same across the transition when the students were hoping for something new. On the other hand our experience suggested that there needed to be some consistency in the way teachers worked.

We decided to 'bite the bullet' and try a hands-on problem solving unit in one strand. We selected two menus of twenty hands-on tasks, one for the primary and one for the secondary, that became the core of the unit. We deliberately overlapped some tasks that we knew were very rich and added some new ones for the high school.

Class lessons and investigation sheets were used to extend the tasks, within a three week model.

It is important to note that although these teachers structured a 3 week unit for the students, they strongly advised an additional *Week Zero* for staff preparation. The units came to be called Replacement Units.

Week Zero - Planning



Staff familiarise themselves with the material and jointly plan the unit. This is not a model that can be 'planned on the way to class'.

> *Getting together turned out to be great professional development for our group.*

Week 1 - Introduction

Students explore the 20 tasks listed on a printed menu:

- students explore the tip of the task, as on the card
- students move from task to task following teacher questioning that suggests there is more to the task than the tip
- in discussion with students, teachers gather informal assessment information that guides lesson planning for the following week.

> *We gave the kids an 'encouragement talk' first about joining us in an experiment in ways of learning maths and then gave out the tasks. The response was intelligent and there was quite a buzz in the room.*

Week 2 - Formalisation

> *It was good for both us and the students that the lessons in this week were a bit more traditional. However, they weren't text book based. We used whole class lessons based on the tasks they had been exploring and taught the Working Mathematically process, content and report writing.*
>
> *Assessment was via standard teacher-designed tests, quizzes and homework.*

Week 3 - Investigations

> *We were most delighted with Week 3. Each student chose one task from the menu and carried out an in-depth investigation into the iceberg guided by an investigation sheet. They had to publish a report of their investigation and we were quite surprised at the outcomes. It was clear that the first two weeks had lifted the image of mathematics from 'boring repetition' to a higher level of intellectual activity.*

Story 5: Curriculum shift

> *I think our school was like many others. The syllabus pattern was 10 units of three weeks each through the year. We had drifted into that through a text book driven curriculum and we knew the students weren't responding.*

Our consultant suggested that there was sameness about the intellectual demands of this approach which gave the impression that maths was the pursuit of skills. We agreed to select two deeper investigations to add to each unit. It took some time and considerable commitment, but we know that we have now made a curriculum shift. We are more satisfied and so are the students.

The principles guiding this shift were:

- Agree

  The 20 particular investigations for the year are agreed to by all teachers. If, for example, Cube Nets is decided as one of these, then all the teachers are committed to present this within its unit.

- Publish

  The investigations are written into the published syllabus. Students and parents are made aware of their existence and expect them to occur.

- Commit

  Once agreed, teachers are required to present the chosen investigations. They are not a negotiable 'extra'.

- Value

  The investigations each illustrate an explicit form of the Working Mathematically process. This is promoted to students, constantly referenced and valued.

- Assess

  The process provides students with scaffolding for their written reports and is also known by them as the criteria for assessment. *(See next page.)*

- Report

  The assessment component features within the school reporting structure.

A Final Comment

Including investigations has become policy.

Why? Because to not do so is to offer a diminished learning experience.

The investigative process ranks equally with skill development and needs to be planned for, delivered, assessed and reported.

Perhaps most of all we are grateful to our consultant because he was prepared to begin where we were. We never felt as if we had to throw out the baby and the bath water.

Assessment

Our attitude is:

stimulated students are creative and love to learn

Regardless of the way you use your **Maths With Attitude** resource, a variety of procedures can be employed to assess this learning.

Where these assessment procedures are applied to task sessions and involve written responses from students, teachers will need to be careful that the writing does not become too onerous. Students who get bogged down in doing the writing may lose interest in doing the tasks.

In addition to the ideas below, useful references are:

- http://www.mathematicscentre.com/taskcentre/assess.htm
- http://www.mathematicscentre.com/taskcentre/report.htm

The first offers several methods of assessment with examples and the second is a detailed lesson plan to support students to prepare a Maths Report.

Journal Writing

Journal writing is a way of determining whether the task or lesson has been understood by the student. The pupil can comment on such things as:

- What I learned in this task.
- What strategies I/we tried (refer to the Strategy Board).
- What went wrong.
- How I/we fixed it.
- Jottings - ie: any special thoughts or observations

Some teachers may prefer to have the page folded vertically, so that children's reflective thoughts can be recorded adjacent to critical working.

Assessment Form

An assessment form uses questions to help students reflect upon specific issues related to a specific task.

Anecdotal Records

Some teachers keep ongoing records about how students are tackling the tasks. These include jottings on whether students were showing initiative, whether they were working co-operatively, whether they could explain ideas clearly, whether they showed perseverance.

Checklists

A simple approach is to create a checklist based on the Working Mathematically process. Teachers might fill it in following questioning of individuals, or the students may fill it in and add comments appropriately.

Pupil Self-Reflection

Many theorists value and promote metacognition, the notion that learning is more permanent if pupils deliberately and consciously analyse their own learning. The

deliberate teaching strategy of oral questioning and the way pupils record their work is an attempt to manifest this philosophy in action. The alternative is the tempting 'butterfly' approach which is to madly do as many activities as possible, mostly superficially, in the mistaken belief that quantity equates to quality.

> *I had to work quite hard to overcome previously entrenched habits of just getting the answer, any answer, and moving on to the next task.*

Thinking about *what* was learned *how* it was learned consolidates and adds to the learning.

When it follows an extensive whole class investigation, a reflection lesson such as this helps to shift entrenched approaches to mathematics learning. It is also an important component of the assessment process. On the one hand it gives you a lot of real data to assist your assessment. On the other it prepares the students for any formal assessment which you may choose to round off a unit.

Introduction

Ask students to recall what was done during the unit or lesson by asking a few individuals to say what *they* did, eg:

> *What did you do or learn that was new?*
> *What can you now do/understand that is new?*
> *What do you know now that you didn't know 1 (2, 3, ...) lesson ago?*

Continuing Discussion

Get a few ideas from the first students you ask, then:

- organise 5 -10 minute buzz groups of three or four students to chat together with one person to act as a recorder. These groups address the same questions as above.
- have a reporting session, with the recorder from each group telling the class about the group's ideas.

Student comments could be recorded on the board, perhaps in three groups.

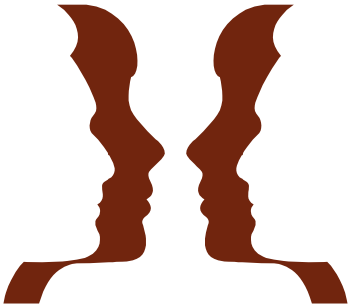
Ideas & Facts	Maths Skills	Process (learning) Skills

If you need more questions to probe deeper and encourage more thought about process, try the following:

> *What new things did you do that were part of* how *you learned?*
> *Who uses this kind of knowledge and skill in their work?*

Student Recording

Hand out the REFLECTION sheet (next page) and ask students to write their own reflection about what they did, based on the ideas shared by the class. Collect these for interest and, possibly, assessment information.



REFLECTION

me looking at me learning

NAME: CLASS:

Reproducible Page

Working With Parents

Balancing Problem Solving with Basic Skill Practice

Many schools find that parents respond well to an evening where they have an opportunity to work with the tasks and perhaps work a task together as a 'whole class'. Resourced by the materials in this kit, teachers often feel quite confident to run these practical sessions. Comments from parents like:

> *I wish I had learnt maths like this.*

are very supportive. Letting students 'host' the evening is an additional benefit to the home/school relationship.

The 4½ Minute Talk

Charles Lovitt has considerable experience working with parents and has developed a crisp, parent-friendly talk which he shares below. Many others have used it verbatim with great success.

Why the Four and a Half Minute Talk?

When talking with parents about Problem Solving or the meaning of the term Working Mathematically, I have often found myself in the position, after having promoted inquiry based or investigative learning, of the parents saying:

> *Well - that's all very well - BUT ...*

at which stage they often express their concern for basic (meaning arithmetic) skill development.

The weakness of my previous attempts has been that I have been unable to reassure parents that problem solving does not mean sacrificing our belief in the virtues of such basic skill development.

One of the unfortunate perceptions about problem solving is that if a student is engaged in it, then somehow they are not doing, or it may be at the expense of, important skill based work.

This Four and a Half Minute Talk to parents is an attempt to express my belief that basic skill practice and problem solving development can be closely intertwined and not seen as in some way mutually exclusive.

(I'm still somewhat uncomfortable using the expression 'basic skills' in the above way as I am certain that some thinking, reasoning, strategy and communication skills are also 'basic'.)

Another aspect of the following 'talk' is that, as teachers put more emphasis on including investigative problem solving into their courses, a question arises about the source of suitable tasks.

This talk argues that we can learn to create them for ourselves by 'tweaking' the closed tasks that heavily populate out existing text exercises, and hence not be dependent on external suppliers. (Even better if students begin to create such opportunities for themselves.)

The Talk

In preparation, write the following graphic on the board:

CLOSED	OPEN	EXTENDED INVESTIGATION
		How many solutions exist? How do you know you have found them all?

I would like to show you what teachers are beginning to do to achieve some of the thinking and reasoning and communication skills we hope students will develop. I would like to show you three examples.

Example One: *6 + 5 = ?*

I write this question under the 'closed' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 —		How many solutions exist? How do you know you have found them all?

And I ask:

What is the answer to this question?

I then explain that:

We often ask students many closed questions such as 6 + 5 = ?

The only response the students can tell us is "The answer is 11." … and as a reward for getting it correct we ask another twenty questions just like it.

What some teachers are doing is trying to *tweak* the question and ask it a different way, for example:

I have two counting numbers that add to 11. What might the numbers be?

[Counting numbers = positive whole numbers including zero]

I write this under the 'open' label on the diagram:

CLOSED	OPEN	EXTENDED INVESTIGATION
6 + 5 ___	? + ? 11	How many solutions exist? How do you know you have found them all?

What is the answer to the question now?

At this stage it becomes apparent there are several solutions:

The question is now a bit more open than it was before, allowing students to tell you things like 8 + 3, or 10 + 1, or 11 + 0 etc.

Let's see what happens if the teacher 'tweaks' it even further with the investigative challenge *or* extended investigation question:

How many solutions are there altogether?

and more importantly, and with greater emphasis on the second question:

How could you convince someone else that you have found them all?

Now the original question is definitely different - it still involves the skills of addition but now also involves thinking, reasoning and problem solving skills, strategy development and particularly communication skills.

Young students will soon tell you the answer is 'six different ones', but they must also confront the communication and reasoning challenge of convincing you that there are only six and no more.

Example Two: *Finding Averages*

Again, as I go through this example, I write it into the diagram on the board in the relevant sections.

The CLOSED question is: *11, 12, 13 - find the average*

Tweaking this makes it an OPEN question and it becomes:

I have three counting numbers whose average is 12. What might the numbers be?

Students will often say:

10, 12, 14 ... or 9, 12, 15 ... or even 12, 12, 12

After realising there are many answers, you can tweak it some more and turn it into an EXTENDED INVESTIGATION:

How many solutions exist? ... AND ...

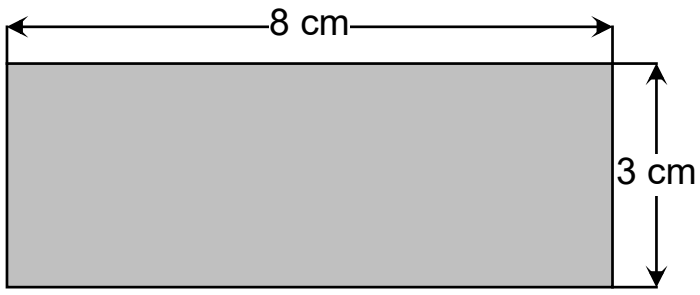
How do you know you have found them all?

Now the question is of a quite different nature. It still involves the arithmetic skill, but has something else as well - and that something else is the thinking, reasoning and communication skills necessary to find all of the combinations and convince someone else that you have done so.

By the time a student announces, with confidence, there are 127 different ways (which there are) that student will have engaged in all of these aspects, ie: the skill of calculating averages, (and some combination number theory) as well as significant strategy and reasoning experiences.

Example Three: *Finding the Area of a Rectangle*

A typical CLOSED question is:



Find the area. Find the perimeter.

The OPEN question is:

A rectangle has 24 squares inside:

What might its length and width be?

What might its perimeter be?

The EXTENDED INVESTIGATION version is:

Given they are whole number lengths, how many different rectangles are there? … AND …

How do you know you have found them all?

In summary, mathematics teachers are trying to convert *some* (not all) of the many closed questions that populate our courses and 'push' them towards the investigation direction. In doing so, we keep the skills we obviously value, but also activate the thinking, reasoning and justification skills we hope students will also develop.

This sequence of three examples hopefully shows two major features:

- That skills and problem solving can 'live alongside each other' and be developed concurrently.
- That the process of creating open-ended investigations can be done by anyone - just go to any source of closed questions and try 'tweaking' them as above. If it only worked for one question per page it would still provide a very large supply of investigations.

In terms of the effect of the talk on parents, I have usually found them to be reassured that we are not compromising important skill development (and nor do we want to). The only debate then becomes whether the additional skills of thinking, reasoning and communication are also desirable.

I've also been told that parents appreciate it because of the essential simplicity of the examples - no complicated theoretical jargon.

A Working Mathematically Curriculum

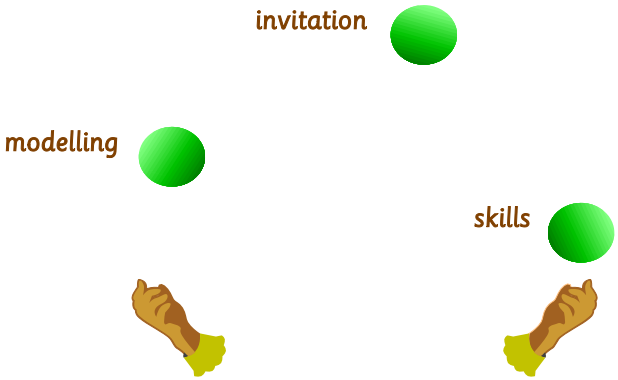
An Investigative Approach to Learning

The aim of a Working Mathematically curriculum is to help students learn to work like a mathematician. This process is detailed earlier (Page 8) in a one page document which becomes central to such a curriculum.

The change of emphasis brings a change of direction which *implies and requires* a balance between:

- the process of being a mathematician, and
- the development of skills needed to be a *successful* mathematician.

This journey is not two paths. It is one path made of two interwoven threads in the same way as DNA, the building block of life, is one compound made of two interwoven coils. To achieve a Working Mathematically curriculum teachers need to balance three components.



The task component of **Maths With Attitude** offers each pair of students an invitation to work like a mathematician.

The Maths300 component of **Maths With Attitude** assists teachers to model working like a mathematician.

Content skills are developed in context. They *are* important, but it is the application of skills within the process of Working Mathematically that has developed, and is developing, the human community's mathematical knowledge.

A focus for the Working Mathematically teacher is to help students develop mathematical skills in the context of problem posing and solving.

> *We are all 'born' with the same size mathematical toolbox, in the same way as I can own the same size toolbox as my motor mechanic. However, my motor mechanic has many more tools in her box than I and she has had more experience than I using them in context. Someone has helped her learn to use those tools while crawling under a car.*

Afzal Ahmed, Professor of Mathematics at Chichester, UK, once quipped:

> *If teachers of mathematics had to teach soccer, they would start off with a lesson on kicking the ball, follow it with lessons on trapping the ball and end with a lesson on heading the ball. At no time would they play a game of football.*

Such is not the case when teaching a Working Mathematically curriculum.

Elements of a Working Mathematically Curriculum

Working Mathematically is a K - 12 experience offering a balanced curriculum structured around the components below.

Hands-on Problem Solving Play

Mathematicians don't know the answer to a problem when they start it. If they did, it wouldn't be a problem. They have to play around with it. Each task invites students to play with mathematics 'like a mathematician'.

Skill Development

A mathematician needs skills to solve problems. Many teachers find it makes sense to students to place skill practice in the context of *Toolbox Lessons* which *help us better use the Working Mathematically Process* (Page 8).

Focus on Process

This is what mathematicians do; engage in the problem solving process.

Strategy Development

Mathematicians also make use of a strategy toolbox. These strategies are embedded in Maths300 lessons, but may also have a separate focus. Poster Problem Clinics are a useful way to approach this component.

Concept Development

A few major concepts in mathematics took centuries for the human race to develop and apply. Examples are place value, fractions and probability. In the past students have been expected to understand such concepts after having 'done' them for a two week slot. Typically they were not visited again until the next year. A Working Mathematically curriculum identifies these concepts and regularly 'threads' them through the curriculum.

Planning to Work Mathematically

The class, school or system that shifts towards a Working Mathematically curriculum will no longer use a curriculum document that looks like a list of content skills. The document would be clear in:

- choosing genuine problems to initiate investigation
- choosing a range of best practice teaching strategies to interest a wider range of students
- practising skills for the purpose of problem solving

Some teachers have found the planning template on the next page assists them to keep this framework at the forefront of their planning. It can be used to plan single lessons, or units built of several lessons. There are examples from schools in the Curriculum & Planning section of Maths300 and a Word document version of the template.

Unit Planning Page

Class

Topic

Pedagogy	Problem Solving In this topic how will I engage my students in the Working Mathematically process?	Skills
How do I create an environment where students know what they are doing and why they have accepted the challenge?		Does the challenge identify skills to practise? Are there other skills to practise in preparation for future problem solving?

Notes

As a general guide:

- Find a problem(s) to solve related to the topic.
- Choose the best teaching craft likely to engage the learners.
- Where possible link skill practice to the problem solving process.

More on Professional Development

For many teachers there will be new ideas within **Maths With Attitude**, such as unit structures, views of how students learn, teaching strategies, classroom organisation, assessment techniques and use of concrete materials. It is anticipated (and expected) that as teachers explore the material in their classrooms they will meet, experiment with and reflect upon these ideas with a view to long term implications for the school program and for their own personal teaching.

Being explored 'on-the-job' so to speak, in the teacher's own classroom, makes the professional development more meaningful and practical for the teacher. This is also a practical and economic alternative for a local authority.

Strategic Use by Systems

From Years 3 - 10, **Maths With Attitude** is designed as a professional development vehicle by schools or clusters or systems because it carries a variety of sound educational messages. They might choose **Maths With Attitude** because:

- It can be used to highlight how investigative approaches to mathematics can be built into balanced unit plans without compromising skill development and without being relegated to the margins of a syllabus as something to be done only after 'the real' content has been covered.
- It can be used to focus on how a balance of concept, skill and application work can all be achieved within the one manageable unit structure.
- It can be used to show how a variety of assessment practices can be used concurrently to build a picture of student progress.
- It can be used to focus on transition between primary and secondary school by moving towards harmony and consistency of approach.
- It can be used to raise and continue debate about the pedagogy (art of teaching) that supports deeper mathematical learning for a wider range of students.

Teachers in Years K - 2 are similarly encouraged in professional growth through **Working Mathematically with Infants**, which derives from Calculating Changes, a network of teachers enhancing children's number skills from Years K - 6.

In supporting its teachers by supplying these resources in conjunction with targeted professional development over time, a system can fuel and encourage classroom-based debate on improving outcomes. There is evidence that by exploring alternative teaching strategies and encouraging curriculum shift towards Working Mathematically, learners improve and teachers are more satisfied. For more detail visit Research & Stories at:

- http://www.mathematicscentre.com/taskcentre/do.htm

We would be happy to discuss professional development with system leaders.

Web Reference

The starting point for all aspects of learning to work like a mathematician, including Calculating Changes, and the teaching craft which encourages it is:

- http://www.mathematicscentre.com/mathematicscentre

Appendix: Recording Sheets

Win At The Fair: On-going Results Sheet

Sheet Number ....

One row per pair. Enter your initials in the 'Tally for...' box.

	20¢	50¢	$1	$2	$3	$4	$5
Tally for							
Your totals							
Totals so far							

One row per pair. Enter your initials in the 'Tally for...' box.

	20¢	50¢	$1	$2	$3	$4	$5
Tally for							
Your totals							
Totals so far							

One row per pair. Enter your initials in the 'Tally for...' box.

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Tally for							
Your totals							
Totals so far							