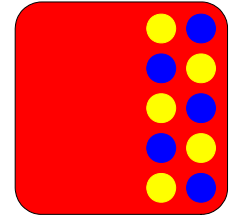# Aha! Teaching Maths is Simple

Doug. Williams

Simple But Subtle

It's time we realised that teaching maths is simpler than government documents, text books and even many professional development programs would have us believe. Teaching maths is simple ... but subtle. In this paper I will draw on Number examples from several levels of primary school to explain what I mean. The story I tell is based on the experience of teachers who, since around 1996, have been working in a network titled Calculating Changes (HREF1).

Big Bang theories aside, nothing comes from nothing. So it is with Calculating Changes. Although originating in Australia, two key elements of its heritage link to Britain.

1. In 1993 I asked Geoff Giles, DIME Projects, Scotland, to help me with the final design detail of a resource called Poly Plug.

2. Initially from 1986-89, but with later extensions, the Calculator Aware Number (C.A.N) Project led by Hilary Shuard, Cambridge University, and funded by the British Government, explored the free use of calculators with thousands of primary children in classrooms where formal calculation algorithms were not taught. The remarkable success CAN had in enhancing children's number sense was later repeated in Australia in the Victoria College Project.

Calculating Changes teachers focus on engineering 'aha' moments in their classroom and as a result enhance children's number sense beyond what is usually expected for the age. That is, concentrating on the richness of the teaching environment, rather than the

detail of syllabus content, has led to better number skills.

Now on with the stories.......

Ten Friends

(Suitable for school years R - 1. See HREF2)

*   Person A pushes out a row of five red plugs and puts the plugs in the plastic bag which is part of the equipment. They are not used again and can't get lost in the bag. Person B pushes out, and stores, a parallel row of five plugs to complete a Tens Frame.
*   Person A rolls one spot dice and plugs in that many plugs of their chosen colour (yellow or blue). The dice stays where it landed.
*   Person B *does not* touch the dice. Their job is to look really hard at the frame and *guess* how many of the other colour it will take to complete it.
*   Person B tells their guess, then proceeds to check this hypothesis by counting in their colour.

It doesn't matter whether the hypothesis is right or wrong. The counting makes the activity self-correcting. I know kids who have screwed up their brows in great furrows studying the Tens Frame and announced, *"Five, 'cos I'm five."* They proceed to count in their plugs and if they get to five and have gaps left, they shrug their shoulders and keep plugging and counting.

Not having to be right or wrong takes a lot of stress out of the classroom. On the other hand collecting data about a problem and using it to test your own hypotheses adds a lot of learning to the classroom.

Once the frame is filled we reach the reporting/recording stage. At the very least the players tell each other what they made. *"I rolled six and you added four. So six plus four equals ten."* They can also record the event on their calculator - a must, so the number symbols are partnered with their visual/kinaesthetic representations - and in their journal or on Poly Plug paper. (HREF3). When some form of recording has been completed players swap roles and play again.

Little kids love this activity. Don't ask me why, I'm no longer a little kid. It couldn't be simpler could it? Easy to state, easy to start and, when you take a second look, heaps of maths. Think it through. Can you see:
• 1:1 correspondence
• addition facts beyond 10
• addition facts to 10
• complementary addition
• conservation of number
• counting
• estimating number
• group counting
• mathematical conversation
• problem solving
• recording - calculator
• recording - written
• subitising
• visual & kinaesthetic representation of number

Not addition facts beyond 10 you say? Well what about when a five year old asks, *"Can we play with three rows out of the red board?"* Do we say *"No, the curriculum says that at this level you use numbers up to 10"*?

This is where subtle begins to kick in. Each of these content bullet points could be the focus of a teaching slot. *"What does the curriculum say I have to teach today. Mmm, 1:1 correspondence. Now what can I do for that?"* The risk in beginning our teaching from content statements like this is that we come up with dry, repetitive, unconnected, decontextualised, trivial, challengeless learning experiences. The bigger risk is that we get so used to thinking these are what maths is about that we can't even see how a rich activity like this can be used to develop several content elements simultaneously and meaningfully.

What the teacher does with Ten Friends is as important as what the children do. Every completed Ten Friends round is an opportunity for much more teaching. It might be as the teacher sits down with that pair, or it

might be that the teacher selects a completed board such as:
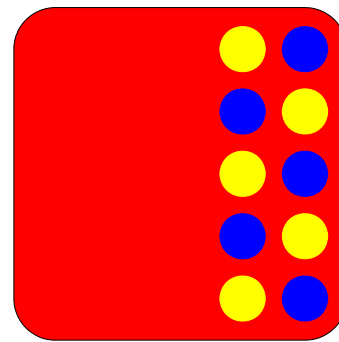
and uses Jamie and Abdul's as a model for the day. What follows is a Touch & Tell segment that involves saying, touching and recording as many ways of looking at the completed board as possible. For example, in the board shown, children have found and demonstrated:

• 5 + 5 = 10 (two zig-zags)
• 10 - 5 = 5 (cover one zig-zag)
• 10 - 5 - 5 = 0 (cover one zig-zag then the other)
• 2 + 2 + 2 + 2 + 2 = 10 (touch the yellow/blue pairs)
• 5 × 2 = 10 (same way)
• 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 10 (touch each plug as you count)
• 10 × 1 = 10 (same way)
• 3 + 3 + 2 + 2 = 10 (first 3 blues and first 3 yellows each make a triangle and the twos are the arms of the remaining cross)

and, of course, if the board is visually different the Touch & Tell outcomes will be different too.

That leads us to the next subtlety. This is not an activity to be used once and discarded. We *don't* have to find something new for maths tomorrow. We use Ten Friends again. The structure is the same so the children don't have to waste discernment on the rules of new activity; but the challenge is bound to be different because the dice will fall differently and Person A will place differently and both players might be seeing things differently. But we don't use it for a full lesson; just a few minutes a day for two or three or four days this week, then perhaps two days next week and so on for a number of weeks. It's called Threading because the activity appears as a set of stitches in the fabric of the curriculum.

Threading:
• Rich tasks
• Familiar structure
• Fresh challenge
• Short, frequent visits

It gives children time to construct their own learning in a non-threatening situation *which they own.*

Nicholas Dale, an upper primary teacher at Winkie Primary School, South Australia knows Threading Works (HREF4). In this article he writes: *Students have been more focused, engaged and interested in mathematics. They now enjoy doing maths everyday and look forward to the tasks I present to them. (Our curriculum...) has moved away from the textbook oriented lessons and into creativity in maths. Lessons in which I used to struggle to engage students for 30 minutes, now engage them for sometimes up to two hours. Therefore, this idea we have been presented with WORKS!*

Poly Plug, Proportion & Percent

(Suitable for school years 3 - 6. See HREF5)
Kate Thureau was teaching a composite Year 5/6 class at Gagebrook Primary School when she was introduced to Threading at a professional development (INSET) program. Participants were challenged to use any Calculating Changes activity and try threading it two or three times a week for about 4 weeks. I'll let Kate tell the story:

I selected Poly Plug, Proportion & Percent and approached it with a certain amount of trepidation. Firstly I felt the topics of proportion and percentages would cause some problems for my 5/6. Secondly I was using the Poly Plugs for the first time - with an older group that would be interesting.

I started the first lesson and put a yellow/blue Poly Plug board on the desks and asked the children to work in pairs. For the first 10 minutes the children were asked to experiment with these previously unseen boards. They enjoyed this exploration time and were well occupied. They were then happy to proceed with the lesson.

I followed the instructions described in the activity. Each pair turned over 12 in 25, then a group came up the front and we recorded it on the board. When a second group came up we recorded 24 in 50, then 36 in 75,

48 in 100 and so on. We got a little carried away and continued till we reached 400, it was all recorded.

The next lesson the children turned over 6 plugs and we did it as a class activity and then they were asked to fill in the missing spaces, eg:

<div align="center">6 in every 25 are turned over</div>
<div align="center">12 in every ___ are turned over etc.</div>

Soon I was getting explanations such as:

<div align="center">All you need to do is double both numbers to get the next numbers.</div>

To get the answer for 100 all you need to do is to times by 4.

By this stage the children were more easily able to visualise the problem and were doing away with the Poly Plugs and the calculators. It was great seeing the children's reactions when all of a sudden they had the 'aha' moment when everything fell into place.

Once they were able to visualise the problems, looking at equivalent fractions and percentages became remarkably easy. It was like a journey that we all approached together because we were all students; it became extremely enjoyable and a very valuable learning experience. The children were keen to do these activities and even the repetition made them feel more secure and none of them complained.

They were asked every day when a certain number of plugs were inverted what percentage was turned over. It was amazing how quickly they were able to work out the equivalent fractions and determine the percentage.

I am amazed at how a topic that I thought would cause a huge amount of difficulty turned into an extremely valuable and enjoyable unit. The children did

not feel threatened and it was wonderful to observe them when they understood the concepts and were happy to help each other.

See, teaching maths *is* simple. Let the kids do it. The subtlety is in being the teacher who can facilitate this. It's a teaching model that we may not have experienced as a learner, so simple as it sounds, it may not be so simple to enact. That's why Calculating Changes exists. It is a network of teachers teaching each other to teach kids to teach each other maths!

So how about one more example to help you find your way to a more exciting maths curriculum ... for you and the kids.

Six Plus

(Suitable for school years R - 6. See HREF6)
Each player has a calculator. Player A enters 6 + and a secret number and presses =. Player A's screen is shown to Player B.

Player B knows how to start - enter 6 +. Now they have to guess a secret number, enter it, press = and hope to get the same screen as Player A. One point for each guess. Try again until you get it. Each try is a new piece of data to support a growing hypothesis. Play a few rounds and the player with the lower total score wins, 'cos they are the better guesser.

Another simple idea. Easy to state, easy to start and heaps of maths. Today it's 6 Plus and of course we use our plugs to make a picture explaining how we worked it out. And of course we record that explanation in our journal. Then tomorrow the game is 7 Plus, the next day 4 Plus, then 21 Plus then ...

...how about 9 Times or 99 Minus or... This activity could be threaded into the curriculum every year and still be a fresh challenge!

But in essence Six Plus is 6 + ___ = 15. Mathematically the two presentations are the same. Why is it that children will do dozens of these 'sums' as part of the game, but baulk, and even think they can't do them, when ten of these 'sums' are presented on a sheet or from a textbook?

I posed this question to a group of superintendents of mathematics at a conference in the USA some years ago. Their responses were:
- Empowerment –
students are in charge of the numbers
- It's yours – you are involved
- Recreational – this is 'stuff we can have fun with'
- Learning together - student/student and student/teacher - everyone has a part
- Self-checking of the reasonableness of answers

- Provides reason to learn the skill - ie: to play the game better
- Immediate response
- Non-threatening
- Mathematical conversation
- Writing doesn't get in the way
- Potential to link concrete, symbolic and personal recording
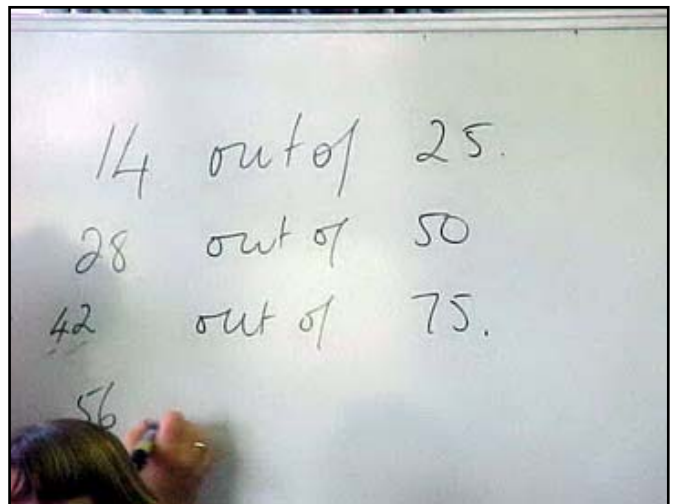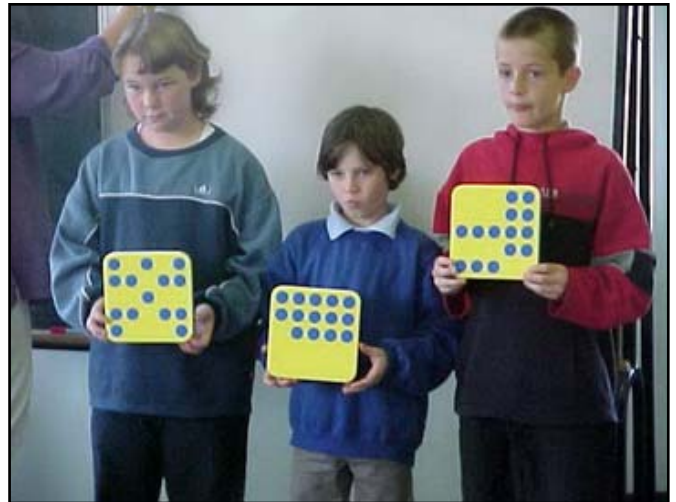
The Bigger Picture

The Calculating Changes network has developed over sixty activities like these. Taken a couple at a time and threaded into the curriculum learning looks and feels different. Number sense is enhanced and children and teachers both feel better about themselves. But this is not a complete curriculum for at least two reasons:

1. There's more time in each lesson than is filled by a couple of threaded activities for 10 minutes or so each.
2. We have yet to put these developing skills to work and for that we need investigations. Why, because we are learning to work like mathematicians, not just learning to practise the skills of a mathematician.

When we asked some professional mathematicians to describe their work they responded:

First give me an interesting problem.

So, a Working Mathematically curriculum would include problems and our teaching craft would be called on to interest learners in them. That's very different to a content-driven curriculum.

But how do you approach one of these interesting problems? There is no promise of solution, there is only hope that there might be one, so what do you do? Mathematicians went on to describe their 'ways of knowing' as follows:

When mathematicians become interested in a problem they:

- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

Questions which help mathematicians learn more are:

- Can I check this another way?
- What happens if …?
- How many solutions are there?
- How will I know when I have found them all?

A mathematician's strategy toolbox includes:

- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Break the problem into smaller parts
- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- …
- *If one way doesn't work I just start again another way.*

This is the simple, yet subtle, description that has brought success to hundreds of classrooms from K to 12. It provides a higher order curriculum focus and a language for exploring it. Schools having most success use this language consistently across the school - yes even in reception years. And, perhaps because it makes sense, students don't ask *"Why do we have to do this?"*

Calculating Changes draws on this bigger picture by using activities that provide time for children to gather data in game-like situations, make and test their own hypotheses and ask key questions such as *Can I check this another way?* and *What happens if…?* In return it offers to the bigger picture - the Working Mathematically classroom - children with sound number concepts and an effective skill toolbox. There are hundreds of suitable investigations, but that's another story. For now, I invite you to explore the ideas in this article and contribute to the Network. When you are ready to look for investigations there are at least 100 in the Task Cameo link of the Mathematics Task Centre (HREF7).

References

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