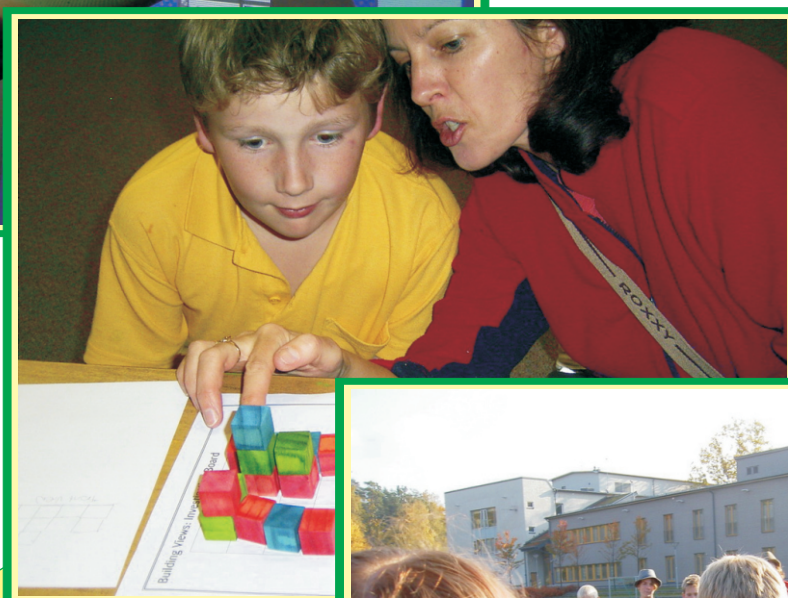
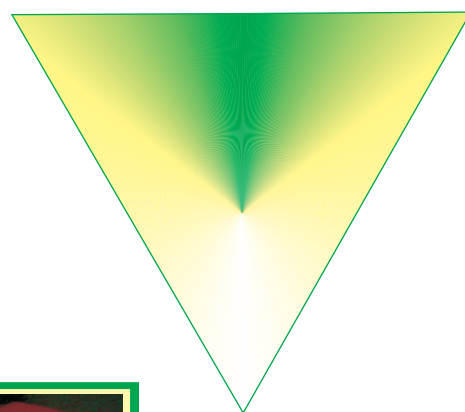
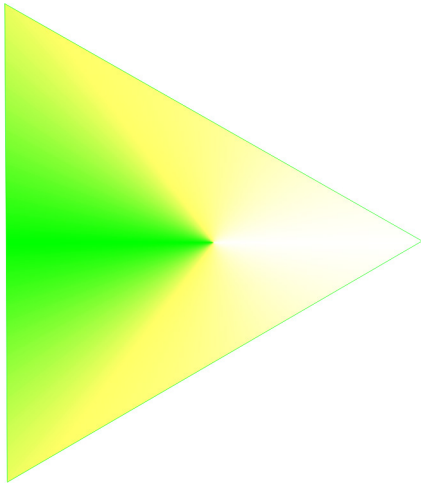


Maths300 ETU TE

a framework for exploring maths300



DOUG
WILLIAMS



Maths300

ETUTE

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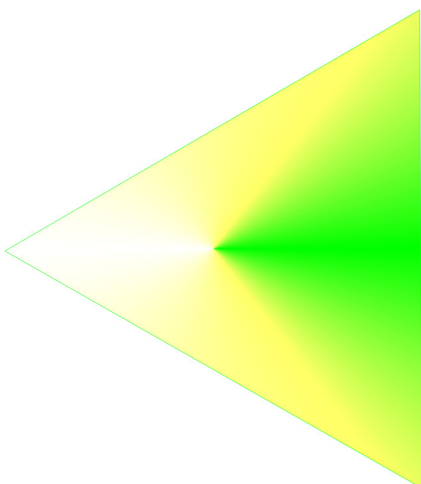
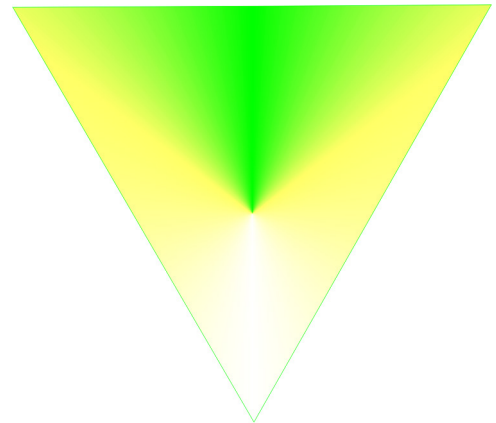
Author ... Doug. Williams ... © 2010.

First published 2010. Revised 2015.

ISBN: 978 0 9807630 1 0

Design and Desktop Publishing:

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Introduction

History

Maths300 began in early 2000 to:

- ♦ engage teachers in debate about teaching craft that fascinates, captivates and absorbs students of all ages in learning to work like a mathematician.

It was released with 25 lessons to begin a collection that might one day reach 300.

- ♦ Each lesson is so rich that it might take several periods to explore.

A mathematician is never finished with a problem. They simply choose to end their work on it for now.

- ♦ Each lesson helps teachers model what it means to work like a mathematician. (See Page 4)

When learning to work like ... anything at all really ... modelling on its own is not enough. You also need the invitation to try the process independently. Tasks from the Mathematics Task Centre offer such invitations and, in addition, each task has a life as a whole class investigation. Consequently, about half of Maths300 lessons develop from tasks.

- ♦ Each lesson addresses content found in any official curriculum document.

Maths300 content statements are generic. It is not tied to any particular syllabus document; rather, it is a resource for all. Consequently Maths300 soon began receiving wide recognition.

By the beginning of 2004 the Maths300 collection had expanded to 135 lessons and the site had 936 members. (The 1000th member joined in April of that year.)

- ♦ New members were asking how to find their way through such an extensive resource.
- ♦ Longer term members were finding they had settled into a small set of favourite lessons and knew that there was other 'great stuff' they hadn't yet tried.

Maths300 ETuTE began in February 2004 as a professional development tool in the form of:

- ♦ "...a regular email to guide you deeper into the richness of Maths300. An electronic tutorial to help you find out more about your site."

It continued until April 2009.

Each issue began with a question and, as shown by the emails below that were received following the distribution of the final issue, it did fulfil its professional development purpose.

Thanks for all your hard work over the years. I know it has made a huge difference to the way maths is taught in schools.

The etutes have been great ... We regularly use Maths 300 in our school - it makes sense to students and the teachers!

Well done with all the work (...on...) Maths300 etc. over the years. It certainly is a great resource.

Have enjoyed your posts over the years. Loved the way you signed off with a story about the just-arrived generation!
(See April 24th 2009)

Thank you for your ever interesting etutes for the last 5 years. I have found them useful, informative and some inspiring.

Since 2009 membership of Maths300 has continued to expand and members are still seeking to make best use of the breadth and depth of the resource.

- ◆ Consequently the first five years of ETuTE have been published in this book to support in-house Maths300-based professional development.

With only minor editing, the original text proved as relevant today as it was when first written. Used in any of the ways described below it will help you expand your use of Maths300 in a rich and rewarding way.

Since 2009 Maths300 lessons have continued to expand. At time of writing there were 189. Also many teachers have expanded existing lessons with exciting contributions from their classrooms.

- ◆ So, as you work through these tutorials, keep in mind that there is more to Maths300. The site continues to grow.

Use the choices in Search Maths300 to hunt for lessons to complement or extend each tutorial you work with.

Professional Development

Use this book to help yourself or your staff explore Maths300. You could:

- ◆ Flick the pages and randomly find questions that interest you. Then trial what is suggested and return to faculty meetings to compare experiences.
- ◆ Start at the first ETuTE and follow in sequence on a regular basis, trialing what is suggested before beginning the next ETuTE. With this approach, you will have material for at least five years of faculty meetings.

- ◆ Use the Year Level Finder (Page 58) to select those tutorials relevant to your current teaching load. Then either select at random or follow in sequence.
- ◆ Use the Content Finder (Page 60) to select content relevant to your current syllabus demand, trial what you find and evaluate it against the approach 'we would have normally used'.

Maths With Attitude: the next step

Long before reaching the end of this manual it is likely that you will recognise the learning benefits accruing to your students; perhaps also a new enthusiasm in your own teaching. Exploration leads to questions such as:

- ◆ How do we write a curriculum document that ensures these lessons are included?
- ◆ How do we choose and use Maths300 lessons so that students aren't exposed to the same investigation year after year?
- ◆ How do we prepare a curriculum that builds from year to year on the learning these lessons provide?

Maths With Attitude, addresses these issues and more in a set of kits that make Working Mathematically the core of mathematics education from Years 3 - 10.

- ◆ Hands-on tasks
- ◆ Whole class investigations
- ◆ Skill practice in context
- ◆ Weekly planners
- ◆ 25 weeks of investigation at each year level

Working Mathematically with Infants, available only to Calculating Changes members, prepares children in Years K - 2 to work like a mathematician.

- ◆ Poly Plug
- ◆ Threaded activities
- ◆ Whole class investigations
- ◆ Weekly planners
- ◆ 20 weeks of number sense, concepts and skills at each year level

When used together, WMI and MWA provide a complete core curriculum in learning to work like a mathematician for all compulsory years of schooling.

There is no reason for mathematics to be taught the way it always has been.

For more detail:

- ◆ Visit Maths With Attitude at:
<http://www.mathematicscentre.com/taskcentre/mwa.htm>
- ◆ Visit Working Mathematically with Infants at:
<http://www.mathematicscentre.com/calchange/index.htm#wmi>

Working Mathematically

First give me an interesting problem.

When mathematicians become interested in a problem they:

- ◆ Play with the problem to collect & organise data about it.
- ◆ Discuss & record notes and diagrams.
- ◆ Seek & see patterns or connections in the organised data.
- ◆ Make & test hypotheses based on the patterns or connections.
- ◆ Look in their strategy toolbox for problem solving strategies which could help.
- ◆ Look in their skill toolbox for mathematical skills which could help.
- ◆ Check their answer and think about what else they can learn from it.
- ◆ Publish their results.

Questions which help mathematicians learn more are:

- ◆ Can I check this another way?
- ◆ What happens if ...?
- ◆ How many solutions are there?
- ◆ How will I know when I have found them all?

When mathematicians have a problem they:

- ◆ Read & understand the problem.
- ◆ Plan a strategy to start the problem.
- ◆ Carry out their plan.
- ◆ Check the result.

A mathematician's strategy toolbox includes:

- ◆ Do I know a similar problem?
- ◆ Guess, check and improve
- ◆ Try a simpler problem
- ◆ Write an equation
- ◆ Make a list or table
- ◆ Work backwards
- ◆ Act it out
- ◆ Draw a picture or graph
- ◆ Make a model
- ◆ Look for a pattern
- ◆ Try all possibilities
- ◆ Seek an exception
- ◆ Break a problem into smaller parts
- ◆ ...

If one way doesn't work, I just start again another way.

Have you found the Palindromes lesson?

Years: 4 - 10

February 16th 2004

A palindrome reads the same forwards and backwards. For example: MUM, NOON, MADAM and ETuTE. A Palindromic Numeral also reads the same backwards and forwards. For example: 33 and 34543. Lots of kids get hooked on Palindromes, both numeric and linguistic.

For example a week or so back my adult daughter sent me this email:

*Hi Dad,
I just realised there will be no more palindromic dates until 11/1/11 - well,
I'll try to remember to email you then (or use whatever form of
communication is normal then!).
Love,
Mouse*

Is she correct?

Perhaps this is the right time of the year for the Palindromes lesson. Perhaps even a brief numeracy/literacy unit.

The curious thing about Palindromes in mathematics is that, starting with any number and following with a sequence of reversing the digits, adding, reversing the digits, adding ... no one has ever found a sequence that doesn't end with a palindrome! On the other hand, no one has been able to prove that this will always happen. For example:

$$78 + 87 = 165$$

$$165 + 561 = 726$$

$$726 + 627 = 1353$$

$$1353 + 3531 = 4884$$

The lesson is an investigation into this curiosity and it is supported by software. Visit:

Maths300 Lesson 103, *Palindromes*

Content

- arithmetic skills
- place value skills

What value do you place on Place Value?

Years: K - 11

March 16th 2004

Understanding the concept of, and being able to operate within, the base 10 place value system is an important tool in the mathematician's toolbox. For example you can't get very far with a problem like *Number Tiles* (Lesson 28) without it. It is crucial to the arithmetic used to find even one solution, and essential to the general proof of the conditions that govern all solutions.

Might your students benefit from further development of their place value concepts and skills?

If so, consider Lesson 35, *Nine & Over*. Consider too the teaching strategy of 'Threading' which is embedded in the lesson. This is a teachers' term developed in the Calculating Changes network to describe rich, revisitable activities that can be included for short periods within many lessons over a long period of time, eg: 10 -15 minutes, twice a week for 10 weeks. The structure of the activity remains the same - and in fact becomes an old friend to the students - but the challenge is made fresh each time by changing the numbers involved.

Nine & Over is based around a structured abacus game and software that stimulates, challenges and extends. The game is presented using Poly Plug, but it is easily adapted to a printed board and screw caps from soft drink bottles (easy to pick up and move), or counters. The lesson even provides a print master for the board. Once the structure of the game is learned it is easily threaded into the curriculum.

On the one screen, the software represents numbers as an abacus (0 - 9), calculator symbols, an odometer (the device that measures distance travelled in a car) and in words. The software also challenges students to count, add and subtract within these multiple place value representations.

- American users beware. We say and write 153 as 'one hundred AND fifty-three'. Most of you say and write 'one hundred fifty-three'. A difficulty, or an opportunity for further developing awareness of the world 'outside'? The software allows either system to be selected.

Threading the software and the game develops understanding over time in a manner that parallels the development of place value in the history of mathematical thinking. It also provides the time and personal challenge that Constructivists argue is necessary for learners to construct their own understanding. Kindergarten or Year 8; there is something in this lesson for all levels. Visit:

Maths300 Lessons 28, *Number Tiles* and 35, *Nine & Over*.

Content

- addition & subtraction
- arithmetic skills
- concept of proof
- counting
- place value concepts
- place value skills

Years: K - 12

How can Maths300 help me identify and articulate when I'm doing a good job?

May 17th 2004

Editor's Note: This issue was written by Charles Lovitt.

If I had an ambition about Maths PD it would be to enable every teacher across the country to talk with each other - seriously and deeply - about what it means to create a classroom we would all be proud of. So what can we put on the table as fuel for such discussions? The best starting point seems to be what we do all day every day - teach lessons.

So that's what I see in Maths300; lots of lessons from classrooms, the purpose of which is to create a climate of discussion and reflection about desirable teaching and learning features.

For example, *Cube Nets* (#116) and *Temperature Graphs* (# 83) both beautifully set up a contrast between the investigative approach and the more directed 'text approach'.

Trialing the lesson (which came from someone else's classroom) enables us to 'have a conversation' with that teacher about the relative merits of each approach.

Similarly I hear much comment in the Numeracy Debate about 'Informed Citizenship' and 'Relevant Contexts'. So what do these words mean when they come alive in classrooms? Lessons such as *Win At The Fair* (#1) and *Sporting Finals* (#149) allow me to consider and discuss what these words might really mean in practice.

The 139 lessons currently available multiply the above examples many times over in connection with:

- effective use of technology, or
- multiple intelligence teaching, or
- effective use of group work, or
- mathematical conversation, or...

so the opportunity exists for me to get serious about thinking through many issues of classroom practice.

Some of these are discussed in depth in the context of the popular Lesson 5, *Greedy Pig*, which has a special link to a document exploring the multiple features of the lesson.

As I trial these lessons and reflect on their features the real benefit of Maths300 starts to shine through. The lessons become guideposts, or templates, to support me in creating rich lessons of my own design.

Content

- curriculum planning

I am an infant teacher. What's in Maths300 for me?

Years: K - 2

June 16th 2004

Firstly, the same thing as for all other teachers - the framework of learning to work like a mathematician. This one page document (Page 4) captures a vision, a language and a practice which can be successfully introduced to children from the day they enter school.

Mathematicians solve problems. So do little kids - every day. Some of those problems can be presented to them through the mathematics curriculum; in that way, learning to work like a mathematician is something that grows through school, rather than something left until the older grades.

The Working Mathematically process is consistent with all district, state and national guidelines throughout the world, so in using it as your framework you are placing the formal outcomes expected by your system within a context, and not burdening yourself with extra work. This is illustrated by trial school teachers who have recorded their classroom experiences in specific Maths300 lessons. Access these lessons through:

Search Engine

Use the 'Advanced Lesson Search' link and set the drop down box under the heading Year Level to Lower Primary. If you don't change any other setting, but simply scroll down the page and click Search, the engine will return all the lessons it currently holds which have a Lower Primary component. Each lesson is returned with a paragraph of summary information. The first line of the summary is the Year Levels at which the lesson has been used (0 = Year K). You will notice some that simply state K, 1, 2; whereas others add more year levels. That's one of the great things about Maths300 lessons - they are rich enough to use at a range of levels, resulting in new learning each time they are visited.

Lesson Library

This is another form of the Search Engine, but it doesn't supply the summary information. When you enter the Lesson Library it lists all available lessons in alphabetical order. However there are sorting arrows at the top of each column. If you click the up arrow above the last column the list will resort by Year Level. All the K+ lessons will come to the top. From here, once again, the choice is yours. All are framed within the Working Mathematically process. All are designed to continue professional debate about the features of happy, healthy, cheerful, productive, inspiring classrooms. Perhaps my favourites with very young kids are:

- Lesson 15, *Calculator Walk*, physical involvement & calculators
- Lessons 73, *Halving Squares*, cutting kinder squares in half & joining them in new ways
- Lesson 91, *Around Our Neighbourhood*, wide content coverage & cross-curricula links
- Lesson 141, *Jumping Joey*, problem solving based on a children's story
- Lesson 156, *Chart Strategies*, concrete materials lead to software representation

Content

- | | | |
|-----------------------|-----------------------------|-----------------------|
| • numeral recognition | • spatial & number patterns | • concept of fraction |
| • counting | • addition & subtraction | • language of shapes |

With the Olympics coming up, what can I find in Maths300?

Years: K - 12

July 13th 2004

Potato Olympics has to be the first response. The Classroom Contribution in this lesson from Kingston Primary School begins:

*There was a buzz of excitement in the class when I produced a bag full of something. I removed one of the bag's contents and held it up.
"It's just a potato," said Brad.
I replied, "This is not just a potato! It's a special potato. I'm going to give you all a potato and you're going to draw in its features and give it a name."
Each child was given a potato and a permanent marker - permanent because in the later water experiments the drawing won't wash off. They then proceeded to put in the features on their potatoes and name them.*

You can find photos of some character potatoes in both the lesson plan and the Classroom Contributions.

The lesson works beautifully in primary school, but adventurous secondary teachers might also give it a go. The amount of mathematics that teachers have teased out of the lesson is amazing, and another advantage is its strong cross-curricula links.

Anything else? ... Well...

- Athletes have to be fit. How about examining fitness using the lesson *Pulse Rates* (#68)?
- The Olympic rings might draw our attention to the lessons *Circumference of a Circle* (#45) and *Area of a Circle* (#43). Some creative teachers will also, no doubt, be able to weave in the circle-based options of *Fraction Estimation* (#33) to build a mini-unit around the circle.
- Suppose the area of Greece is worth 1. What is the area of France, Germany, Holland, Sweden... or any of the other competing countries? The lesson *Country Maps* (#50) can easily be adapted.
- If you are an infant teacher you could adapt the lesson *Where Do We Sit?* (#8) to investigate the arrangements in which a family of three can sit to watch the swimming finals on TV.
- Greece is the historic home of Western Mathematics. Could you make a link with the lesson, *Pythagoras & Other Polygons* (#157)? ...or the lessons *Gauss Beats the Teacher* (#12) for the middle years and *Pyramid Puzzle* (#138) for the senior years which both include a geometric approach to number in the Greek tradition.
- You could adapt the lesson *Famous Mathematicians* (#124) to be Famous Athletes, make your own cards and investigate the mathematics of collecting a complete set of cards.

And, if you have hands-on tasks from the Mathematics Task Centre you can organise your own Puzzle Olympics: <http://www.mathematicscentre.com/taskcentre/puzzolym.htm>

Content

- | | | |
|----------------------------|----------------------|-------------------|
| • algebraic generalisation | • combination theory | • measurement |
| • area & perimeter | • concept of proof | • number patterns |

Are you ready for the AFL finals?

Years: 1 - 12

August 12th 2004

The AFL football final series is almost upon us, so three Maths300 lessons seem very timely! A huge feature of each of these is the range of grade levels at which they can be effectively used!

1. Sporting Finals (#149) Years 2 to 12
2. Dice Footy (#161) Years 2 to 12
3. Football Ladder (#69) Years 1 to 10

Sporting Finals (Final Eight)

This lesson has become extraordinarily popular. The central aspect is a simulation of the structure of the final eight and the concept of 'fairness'. Eight students in a group 'become' the finalists and simulate the final series using dice. The data collected highlights the mathematical structure - and how one team in particular could see the structure as 'unfair'. The supporting computer software allows students to enter or randomly choose the AFL teams and also simulate the final series many thousands of times. The context is extremely rich mathematically, but another important outcome is what could be described as 'socially useful knowledge' as students come to understand how this real event is constructed.

Editor's Note: In 2010 Lesson 179, *Sporting Finals - NRL Rugby League*, was added to Maths300 to extend this investigation to Australia's second major football code.

Dice Footy

A delightfully simple game to play - roll two dice, one for goals and one for behinds. For example, Ben rolls a '4' then a '5'. This means Ben scores 4 goals 5 behinds = 29 points in the first quarter of a game against Sarah. Sarah rolls 6 goals 2 behinds for 38 points. They then play 4 quarters and aggregate the scores for a full game.

Collecting class data on the board then becomes a starting point for all sorts of number and probability opportunities such as the expected or average score [Answer $14 \times 14 = 98$ points], or the chances of a draw. Computer software allows the game to be simulated and thousands of games to be played to further explore the underlying mathematics

Football Ladder

This logic and number lesson uses the context of the AFL football ladder to provide a series of logic puzzles. Group problem solving is the highlight and major purpose as students develop strategies to make sense of all the clues. An additional benefit comes when students are challenged to design a similar puzzle - in this students confront the need to provide 'sufficient' clues without being 'contradictory'.

The lesson contains a range of puzzles at various difficulty levels which ensures there is 'something for everyone' to enjoy and potential success for all.

Content

- | | | |
|------------------------------|----------------------|--------------------------------|
| • collecting/tabulating data | • percentages | • average/expected result |
| • long run frequency | • combination theory | • probability |
| • calculating probabilities | • arithmetic skills | • language of position & order |

Do you mean I could teach from Crossing The River for a whole term?

Years: 5 - 8

September 15th 2004

With a sense of excitement mixed with amazement, this question was asked recently by a participant in the professional debrief session following a Discussion Lesson. The Year 7 class had explored Lesson 23, *Crossing The River*, for 90 minutes and would have happily continued. In the 90 minute debrief the teachers were exploring the pedagogy of the lesson and the mathematical extensions of the problem.

The question didn't really need to be answered. It was going to be possible for that teacher to teach from *Crossing The River* for a whole term because he could see/sense that the problem was rich enough to support that.

Interestingly, on reflection, some teachers realised that when the Year 7 lesson had started they didn't think there was enough in it to last the assigned hour and a half.

Perhaps the more important question was the implied one:

What would the curriculum look like - how would it 'feel' to students and teachers - if it were constructed around Pivot Problems such as this which could be visited and revisited to 'milk' their richness?

- Maths would be less likely to look like a passing procession of changing chapters.
- Maths would be less likely to appear as a set of skill hurdles stretching on to educational infinity.
- Skill and knowledge development might have more sense in the context of problems that require or generate their need.
- Maths might more closely reflect the way mathematics professionals work.
- More teachers and students might find maths more intellectually and emotionally satisfying.

One teacher has looked at the lesson with new eyes. You are invited to review it too. There is even a Classroom Contribution adapting the lesson to younger children.

Further, it was the professional development context which enabled the teacher's insight. In what ways could a PD program, delivered over time, support curriculum shift in your school, cluster, district, system? Consider options at Mathematics Centre Professional Development: <http://mathematicscentre.com/taskcentre/pdffrommc.htm>

The lesson involves a huge range of content, a summary of which is:

Content

- | | | |
|----------------------------|--------------------------|-------------------------|
| • algebraic generalisation | • graphing ordered pairs | • number patterns |
| • concept of variable | • interpreting graphs | • ordered pairs |
| • functions | • linear graphs | • solution of equations |
| • gradient | • mental arithmetic | • substitution |

Will you help us to continue developing Maths300?

Years: K - 12

October 13th 2004

Editor's Note: This issue was distributed in advance of a major review meeting to review Maths300. Considerable parts of the ETuTE text are no longer relevant because they have (or have not) since been included in Maths300. However, our work in all parts of Mathematics Centre is driven by successful experiences reported by teachers. Therefore, parts of this issue have been included in this book because it is always the case that we want you to help us continue to develop.

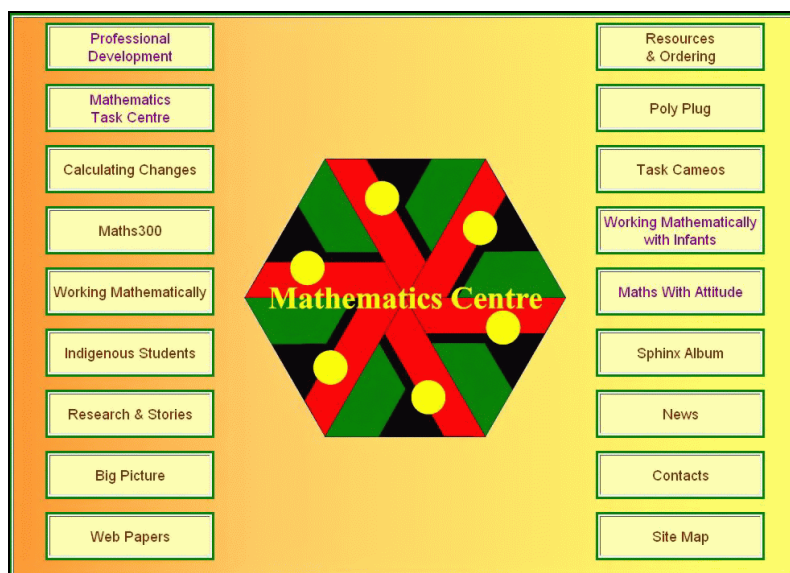
We are approaching our 150th lesson and five years of development. It is appropriate that we review our service to date as we continue the journey towards the 300th lesson. Maths300 is a 100% professional development service derived from teachers' experience and intending to support the continuing search for better mathematics education.

Only you can tell us if we are on track and guide us in our growth. Please take a few minutes to write your thoughts on the following points and send them to Doug.

Williams, doug@blackdouglas.com.au, Project Manager, Mathematics Centre

- Has Maths300 influenced your teaching practice? How?
- How much use do you make of the Maths300 site?
- Tell us what you like about the site.
- What are the issues in your school, if any, which get in the way of better use of the site?
- Any thoughts about improving the site?

If you wish, please extend your comments to include any other part of Mathematics Centre.



<http://www.mathematicscentre.com>

Are you making good use of the Curriculum & Planning link?

Years: 3 - 10

November 11th 2004

All the feedback we get indicates that teachers believe there are great lessons on Maths300. *The kids love 'em* is a common message we receive.

Perhaps now is the time of the year to think about how you have been using the resource. Could you use the lessons in a more integrated way next year? Is it time to seriously consider a curriculum shift away from teaching mathematics and towards learning to work like a mathematician?

Maths With Attitude is an excellent resource to support the Working Mathematically curriculum shift. From 2005 kits will be available for Years 3 - 10 with 25 weeks of problem solving resources in each kit. MWA kits are a planned and purposeful way to begin membership of Maths300, or a supportive and detailed way to get more from it. But you have to spend money to go that direction.

Perhaps the Curriculum & Planning link on the Member's Page can guide you into integrated use of Maths300. The sample units it provides (within its Resources link) have been prepared by your colleagues. It is also a place where you can contribute units for others to use.

In this link you will find:

- detailed, yet succinct planning advice
- a Unit Bank with units already prepared in:
 - ...Creating and solving equations
 - ...Fractions: understanding and estimating
 - ...Perimeter & area
 - ...Angles, triangles and shapes
- a section of other ideas and examples from teachers who are working on changing their curriculum direction.

Whichever way you choose to approach your curriculum shift, professional development is available to assist if you wish. Best of luck.

Content

- curriculum planning

How do I find what I need on Maths300?

Years: 7 - 12

February 3rd 2005

Well actually, the question had two parts and came from Ray Peck, President of the Mathematical Association of Victoria. It arrived in this form:

- ◆ How can a secondary teacher easily and quickly find and select a Maths 300 open-ended investigation that links with the curriculum area being studied (or studied last month)?
- ◆ Would you describe all Maths 300 investigations as open-ended?

We thought our response may be useful to other members, so here it is:

Answering in reverse order:

Open-ended?

I wouldn't describe all Maths300 lessons as open-ended, but many are. The purpose of Maths300 is simply to engage teachers in debate about the features of better maths lessons and 'open-endedness' is only one feature to consider. Group work, integrated software, estimation, first hand data, and range of others take the focus in various lessons. If by open-ended you mean students choosing their own direction within a problem, then take a look at the new lesson *Cars in a Garage* (#128).

Finding lessons?

There are a couple of ways to use the Maths300 search facilities. If you go to the Lesson Library from the Members' Page the last column is Year Level. You can resort the list based on this column using the sorting arrows at the top. The default setting for the list is the alphabetic order of the lesson names. If you want the more 'senior lessons' first, click the down arrow above the second column. Then use the code for Strand and the name of the lesson to make your choices of lessons to review.

The other facility is the search engine. Choose Advanced Lesson Search from the Members' Page. Set the drop down boxes and tick boxes to what you require and click Search. I just did a search with only the 'open ended' box ticked and it turned up 63 lessons. Each of them is presented with a couple of paragraphs of summary information to help you decide which ones to review. I then did the same search but added the keyword 'fractions' and the engine offered me 10 lessons.

Through the Free Sample Tour link non-members can access a complete list of currently available Maths300 lessons with summaries, but they do not have access to search facilities.

Content

Editor's Note: Based on *Cars in a Garage*.

- | | | |
|----------------------|---------------------------|-------------------|
| • 1:1 correspondence | • curriculum planning | • visual patterns |
| • combination theory | • multiplication | |
| • counting | • multiplicative thinking | |

Are you starrng with your maths classes?

Years: 3 - 10

February 15th 2005

Hunting For Stars, Lesson 51, is a gem. Good value everywhere from Year 3 to Year 10. Easy to begin in a practical manner in mixed ability classes. Small group work is a feature. The physically involving beginning leads gently into further pencil and paper investigation supported by the Investigation Sheets supplied. The companion software offers additional opportunities for small group and individual self-directed investigation.

The lesson begins with groups of eight students sitting in a circle and passing a ball of wool to each other according to a rule. The challenge is to predict which rules produce stars. In all the classrooms I have used it, the kids just seem to get hooked, and at the end of a couple of lessons we can look back with pride at what we have accomplished as mathematicians.

The lesson presentation is full of photos and diagrams and the mathematics involved ranges from properties of polygons, through angles, to basic number facts, factors, multiples and primes, into algebraic generalisations and fractions. There is even some neat Euclidean Geometry and a dose of Trigonometry for those using the lesson with older students.

But be warned. You will need to raid someone's knitting basket to get started!

Editor's Note: Greg Lee, Rosebud Secondary College, made use of *Hunting For Stars* as part of our six day Maths on the Move program titled *Engineering 'aha' Moments in Algebra*. He has produced a photo slide show of his class at work, an additional Investigation Guide and a test to follow a *Hunting for Stars* unit. Greg's extra work is available to teachers through the Research & Stories link of Mathematics Centre.

- <http://mathematicscentre.com/taskcentre/do.htm>

The article is titled: *Engineering 'aha' Moments in Algebra: Reflections on a 6 Day PD Program*. It includes the work of several teachers in the program and Greg Lee's work is easily found from a link near the beginning of the article.

Content

- | | | |
|----------------------------|----------------------|------------------------------|
| • angles & polygons | • Euclidean geometry | • multiples, factors, primes |
| • algebraic generalisation | • language of shapes | • symbolic representation |
| • arithmetic skills | • measurement | • trigonometry |

Years: 3 - 10

How many times have you changed your text book since 1992...?

March 18th 2005

... or even since 2000 when Maths300 began?

This question was asked by Per Berggren, one of our Swedish members who was visiting at the beginning of March. Per has been teaching with tasks since the mid-90s and has been using Maths300 since its inception. His point was two-fold:

- Tasks and other problems rich in mathematical content and teaching craft possibilities haven't changed in all that time; other than becoming richer through the involvement of more and more teachers.
- It is tasks and Maths300, not text books, which can link teachers, schools and students internationally in jointly solving problems in the way mathematicians do.

It is worth thinking about.

Professor Dick Evans, Plymouth State College, New Hampshire, USA, once called Maths300 *the best math resource in the world*. The deep, broad and stable nature of both Maths300 and the Mathematics Task Centre resources can provide - worldwide - the core of a curriculum which helps all students to learn to work like a mathematician in happy, healthy, cheerful, productive, inspiring classrooms.

In the quicksand-based world of teaching where everything sometimes seems to change just for the sake of change, isn't it important to have a rock solid core in the curriculum?

Maths With Attitude goes even further in this direction. Teachers have selected tasks, and lessons from Maths300, for these kits which match Year levels and curriculum strands. They have mapped them into a week by week program to build structured core units from Years 3 - 10 which still leave many weeks to do the best of what you already do. Everything you need to build a Working Mathematically curriculum is 'in the box'.

Of course there is still preparation to do if you want the best from your kit. You do have to assign the weeks in the planner to actual terms and weeks in your school year. You do have to read lesson plans and try a few tasks yourself to grow your knowledge of the mathematician's problems involved.

But you don't have to:

- go to web to find what you need
- make choices about which tasks might be best for kids of the age you teach
- mine through the incredibly rich Maths300 mother lode to find nuggets which work with the kids you teach
- plan a sequence or unit to integrate the use of your choices.

Your colleagues have already done that work!

Find out more about *Maths With Attitude* and how it links Maths300 and tasks through the concept of learning to work like a mathematician from Mathematics Centre at:

<http://www.mathematicscentre.com/taskcentre/mwa.htm>

Content

- curriculum planning

How can I induct new staff into Maths300?

Years: 4 - 11

April 15th 2005

Of course there are many ways, but here is one you might consider:

At this school Maths300 is the core of our curriculum. It's an exciting professional development support program. In many ways it is like having an online training course in mathematics education. It is designed to stand beside the best of what you do already, rather than substitute as an alternative curriculum. The objective is to encourage teachers to research, plan and try one lesson (as a starting point) which is consistent with the content of the curriculum, but presented in a different way.

If you take this step you will begin engaging in the debate about features of mathematics lessons which make learning better for students. You can find a starting point lessons by using the Maths300 search engine.

Come on I'll show you.

Perhaps it is wise at this point to choose a lesson which is easy to state, easy to start and has heaps of maths. One candidate is Lesson 101, *Odds & Evens*. It is based on two simple rules:

- if the number is odd, multiply it by 3 and add 1
- if the number is even, halve it.

Then consider 13 as the first number. It produces this chain:

13 --> 40 --> 20 --> 10 --> 5 --> 16 --> 8 --> 4 --> 2 --> 1

The chain for 17 is:

17 --> 52 --> 26 --> 13 --> ...then we join the 13 chain.

Could it be that all numbers chain to 1 in this way?

Thus begins an extensive investigation reflecting many aspects of the work of a mathematician and offering opportunity for group work and class display. The companion software allows considerably more investigation of hypotheses in real time than could be possible by hand.

Finally, the time may be fast approaching when you won't need to induct new staff. Several teacher training institutions are already using Maths300 to help new teachers develop their mathematical skills and concepts, their problem solving skills and their understanding of best practice techniques.

Content

- arithmetic skills
- odd & even numbers
- prime numbers
- powers of 2

What's your area?

Years: 6 - 12

May 16th 2005

A circle should answer:

Pi times R squared.

...But you are a circle, where is your square?...

Down there, in the corner. It's my Corner Square.

...Yeah, so?

Well you can see that my area is less than four of my Corner Squares?

...Yeah... and my guess is that it is bigger than 3 of your Corner Squares.

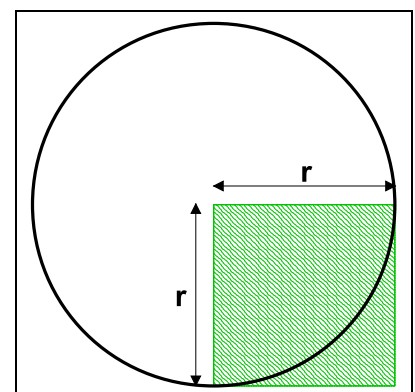
Correct, and that's where the Pi comes in. Pi is a special number worth three and a bit and its exact value just happens to be the number of my Corner Squares that it takes to make my area. So, my area is Pi times R squared. R is my radius.

If you would like to learn more about this scenario visit Lesson 43, *Area of a Circle*.

If you want to build up a unit on area you might also visit:

- Lesson 33, *Fraction Estimation*, (all the software options are based on areas of rectangles or circles)
- Lesson 44, *Area of a Triangle*
- Lesson 50, *Country Maps*
- Lesson 152, *Area Functions*, (for Years 11 & 12)

...or you could type the word area into the keywords box of the Maths300 Search Engine and sift the responses to find the lessons which best suit you.



Content

- | | | |
|----------------------------|-------------|-----------------------|
| • algebraic generalisation | • decimals | • measurement of area |
| • area functions | • iteration | • sketch graphs |
| • arithmetic skills | | |

What's in the bag?

Years: 3 - 12

July 11th 2005

That's the title of Lesson 125; a great lesson for primary and secondary school which is built, as is the case with many of our lessons, on the iceberg of a task with the same name. For those with a task collection, it's Task 198. Those with Maths With Attitude kits will find it in the Years 3 & 4 Chance & Measurement kit or the Years 9 & 10 Chance & Measurement kit.

Secretly one player puts ten mixed coloured cubes in a bag. Player B is allowed to take a sample of four cubes and record the result. The sample is put back in the bag; and another sample of four is taken. This is repeated one more time. Player B now has three clues to '*What's In The Bag?*'. How many cubes of each colour are likely to be in the bag?

Easy to state, easy to start and heaps of maths.

The lesson plan presents the problem with a degree of mystery which tends to engage students at all levels. As a starting point it uses the cunning device of coloured plastic clothes pegs being withdrawn from a container. The enticement is that as each peg is withdrawn it is dramatically clipped to the rim of the container in full view. Similar expectation can be developed using coloured cubes placed dramatically on the table.

The lesson gradually introduces (in context) statistical language such as sample, population and confidence limits.

Following the whole class introduction, students have to complete several experiments in a game-like atmosphere to contribute to the overall problem of developing a set of rules which will help others learn how to predict accurately. The examples in the lesson suggest a wide range of students are capable of tackling this problem.

There is a direct link between the problem and the 'real world' work of a mathematician. The lesson is supported by Investigation Sheets which encourage deeper individual and partner exploration.

When you do try this lesson, remember we are always delighted to receive comment, photograph and the like which could be included in the lesson. Greg Port, King's College, Auckland, New Zealand has already contributed a spreadsheet simulation of the activity which works for populations of two colours. You will find this addition in the Classroom Contributions.

Content

- | | | |
|------------------------------|----------------------|--------------------------|
| • collecting/tabulating data | • natural variation | • statistical confidence |
| • language of chance | • ratio & proportion | • statistical inference |

Do you know the truth?

Years: 2 - 8

August 11th 2005

Truth Tiles that is. It's Lesson 30 and it has application from Years 2 to 8 (at least). The lesson is based on the iceberg of Task 30 from the Task Centre Project, and if you have Maths With Attitude kits you will find the task in Number & Computation 3 & 4. The Maths300 lesson version is included in this kit and Number & Computation 7 & 8.

The task is one of a family which use the digits 1 to 9. You can print these and the puzzle board from the lesson, or you could use Ace to 9 from packs of cards, or you could simply ask students to tear up scrap paper into nine pieces. The initial puzzle is to place the digits into the nine empty spaces in these three equations so that all three are true simultaneously:

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

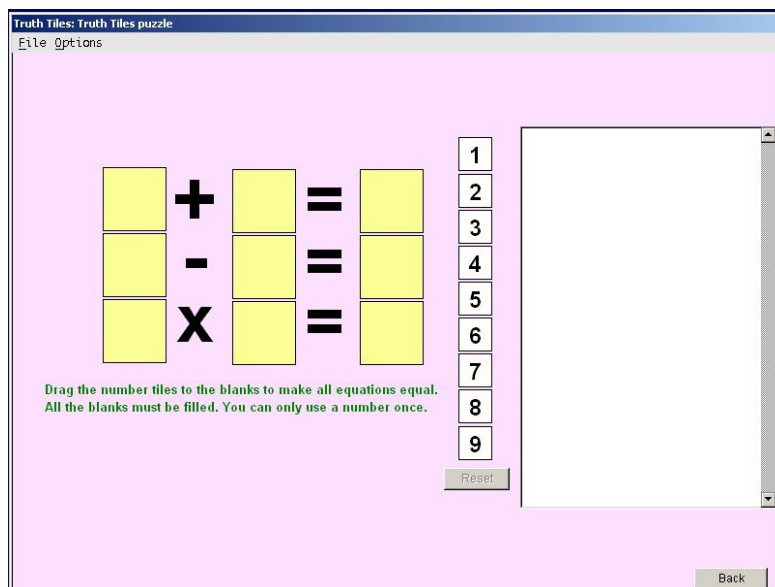
$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

It is almost certain that your students will find more than one answer in a reasonably short time, and that opens the door to the mathematician's questions:

- How many solutions are there?
- How will I know when I have found them all?

This is the real investigation and it can be tackled at a range of levels, all of which touch on, or formally call on, key arithmetic laws such as commutativity, inverses and identity. To answer the second question also requires discussion and agreement about what makes solutions unique in this problem. The arithmetic content is not difficult but the higher order thinking involved is equivalent to the reasoning of a professional mathematician.

The lesson is supported by software and has a sweet Classroom Contribution from a Year 7.



Content

- arithmetic skills
- combination theory
- inverse operations
- laws of arithmetic
- number patterns

Are you spot on with your Pattern & Algebra work?

Years: 2 - 8

September 14th 2005

Colour Spots on a Number Line, Lesson 117, is remarkable because it only requires an unnumbered number line (supplied) and a coloured marker or pencil for each student, but it grows from colouring a few spots to discovering and understanding key concepts in the graphs of straight lines. In between there are many number pattern explorations and the opportunity to express hypotheses in algebraic symbols in a meaningful context.

The lesson is very easy to start - anyone can put a spot of colour in a dot that is already drawn - so it is immediately accessible. However while spot colouring (no time wasting here) the students are asked to look for a way of describing where their colour spots are being placed. The dots look like they are in a pattern, so how do we describe and predict their placement?

One of the key points of the lesson is students all start with the same task on the unnumbered line, but because they then add their own numbers to the number line, a different data set is provided by each student. The lesson builds on this personal (and therefore more meaningful?) data and the visual pattern of spots can now be linked to a number pattern. The number pattern is governed by a rule. The rule can be expressed algebraically. A table can be constructed linking Spot 1, 2, 3, ... each to its own number on the line. And the pairs in this table can be graphed and explored. The possibilities just don't stop.

Moreover, the further you press into the lesson (which is now of course much more than one time slot) the more you discover its links with Lesson 34, *What's My Rule?*

Together they make a nice unit, and students can learn how to solve any problem in the *What's My Rule?* software, by doing a couple of simple calculations.

What's My Rule?: Lesson 34

File

In	Out
x	y
0	-4
1	-3
2	-2
3	
4	
13	
21	
	20
27	
31	

Fill in the table, or write your rule

New rule
Answer
Check

Level 1
Level 2
Level 3

Type your rule in words

Type your rule using symbols

Content

- algebraic generalisation
- arithmetic skills
- concept of variable
- equivalent expressions
- gradient
- graphing ordered pairs
- group counting
- linear equations
- linear graphs
- mental arithmetic
- number patterns
- substitution

Have you trialed a well-trialed trial lesson?

Years: 1 - 9

October 12th 2005

Trial, Record & Improve, Lesson 94, is named after one of the strategies in a mathematician's toolbox. The lesson is not so much a full lesson plan as a smorgasbord of ideas which can be used and reused. It begins with examples of quite young children investigating equations which, under traditional teaching methods, could cause difficulties for students in the early years of high school. The difference in success appears to stem from an approach which:

- gives students ownership of the calculation,
- values the guesses they make as information leading to a better guess,
- removes rightness and wrongness from the learning environment and replaces it with a desire to become progressively more accurate.

This contrasts with the traditional teaching approach which provides recipes for solving particular types of equations.

The lesson continues by suggesting that a key aspect of understanding how to solve equations is to first become a participant in creating them. In this sense it has a strong link with Lesson 19, *Backtracking*, and between them the pair build a consistent approach from Years 1 to 9. Games/activities are suggested to support creating equations, and these are followed by activities which use the equations the children create and turn them into equations which have to be solved.

Editor's Note: A calculator programmed with the correct order of operations is an important tool for supporting students in their trial, record and improve experiments. There are only a limited number of simple four function machines available with this feature. The difference between these and one without the feature is giving correct answers to calculations involving mixed operations.

- With the feature: $2 + 3 \times 5 = 17$, which is correct.
- Without the feature: $2 + 3 \times 5 = 25$, which is *not* correct.

Check the machines in your school before using them.

In the Free Tour section of Calculating Changes:

<http://www.mathematicscentre.com/calchange>

under Materials, you can find information about available machines. Also, Calculating Changes members will find strong links between this lesson and the activity *Squares & Square Roots*.

Content

- | | | |
|----------------------|--------------|-----------------------|
| • arithmetic skills | • decimals | • order of operations |
| • creating equations | • estimation | • solving equations |

Do you really know how to subtract?

Years: 1 - 6

November 21st 2005

In the last fortnight we have run our Maths on the Move workshop titled *Engineering 'aha' Moments in Number* for groups of teachers in two states. They all thought they knew how to subtract, and how to teach how to subtract. Then they discovered what children are able to do if the classroom environment encourages children to learn for themselves how to subtract.

There are at least six different ways to subtract. All are accurate. All are valid. Each has most meaning to the person who chooses to use it - an important point, because if a child is comfortable and accurate with a method they have developed for themselves, what damage might it do if they are forced to demonstrate the process in a particular (adult approved) way?

For example, I never subtract. I always add. Why learn two sets of facts when one will do? I start my subtractions from the lower number and add on using a mental image of the odometer in a car or a video counter. If I have to work out $137 - 48$, I start with the units 'wheel' on 8 and count forward until the 7 shows. That is 9 'clicks' and in the process of passing from 9 to 0, the tens 'wheel' has clicked up one. So, I start the tens 'wheel' on 5 and count forward until the 3 shows. That is 8 'clicks' and in the process the hundreds 'wheel' has clicked up one. So, I start the hundreds wheel on 1 and count forward until the 1 shows, which, of course, is zero clicks. Therefore:

137

- 48

89

You won't find that method recorded in Lesson 96, *Take Away of the Day*, because it is mine, and I am an adult. But you will find several children's methods including using negative numbers - an approach always discovered by some children in classes that have free access to calculators. Using the subtracting by negatives method on the example above:

$7 - 8$... you can. It's -1 .

$30 - 40$... you can. It's -10 .

$100 - 0$... you can. It's 100.

So $137 - 48 = 100 - 10 - 1 = 89$!

If nothing else, if you dig around in all the parts of *Take Away of the Day*, you will find food for thought.

Content

- arithmetic skills
- place value concept & skills
- subtraction skills

Do you make good use of the Investigation Sheets?

Years: 2 - 12

March 15th 2006

All Maths300 lessons are rich enough to be able to go in a range of directions depending upon the age of your students and the demands of your curriculum. With such extensive ground available to explore, teachers sometimes write an Investigation Sheet to guide students in a certain direction. Do you make use of the Investigation Sheets your colleagues have already prepared? They can be a useful tool for differentiating learning to allow for the range of student differences; or they can be a guideline for substantial project work.

If you tick the Investigation Sheet box in the Advanced Lesson Search and keep all other items at default, you will discover that 85 lessons currently on site are supported in this way. If you refine your search to select say a particular Year Level or Curriculum Strand, you will find for example that:

- 31 of the 85 are suitable for middle primary, or
- 16 of the 85 address content from the Space strand.

When you find an Investigation Sheet that will work for you, you can print it as many times as you wish. If you don't want to use all you see, you can cut and paste what you do like AND massage it to suit yourself and your students.

Many Maths300 Investigation Sheets were originally developed in the Mathematics Task Centre Project through its Replacement Units. Tasks have three lives:

- as an invitation for two students to work like a mathematician
- as a whole class investigation to model how a mathematician works
- as a deeper investigation led by an Investigation Guide.

There are over 240 tasks in the collection and a few sample Investigation Guides are provided in the Task Cameo section of Mathematics Centre. Visit:

<http://www.mathematicscentre.com>

and select Task Cameos.

Each Task Cameo outlines the possibilities offered beyond the task card. From this information, and guided by the samples, teachers write their own Investigation Guides. These are an important part of students coming to realise that tasks are learning tools representing the work of a mathematician, not just 'games' to 'finish'.

Content

- curriculum planning

Have you thought of going to the country?

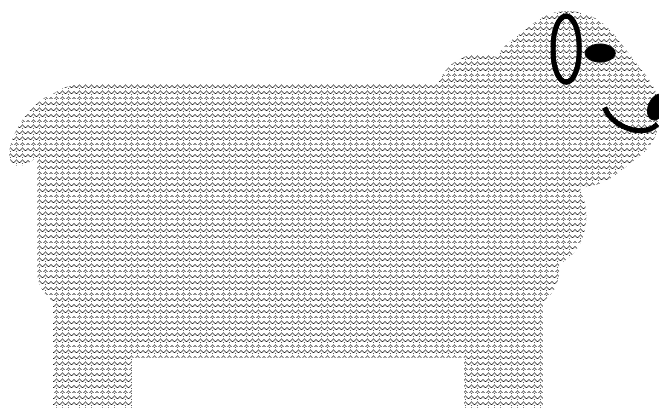
Years: K - 8

April 13th 2006

Perhaps it's because I have been working in the bush for the last week, but have you realised that organising a mathematics unit around a theme such as Farmyard Maths is another way to explore, and make better use of, the richness of your site?

Lesson 47, *Farmyard Friends*, could be a good place to begin. There are aspects of this lesson applicable from Years K - 11. It begins with a non-threatening, easy to solve logic puzzle and has mathematical content that includes logical reasoning, problem posing and solving and combination theory.

Lesson 14, *The Farmer's Puzzle*, and Lesson 41, *Heads & Legs*, are also focussed on the farmyard and are suitable for a wide range of ages. Lesson 17, *Eric The Sheep*, is another obvious choice for the unit and within these three there is heaps of work involving counting, addition and multiplication. Lesson 16, *Garden Beds*, would also have a place if you wanted to expand the number work into a little generalisation and algebra. For the younger students it might be *Jumping Joey*, Lesson 141, which involves lots of counting, pattern and addition in the context of a 'runaway' baby kangaroo, that integrates with the theme.



It's easy to push the unit into measurement areas too. Farmers are always interested in weather - drought and rain being a major determinant of their lives - so Lesson 83, *Temperature Graphs* is a definite starter. When the rain comes it is stored in tanks, always cylindrical, so *Cylinder Volumes*, Lesson 80, also has a place.

Well stone the crows Dave, it's starting to look like we could design a term of integrated mathematics around our theme. You can use our ideas or design your own integrated use of Maths300 lessons. In either case please let us know how it all works so we can share your successes with others.

Content

- curriculum planning

How's things at your place?

Years: K - 6

May 15th 2006

This month ETuTE is the composite of a couple of emails from Kerryn Andersen, Canterbury Primary School. Nothing better for a classroom based project than words from the classroom. For members in other places, a little translation will be necessary:

- VELS = Victorian Essential Learning Standards
- PoLT = Principles of Learning & Teaching

Kerryn also mentions some photos of the Chocolate Cake lesson. They have been included in the Classroom Contribution section of the lesson.

Hi Doug,

I feel dreadful not sending you anecdotes of all our successes with Maths300. Suffice to say I have found it inspirational and have promoted it at every available moment and situation. All teachers in the school are expected to use it and most do so willingly. The Year 4 team plan the maths program using as many lessons as possible from it, as it encompasses all aspects of VELS and PoLT thinking as far as we are concerned. I have especially loved the following lessons but this is not a complete list of fantastic lessons as I haven't had a bad one yet.

Chocolate Cake, Palindromes, Sphinx, Take Away of the Day, Crazy Animals, Fraction Estimation and Halving Squares. Included here are some photos of the kids yesterday enjoying Chocolate Cake. We had to do it twice so that all could have a go at being invited to the party!

As you can tell I just get excited with the concepts in Maths300. But I'm not alone. There is always talk in the staffroom about what happened in this or that lesson. That doesn't happen with Signpost stuff!!!

The search for lessons is so handy when planning a topic of work as it leads you to lots of lessons. At the start they look difficult but the kids are now used to me teaching with the notes beside me and referring to them during the session. I don't want to forget an important thing! Maths With Attitude has been purchased and we have nearly all the Year 3/4 kits now. It is great, but I find it needs me there constantly or the kids think they've solved it when they haven't. We are trying to instil Habits of Mind, especially perseverance, but it'll take time. I am also constantly nagging to make sure nothing is left out when packing up!!! All in all, slowly goes it.

The biggest thrill is of course the kids who were previously scared of maths, now realise they are quite capable of using their thinking skills and having a go, when they found it hard to remember rules etc.. Confidence and fun is the key!

So thanks for all your hard work, we are reaping the benefits!

Regards, Kerryn Andersen

Well Kerryn your thanks go to all the teachers who have contributed to the development of the lessons already on site, and to teachers like yourself who confirm and refresh the lessons in the pursuit of an excellent mathematics curriculum for their students.

Content

Editor's Note: Based on, Lesson 135, *Chocolate Cake*.

- comparing fractions
- concept of fraction
- equivalent fractions

Years: K - 12

So you are having success, but have you planned for succession?

June 22nd 2006

Based on the feedback we receive there are many teachers and schools having success with Maths300 lessons. To a person the measure of success is always related to how the students respond. Teachers are finding that the challenges and teaching craft embedded in Maths300 lessons are features which encourage more engagement and more learning. Objectives which we all hold dear.

But you know what it is like in schools. Things change - new staff, new official documents, new administration, new extra duties - sometimes it is almost like forces are aligning to prevent us from practising the teaching which brings out the best in our kids.

What plans have you made so that the success you are experiencing will continue in successive years, for successions of students who may be in the care of your successors?

The Mathematics Task Centre Project has advice about the principles of continuing use of best practice.

Framework: Accept and publish a framework for your teaching. Are your students learning to work like a mathematician or just trying to reproduce the skills of a mathematician? Ensure that the administration, kids and parents are clear about your curriculum's purpose. By default, requiring students to purchase a text book is a statement of your framework. There are alternatives - class sets, teacher reference copies, no text at all or Maths With Attitude kits.

Document: Feature the use of Maths300 lessons and Task Centre tasks in the written syllabus material. Try to build in an opportunity for teachers to choose from a number of investigations assigned to each part of the syllabus document. Different teachers will find different investigations more or less satisfying to manage. Where possible prepare together (with parent help?) easily accessible resource kits for each investigation.

Induct: In-coming teachers (...and students and parents) need to know why and how you use investigations. Make time to work with them on understanding the depth of the investigations and the principles and practice behind their use.

Communicate: Keep the framework and the documentation in front of the school population. Perhaps the most important way to do this is to look for and celebrate student work which reflects your objectives. Consider using the school newsletter and/or web site and relaxed but purposeful 'maths nights'. Maths displays in hallways can be very powerful.

Assess: Don't just talk about the Working Mathematically principles detailed in your framework. Build in appropriate assessment to show how kids are in fact learning to work like a mathematician.

Re-enthuse: Build in-house and external professional development into your regular professional learning program. Show each other kids' work. Play with some tasks together.

Revitalise: Make a feature of adding new investigations to your syllabus - just a couple each year is enough. The same content objectives can be achieved through a wide range of investigations. For example, if last year you used Match Triangles to kick off your algebra unit, then use Garden Beds this year.

This advice is from the Principles section of the Mathematics Task Centre:

<http://www.mathematicscentre.com/taskcentre/prin.htm>

Content

- curriculum planning

Have you looked at Domino Trails?

Years: K - 6

July 14th 2006

Mary & Ursula, teachers of Years 3/4 at Footscray North Primary School, have and they are delighted with the opportunities it has opened up in their classrooms. The lesson has aspects for all learners from K - 6, with possibilities for older students, and is a partner with the lesson Dominoes which certainly contains secondary level challenges.

In essence the task is to choose a total and then make a domino trail which sums to that total. It grows from a task of the same name in the Task Centre collection. The lesson suggests variations such as:

- What happens if we use fewer dominoes?
- What happens if the dominoes in the trail have to match end to end?

However Mary, Ursula and their students were soon creating problems of their own.

- What happens if you are not allowed to use any domino with a zero?
- What happens if we cover up 1 or 2 dominoes in the trail? Can we work out possibilities for the covered dominoes?

The students created several games of their own, all of which involved plenty of skill practice in problem solving contexts. They also devised a range of ways to record their dominoes and began to move from using the objects to using symbolic representations of the dominoes.

These ICT-literate teachers also used digital technology to video or audio record their students and found ways to make use of the domino tool in their electronic whiteboard.

Mary & Ursula have a broad range of students with a range of immigrant backgrounds, but were delighted to find all their students were engaged. When asked about the key features which generated this participation and learning they identified:

- free and guided play with the dominoes
- the sense of a game environment
- non-threatening atmosphere where it is okay to 'have a go'
- conversation and informal peer teaching
- concrete materials
- kids having control of the problems
- challenges scaffolded for success

Interestingly, although there is a printable set of dominoes in the lesson plan, the teachers thought that the tactility of the real dominoes made a difference.

Content

- | | | |
|-----------------------|----------------------|-----------|
| • 1 :1 correspondence | • counting | • sorting |
| • arithmetic skills | • number recognition | |

What's your angle on learning about angles?

Years: 3 - 9

August 15th 2006

Is it practical and concrete, or too much from the text book? Does your unit of work about angles contain content, skills and applications? Enter the word angle in the Maths300 search engine and you will get at least fifty responses. Some are there because that word is part of the words triangle or rectangle, but there are still plenty of responses which relate to learning about angles.

Start by looking at Lesson 133, *Angle Estimation*. A practical experience of estimating angles by tying a rope to the netball pole, setting a base line, and going for a walk around the pole. Students learn about the concept of an angle, naming an angle, the relative sizes of angles and all estimates can be checked with a blackboard protractor. Back inside the classroom, software backs up and extends the experiences, and does so in a way that can never be achieved by text book examples. The software alters the orientation of the angles at random, so misconceptions about 'one line always being horizontal' can never develop.

The lesson also highlights a little device called a Rotagram which is used for comparing angles.

Editor's Note: Rotagrams are now available as a Mathematics Centre resource.

<http://www.mathematicscentre.com/taskcentre/resource.htm#rotagrams>

Lesson 33, *Fraction Estimation*, contains software which includes circles. Estimating the 'pie' fractions in these options calls on an understanding of angles as much as it does an understanding of fractions. Further, if you set the fractions used to be twelfths there is a strong connection with the angles of a clock face.

Another lesson involving physical activity and the concept of angle is Lesson 48, *String Shapes*. This time the students are using loops of string to create various polygons and explore their properties.

Lesson 123, *Mirror Bounce*, Lesson 51, *Hunting For Stars*, and Lesson 55, *Billiard Ball Bounces*, each offer applications of the concept and measurement of angles in problem solving situations. The latter two also have software support which helps to build a visual image in the student's mind of the relative sizes of angles and how they are formed from two rays.

Lesson 134, *Pentagon Triangles* involves the angle sum of a triangle and properties of similar triangles, and Lesson 108, *Trigonometry Walk*, introduces one of the most widely used applications of relationships between triangles and angles.

With well documented, extensively trialed and successful lessons such as these so readily available, the text book can be seen far more in its rightful place as a source of skill practice, rather than as the definition of curriculum.

Content

- | | | |
|---------------------|-----------------------|--------------------|
| • angles & polygons | • estimating angles | • measuring angles |
| • concept of angle | • fractions of a turn | • trigonometry |

Have you visited the Land of ET?

Years: 3 - 10

September 13th 2006

Okay ET might be a little 70s, and you might be over that, if you are thinking ET means extra terrestrial. However, the 'sting in the tail' of this lesson is that ET means equilateral triangle. The lesson doesn't require too much equipment (just some blue and yellow counters - Poly Plug springs to mind) and a supply of equilateral triangles (perhaps students could make them in an introductory lesson using compasses and card, but there is a template in the lesson). The lesson is easy to state and easy to start. It is non-threatening because its introduction is through a language-based logic puzzle, however, the lesson goes to the heart of number theory and investigates the symmetry properties of an equilateral triangle into the bargain.

The puzzle is about a 'written' language from the Land of ET which uses only two letters - B and Y. Words in this language can be as long as you wish, but they can only be sequences of B and Y. There are, however, some 'spelling' rules in the language.

- BBB can be added to, or removed from, a word without changing its meaning.
- YY can be added to, or removed from, a word without changing its meaning.
- BYB can be replaced by Y and vice versa.

The first set of puzzles is to find the shortest version of any word ... and then, after shortening, to find just how many words there are in this language?

Once it is agreed that there are only six possible words (yes 6!) using these spelling rules, the students try adding the key words together. The table of combinations is compared to a similar table for adding whole numbers and this process highlights the laws underpinning arithmetic.

And triangles? Well it turns out there are only six symmetries of an equilateral triangle, and 'adding' them in pairs produces a set of outcomes that behaves exactly like the words in the ET language. Mathematics does have structure and seemingly different systems can have the same structure.

In fact, although not mentioned in the lesson, it is exactly this connected behaviour of apparently unconnected systems which led to the solution, in 1994, of the world's hardest problem, Fermat's Last Theorem, which had defied solution since 1637. Andrew Wiles, the architect of the solution, had to first prove that systems called Elliptic Curves behaved in the same way as systems called Modular Functions.

Visit Lesson 65, *Land of ET*, for more information.

Content

- | | | |
|----------------------|-------------------------------------|----------------------------------|
| • laws of arithmetic | • mathematical systems & structures | • symmetry: equilateral triangle |
|----------------------|-------------------------------------|----------------------------------|

How do we get those brains into gear?

Years: 4 - 8

October 11th 2006

That's what we are aiming at isn't it? Kids pondering over a problem and actually wanting to call on the processes, strategies and skills of a mathematician to work it out? But it's teaching craft rather than the problem itself which is likely to activate those brains, and one of the techniques which often captures students is a story shell. The Maths300 Search Engine allows you to look for lessons which have a story shell. If you tick that box only, you will find many lessons with that feature.

Lesson 61, *Doctor Dart* is one example. It's one I love, perhaps because I am a classroom Thespian. The lesson summary sets the scene:

Doctor Dart is on an adventure. She must get through the anti-gravity door to save the universe, but the Evil Professor has locked the door with an electronic key pad. He explains the rules for operating the pad, then challenges her, with appropriate melodramatic cackling, to find the score that opens the door.

One doesn't have to present the problem as a cameo play with appropriate cackling, but, as explained in the lesson plan, it certainly makes the kids sit up and take notice if you do. Mathematically, whether presented this way, or more calmly as an oral description, one of the neat mathematical outcomes is that the lesson offers meaning and application for Tree Diagrams. The initial problem will be solved in 10-15 minutes, but that only opens the (anti-gravity) door to more mathematical adventures, which are supported by the companion software.

Now, if *Doctor Dart* has started your brain working, then you might also consider incorporating current theory about how the brain works into your teaching practice. Findings from recent studies about the functioning of the human brain have been generating a buzz amongst teachers across Australia. How do these findings affect your teaching practice?

To find out more about brain based learning, enter that phrase into a web search engine.

Content

- arithmetic skills
- number patterns
- tree diagrams

Years: K - 10

Have you planned your professional development program for next year?

November 17th 2006

Our work has only one purpose. Everything we do is simply to encourage teachers to continue debating the features of lessons which lead to better mathematics learning for students at all levels. Whether it is a Maths300 lesson, a hands-on task, a face to face workshop, a class set of materials, a published article - or anything else - we are retelling a success story from a classroom. Our work invites you share those successes and to participate in the debate by re-enacting them in your classroom.

Much of this professional learning you can do yourself. If you are prepared to create time to delve into the extensive web pages linked to Mathematics Centre, for example:

- Maths300
- Calculating Changes
- Mathematics Task Centre

or prepared to sit down with a Maths With Attitude kit and explore its manual, you will find a huge anthology of inspiring stories.

But sometimes it is the presentation from outside which stimulates more interest, so we have developed a wide range of workshops which support students learning to work like a mathematician in best practice classrooms.

- We have 'off-the-shelf' sessions which are tried and true in our Maths on the Move library.
- We run Discussion Lessons in which we take your class and re-enact a story so that you have time to observe your students learning.
- We can draw on a wide range of resources and experience to 'tailor make' a program to your objectives.

We work with schools, clusters, districts, regions and systems. We work anywhere in Australia or overseas. BUT there are only so many slots in the diary. If you would like us to work with your staff, then now is the time to plan - and book - for next year.

Begin your exploration of possibilities at the Professional Development link of Mathematics Centre.

<http://www.mathematicscentre.com>

Content

- curriculum planning

Is your class settled yet?

Years: K - 9

February 14th 2007

In Australia, following the summer holiday, students have only recently returned to school. So, if the class is not settled, some students may still be asking "Where do we sit?". Now there's an opening for a lesson. Lesson 8, which has exactly that title.

The lesson is written as if for infant teachers (it has the context of Goldilocks and her bear friends), but, if you teach older children, don't let that put you off. The mathematics is about arranging 3 people in three seats.

- How many ways?
- How do we know if we have found them all?
- How do we record the different ways?
- What happens if there are 4 people?
- What happens if there are more seats than people?
- ...

Another lesson which can be related to this time of the year is Lesson 21, *Lining Up*. Often, that is exactly what students are asked to do before they enter class. Let's assume they line up in single file:

Sharlene, I notice you are 12th from either end. How many people are there in the line?

That is the type of question at the core of the lesson. The mathematician's question around which it then revolves is:

- Can I check this another way?

The focus shifts to how many different ways we can check that there are 23 in the line. Then, of course, there are *What happens if...* questions which extend the investigation.

And if your class is sufficiently settled, while the weather is fine, why not plan an outdoor lesson. *Estimation Walks*, Lesson 13, is a good place to begin. It's easy to set up (just a 50m measuring tape and a few metre rulers) and includes content in measurement, decimals and data.

Whichever lesson(s) you choose, we wish you well for the new year. We will keep on trying to support better mathematics education, and we hope you will let us know some of your stories of success.

Content

- | | | |
|------------------------------|--------------------------------|---------------------------|
| • algebraic generalisation | • equivalent expressions | • measurement |
| • counting | • estimating length | • number patterns |
| • collecting/tabulating data | • language of position & order | • statistics |
| • decimals | | • symbolic representation |

What are you eating for Maths this week?

Years: 5 - 12

March 14th 2007

How about Chocolate Chip Cookies? Coming up to Easter, chocolate becomes part of the daily scene. We don't have any lessons on chocolate eggs, but we do have Lesson 58, *Chocolate Chip Cookies*. The students will soon be munching their way through a cookie and, apart from providing a brain snack, the objective is to try to count how many chips in this mini-meal.

First though we develop a bit of mystery at the beginning of a problem - a real problem in fact for biscuit manufacturers.

- How many chocolate chips have to go into a mixture to be fairly certain that each cookie will have at least ... chips?

For example, if 10 chips were stirred into a batch which made 10 cookies, do you think each cookie would get one chip? ...Would eleven chips be enough?

We can't make and bake heaps of test batches in order to decide how many chips to include, so we have to find a way to model the variables in the problem to help us predict a satisfactory mix. The lesson plan tries to engage the students in designing the model in a manner similar to the way mathematicians work. An important part of the modelling process is trying to clarify the problem so that it *can* be tackled using mathematical methods, therefore discussion begins with identifying the variables. Then the real cookies are munched and the number of chips in each cookie is recorded to get some idea of how real world data varies.

With student help, investigation into particular cases, for example, 10/7 - 10 cookies with at least 7 chips in each - and 6/3 develops. Software is used to explore a wider range of conditions. Stem and leaf graphs feature as a useful tool.

And, for those who are game, the best way of checking an agreed prediction (batch size, no. of chips) from the model is to make it, bake it and munch it.

Best of luck with this lesson. We hope it at least provides food for thought!



Content

- mathematical simulation
- natural variability
- probability
- statistical confidence
- statistical inference
- statistics
- stem & leaf graphs

Have you been out walking lately?

Years: K - 10

May 18th 2007

Many Maths300 lessons involve walking, so why are you still sitting in the classroom doing maths every day? Drought's broken, weather's not so good any more? Well you can still use most of them in the multi-purpose room and add a whole lot of refreshing qualities to your maths lesson.

Consider:

Estimation Walks

- which involves length estimation, decimals, average and graphical representation.

Calculator Walk

- where infants go hunting for numbers around the school and record them on their calculator.

Algebra Walk and Walking With Children

- which involve students being points of physical graphs of straight lines and parabolas.

Walk The Plank

- which investigates the behaviour of positive and negative numbers.

Fraction Estimation

- part of which suggests walking around perimeters of large objects and spaces.

Circumference of a Circle

- you can guess what that is about.

Around Our Neighbourhood

- which involves young students in all sorts of location skills and informal measurement in their local area.

Trigonometry Walk

- a physically involving introduction to a major middle secondary school concept.

Angle Estimation

- which involves walking around a basketball post or similar.

Maths in Motion

- a suite of actively involving number-related lessons for infant students.

Speed Graphs

- there is a bit of jogging in this one too.

Not a bad collection really. But how do you find these lessons to consider which ones you will build into units of work? Easy. By default, the Lesson Library lists lessons in alphabetical order of name. Just scroll down to find the ones which interest you.

Content

- curriculum planning

Years: 4 - 12

Have you noticed that so many simple starting points produce surprisingly deep mathematics?

June 14th 2007

Perhaps that's part of what makes Maths300 such an exciting resource for so many teachers - investigations which are easy to state, easy to start and contain heaps of maths.

Two examples in point, applicable to a wide range of levels, are:

- *Addition Totals*, Lesson 18 for Years 4 - 8
- *Same Or Different*, Lesson 153 for Years 5 - 12

All you need to start *Addition Totals*, is 12 sheets of paper displaying the numbers 0 - 11 in large print; handwritten with a big marker is fine. Hand out the sheets to 12 students and ask them to sort themselves into pairs which sum to 11.

- How many pairs?
- How do we know when we have found them all?
- What if we used the numbers 0 - 12? How many pairs?

Now comes the twist. We ask students to predict the number of pairs for 0 - 13. Their hypothesis is often incorrect and this starts a search for more data.

- Given any number can we predict the number of pairs which can be formed from it?

If we predict in words, can we translate those words to symbols? Interestingly the answer has to be a two part formula depending on whether the given number is odd or even. A simple start leads to an algebraic experience far deeper than atomised exercises in most text books.

- And ...what happens if we explore all the triples which can be made with numbers from 0 - N?

For *Same Or Different*, you will need a tub of coloured cubes. Each student chooses 6 of the same colour and pairs up with someone whose selection is a different colour. Model the game first by hiding two reds and one blue in a container. Ask two students to select a block each. Are the blocks the same, or different? Model again and record the results on the board each time. Invite the class to add more data - they can hide their blocks under books or jumpers. As the data builds it becomes clear that this game is unfair - selections will more often be different than the same.

- Is there a way of placing a combination of blocks in the container so that the game is fair?

It turns out that there is; in fact there are many fair combinations and even though this is a random event the fair combinations belong to the very non-random set of Triangle Numbers! Proving that is the case requires application of probability theory leading to an understanding of the discriminant of a quadratic. Complex yet beautiful stuff and within the grasp of many students in Year 10 and above.

Content

- | | | |
|----------------------------|---------------------------|---------------------------|
| • algebraic generalisation | • conditional probability | • sample space |
| • arithmetic skills | • long run frequency | • statistical confidence |
| • concept of variable | • number patterns | • statistical inference |
| • concept of fairness | • probability | • symbolic representation |

How are Tasks and Maths300 related?

Years: K - 10

July 13th 2007

Actually the question was asked at the AAMT (Australian Association of Mathematics Teachers) conference last week. The answer is that each offers a different, but complementary, component of learning to work like a mathematician.

Think of it as similar to a 16 year old - at least in my state - who has just got their L-Plates. The learner comes home rejoicing ... and the parents have a choice.

They could say:

There's the keys, there's the car, you've got two years go and figure out how to do it.

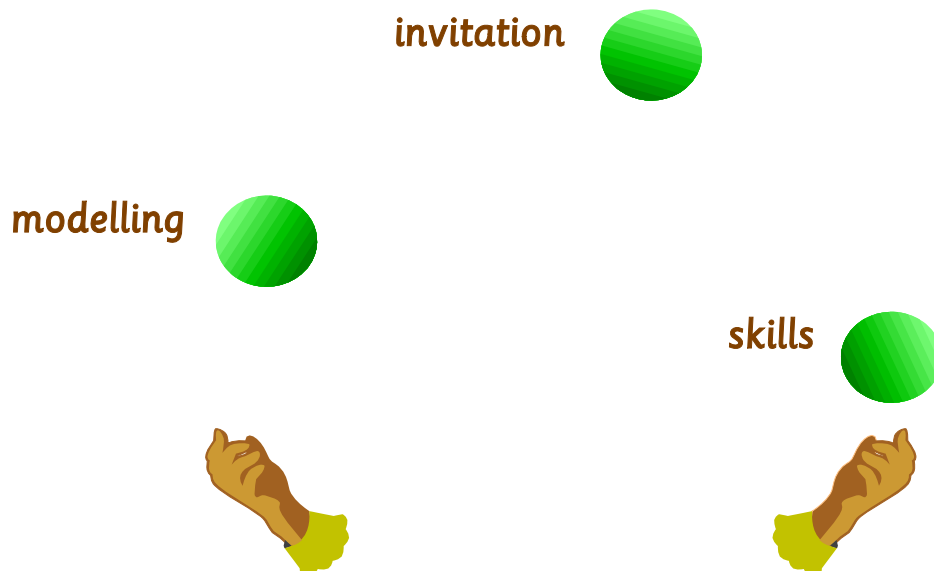
The invitation to drive.

They could say:

Mate, I have been doing this for 30 years. Sit here next to me for the next two years and I'll show you how it's done."

Modelling how to drive.

Hopefully they won't say either exclusively, but rather will juggle the balance of the two opportunities over the learning period, and include lessons focussing on particular skills such as checking the oil, water and tyres, and reverse parking.



Three components in learning to drive - invitation, modelling and skill practice - and three components in learning to work like a mathematician. Tasks are the invitation to try for yourself. Maths300 supports teachers (regardless of their own perceived ability) to model how a mathematician works. Both offer skill practice in context, and beyond that we all know where to find plenty (too many?) skill practice exercises.

If you are constructing a Working Mathematically curriculum, the diagram is worth including in your syllabus document.

It was developed to explain how teachers in the INISSS project (Improving Numeracy for Indigenous Secondary School Students) changed their curriculum to achieve such outstanding results for Indigenous and non-Indigenous students alike. Students with better problem solving skills, better standard maths skills and better literacy (yes literacy!) than students whose teachers had not been embedded in the practice of learning to work like a mathematician.

In the closing address at the AAMT conference, Professor John Mason, Centre for Mathematics Education, Open University, UK, taunted:

*Would you employ an English teacher who didn't read?
Would you employ an Art teacher who didn't art?
Would you employ a mathematics teacher who didn't mathematic?*

Tasks are clearly designed for students to do the work of a mathematician on their own (well, with a mate). Students develop both their perception of what this means and their ability to investigate, through the modelling lessons offered in Maths300. Juggling the opportunities to do both and including lessons with a skill focus becomes the art of teaching in a Working Mathematically curriculum.

In Maths With Attitude kits these three aspects are drawn together lesson by lesson from Years 3 - 10. To learn more about MWA kits take the Maths With Attitude link at Mathematics Centre:

<http://www.mathematicscentre.com>

Editor's Note: Since this ETuTE was written, the *Working Mathematically with Infants* kit, Years K - 2, has been added to the suite of resources supporting a curriculum centred on learning to work like a mathematician in best practice classrooms. To explore this resource further take the Working Mathematically with Infants link at Mathematics Centre.

It is now possible to resource the teaching of a Working Mathematically curriculum through all compulsory years of schooling, so there is no reason for maths to be taught the way it always has been.

Content

- curriculum planning

What the heck is a forensic accountant?

Years: 7 - 12

August 22nd 2007

Last Tuesday evening on 'Temptation' a contestant mentioned they were a forensic accountant. The compere noticed the comment but didn't take the time to explore further, therefore, our question of the month.

Maths300 is not usually a careers information centre (except perhaps, regarding the career of mathematician) but it happens to give quite a good indication of the work of a forensic accountant through Lesson 90, *The Grubby Pages Effect*.

The lesson has a background sheet entitled:

So You Want to be a Forensic Accountant

which leads into Benford's Law, which is the focus of the lesson and one of the underlying mathematical techniques applied by forensic accountants to identify fraudulent entries in company account records.

In the classroom the mathematical understanding is built through exploring digits on newspaper and magazine pages and applying percentage calculations. At this level it has application from Year 7. However, the underlying function is a logarithmic one (to Base 10), so the lesson clearly has a place in Years 11 and 12; certainly in alternative courses such as the Victorian VCAL, as well as in academic courses.

Content

- collecting/tabulating data
- logarithmic function
- percentages

Editor's Note: Benford's Law states that the leading digit in a base b ($b > 2$) number occurs with probability

$$\begin{aligned}P(d) &= \log_b(d + 1) - \log_b(d) \\ &= \log_b(1 + 1/d)\end{aligned}$$

Do you talk the talk?

Years: K - 12

September 14th 2007

I used Maths300 Lesson 44, *Area of a Triangle*, in a Year 5 Discussion Lesson this week. Strathaird Cluster has organised a series of these professional development sessions across its primary and secondary schools to stimulate the use of Maths300. It will be a long time before I forget the shining face of the young girl who wouldn't go out to lunch after the lesson until she excitedly bubbled out:

<i>My dad always tells me I can do maths, but I always thought I couldn't. After this lesson, I know I can.</i>

We are no longer teaching mathematics. We are creating classroom experiences in which students are learning to work like a mathematician. These are meaningful words when you explore any Maths300 lesson. Each is built around a process, the Working Mathematically process, which has been identified by professional mathematicians as the core of their work.

Professional mathematicians simply want to fall in love with a problem, then apply reasoning processes and mathematical skills to explore it. They can't even guarantee to solve it. Those reasoning processes are linked to (or implied in) every Maths300 lesson and expressed in a one page document titled Working Mathematically.

Traditional mathematics teaching, with its emphasis on decontextualised skill practice in atomised, unconnected small bits, has failed to engage huge numbers of students. Perhaps worse, it has failed to interest many of its teachers.

Working Mathematically is now an accepted component of every state curriculum document and something like it appears in most international documents. A refreshed vision of the meaning of mathematics education, with Working Mathematically as its core, is now possible. Maths300, and all our related work, is designed to help you make that curriculum shift in happy, healthy, cheerful, productive, inspiring classrooms.

Consider the Working Mathematically process on Page 4. Within the context of whole class Maths300 lessons and within the students' personal investigations of Maths Tasks every word makes sense - mathematical common sense. It can be used as a teaching framework, a planning focus, an assessment tool. And when you learn to talk its talk in genuine, contextual, problem solving situations, the students do better and feel better about themselves.

Andrew Wiles who solved Fermat's Last Theorem, which could reasonably be considered the world's hardest maths problem because it defied mathematicians for 350 years, describes doing mathematics as finding your way through a mansion of dark rooms. You enter the first room and stumble around in the dark bumping into the furniture, getting a sense for where things are. Then you find the light switch, turn it on, and you see everything fit into place. Including the door to the next dark room.

In what ways does your teaching reflect the work of a professional mathematician?

Content

- curriculum planning

Does division have you boxed in?

Years: 4 - 12

October 8th 2007

For the most part, where division is part of Maths300 lessons it is the context of the inverse operation to multiplication. For example, Lesson 156, *Chart Strategies*, and Lesson 84, *Number Charts*, both have multiplication options and in both, there are multiplication situations where the answer and one of the factors is given. Finding the missing factor is, in essence, a division problem.

As you would expect in a site like ours, there is certainly no lesson developing a recipe for something like a long division algorithm. However, Lesson 162, *Multiplication in a Table Format*, develops a visual image of long multiplication, and again, you only need to ask the backwards question, something a mathematician might do, to create the equivalent of a division situation.

One investigation which does focus on division is Lesson 146, *Division Boxes*. It applies Divisibility Rules to a fascinating situation of looking for numbers such that:

The first digit is divisible by 1,
AND

The first two digits are divisible
by 2, AND

The first three digits are divisible
by 3, AND

...

The first ten digits are divisible by
10, AND

...

It's a wonderful investigation with a worthy sub-problem appearing at the 3-digit level. Software can be used to extend into larger numbers. Enjoy!

Content

- arithmetic skills
- divisibility tests
- tree diagrams
- combination theory

How do you build a skill toolbox lesson?

Years: 4 - 12

November 16th 2007

In the process of investigating a problem, a mathematician reaches for two toolboxes - their skill toolbox and their strategy toolbox. If your curriculum is based around students learning to work like a mathematician - rather than endlessly drilling the skills of a mathematician - you will create a balance between lessons that model how a mathematician works (Maths300 is great source), lessons where the students are invited to apply the model for themselves (the Mathematics Task Centre is a great source) and lessons in which students practise the skills of a mathematician (but is a textbook the only source of these???).

Painted Cubes is a task, Task 160. In this form it contains enough wooden cubes of various sizes for two students to take on the challenge. But all tasks have been chosen because they have a life as a whole class lesson, so if you have a larger collection of wooden cubes you can convert the challenge to a whole class investigation. Lesson 38, *Painted Cubes*, of Maths300 explores the problem in this way.

In the process of developing this whole class investigation you might hold up a $3 \times 3 \times 3$ cube and ask the seemingly innocent question:

How many unit cubes did it take to make this?

If you are satisfied with the answer 27 and move on, you might miss an opportunity.

How do you convince me there are 27?

There are three layers and each layer has 9.

Good, can anyone convince me another way?

Each tower has 3 and there are 9 towers.

Excellent. A mathematician always asks 'Can I check this another way?' and you have. In the process you have shown that 3×9 and 9×3 give the same answer, but are not necessarily the same thing.

Now move on with the lesson ... but this little informal moment is the germ of the next day's skill toolbox lesson. It could be presented in several ways, including as a worksheet. It begins with challenges that call up images of cubes...

I have a $4 \times 4 \times 4$ cube, how many unit cubes? Convince me in two ways.

The cube is made of layers of 25. How many unit cubes in the whole?

The cube is made of columns of 2. How many unit cubes in the whole?

...and develops into cuboids...

My cuboid is $2 \times 3 \times 4$, how many unit cubes? Convince me in two ways.

The cuboid is made of 6 layers, each 3×10 . How many unit cubes in the whole?

There is another way this cuboid could be made of 6 layers of 30. Explain.

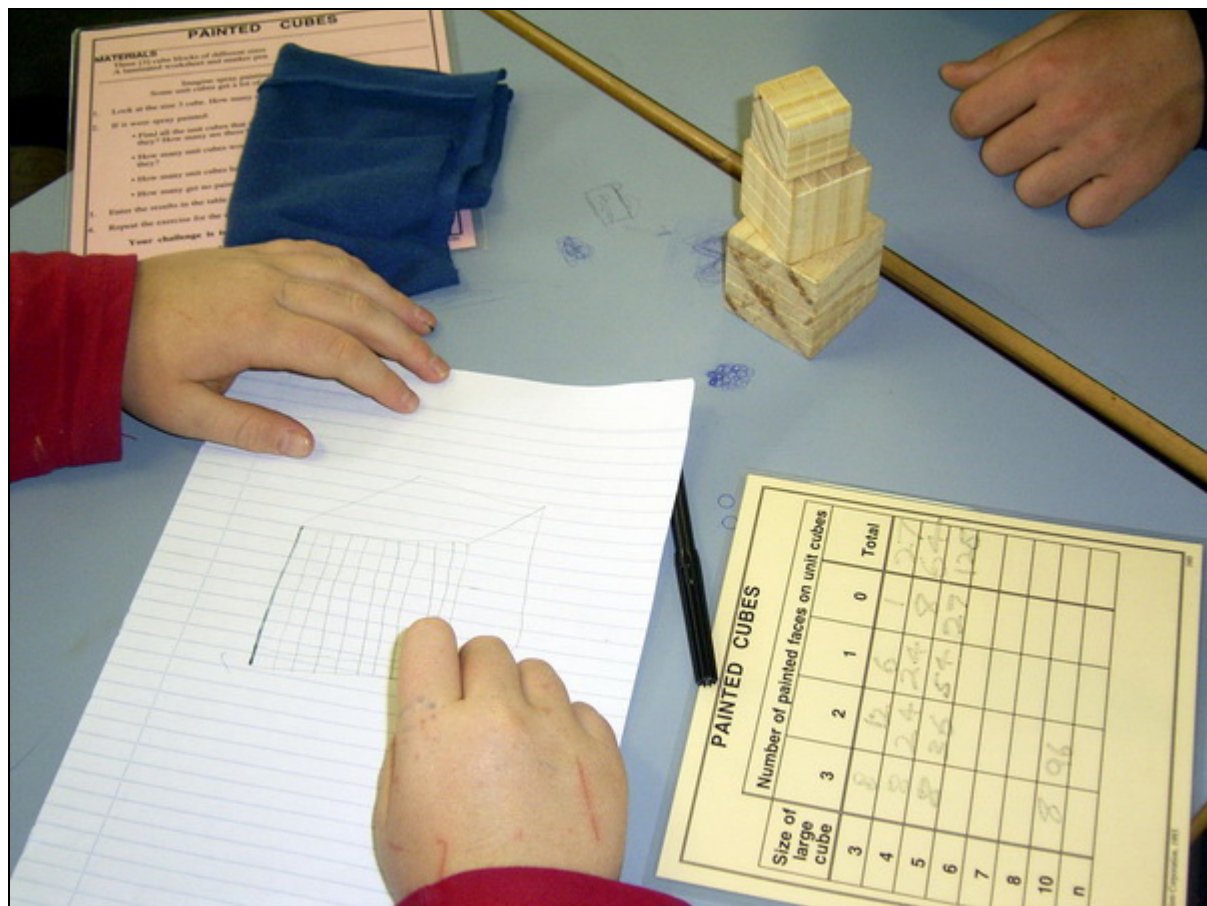
Actually there is more than one way if one side of the layer is something and a half units long. Explain.

Find all the ways to build a cuboid with a total of 144 unit cubes. (Only use whole numbers.)

...rounding off with some classic text book problems like...

Evaluate the following expressions...

In what ways has value been added to the practise of mathematical skills through this approach?



Content

- arithmetic skills
- linear functions
- quadratic functions
- cubic functions

Have you interested enough students in maths this year?

Years: K - 12

December 13th 2007

Last week, for a broad range of mathematics education leaders from across the country, Charles Lovitt led a two day workshop built around Maths300. It was his final official engagement in forty plus years of teaching, most of which have been deeply concerned with encouraging debate about the big ideas in mathematics education. Charlie's practice has always been to centre that debate around captured classroom images, hence his deep involvement in Maths300, the Mathematics Task Centre Project and a heap of other professional learning initiatives.

Throughout the workshop, discussion returned to features of a classroom which interest kids; kids at all levels. It seems engaging students in their mathematics learning is a big issue in mathematics education classrooms across the nation. Participants were encouraged to explore examples of successful lessons and identify features likely to encourage learning.

Since the session, these common thoughts have been massaged into the list below. Have you interested enough students in maths this year? If not you might find the document useful in your preparation for next year. The document is open for discussion, so please let us know what it stimulates where you are.

A checklist for encouraging learning

- | | |
|---|----------------------------------|
| • application focus | • kinaesthetic |
| • assessment opportunities | • links to learning theory |
| • builds on personal student experience | • mathematical modelling |
| • communicating mathematics | • mixed ability |
| • concept focus | • multiple entry & exit points |
| • concrete materials | • non-threatening |
| • concurrent teaching of topics | • open-ended |
| • differentiation for ability range | • outdoor |
| • easy to state/easy to start | • ownership |
| • estimation | • recording & publishing |
| • first hand data | • skill development in context |
| • game context | • social issues |
| • group work | • story shell |
| • history of mathematics | • technology (calculators) |
| • home/school links | • technology (software) |
| • inclusive | • physical involvement |
| • informal or incidental learning | • visual (visualisation) |
| • interdisciplinary connections | • whole class |
| • investigative process | • working mathematically process |
| • | • |

Content

- curriculum planning

How does your pulse rate?

Years: 5 - 8

February 27th 2008

We are about four weeks into the year in Australia so with luck it has settled down to a steady, relaxed beat. In fact, it should be the students' pulse rates that are rising as they are becoming more and more excited about their mathematics learning. There is a way to find out about student pulse rates. Lesson 68, *Pulse Rates*, will lead you in and, if you are up to it, you can be part of the data collection yourself.

In essence the lesson is based around using as a measure of fitness, the time it takes a pulse to return to its normal state after exercise. Mathematics involved includes measuring time in seconds and minutes, collecting and recording data in tables, graphs and spreadsheets and interpreting recorded data. The teaching craft involved includes:

- estimation
- group work
- cross-discipline study including science and health possibilities
- first hand personal data
- informal teaching of related mathematical content such as calculating the slope of a line
- peer discussion
- use of technology (including possibilities for a data projector and interactive whiteboard)

There is also an example of gathering assessment information using a self-assessment sheet.

The lesson is easy to start because student interest in sport can quickly generate a discussion of fitness and how it might be measured. The test suggested as the core of the data collection is also easy to set up because it only needs a step for each pair which is about 30cm high, a watch or timer for each pair and some form of notepad for recording. The lesson can be used outside, or inside, depending on facilities and the weather.

This is one lesson that doesn't have a Classroom Contribution yet. As you use it with your class, keep your eye out for 'tweaks', successes and suggestions you could share with your colleagues in this way.

Content

- collecting/tabulating data
- graphing ordered pairs
- interpreting graphs
- measurement: time
- spreadsheets

Have you been to the racetrack to catch up?

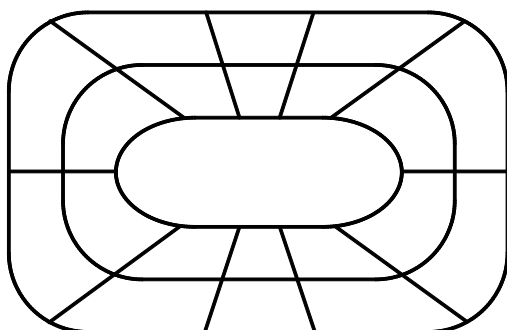
Years: 2 - 8

March 27th 2008

Well, if you want to, you need go no further than Maths300. Gambling is not required.

Racetrack and **Catch Up** are skill practice games supplied in Lesson 97, *Tackling Times Tables*. Of course students need automatic response in times tables, and lots of other skills too. A mathematician can't solve problems without a toolbox of skills. But each of these drill and practice games is different. A major difference is that they can be prepared once and used many times. They can also be used to practise skills other than times tables.

Racetrack



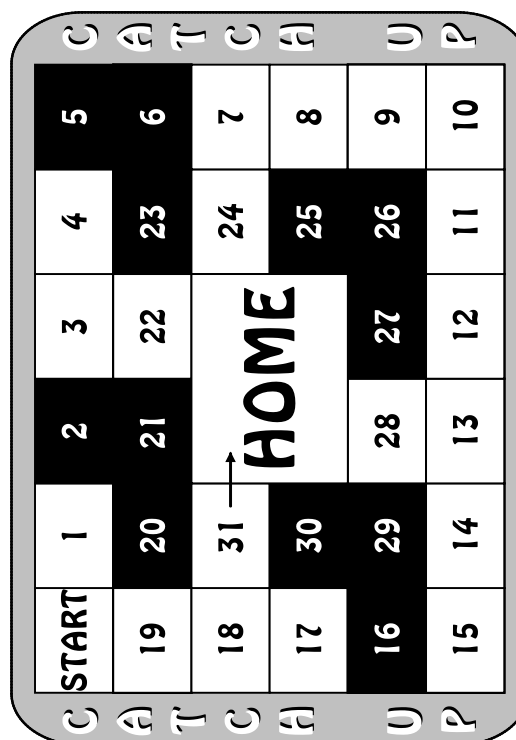
is a class game which is easy to state and easy to start. Students choose their own numbers, but the teacher chooses the operation. Use it 'a little bit often' in a supportive environment and watch the students' automatic response improve.

Explore the lesson plan for *Tackling Times Tables* to find more information about the rules of these games.

Content

- arithmetic skills

Catch Up



is a small group game in which the students choose the 'sums' they will practise - ownership, group work, game context and mathematical conversation are all valuable learning features.

Have you had a crack at Cracked Tiles?

Years: 5 - 10

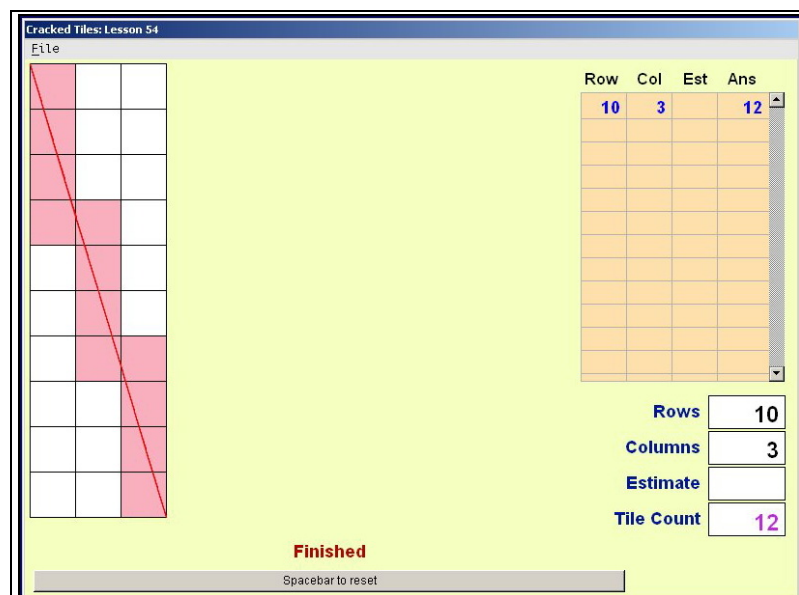
April 29th 2008

Matt Skoss has. Matt is Mathematics Project Officer in the Northern Territory. He used it recently in a remote presentation to the Mathematical Association of South Australia (MASA) conference.

The context of the lesson has a certain reality. A builder has just finished laying a rectangular tiled floor when the electrician arrives and declares a cable has to be laid diagonally under the floor. Frustrated, the builder asks; "How many tiles have to be replaced?" The general problem that develops is, given any size floor, can we predict the number of tiles that would have to be replaced to lay a diagonal cable?

Using grid paper, students begin by exploring cases where the length and width are relatively prime and soon find a relationship. But what happens if we try some other situations, for example where the length and width have a common factor?

The work of exploring a broader range of cases can be shared around the class - this group explore cases where the longest side is 12, this one 11, this one 10 and so on - and the data recorded on the whiteboard. But this process is limited by what can be drawn on a grid page and as hypotheses develop software allows them to be more efficiently tested. For many cases, the visual representation of the cracking and the developing list of data stand side by side on the software screen. For cases too big to draw the computer will calculate the result up to a rectangle 1000 x 1000. Cleverly though, the software won't do a calculation without the student first entering a prediction. This is a tool to support students learning to work like a mathematician; not a tool to do the mathematics for them.



Once a generalisation has been discovered, the next challenge is to explain why it works. The lesson involves spatial aspects, factors and an algebraic component.

Editor's Note: See October 2008 for more on Lesson 54, *Cracked Tiles*.

Content

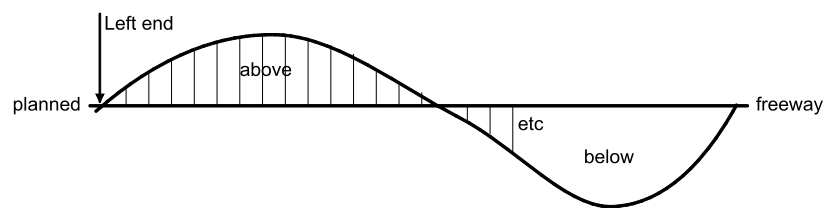
- algebraic generalisation
- arithmetic skills
- multiples, factors & primes
- number patterns
- visual patterns

Are you looking for a way to introduce integration?

Years: 11 - 12

May 28th 2008

This month's message is for Year 11 and 12 teachers. Lesson 152, *Area Functions*, introduces the concept of developing a function to describe an area under a curve. The story shell used is that of building a freeway through hilly country. Students are familiar with the idea that the freeway is made 'level' by removing soil from the hills and placing it in the valleys. Construction engineers need some way to measure the amount of soil involved and it is this need that leads into the lesson.



Having focussed on the idea of the area 'under' a function, students are invited to explore the relationship between the x-value of the function and the area up to the point (x, y) , firstly by applying strategies such as try a simpler problem, then with specially prepared Excel spreadsheet software. Using the software is supported by a detailed Investigation Guide. Answers and discussion are included in a separate link for teachers.

Secondary goals of the lesson include:

- a growing ability to visualise the shape of an area function just by looking at a graph;
- recognition of the types of functions that area functions can be;
- how this is related to the type of initial function; and
- awareness that continuity is an important feature of a graph for an area function to exist.

Perhaps this lesson will take you into a new area of teaching.

Content

- | | | |
|------------------|--|-----------------|
| • area functions | • functions: polynomial, trigonometric, exponential, logarithmic | • sketch graphs |
| | | • spreadsheets |

Editor's Notes:

1. There was no ETuTE for June 2008 because the author took long service leave and went bush.
2. July 2008 ETuTE was a repeat of July 2004 because the Olympics came around again. However two new lesson had been added in the interim that were relevant to the Olympic theme.

Lesson 122, *How Many People Can Stand?* Lesson 145, *Estimating Averages*.

Are you primed and ready for a challenge?

Years: 4 - 10

August 19th 2008

Goldbach's Conjecture is a lesson which is easy to state and easy to start. It also has a wonderful thread of mathematical history and a link to modern literature.

On June 7th 1742, Christian Goldbach (a Prussian mathematician) wrote to the famous mathematician Leonhard Euler suggesting that every integer greater than two can be written as the sum of three primes. Goldbach was counting 1 as a prime - a convention since abandoned. Euler became interested in the conjecture and rephrased it as every EVEN integer greater than 2 can be written as the sum of two prime numbers. Euler also commented that this was a certain proposition, even though he was unable to prove it. Trouble is, since that time, no one has been able to prove it. Nor has anyone been able to find an even number for which the conjecture isn't true - a counter example.

When the book *Uncle Petros and Goldbach's Conjecture* by Apostolos Doxiadis was published by Faber & Faber, the publishers offered a million dollar prize to anyone who could prove (or disprove) Goldbach's Conjecture. The prize is no longer available, but the problem remains unsolved. Perhaps in your lesson there will be someone who can find the counter example.

The lesson is essentially about recognising primes and it contains plenty of addition practice, but the problem solving context and the historic and literary connections add interest. It also partners well with lessons like Odds & Evens and Palindromes and it has software to support the investigation.

Apology: There are some external links in the lesson that no longer work because they have been removed from the source site. However, those that do work provide very good additional information.

Goldbach's Conjecture: Enter your number and search range

File Options

Start:

Finish:

Every even number, greater than two, can be written as the sum of two primes.

Number	Pairs
	23 71
	41 53
	47 47
96	7 89
	13 83
	17 79
	23 73
	29 67
	37 59
	43 53
98	19 79
	31 67
	37 61
100	3 97
	11 89
	17 83
	29 71
	41 59
	47 53

Finished

Spacebar to reset

Back

Content

- arithmetic skills
- combination theory
- prime numbers

Does a theme seem right for you?

Years: 2 - 8

September 17th 2008

The question generates from a recent email. Mal Closter, Kyneton Secondary College, wrote:

We are trialing a flexible learning space for Year 7s next term. The topic we are attempting is Pirates. Do you have any ideas, tasks, assignments that apply to maths and pirates? Keep up the good work with the newsletters and the tasks. We have about 50 that are used in the Year 7, 8 areas.

Perhaps our answer below will give you food for thought too.

For your theme, an obvious task that springs to mind is *Walk The Plank*, Task 131. It is also explored in Maths300 Lesson 32. Given this is a model for integer arithmetic, it fits well with the Year level. After that you have to get a bit imaginative. For example:

Pirates are always after gold. Adapt Task 163, *Eureka*. This is also explored in Maths300 Lesson 71. There is a comment in the Maths300 summary from a Year 8 student that will confirm its age appropriateness.

Pirates sail under the skull and crossbones. Adapt Task 35 *Crosses* (M300 #112) so they are sailing under the skull and mathematical crossbones, eg: every ship in the pirate fleet sails under a different skull and mathematical crossbones

- How many ships are there in the fleet?
- How do you know when you have sunk them all?

When on land, Pirates have to hunt treasure. Surely they would have to cross a river in doing that. Adapt Task 173, *Crossing the River* (M300 #23). They could perhaps cart one small landing craft with them into the jungle in case they needed it for such purposes and two cabin boys as servants.

They would also have to follow treasure maps which means they would have to be good at estimating angles. There's an opening to Task 214/Lesson 133, *Angle Estimation*.

There will be more. Can I also suggest that to structure the unit you include a Self-directed Maths Journey (SMJ) as an excellent way of integrating with other disciplines. This model is described in the Integrating Tasks link of the Maths Task Centre and is a key structure in some Maths With Attitude kits.

<http://www.mathematicscentre.com/taskcentre/plans.htm>

Content

- curriculum planning

Do you use the three step planning method?

Years: 5 - 10

October 22nd 2008

When using text books to drive the curriculum, these three steps are often the last three before the classroom door.

When using Maths300 or Maths With Attitude it can't be that way. The three steps are:
Research, Reflect, Collect

Research

... by reading your chosen lesson through and making notes about the key steps.

Reflect

... on your students, the room space and the teaching craft suggested in the lesson and make the appropriate adjustments to your planning.

You will be better prepared to think on your feet, if you have thought on your bottom first.

Collect

... what you need - which is often not very much - and go for it.

I was reminded of the importance of this process recently when I decided to use Lesson 54, *Cracked Tiles* with a Year 9. On reading the lesson through again I realised that it bounced straight in with drawing a rectangle on graph paper to represent the tiled floor. Reflecting on successes with other lessons, such as Garden Beds, when I have introduced the story shell with a floorboard model I wondered if this lesson could be improved with the same approach. I collected thirty 20cm square pieces of card and a length of string to add to my graph paper. As I introduced the story about the builder tiling the 10x3 floor I gave the cards out to students. They had to build the tiled floor at the front of the room. The piece of string, stretched diagonally by two students was the line where the electrician wanted to rip up tiles to lay a cable.

In my judgement this addition to the lesson mattered a great deal in this group. It influenced who became involved, how quickly and with what level of understanding.

After recording that experiment on graph paper, the materials also allowed us to try a 6x5 floor, and, although we didn't need to, any other rectangle that could be made with up to 30 tiles.

Content

- curriculum planning

Editor's Note: See April 2008 for more on *Cracked Tiles*

What will you get for Christmas?

Years: 4 - 12

November 20th 2008

It's a relevant question since this will be your last ETuTE before Australian schools start up again in February 2009.

The answer, of course, is gifts! And would you believe Santa had them all organised in groups for delivery to different areas - all beautifully decorated, numbered and everything - ready to load into the sleigh, when a huge wind from the North Pole scattered the lot and mixed up all the groups!! Oh no!! Yuletide disaster.

Fortunately Santa's #1 Elf remembered the rules for sorting the gifts and the team was able to get them back in the correct piles ... but were they??? What Ol' #1 didn't know was that the rules he remembered resulted in more than one answer.

- What are the possible arrangements using the remembered rules?
- What is the one rule Ol' #1 needed to remember so there was a unique answer.

Now you have the plot for Lesson 70, *Pick A Box*, which is appropriate for students from Years 5 - 8 (at least). But don't try it unless you have access to a colour printer. It simply doesn't work in black and white. (Imagine a grey scale Christmas!). The maths content is number work related to consecutive numbers, odds and evens and problem solving strategies.

If you have tasks from the Mathematics Task Centre check whether you have Task 91. It's the same problem and it has a great set of wooden 'boxes' as its material.

For more Christmas, take a look at Lesson 167, *Twelve Days of Christmas*. This is suitable for Years 4 to 12. It includes visual and number patterns, all of which are generalisable in words and symbols (which bit of that would be the algebra then?), equivalent algebraic expressions and quadratic expressions.

And while we are ending the year with a bit of fun, many thanks to Greg Lee from Rosebud Secondary College who sent in the following information.

Lesson 103 is about *Palindromes* and, in fact, so was the first ETuTE back in February 2004. Many years before that Bob Dylan did a classic video called *Subterranean Homesick Blues*. Greg has found a Weird Al Yankovic parody of it on You Tube in which every display card is a palindrome. It's very clever, especially when you can compare it with the original which is also on that site. Better still, not to be outdone, Ian Wilson has parodied Weird Al with his own set of palindromes which include "I PREFER PI". Over to you on how you use this information in your Palindromes lesson.

Content

- | | | |
|----------------------------|--|----------------------|
| • algebraic generalisation | • equivalent expressions | • quadratic function |
| • arithmetic skills | • number patterns: natural, triangle, square | • visual patterns |
| • consecutive numbers | • odd & even numbers | |

Can you match this?

Years: 4 - 8

February 26th 2009

Match Triangles, Lesson 164 and Task 178, has that name because when classes first started exploring it they used matches. Icy-pole sticks (popsticks) are much less fiddly and at around \$10 per 1000 at your local craft shop are hardly an expensive budget item. The lesson actually begins without them anyway. Students roll up newspaper into tubes, use bits of masking tape to prevent them from unrolling, then everyone contributes their roll to a floorboard model of triangle chains similar to those often seen providing strength as struts in buildings. As a community of learners, a growing set of challenges is explored and alternative views of the pattern are validated.

It is a thoroughly absorbing visual algebra lesson for Years 4 to 8 that is rich in content. We're talking a week working on the same problem, which is why it is a core example in Maths With Attitude Pattern & Algebra Years 7/8. But like all Maths300 lessons and all tasks it can take a new direction when a teacher responds to student suggestions.

Greg Muir, Rosebud Secondary College, is participating in our 6 day *Engineering 'aha' Moments in Algebra* course organised by a small cluster of Victorian secondary schools through an ASISTM grant. His start to the lesson was a discussion with Year 8 students that began with trusses obvious in the school building and led into exploring the pattern using popsticks. Students soon identified and confirmed a rule relating the total number of sticks to the required number of triangles with rules such as:

$$2 \times \text{triangles} + 1 = \text{icy-pole sticks}$$

which they soon shortened to:

$$2T + 1 = i$$

The lesson diverged into a new rich field when a student asked if ...*we could make squares like this?* Other shapes followed and soon (quoting Greg):

All results were recorded on the board and ordered pairs were made for triangles, squares, pentagons and hexagons. These were entered onto an Excel spreadsheet and then graphed.

To round off, Greg discussed with the students what they thought would be necessary to include in a report that explained their investigation to someone else. Then he set the task of preparing such a report and including spreadsheet information as part of it.

But did the students learn any algebra from all this playing around in maths?

Again, quoting Greg:

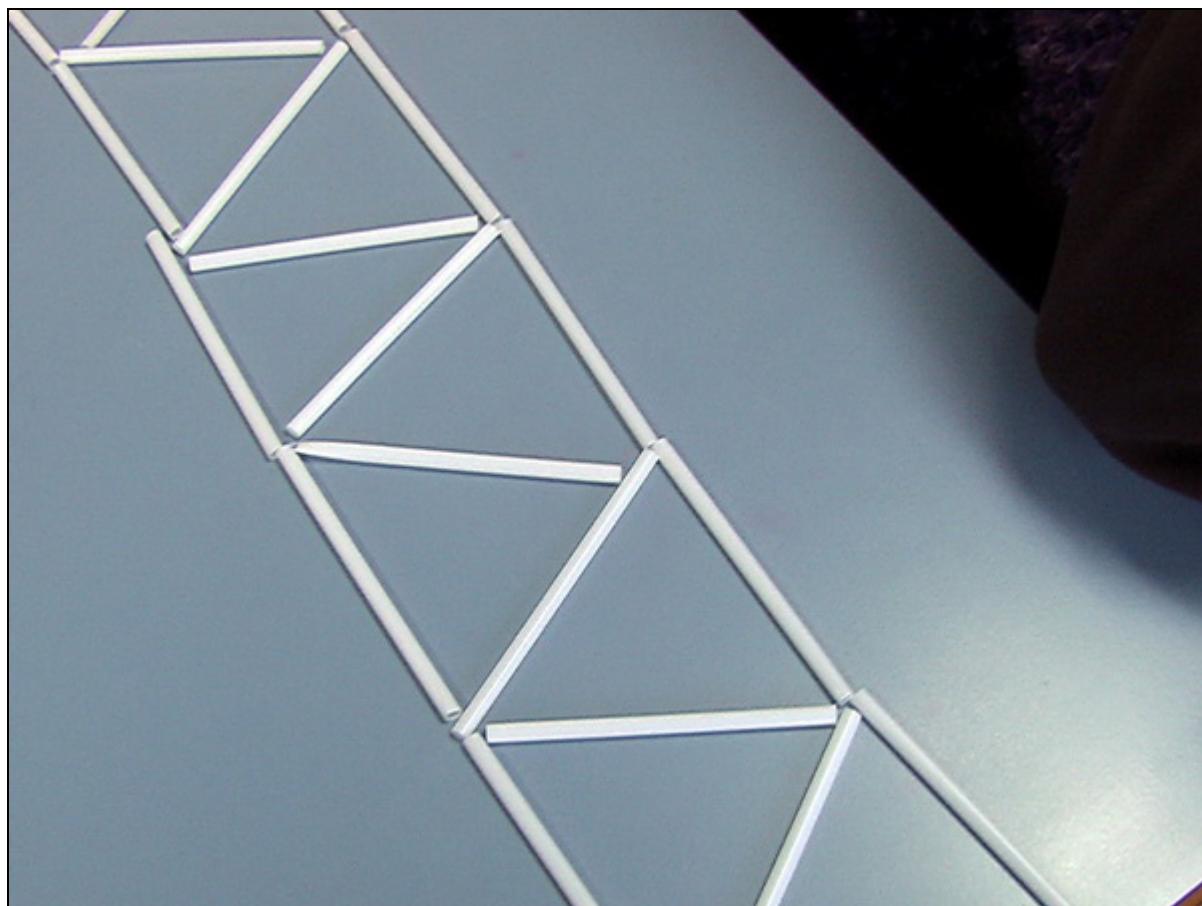
A group of girls came running across the room very excited with several formulas relating to different shapes and they said, "We've found a pattern with the shapes. If you have a shape you multiply the number of shapes you want by one less than the sides of the shape and then add one. Just like the triangles."

Three boys who usually work like a frozen polar bear actually finished all work required and sought help out of class to make sure that they were on the right track.

One boy who was a reluctant participant made two graphs and compared them saying that the graphs were steeper as the number of sides went up. So the squares graph was steeper than the triangles and so on. (Meaning of course that student work could now be the source of future investigation into gradients, intercepts and the properties of linear graphs.)

After these hands-on lessons were finished the students completed some exercises out of the text book and the statement was made that a lot of the questions were like the icy-pole stick questions and they could understand the questions a lot more now.

Can your students match that understanding?



Content

- algebraic generalisation
- arithmetic skills
- equations: solving
- equivalent expressions
- graphing ordered pairs
- spreadsheets
- substitution
- symbolic representation

G'day mate. Nice to meet you. What's your birth month?

Years: 4 - 12

March 25th 2009

No it's not the opening line at a party of astrologers, but it does raise an interesting mathematical question. If, say, a group of 5 people meet at a party, what's the chance that they share the same birth month?

First level reasoning might argue that there are 12 months in a year, the 5 people only represent 5 out of 12, so the chances are less than 50%. At the next level, a mathematician might use the strategy of 'act it out'.

Okay kids, move around the room and when I say NOW you quickly make groups of 5.

Then we check how many groups have matching birth months. That would give us more data and we might have to alter our first estimate of the chance. But perhaps it doesn't give us enough data to be sufficiently certain, so a mathematician might use the strategy of 'make a model'.

Each person in our groups of 5 has a set of cards from 1 to 12 (1, 2, ..., 9, 10, J, Q). Shuffle the cards and show your top one. Record how many trials and how many of those have matches. Mmmm, but perhaps still not enough data to be sure of the chance, so the mathematician might use the strategy of designing software to carry out many simulations very quickly - another form of model. We would be able to tell what the real chance is likely to be by the way the average match rate 'settles down'.

The actual result is counter-intuitive. To find out more you would need to visit Lesson 148, *Birth Month Paradox*, lesson. As a bonus, you might be able to use the attached Investigation Guide contributed by Bill Chapman from Manjimup Senior High School. He designed it to support this investigation with his Year 11 class and has donated his work to Maths300 Members. Thanks Bill.

Do I hear you say, *Year 11! I can use that investigation with my Year 5s.*

You're right. Isn't it wonderful to have such a rich resource to support all students to learn to work like a mathematician?

Editor's Note: Bill's Investigation Guide was distributed as an electronic attachment in the original ETuTE and has not been attached to this book.

Content

- cumulative probability
- long run frequency
- mathematical simulation
- probability
- statistical confidence
- statistical inference

Where to from here?

Years: K - 12

April 24th 2009

It's an appropriate question for a number of reasons. Firstly, this will be the last ETuTE I write. Responsibility for continuing this service passes to Curriculum Corporation. My future energy will be directed towards managing all professional learning aspects of the Mathematics Task Centre and Calculating Changes. Of course, I will be drawing on Maths300 as part of my continuing work, as it contributes so much to the big picture of students learning to work like a mathematician. But from the end of April I will no longer be responsible for organising and managing Maths300.

I began this monthly ETuTE service in February 2004 with a question raised in an email from my daughter. Call it self-indulgent if you will, but I am going to finish with an email from my son. He tells this story about my grandson who was three and a bit at the time they were driving home from somewhere:

"That's a mail truck Daddy", he said from the back seat. "Will we have mail when we get home?"

Reasonable question I thought.

"We'll have to check mate. There might be big square envelopes in the mail box then we will know."

"Daddy a square is a shape."

"Yes mate it is." (At this stage I thought what a clever little boy I have.)

"It has four sides all the same."

"You're right mate." (...a REALLY clever little boy)

"But Daddy, an envelope is not a square it is a rectangle. It has two long sides and two short sides, and that is a rectangle."

I laughed out loud at this point thinking Joshua probably chose to avoid saying "Dummy" at the end of that sentence. But I was intrigued by the conversation, so I probed a bit.

"What other shapes do you know about mate?" I prepared myself for an in depth discussion of triangles and circles when Josh said, "Pentagon Daddy." and I nearly drove off the road.

Laughing I asked, "And what do you know about pentagons?"

"They are a shape Dad, they have five sides and look like a house."

"So we live in a house, does it look like a pentagon?"

"No Dad, you silly billy", my beautiful boy said, "a pentagon is a square with a triangle on top, our house does not look like that. A pentagon is just a shape."

That was when I realised my Son was smarter than me, at 3 years old!

- Where to from here for this lad on his journey through school?
- Will school contribute to or hinder the mind-expanding journey he has begun?
- In what ways did the teaching you and I did today expand the minds and souls of the students we taught?

And where to from here with pentagons and triangles? Only one answer to that really.
Lesson 134, *Pentagon Triangles*.

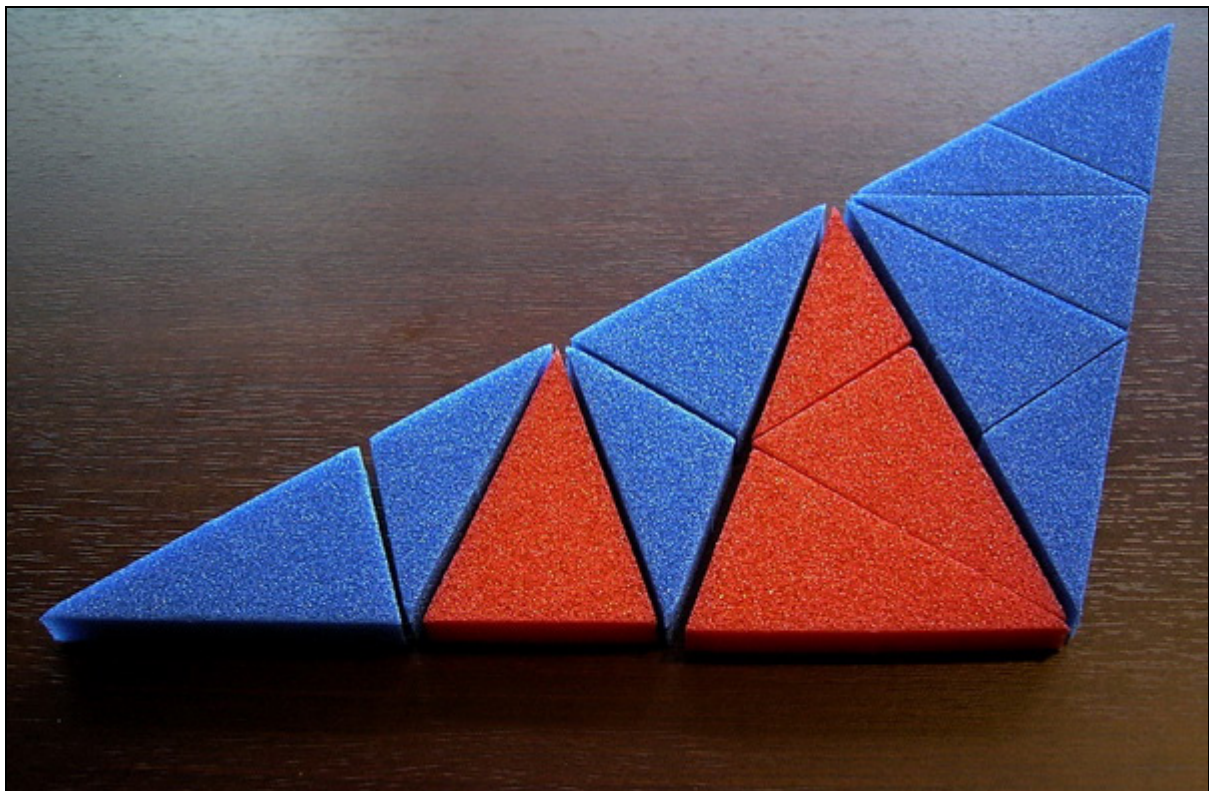
Take a regular pentagon and cut it into three triangles along its diagonals. Easy to state, easy to start and heaps of maths. At one level the three pieces can be used for creating spatial patterns and exploring shapes such as triangles, pentagons and decagons. At another there is work on angles, lengths, areas, fractions, decimals, Fibonacci Numbers and the Golden Ratio. Best of all, the joy of discovering this mathematical content develops in a problem posing and solving environment that reflects the work of a professional mathematician.

The lesson is well supported with photographs and a fabulous Classroom Contribution from Damian Howison.

Keep smiling,
Doug.

Content

- | | | |
|-----------------------|---------------------------|----------------------|
| • curriculum planning | • polygons | • ratio & proportion |
| • number patterns | • properties of triangles | • similar triangles |



Year Level Finder

Two ETuTEs were issued in February 2005, hence reference below to February 2005 (1) and February 2005 (2).

Year Level	ETuTE Dates
K	March 2004, May 2004, June 2004, July 2004, April 2006, May 2006, June 2006, July 2006, November 2006, February 2007, May 2007, July 2007, September 2007, December 2007, April 2009
1	March 2004, May 2004, June 2004, July 2004, August 2004, October 2005, November 2005, April 2006, May 2006, June 2006, July 2006, November 2006, February 2007, May 2007, July 2007, September 2007, December 2007, April 2009
2	March 2004, May 2004, June 2004, July 2004, August 2004, August 2005, September 2005, October 2005, November 2005, March 2006, April 2006, May 2006, June 2006, July 2006, November 2006, February 2007, May 2007, July 2007, September 2007, December 2007, March 2008, September 2008, April 2009
3	March 2004, May 2004, July 2004, August 2004, February 2005 (2), March 2005, July 2005, August 2005, September 2005, October 2005, November 2005, March 2006, April 2006, May 2006, June 2006, July 2006, August 2006, September 2006, November 2006, February 2007, May 2007, July 2007, September 2007, December 2007, March 2008, September 2008, April 2009
4	February 2004, March 2004, May 2004, July 2004, August 2004, February 2005 (2), March 2005, April 2005, July 2005, August 2005, October 2005, November 2005, March 2006, April 2006, May 2006, June 2006, July 2006, August 2006, September 2006, October 2006, November 2006, February 2007, May 2007, June 2007, July 2007, September 2007, October 2007, November 2007, December 2007, March 2008, August 2008, September 2008, November 2008, February 2009, March 2009, April 2009
5	February 2004, March 2004, May 2004, July 2004, August 2004, September 2004, February 2005 (2), March 2005, April 2005, July 2005, August 2005, September 2005, October 2005, November 2005, March 2006, April 2006, May 2006, June 2006, July 2006, August 2006, September 2006, October 2006, November 2006, February 2007, March 2007, May 2007, June 2007, July 2007, September 2007, October 2007, November 2007, December 2007, February 2008, March 2008, April 2008, August 2008, September 2008, October 2008, November 2008, February 2009, March 2009, April 2009
6	February 2004, March 2004, May 2004, July 2004, August 2004, September 2004, February 2005 (2), March 2005, April 2005, May 2005, July 2005, August 2005, September 2005, October 2005, November 2005, March 2006, April 2006, May 2006, June 2006, July 2006, August 2006, September 2006, October 2006, November 2006, February 2007, March 2007, May 2007, June 2007, July 2007, September 2007, October 2007, November 2007, December 2007, February 2008, March 2008, April 2008, August 2008, September 2008, October 2008, November 2008, February 2009, March 2009, April 2009

Two ETuTEs were issued in February 2005, hence reference below to February 2005 (1) and February 2005 (2).

Year Level	ETuTE Dates
7	February 2004, March 2004, May 2004, July 2004, August 2004, September 2004, February 2005 (1), February 2005 (2), March 2005, April 2005, May 2005, July 2005, August 2005, September 2005, October 2005, March 2006, April 2006, June 2006, August 2006, September 2006, October 2006, November 2006, February 2007, March 2007, May 2007, June 2007, July 2007, August 2007, September 2007, October 2007, November 2007, December 2007, February 2008, March 2008, April 2008, August 2008, September 2008, October 2008, November 2008, February 2009, March 2009, April 2009
8	February 2004, March 2004, May 2004, July 2004, August 2004, September 2004, February 2005 (1), February 2005 (2), March 2005, April 2005, May 2005, July 2005, August 2005, September 2005, October 2005, March 2006, April 2006, June 2006, August 2006, September 2006, October 2006, November 2006, February 2007, March 2007, May 2007, June 2007, July 2007, August 2007, September 2007, October 2007, November 2007, December 2007, February 2008, March 2008, April 2008, August 2008, September 2008, October 2008, November 2008, February 2009, March 2009, April 2009
9	February 2004, March 2004, May 2004, July 2004, August 2004, February 2005 (1), February 2005 (2), March 2005, April 2005, May 2005, July 2005, October 2005, March 2006, June 2006, August 2006, September 2006, November 2006, February 2007, March 2007, May 2007, June 2007, July 2007, August 2007, September 2007, October 2007, November 2007, December 2007, April 2008, August 2008, October 2008, November 2008, March 2009, April 2009
10	February 2004, March 2004, May 2004, July 2004, August 2004, February 2005 (1), February 2005 (2), March 2005, April 2005, May 2005, July 2005, March 2006, June 2006, September 2006, November 2006, March 2007, May 2007, June 2007, July 2007, August 2007, September 2007, October 2007, November 2007, December 2007, April 2008, August 2008, October 2008, November 2008, March 2009, April 2009
11	March 2004, May 2004, July 2004, August 2004, February 2005 (1), April 2005, May 2005, July 2005, March 2006, June 2006, March 2007, June 2007, August 2007, September 2007, October 2007, November 2007, December 2007, May 2008, November 2008, March 2009, April 2009
12	May 2004, July 2004, August 2004, February 2005 (1), May 2005, July 2005, March 2006, June 2006, March 2007, June 2007, August 2007, September 2007, October 2007, November 2007, December 2007, May 2008, November 2008, March 2009, April 2009

Content Finder

All Maths300 lessons involve the Working Mathematically process.

All Maths300 lessons involve problem solving strategies, which are part of the Working Mathematically process.

Lessons referenced in ETuTE are samples from Maths300, therefore the list below does *not* represent the full range of content available.

Content	ETuTE Dates
• 1:1 correspondence	February 2005 (1), July 2006
• addition/subtraction	March 2004, June 2004, November 2005
• algebraic generalisation	July 2004, September 2004, February 2005 (2), May 2005, September 2005, February 2007, June 2007, April 2008, November 2008, February 2009
• angles & polygons	February 2005 (2), August 2006
• area functions	May 2005, May 2008
• area & perimeter	June 2004, July 2004, May 2005
• arithmetic skills	February 2004, March 2004, August 2004, February 2005 (2), April 2005, May 2005, August 2005, September 2005, October 2005, November 2005, July 2006, October 2006, June 2007, October 2007, November 2007, March 2008, April 2008, August 2008, November 2008, February 2009
• average/expected result	August 2004
• collecting/tabulating data	August 2004, July 2005, February 2007, August 2007, February 2008
• combination theory	July 2004, February 2005 (1), August 2005, October 2007, August 2008
• comparing fractions	May 2006
• concept of angle	June 2004, May 2006
• concept of fairness	June 2007
• concept of fraction	June 2004, May 2006
• concept of proof	March 2004, July 2004
• concept of variable	September 2004, September 2005, June 2007
• conditional probability	June 2007
• consecutive numbers	November 2008
• counting	March 2004, June 2004, February 2005 (1), September 2005, July 2006, February 2007
• cumulative probability	March 2009
• curriculum planning	May 2004, November 2004, February 2005 (1), March 2005, March 2006, June 2006, November 2006, May 2007, July 2007, September 2007, September 2008, October 2008, April 2009
• decimals	May 2005, October 2005, February 2007
• divisibility tests	October 2007
• equations: creating	October 2005
• equations: solving	September 2004, October 2005, February 2009
• equivalent expressions	September 2005, February 2007, November 2008, February 2009
• equivalent fractions	May 2006
• estimating angles	August 2006
• estimating length	February 2007
• Euclidean geometry	February 2005 (2)
• fractions of a turn	August 2006
• functions	September 2004, November 2007, May 2008
• gradient	September 2004, September 2005
• graphing ordered pairs	September 2004, September 2005, February 2008

• interpreting graphs	September 2004, February 2008
• inverse operations	August 2005
• iteration	May 2005
• language of chance	July 2005
• language of shapes	June 2004, February 2005 (2),
• laws of arithmetic	August 2005, September 2006
• linear equations	September 2005
• linear graphs	September 2004, September 2005
• language: position & order	August 2004, February 2005 (1), February 2007
• logarithmic function	August 2007
• long run frequency	August 2004, June 2007, March 2009
• mathematical simulation	March 2007, March 2009
• mathematical systems	August 2006
• measurement	July 2004, February 2005 (2), May 2005, August 2006, February 2008
• mental arithmetic	September 2004, September 2005, February 2007
• multiples, factors, primes	February 2005 (2), April 2005, April 2008, August 2008
• multiplication	February 2005 (1)
• multiplicative thinking	February 2005 (1)
• natural variation	July 2005, March 2007
• number patterns	June 2004, July 2004, September 2004, August 2005, September 2005, October 2006, February 2007, June 2007, April 2008, November 2008, April 2009
• numeral recognition	June 2004, July 2006
• odd & even numbers	April 2005, November 2008
• ordered pairs	September 2004, September 2005
• order of operations	October 2005
• percentages	August 2004, August 2007
• place value concepts	March 2004, November 2005
• place value skills	February 2004, March 2004, November 2005
• polygons	April 2009
• powers of 2	April 2005
• probability	August 2004, March 2007, June 2007, March 2009
• properties of triangles	April 2009
• quadratic functions	June 2007, November 2007, November 2008
• ratio & proportion	July 2005, April 2009
• sample space	June 2007
• similar triangles	April 2009
• sketch graphs	May 2005, May 2008
• sorting	July 2006
• spatial patterns	June 2004
• spreadsheets	February 2008, May 2008, February 2009
• statistical confidence	July 2005, March 2007, June 2007, March 2009
• statistical inference	July 2005, March 2007, June 2007, March 2009
• statistics	February 2007, March 2007
• stem & leaf graphs	March 2007
• substitution	September 2004, September 2005, February 2009
• symbolic representation	February 2005 (2), February 2007, June 2007, February 2009
• symmetry: equil. triangles	August 2006
• tree diagrams	October 2006, October 2007
• trigonometry	February 2005 (2), August 2006
• visual patterns	February 2005 (1), April 2008, November 2008