A resource theory of quantum nonlocality

(in space *and* time)

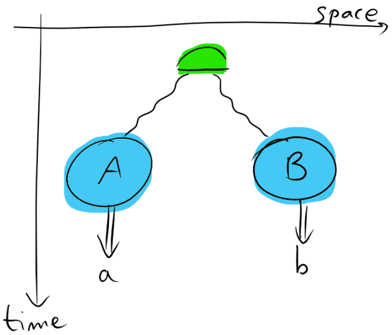
Francesco Buscemi (Nagoya)

Workshop on Multipartite Entanglement
Centro de Ciencias Pedro Pascual, Benasque, Spain
22 May 2018

with Yeong-Cherng Liang (Tainan)  and Denis Rosset (PI) 

Two paradigms for entanglement verification

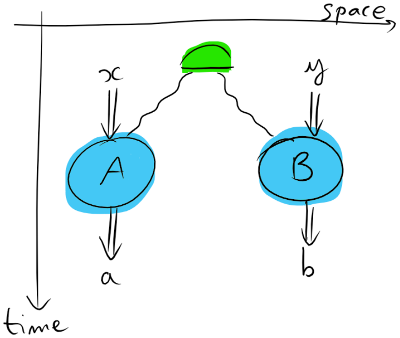
Entanglement witnesses



$$p(a,b) = \mathrm{Tr}\left[(P_A^a \otimes Q_B^b)\ \rho_{AB}\right]$$

- ☺ faithfulness: for any entangled state, there exists a witness detecting it
- ☹ measurement devices need to be perfect

Bell tests



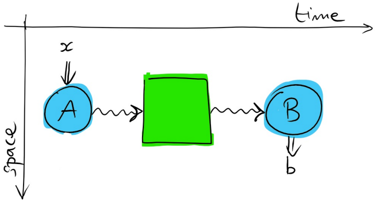
$$p(a,b|x,y) = \mathrm{Tr}\left[(P_A^{a|x} \otimes Q_B^{b|y})\ \rho_{AB}\right]$$

- ☹ hidden nonlocality: some entangled states never violate any Bell inequality
- ☺ device independence

The time-like analogue: quantum memory verification

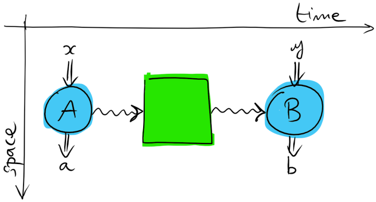
- ✔ the Choi correspondence, $\mathcal{E}_{A\to B} \longleftrightarrow \rho_{AB}$, suggests trying the same approach in time
- ✔ encouraging fact: "classical" (i.e., separable) states correspond to "classical" (i.e., entanglement-breaking) channels

Process tomography



$$p(b|x) = \mathrm{Tr}[\mathcal{E}(\sigma_x)\ P_b]$$

Time-like Bell tests

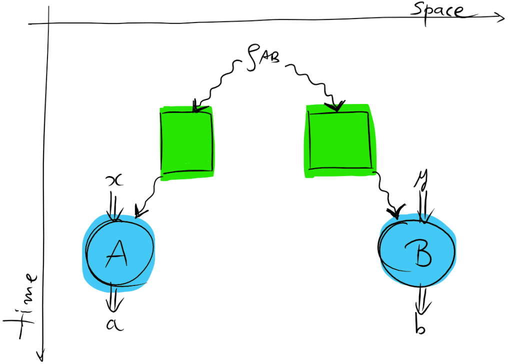


$$p(a,b|x,y) = \mathrm{Tr}\left[\mathcal{E}(\sigma_{a|x})\ P_{b|y,x,a}\right]$$

- ✔ in full analogy with entanglement witnesses, process tomography is faithful (☺) but requires complete trust in the tomographic devices (☹)
- ✔ instead, time-like Bell tests simply trivialize: A can always signal to B

The case of *two* memories

- ✔ however, if *two* quantum memories are available, one can imagine doing the following

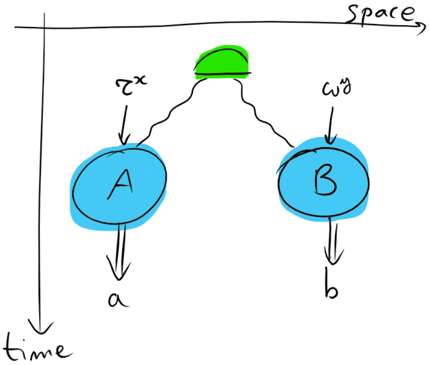


- ✔ here, we need two quantum memories, and the test is assessing *the pair* simultaneously (and it's a Bell test, hence device-independent but not faithful)
- ✔ thus the problem remains: is it possible to certify a single given memory, without using any side-channel?

Let us go back to the space-like setting and try to modify Bell's scenario...

The "semiquantum" Bell scenario

- ✔ in conventional nonlocal games, questions are classical labels; in semiquantum (nonlocal) games, questions are encoded on quantum states
- ✔ the referee chooses questions x and y at random
- ✔ the referee encodes questions on quantum states $\tau^x_{A'}$ and $\omega^y_{B'}$
- ✔ the system A' is sent to Alice, B' to Bob
- ✔ Alice and Bob must locally compute answers a and b



Achievable correlations in the semiquantum scenario are given by

$$p(a,b|x,y,\rho_{AB}) = \mathrm{Tr}\Big[(P^a_{A'A} \otimes Q^b_{BB'})\ (\tau^x_{A'} \otimes \rho_{AB} \otimes \omega^y_{B'})\Big]$$

for varying POVMs

Semiquantum nonlocal games

- ✔ in analogy with quantum statistical decision problems (Holevo, 1973), we also introduce a real-valued payoff function $f(a,b,x,y)$
- ✔ the "utility" of a given bipartite state ρ_{AB} w.r.t. the semiquantum nonlocal game (τ^x, ω^y, f) is then computed as

$$f^*(\rho_{AB}) = \max_{P,Q} \sum_{a,b,x,y} f(a,b,x,y) \underbrace{\mathrm{Tr}\left[\left(P^a_{A'A} \otimes Q^b_{BB'}\right)\left(\tau^x_{A'} \otimes \rho_{AB} \otimes \omega^y_{B'}\right)\right]}_{p(a,b|x,y,\rho_{AB})}$$

Theorem (2012)

Given two bipartite states ρ_{AB} and σ_{CD}, $f^*(\rho_{AB}) \geqslant f^*(\sigma_{CD})$ for all semiquantum nonlocal games, if and only if

$$\sigma_{CD} = \sum_\lambda p(\lambda)\left[\mathcal{E}^\lambda_{A\to C} \otimes \mathcal{F}^\lambda_{B\to D}\right](\rho_{AB}) ,$$

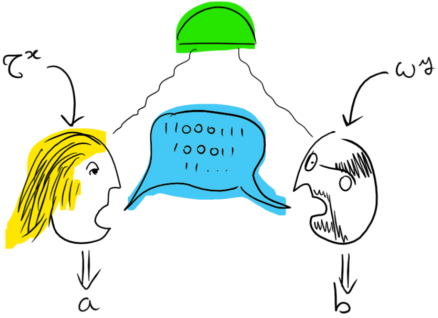
for some CPTP maps $\mathcal{E}, \mathcal{F}$ and normalized probability distribution $p(\lambda)$.

A resource theory of *quantum nonlocality*

- ✔ semiquantum nonlocal games provide a complete set of monotones for local operations and shared randomness (LOSR)
- ✔ it is natural to understand this as a resource theory of quantum nonlocality: free operations are LOSR and hence free states are separable states
- ✔ this is different from a resource theory of nonlocality (without "quantum"): there, being manipulated are correlations $p(a, b|x, y)$ (like, e.g., PR-boxes), not bipartite quantum states ρ_{AB}

Robustness properties of semiquantum nonlocal games

- ✔ semiquantum nonlocal games $\rightsquigarrow$ measurement-device-independent entanglement witnesses
- ✔ in particular, robust against losses in the detectors (losses spoil Bell tests)
- ✔ moreover, robust against classical communication between players (this also spoils Bell tests)
- ✔ this feature is especially welcome in the time-like scenario, where signaling cannot be ruled out and hence *must be assumed*



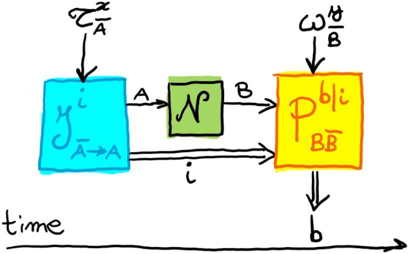
$$p(a,b|x,y) = \mathrm{Tr}\left[(P^{ab}_{\mathrm{LOCC}})\ (\tau^x_{A'} \otimes \rho_{AB} \otimes \omega^y_{B'})\right]$$

(LOCC w.r.t. $A'A \leftrightarrow BB'$)

While we do not have time-like Bell tests, we could have time-like semiquantum tests!

It! Could! Work!

The time-like semiquantum scenario



(here we should think of B as "Alice after some time")

- ✔ give Alice a state τ^x at time t_0
- ✔ wait some time
- ✔ give her another state ω^y at time t_1
- ✔ the round ends with Alice outputting an outcome b

Achievable input/output correlations are computed as

$$p(b|x,y,\mathcal{N}) = \sum_i \mathrm{Tr}\left[P^{b|i}_{\bar{B}B}\ \{\omega^y_{\bar{B}} \otimes (\mathcal{N}_{A\to B}\circ \mathcal{I}^i_{\bar{A}\to A})(\tau^x_{\bar{A}})\}\right]$$

where $\{\mathcal{I}^i\}$ is an instrument, so that any amount of classical communication can be transmitted via the index i

Time-like semiquantum games

- ✔ introduce a real-valued payoff function $f(b,x,y)$
- ✔ the utility of a channel $\mathcal{N}$ is given by

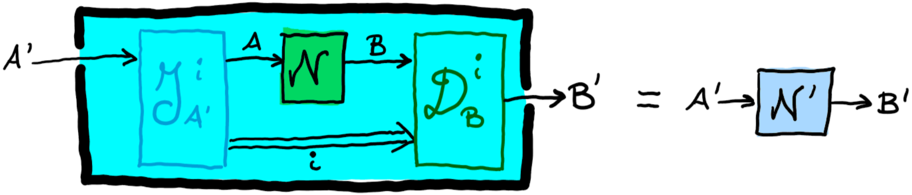
$$f^*(\mathcal{N}) = \max_{\mathcal{I},P} \sum_{b,x,y} f(b,x,y) \underbrace{\sum_i \mathrm{Tr}\left[P^{b|i}_{\bar{B}B}\ \left\{\omega^y_{\bar{B}} \otimes \left(\mathcal{N}_{A\to B} \circ \mathcal{I}^i_{\bar{A}\to A}\right)(\tau^x_{\bar{A}})\right\}\right]}_{p(b|x,y,\mathcal{N})}$$

Theorem (2018)

Given two channels $\mathcal{N}_{A\to B}$ and $\mathcal{N}'_{A'\to B'}$, $f^*(\mathcal{N}) \geqslant f^*(\mathcal{N}')$ for all time-like semiquantum games, if and only if

$$\mathcal{N}'_{A'\to B'} = \sum_i \mathcal{D}^i_{B\to B'} \circ \mathcal{N}_{A\to B} \circ \mathcal{I}^i_{A'\to A}\ ,$$

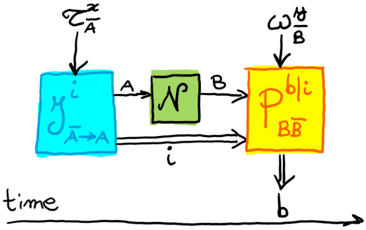
for some instrument $\{\mathcal{I}^i\}$ and CPTP maps $\{\mathcal{D}^i\}$.



- ✔ free operations are given by classically correlated pre/post-processing maps (i.e., quantum combs with classical memory)
- ✔ free "states" are entanglement-breaking channels
- ✔ no shared entanglement or backward classical communication in the case of memories

Other features of time-like semiquantum games

- ✔ as long as the quantum memory (channel) $\mathcal{E}$ is not entanglement breaking, there exists a time-like semiquantum game capable of certifying that
- ✔ assumption: we need to trust the preparation of states τ^x and ω^y, but that is anyway required in the time-like scenario (no fully device-independent quantum channel verification [Pusey, 2015])
- ✔ $\Longrightarrow$ faithfulness with minimal assumptions
- ✔ extra feature: it is possible to *quantify* the minimal dimension of the quantum memory



Conclusions

- ✔ entanglement witnesses: faithful, but complete trust is necessary
- ✔ Bell tests: fully device-independent, but not faithful
- ✔ semiquantum tests: faithful, and trust is required only for the referee's preparation devices
- ✔ semiquantum tests are particularly compelling in the time-like scenario, in which no device-independent quantum channel verification exists anyway
- ✔ $\Longrightarrow$ verification of non-classical correlations among any two locally quantum agents, independent of their causal separation
- ✔ the test is quantitative: a lower bound on the quantum dimension can be given

fin