Information Revival without a Backflow

Non-causal Explanations of Non-Markovianity

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References

- F.B.: *On complete positivity, Markovianity, and the quantum data-processing inequality, in the presence of initial system-environment correlations.* Physical Review Letters 113, 140502 (2014)

- F.B., R. Gangwar, K. Goswami, H. Badhani, T. Pandit, B. Mohan, S. Das, M.N. Bera: *Information revival without backflow: non-causal explanations of non-Markovianity.* Preprint arXiv:2405.05326

Reduced dynamics in the presence of initial system–environment correlations

The problem in a nutshell

textbooks usually begin with the factorization assumption, i.e., $\bullet_Q \otimes \gamma_E$: in this case, the reduced dynamics $\mathrm{Tr}_{E'}\left[U_{QE\to Q'E'}\,(\bullet_Q \otimes \gamma_E)\,U^\dagger_{QE\to Q'E'}\right]$ is always well defined, completely positive and trace-preserving

- 1994: Pechukas' PRL (what if we drop the factorization assumption?) and Alicki's comment on it
- 2004: Sudarshan's group (explicit constructions and examples)
- 2009: Shabani and Lidar's PRL (claim: quantum discord solves the problem)
- 2013: Brodutch et al's counterexample voiding the Shabani–Lidar PRL
- 2014: next slide

Preparable initial conditions

- initial set of possible system-environment states $\sigma_{QE} \ni \mathfrak{S}_{QE} \subset \mathfrak{S}(\mathcal{H}_Q \otimes \mathcal{H}_E)$
- **requirement of preparability**: the set $\mathfrak{S}_{QE}$ is said to be *preparable* if and only if there exists an input system R and a CPTP map $\mathcal{P} : R \to QE$ such that $\mathfrak{S}_{QE}$ is the image of $\mathfrak{S}(\mathcal{H}_R)$ under $\mathcal{P}$, that is,

$$\mathfrak{S}_{QE} = \mathcal{P}(\mathfrak{S}(\mathcal{H}_R)) := \Big\{\mathcal{P}(\varrho_R) : \varrho_R \in \mathfrak{S}(\mathcal{H}_R)\Big\}$$

- **equivalence with steerability**: the set $\mathfrak{S}_{QE}$ is preparable if and only if it is *steerable*, i.e., if and only if there exists a reference system R and a tripartite density operator ω_{RQE} such that

$$\forall \sigma_{QE} \in \mathfrak{S}_{QE} \ , \ \exists \pi_R \geqslant 0 \ : \ \sigma_{QE} = \frac{\mathrm{Tr}_R[\omega_{RQE} \ (\pi_R \otimes \mathbb{1}_{QE})]]}{\mathrm{Tr}[\omega_{RQE} \ (\pi_R \otimes \mathbb{1}_{QE})]}$$

Result (PRL, 2014)

Fact

Let the set $\mathfrak{S}_{QE}$ be a preparable/steerable set of initial system-environment conditions. The following are equivalent:

- $\mathfrak{S}_{QE}$ is **CPTP reducible**: for any interaction $U_{QE \to Q'E'}$, there exists a corresponding CPTP linear map $\mathcal{E}_{Q \to Q'}$ such that

$$\mathrm{Tr}_{E'}\left[U \sigma_{QE} U^\dagger\right] = \mathcal{E} \circ \mathrm{Tr}_E[\sigma_{QE}] \ , \quad \forall \sigma_{QE} \in \mathfrak{S}_{QE}$$

- $\mathfrak{S}_{QE}$ is **Markov-steerable**: there exists a tripartite state ω_{RQE} with $I(R; E|Q) = 0$, such that $\mathfrak{S}_{QE}$ is steerable from ω_{RQE}

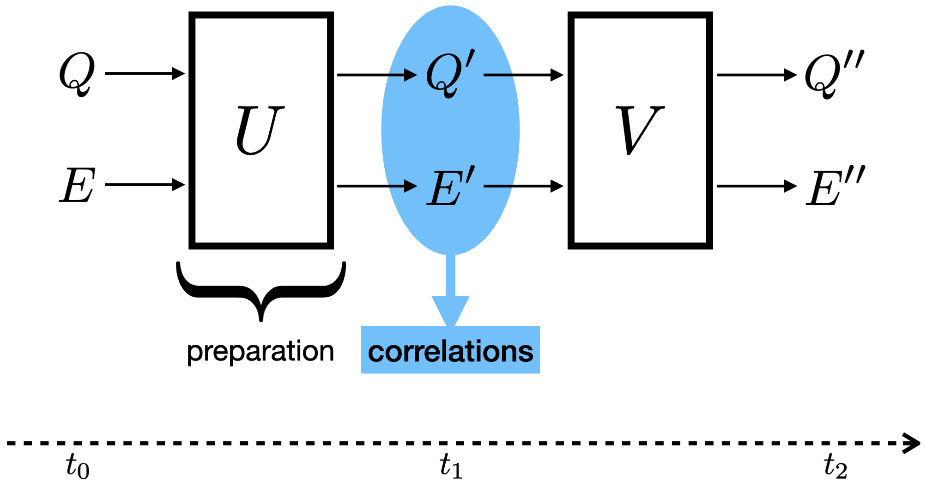
all known examples fall within the scope of the above theorem, which also makes it much easier to verify the CPTP reducibility condition, but many more can be constructed.

What happens when the set of initial assignments is not CPTP-reducible?

Information revival 1/2

let us consider the first interaction step as the "preparation" procedure:

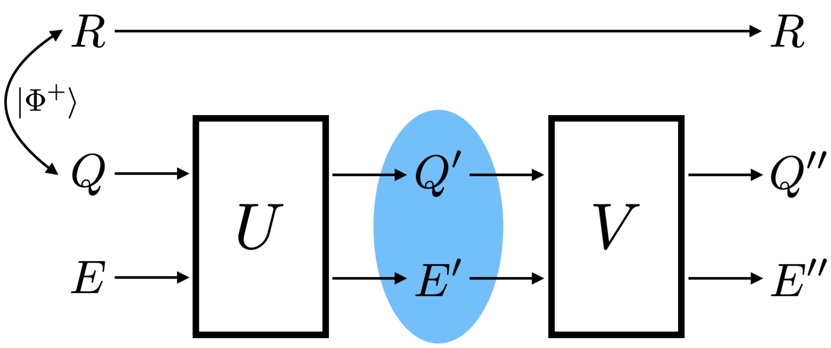
$$\mathfrak{S}(\mathcal{H}_Q) \otimes \gamma_E \xrightarrow{U_{QE}:t_0 \to t_1} \mathfrak{S}_{Q'E'} \xrightarrow{V_{Q'E'}:t_1 \to t_2} \mathfrak{S}_{Q''E''}$$



Information revival 2/2

for convenience, we introduce a reference system $\mathcal{H}_R \cong \mathcal{H}_Q$ and a maxent state $|\Phi^+\rangle_{RQ}$

$$\Phi^+_{RQ} \otimes \gamma_E \xrightarrow{t_0 \to t_1} \underbrace{U_{QE}(\Phi^+_{RQ} \otimes \gamma_E)U^\dagger_{QE}}_{\equiv \sigma_{RQ'E'}} \xrightarrow{t_1 \to t_2} \underbrace{V_{Q'E'}\sigma_{RQ'E'}V^\dagger_{Q'E'}}_{\equiv \tau_{RQ''E''}}$$

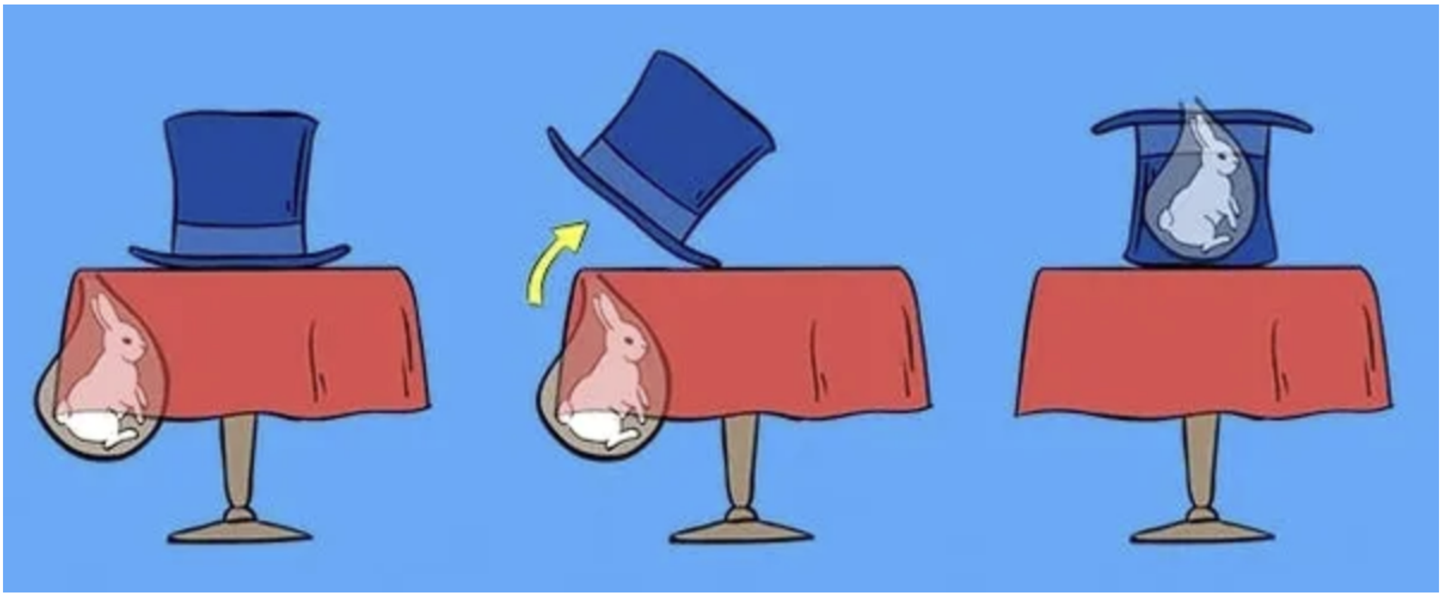


if $I(R;E'|Q') > 0$, a revival, i.e., $I(R;Q'') > I(R;Q')$, may occur

by looking at the system alone, a revival amounts to a violation of locality!

as such, it needs an explanation

Explaining revivals



Explaining revivals

$$\Phi^+_{RQ} \otimes \gamma_E \xrightarrow[t_0 \to t_1]{U_{QE}} \sigma_{RQ'E'} \xrightarrow[t_1 \to t_2]{V_{Q'E'}} \tau_{RQ''E''} \qquad (1)$$

suppose that a revival happens, i.e., $I(R;Q'') > I(R;Q')$

Explanation: compatibly with (1), keep adding parts of the universe to Q', until the revival disappears, i.e., $I(R;Q'\cdots) \geqslant I(R;Q''\cdots)$

Elementary (Dr. Watson's) explanation: just add the environment itself! Indeed, $I(R;Q'E') \geqslant I(R;Q''E'')$

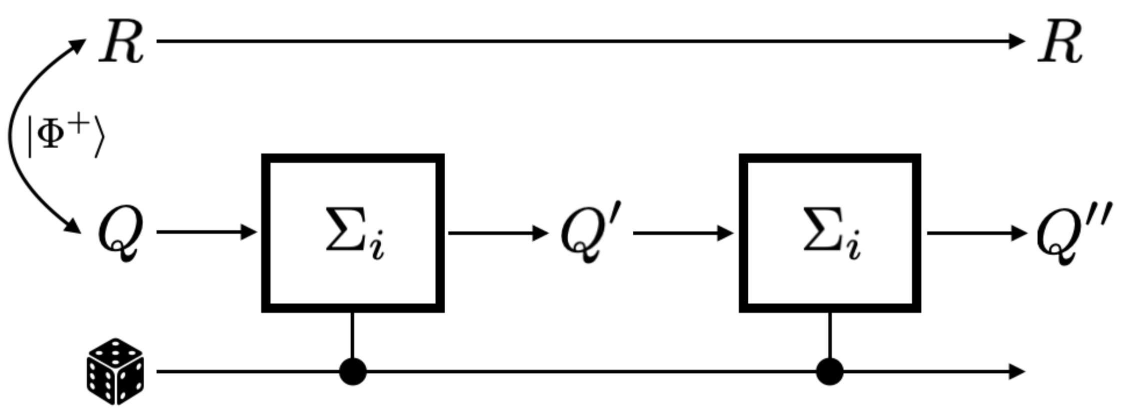
$\Longrightarrow$ **information backflow**

Any revival can be explained as a backflow.

But is a backflow always necessary?

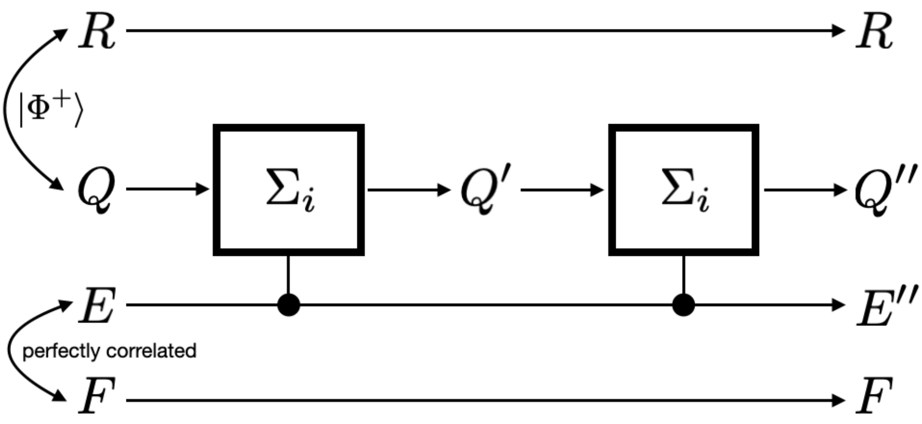
An example 1/2

$\mathcal{H}_R \cong \mathcal{H}_Q \cong \mathbb{C}^2$, $\gamma_E = \frac{1}{4}\mathbb{1}$, $\Sigma^i \in \{\mathbb{1}, X, Y, Z\}$



$$I(R;Q) = 2 \xrightarrow{t_0 \to t_1} I(R;Q') = 0 \xrightarrow{t_1 \to t_2} I(R;Q'') = 2$$

An example 2/2



$$I(R;QF) = 2 \xrightarrow{t_0 \to t_1} I(R;Q'F) = 2 \xrightarrow{t_1 \to t_2} I(R;Q''F) = 2$$

$\implies$ F provides an explanation, even though it never interacts with Q and is causally separated from it at all times!

$\implies$ there **cannot** be any "backflow" from F into Q

Non-causal explanations

- start from

$$\Phi^+_{RQ} \otimes \gamma_E \xrightarrow{U_{QE}} \sigma_{RQ'E'} \xrightarrow{V_{Q'E'}} \tau_{RQ''E''}$$

- take an extension of γ_E using an ancillary system F

$$\Phi^+_{RQ} \otimes \gamma_{EF} \xrightarrow{U_{QE}} \sigma_{RQ'E'F} \xrightarrow{V_{Q'E'}} \tau_{RQ''E''F}$$

- the extension F never interacts with the system: it may reside in a space-like separated (thus, causally separated) region when the first interaction between Q and E takes place
- and yet, we could have $I(R;Q') < I(R;Q'')$ and $I(R;Q'F) \geqslant I(R;Q''F)$
- in this case, the extension F provides a **non-causal explanation**: the information revival **can be** explained without the need for any backflow

Non-causal correlations

$$\Phi^+_{RQ} \otimes \gamma_{EF} \xrightarrow{U_{QE}} \sigma_{RQ'E'F} \xrightarrow{V_{Q'E'}} \tau_{RQ''E''F}$$

$$I(R;Q'F) \geqslant I(R;Q''F) \iff I(R;E''|Q''F) \geqslant I(R;E'|Q'F)$$

non-causal correlations: if there exists a causally separated extension F such that $I(R;E'|Q'F) = 0$, the system-environment correlations present at time $t = t_1$ are called non-causal

Intermediate summary

Correlations: "inert" VS "non-causal"

- start from

$$\Phi^+_{RQ} \otimes \gamma_E \xrightarrow{U_{QE}} \sigma_{RQ'E'} \xrightarrow{V_{Q'E'}} \tau_{RQ''E''}$$

- if $I(R;E'|Q') = 0$, *there may be system-environment correlations*, but **no revival will ever occur** (i.e., correlations exist but are "inert")

- if $I(R;E'|Q') > 0$, look for a causally separated extension F

$$\Phi^+_{RQ} \otimes \gamma_{EF} \xrightarrow{U_{QE}} \sigma_{RQ'E'F} \xrightarrow{V_{Q'E'}} \tau_{RQ''E''F}$$

- if there exists an extension such that $I(R;E'|Q'F) = 0$, *there may be revivals*, but **all such revivals will be non-causal**

The case of a pure-state environment

suppose that the environment's initial state is pure

$$\underbrace{\Phi^+_{RQ} \otimes \psi_E}_{t=t_0} \xrightarrow{U_{QE}} \underbrace{\Psi_{RQ'E'}}_{t=t_1} \xrightarrow{V_{Q'E'}} \underbrace{\Upsilon_{RQ''E''}}_{t=t_2}$$

since in this case all extensions are trivial,

$$I(R;Q'F) \geqslant I(R;Q''F) \iff I(R;Q') \geqslant I(R;Q'')$$

in other words, in this case, a non-causal explanation exists if and only if there is no revival

that is, if the environment is initially pure, any revival of information can only be explained in terms of backflow

Closure under convex mixtures

take the mixture of two processes

$$p\left\{\Phi^+_{RQ}\otimes\gamma^{(a)}_E\to\sigma^{(a)}_{RQ'E'}\to\tau^{(a)}_{RQ''E''}\right\}+(1-p)\left\{\Phi^+_{RQ}\otimes\gamma^{(b)}_E\to\sigma^{(b)}_{RQ'E'}\to\tau^{(b)}_{RQ''E''}\right\}$$

even if both $\sigma^{(a)}_{RQ'E'}$ and $\sigma^{(b)}_{RQ'E'}$ only contain inert correlations, their mixture could allow revivals, i.e.,

$$I(R;E'|Q')_a=0\ \wedge\ I(R;E'|Q')_b=0\ \ \not\Longrightarrow\ I(R;E'|Q')_{pa+(1-p)b}=0$$

instead, any convex mixture of non-causal correlations is automatically non-causal

we can construct a convex resource theory of **genuine** (causal) **non-Markovian backflows**

Conclusion

Today's take-home ideas

- in open quantum systems dynamics, the separation is not only "revival occurs" VS "revival does not occur"
- within revivals, we can further distinguish between "non-causal" VS "genuine"
- further: within the same revival, we can discuss how much of it is "non-causal" VS "genuine"
- such "genuine non-Markovianity" is well-behaved under convex mixtures of processes $\Longrightarrow$ resource theory of genuine non-Markovianity

Thank you