Rubik's cube solving: advanced Fridrich method

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 - o References
- Step 1: Form a white cross
 - edge up
 - edge in front, white sticker up
 - edge in front, white sticker in front
 - o edge down, incorrect spin (white sticker not on face D)
 - edge down front, white sticker front
 - edge down left, white sticker left
 - edge down back, white sticker back
 - edge down right, white sticker right
 - edge on face D, correct spin (white on face D)
 - edge down right
 - edge down left
 - edge down back
 - o edge on equator
 - edge equator front left, white sticker front
 - edge equator front left, white sticker left
 - edge equator front right, white sticker front
 - edge equator front right, white sticker right
 - edge equator back left, white sticker back
 - edge equator back left, white sticker left
 - edge equator back right, white sticker right
 - edge equator back right, white sticker back
- Step 2: First 2 Layers
 - o evict a misplaced pair (corner, edge)
 - o Corner down, Edge in equator
 - corner set, edge unset
 - corner spun clockwise, edge set
 - corner spun clockwise, edge unset
 - corner spun counterclockwise, edge set
 - corner spun counterclockwise, edge unset
 - o Corner down, Edge up
 - corner set, edge front
 - corner set, edge right
 - corner spun clockwise, edge front
 - corner spun clockwise, edge right
 - corner spun counterclockwise, edge front
 - corner spun counterclockwise, edge right
 - o Corner up, Edge in equator
 - corner white front, edge set
 - corner white front, edge unset
 - corner white right, edge set
 - corner white right, edge unset • corner white up, edge set
 - corner white up, edge unset
 - o Corner up, Edge up
 - Corner up with white on front, Edge up
 - edge front, top sticker like front center
 - edge front, top sticker like right center
 - edge left, top sticker like front center
 - edge left, top sticker like right center
 - edge back, top sticker like front center
 - edge back, top sticker like right center
 - edge right, top sticker like front center edge right, top sticker like right center

 - Corner up with white on right, Edge up
 - edge front, top sticker like front center edge front, top sticker like right center
 - edge left, top sticker like front center
 - edge left, top sticker like right center
 - edge back, top sticker like front center
 - edge back, top sticker like right center
 - edge right, top sticker like front center edge right, top sticker like right center

 - Corner up with white on up, Edge up
 - edge front, top sticker like front center edge front, top sticker like right center
 - edge left, top sticker like front center
 - edge left, top sticker like right center
 - edge back, top sticker like front center
 - edge back, top sticker like right center
 - edge right, top sticker like front center edge right, top sticker like right center
- Step 3: Orient Last Layer
 - o 4 unset corners
 - Hub: no edges (bi-symmetric)
 - C12 counterclockwise, C34 clockwise
 - C12 clockwise, C34 clockwise
 - Cross Edge (bi-symmetric)

- C12 counterclockwise, C34 clockwise
- C12 clockwise, C34 clockwise
- <u>Vertical line: 2 edges (symmetric)</u>
 - C12 counterclockwise, C34 clockwise
 - C12 clockwise, C34 clockwise
- Horizontal line: 2 edges (symmetric)
 - C12 counterclockwise, C34 clockwise
 - C12 clockwise, C34 clockwise
- Chevron centered on C1: 2 edges (non symmetric)
 - C12 counterclockwise, C34 clockwise
 - C12 clockwise, C34 clockwise
- Cl2 counterclockwise, C34 counterclockwise
 Chevron centered on C2; 2 edges (non symmetric)

 C34 | Indiana | Indiana
 - C12 counterclockwise, C34 clockwise
 - C12 clockwise, C34 clockwise
 - C12 counterclockwise, C34 counterclockwise
- 3 unset corners
 - Chicken: 2 edges (symmetric)
 C134 counterclockwise

 - C134 clockwise
 - Slash: no edges (symmetric)
 - C134 counterclockwise
 - C134 clockwise
 - <u>Cactus: 2 edges (not symmetric)</u>
 - C134 counterclockwise
 - C134 clockwise
 - <u>Tetris: 2 edges (not symmetric)</u>
 - C134 counterclockwise
 - C134 clockwise
 - <u>Drone: 4 edges (not symmetric)</u>
 - C134 counterclockwise
 - C134 clockwise
 - Square: 2 edges (not symmetric)
 - C134 counterclockwise
 - C134 clockwise
 - <u>Letter L: 2 edges (not symmetric)</u>
 - C134 counterclockwise
 - C134 clockwise
 - Gallow: 2 edges (not symmetric)
 - C134 counterclockwise
 - C134 clockwise
- 2 set corners
 - Butterfly: 4 edges (symmetric)
 - C1 counterclockwise
 - <u>Dice 3 dots: 2 edges (symmetric)</u>
 - C1 counterclockwise
 - <u>Letter Z: 2 edges (symmetric)</u> C1 counterclockwise
 - Letter S: 2 edges (symmetric) C1 counterclockwise
 - Letter V: no edges (non symmetric)
 - C1 counterclockwise
 - C1 clockwise
 - Tree: 4 edges (non symmetric)
 - C1 counterclockwise
 - C1 clockwise
 - Paddle: 2 edges (non symmetric)
 - C1 counterclockwise
 - C1 clockwise
 - <u>Squirrel: 2 edges (non symmetric)</u>
 - C1 counterclockwise
 - C1 clockwise
 - Squirrel reverse: 2 edges (non symmetric)
 - C1 counterclockwise
 - C1 clockwise
 - <u>Letter C: 2 edges (non symmetric)</u>
 - C1 counterclockwise
 - C1 clockwise
 - Letter P: 2 edges (non symmetric)
 - C1 counterclockwise C1 clockwise
 - Letter P reverse: 2 edges (non symmetric)
 - C1 counterclockwise
 - C1 clockwise
 - <u>Letter T: 2 edges (non symmetric)</u> C1 counterclockwise
 - C1 clockwise
 - <u>Letter W (non symmetric)</u>
 - C1 counterclockwise
- C1 clockwise o all corners set

file:///data/dvs/Rubik/Fridrich.html#ref77

- Dice 5 dots: 2 pairs of unset edges (symmetric)
 - 4 unset edges
- Kite: 1 pair of opposite unset edges (not symmetric) 2 opposite unset edges
- Letter H: 1 pair of adjacent unset edges (symmetric)
 - 2 adjacent unset edges

- Step 4: Permute Last Layer
 - 3 corner circular permutations
 - circular corners clockwise C2 fixed
 - circular corners counterclockwise C2 fixed
 - edge circular permutations
 - circular edges clockwise E2 fixed
 - circular edges counterclockwise E2 fixed
 - o 2 binary swaps of 2 pairs of edges
 - swap opposite horizontal E1-E4, opposite vertical E2-E3
 - swap adjacent C2 centered E1-E2, adjacent C3 centered E3-E4
 - o 2 binary swaps: a pair of adjacent corners and a pair of edges
 - swap adjacent horizontal C1-C3, opposite vertical E2-E3
 - swap adjacent horizontal C1-C3, opposite horizontal E1-E4
 - swap adjacent horizontal C1-C3, adjacent C1 centered E1-E3
 - swap adjacent horizontal C1-C3, adjacent C2 centered E1-E2

 - swap adjacent horizontal C1-C3, adjacent C3 centered E3-E4
 swap adjacent horizontal C1-C3, adjacent C4 centered E2-E4

 - 2 ternary permutations: Corner and edge
 circular clockwise corners C2 fix, counterclockwise edges E1 fixed
 - circular counterclockwise corners C2 fix, clockwise edges E1 fixed
 - circular clockwise corners C2 fix, counterclockwise edges E2 fixed
 - circular counterclockwise corners C2 fix, clockwise edges E2 fixed
 - o 2 binary swaps: a pair of opposite corners and a pair of edges

 - swap opposite C3-C2, adjacent C1 centered E1-E3
 swap opposite C3-C2, adjacent C3 centered E3-E4
 swap opposite C3-C2, opposite vertical E2-E3
 swap opposite C3-C2, opposite horizontal E1-E4
 - o 2 binary swaps: 2 pairs of corners
 - swap adjacent horizontal C1-C3, adjacent horizontal C2-C4
- Annexe 1: Permutation Notation
 - Faces
 - right face clockwise
 - left face clockwise
 - up face clockwise
 - down face clockwise
 - fron face clockwise
 - back face clockwise
 - o Slices
 - slice between right and left, following left face
 - slice between up and down, following down face
 - slice between front and back, following front face
 - o Stacked faces and slices
 - stack of middle slice and right face
 - stack of middle slice and left face
 - stack of equator slice and up face
 - stack of equator slice and down face
 - stack of standing slice and front face
 - stack of standing slice and back face
 - Reorientation of the entire cube
 - full cube following right face
 - full cube following up face
 - <u>full cube following front face</u>
- Annexe 2: Summary of Algorithms

version of 2018/12/22

Introduction

Principles

This document describes the Jessica Fridrich's method to solve the 3x3 Rubik's cube efficiently. This method is targeted for people already knowing how to solve the cube in 3 or 4 minutes, wanting to go below the minute.

The Fridrich method involves 4 steps:

- create a white cross: 4 algorithms to use out of 17
- complete 2 first layers: 4 algorithms to use out of 41
- orient the last layer: 1 algorithm to use out of 57
- permute the last layer: 1 algorithm to use out of 21

The method is also called CFOP for Cross/First/Orient/Permute.

The method requires to memorize a set of dozen of algorithms for each step. This document describes a possible implementation based on 119 algorithms from the Ruwix web site (see reference below), but the algorithm themselves do not really belong to the method, since every paper referring to the Fridrich method offers different algorithms. I also added 17 algorithms for the white cross, just for the sake of completness because this step can be done intuitively.

Only 6 algorithms need to be used to solve the 3 last steps.

Illustration

All the algorithms are illustrated with a 3 dimensional exploded cube, that let the 6 faces to be displayed on each view. And also because the resulting images are cool!

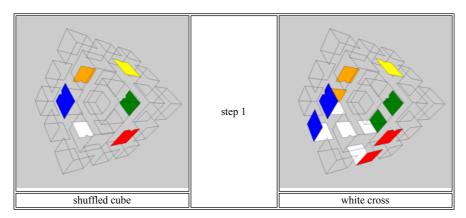
Views are normalized, so every algorithm is described with the white face down, the blue face front and the red face right. Since the cube is seen in perspective, the most visible faces are the blue, red and yellow ones.

Besides, depending on the steps, some irrelevant cubes may be just represented as mesh.

References

- · The initial method described by Jessica Fridrich herself
- Resources including multiple algorithms per case
- Resources for numerous types of methods
- The Ruwix site where most of this paper's algorithms come from
- Fridrich method on official Rubik site
- The online version of this document, with animated GIFs
- Comments and feedback

Step 1: Form a white cross



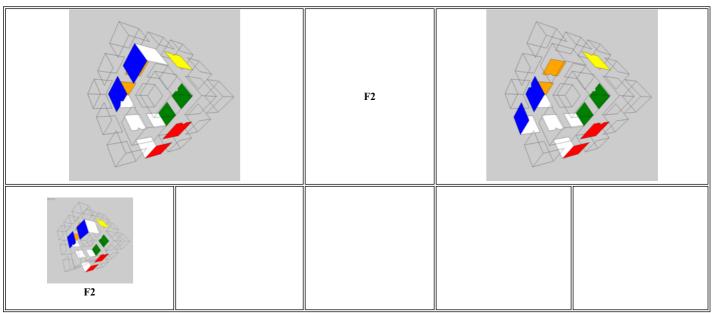
This step should not require algorithms description because setting a cube in place usually does not destroy the rest of the cross, and also because the method is meant to be used by people already knowing how to solve the cube. Still for the sake of completion, here are some algorithms that can be of a certain help:

There are 17 different algorithms for this step.

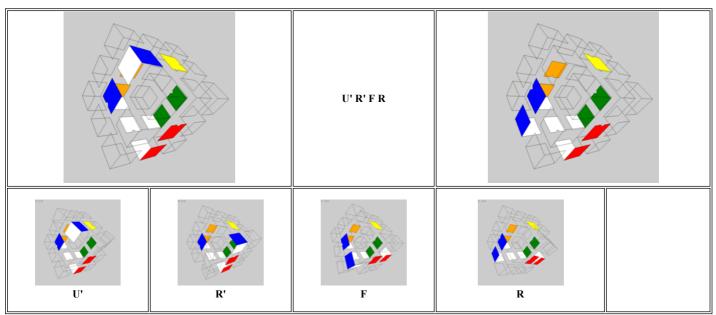
edge up

Only 2 cases are described, because the upper face can rotate to set the edge in front.

edge in front, white sticker up

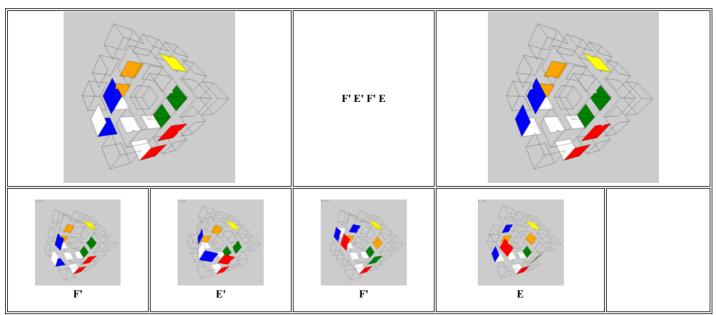


edge in front, white sticker in front

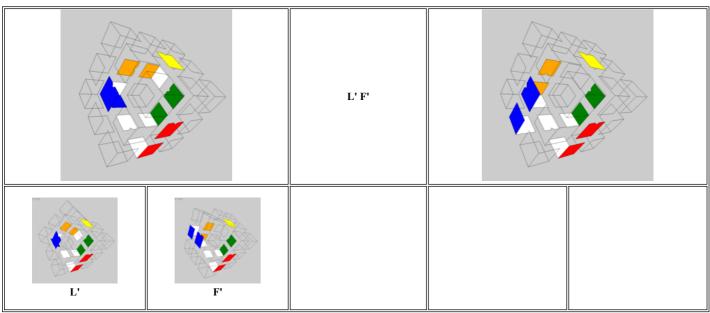


edge down, incorrect spin (white sticker not on face D)

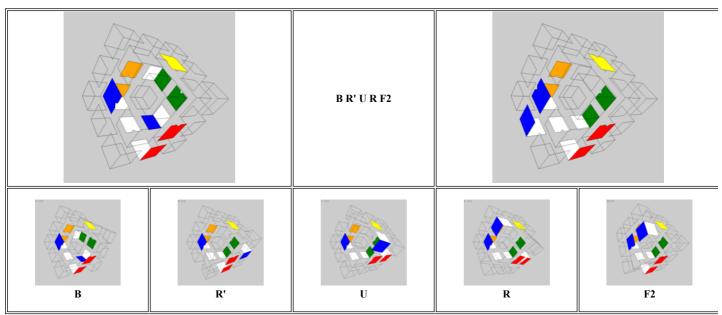
edge down front, white sticker front



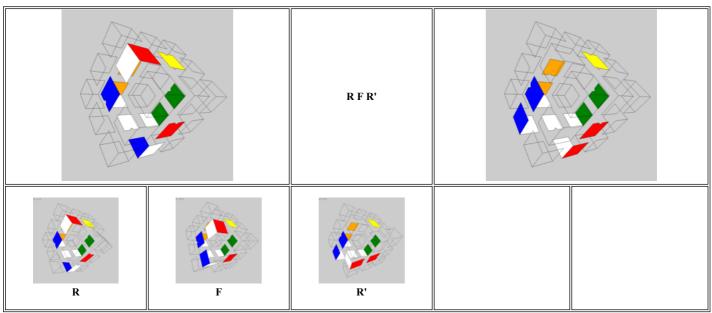
edge down left, white sticker left



edge down back, white sticker back



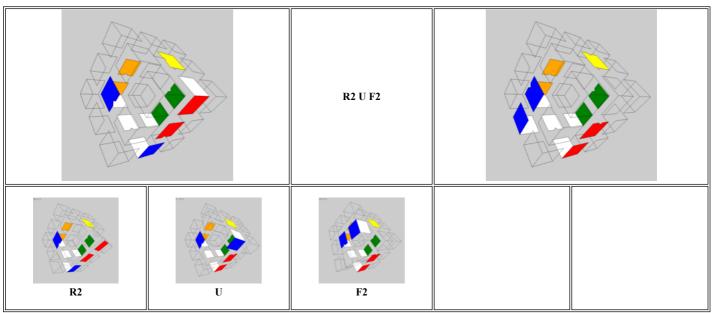
edge down right, white sticker right



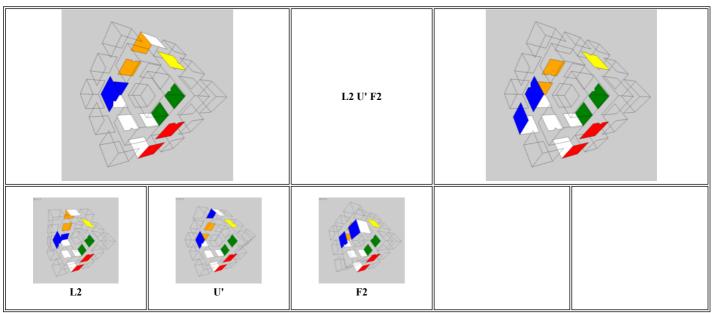
edge on face D, correct spin (white on face D)

There are 4 cases with edge potentially in one of the four rotation position, except that the front with correct spin does not require algorithm.

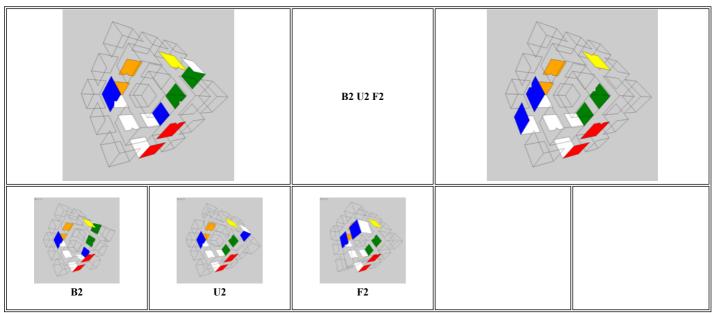
edge down right



edge down left



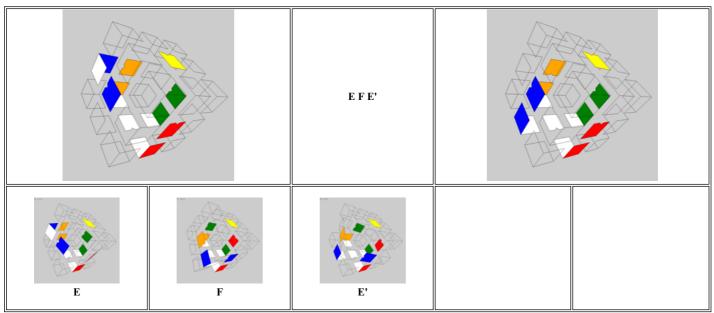
edge down back



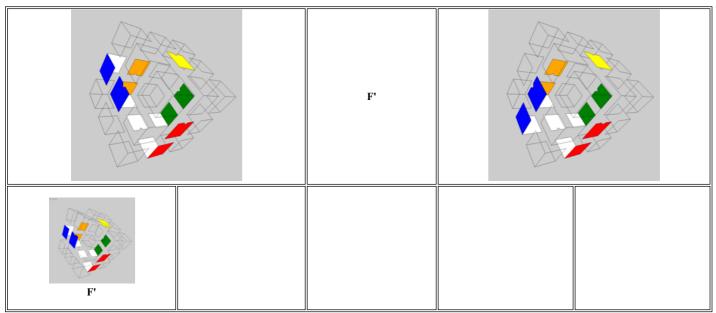
edge on equator

There are 4 positions on the equator, and for each position 2 possible spins.

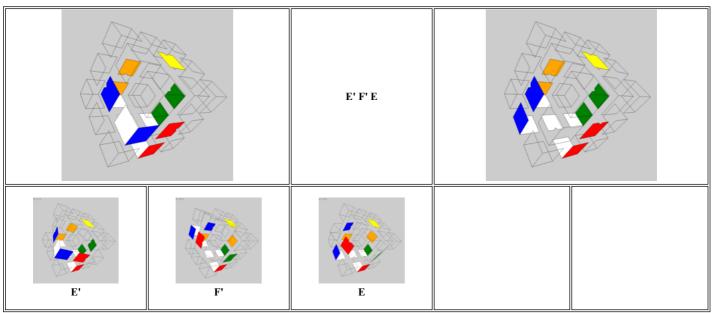
edge equator front left, white sticker front



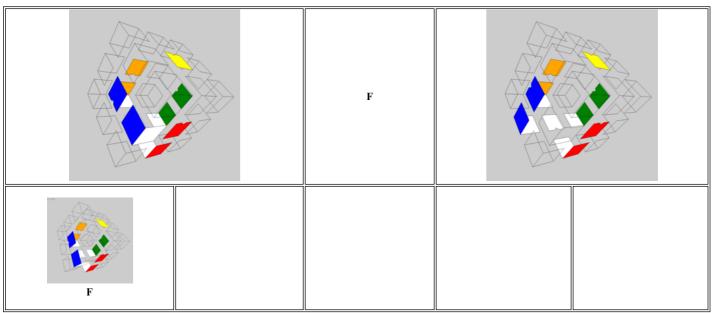
edge equator front left, white sticker left



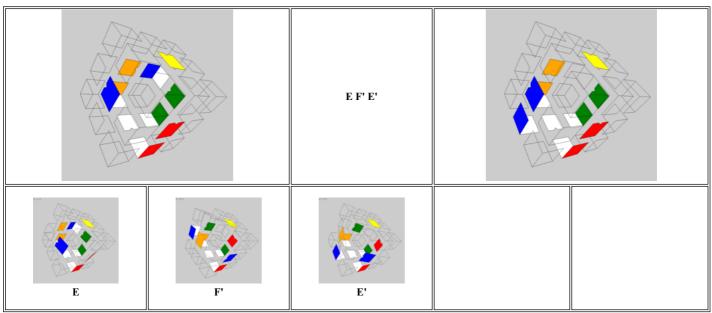
edge equator front right, white sticker front



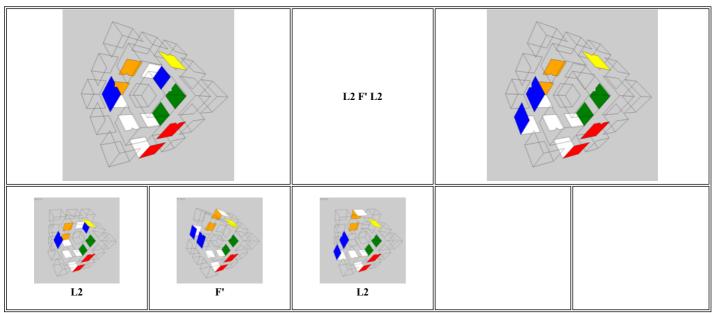
edge equator front right, white sticker right



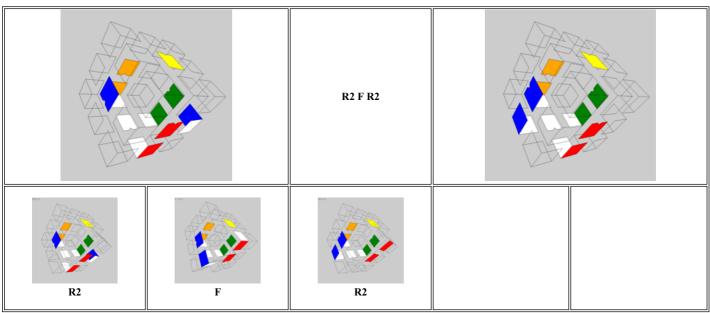
edge equator back left, white sticker back



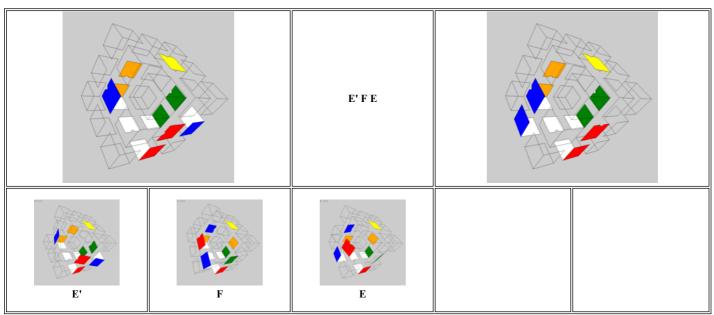
$edge\ equator\ back\ left, white\ sticker\ left$



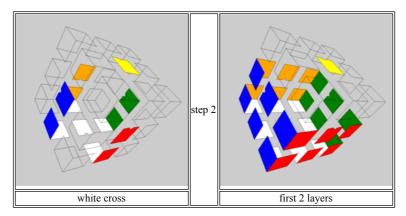
edge equator back right, white sticker right



edge equator back right, white sticker back



Step 2: First 2 Layers



This step consists in completing the 2 layers, started by the white cross, by setting the white face corners and filling the gaps between centers with edges (location and orientation). There are 4 pairs of (corner, edge): blue/red, red/green, green/orange and orange/blue. All the algorithms are illustrated with the setting of blue/red corners and edges. The corner to set are easily identified by one of their 3 faces being white.

• The result is 2 layers ordered and all the cubes containing a yellow face on the last layer.

Corners can have 3 spin positions when they are down: set, spun clockwise or counterclockwise; when they are up we consider the 3 spin positions with respect to the white sticker: up, front or right. Edges have only 2 spin positions so they are either set or unset.

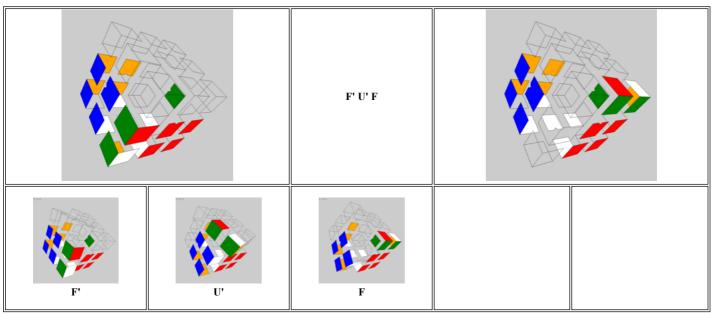
There are 41 different algorithms for this step.

Two tricky cases are worth mentioning:

- a corner is down but on the wrong place
- an edge is on the equator but on the wrong place

A wrong place is for instance for a blue/red cube to be located between the red and green faces. It is possible to reduce the occurrence of these cases by first setting the cubes that pose not problem, and that will mechanically take the places of the misplaced cubes. Nevertheless sometime this is not enough, for instance if all the corner already belong to the white face but are all misplaced. In this case, this simple permutation should solve the problem by putting sending the (unmatched) pair of white/orange/red corner and red/green edge to the upper face:

evict a misplaced pair (corner, edge)



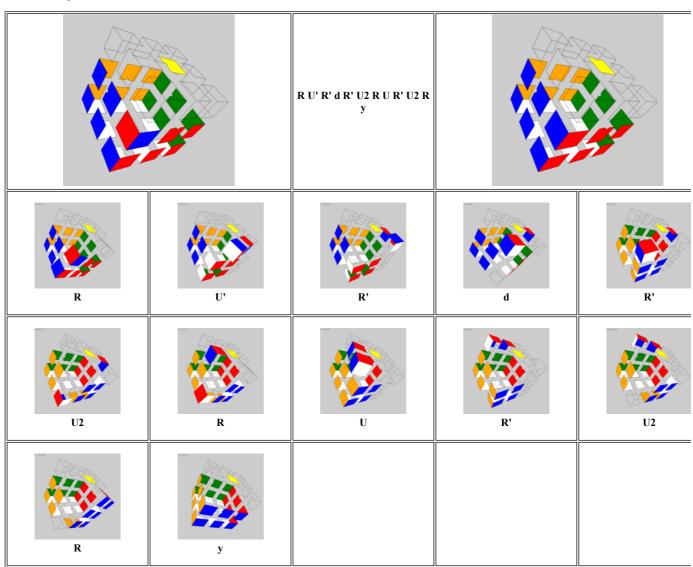
Corner down, Edge in equator

There are 5 different algorithms: 3 positions for the corners and for each corner position 2 positions for the edge, minus the magic position where corner and edge are both set. 3 x 2 - 1.

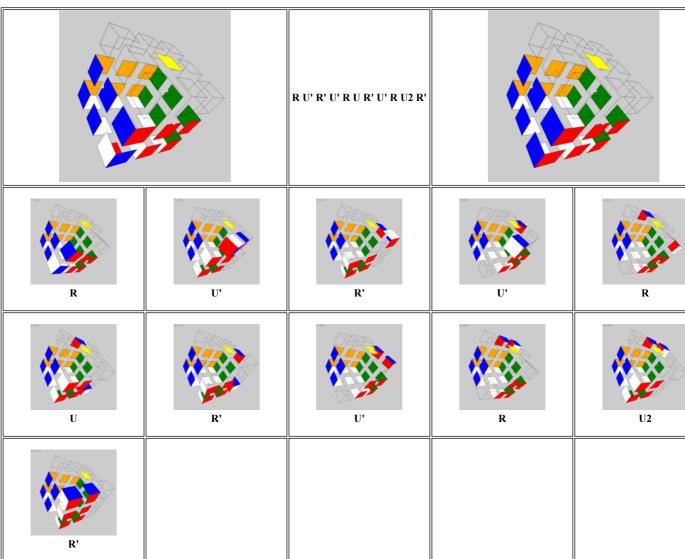
Naming:

- corners: can be set (in position and correct spin), spin clockwise (white face in front) or counterclockwise (white face right)
- edge: can be set or unset (its faces not matching the front and right centers)

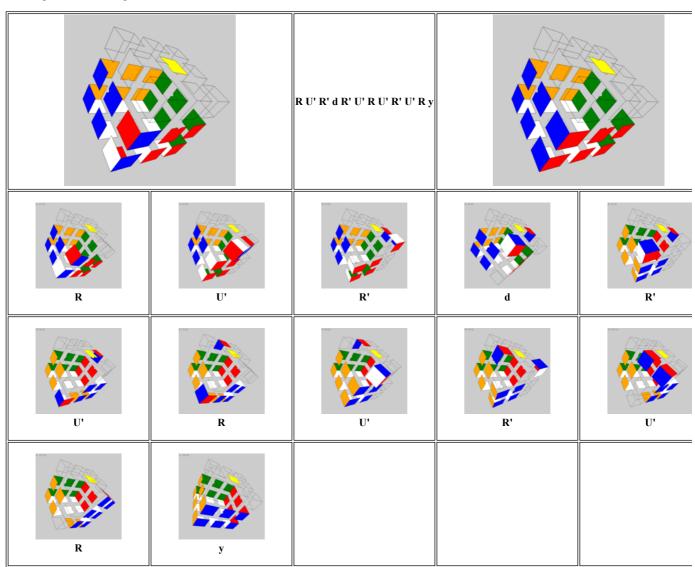
corner set, edge unset



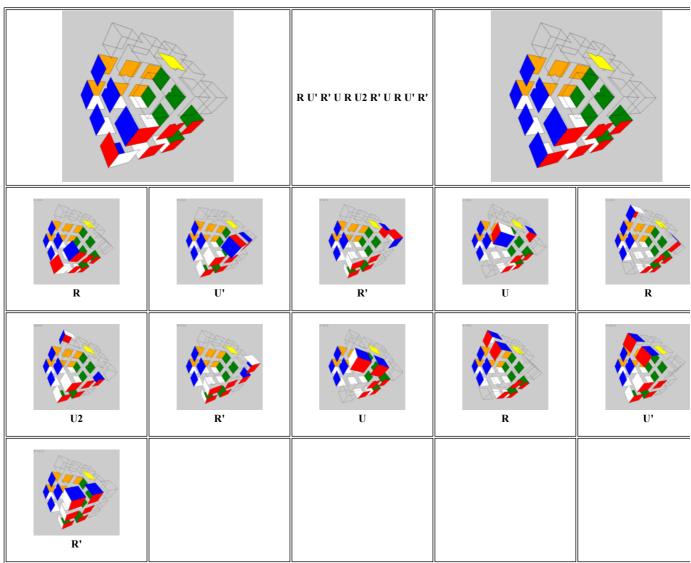
corner spun clockwise, edge set



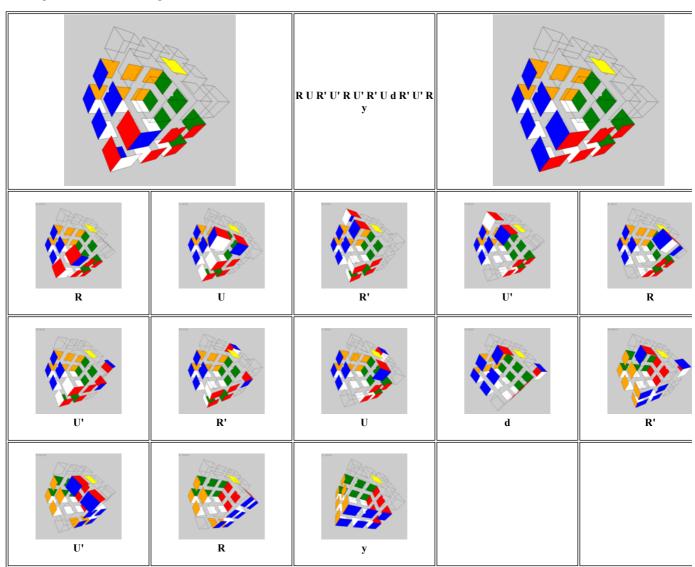
corner spun clockwise, edge unset



corner spun counterclockwise, edge set



corner spun counterclockwise, edge unset



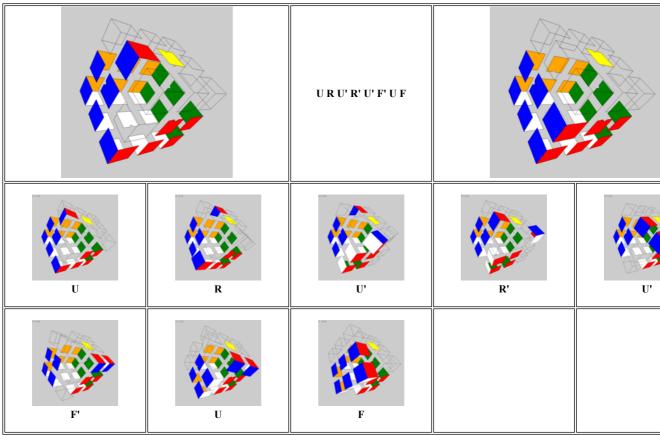
Corner down, Edge up

There are 6 different algorithms: 3 positions for the corner and for each corner only 2 positions for the edge because it can rotate freely with the upper face to be aligned on the front or right face. Indeed when the corner and the edge are on the upper face, they cannot move independently. 3×2 .

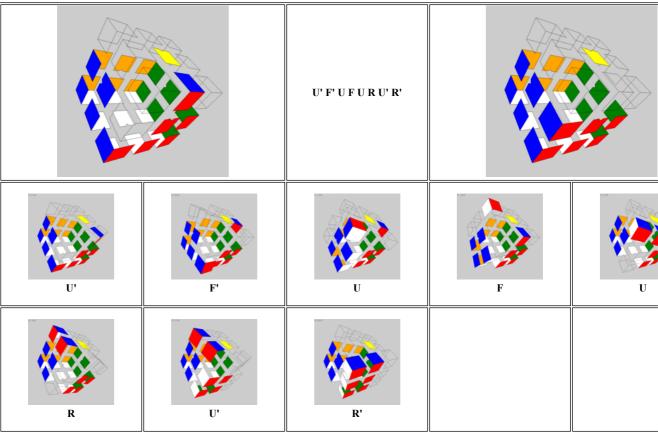
Naming:

- o corners: can be set (in position and correct spin), spin clockwise (white face in front) or counterclockwise (white face right)
- o edge: can be aligned with the front center color of the right center color

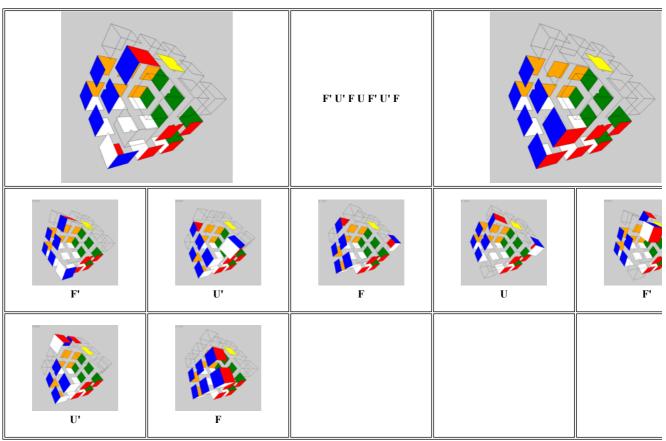
corner set, edge front



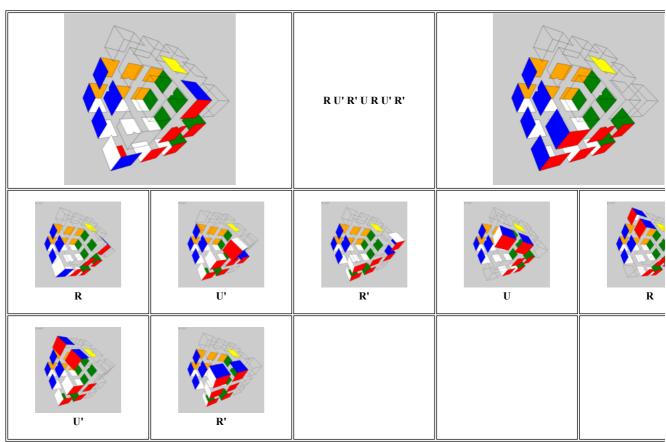
corner set, edge right



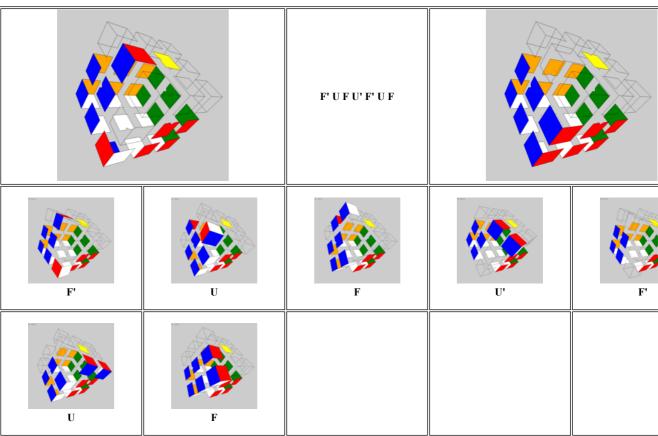
corner spun clockwise, edge front



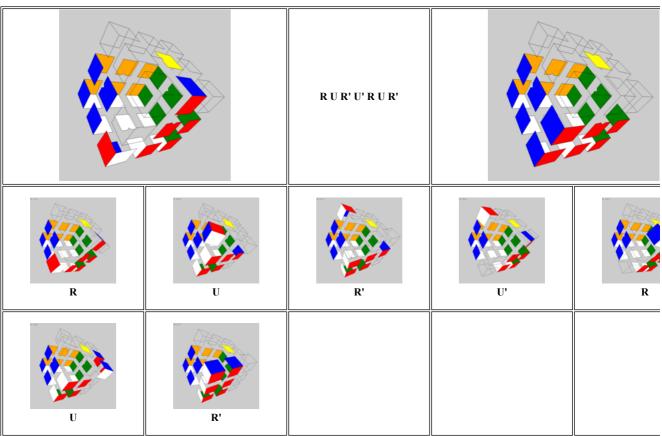
corner spun clockwise, edge right



corner spun counterclockwise, edge front



corner spun counterclockwise, edge right



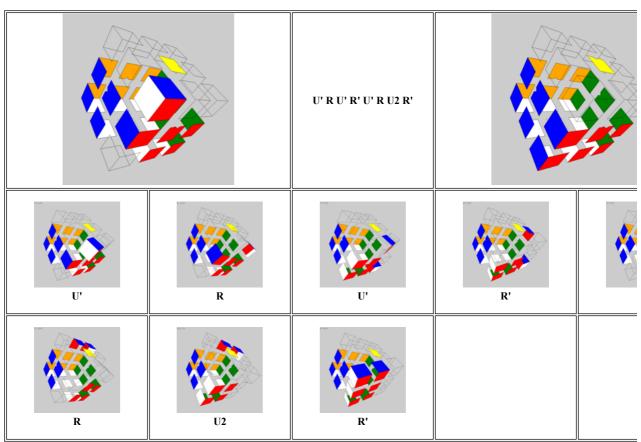
Corner up, Edge in equator

There are 6 different algorithms: 3 positions for the corner and for each corner 2 positions for the edge. 3 x 2.

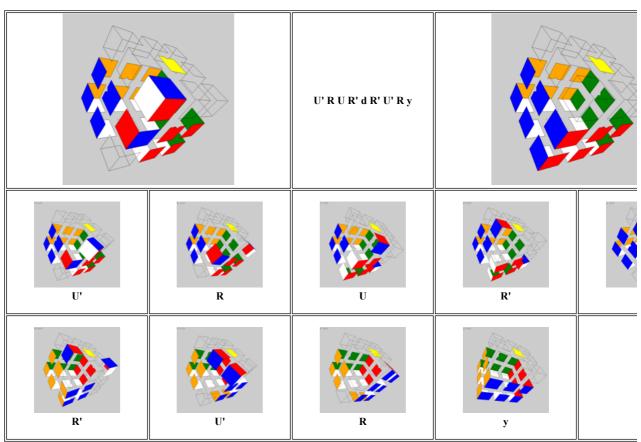
Naming:

- corners: we define its spin with respect to the white face that can be up, right or front
 edge: it can be aligned with the front center color of the right center color

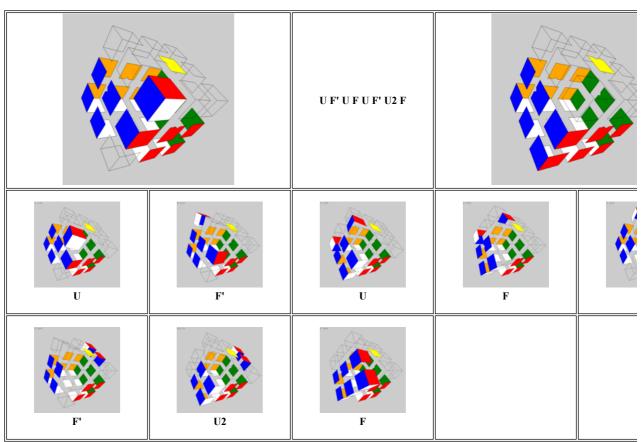
corner white front, edge set



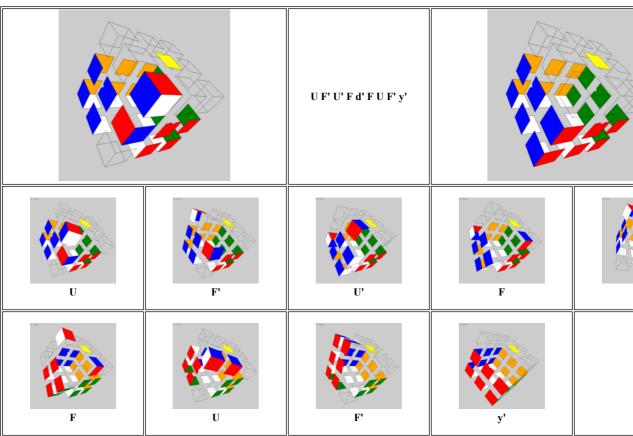
corner white front, edge unset



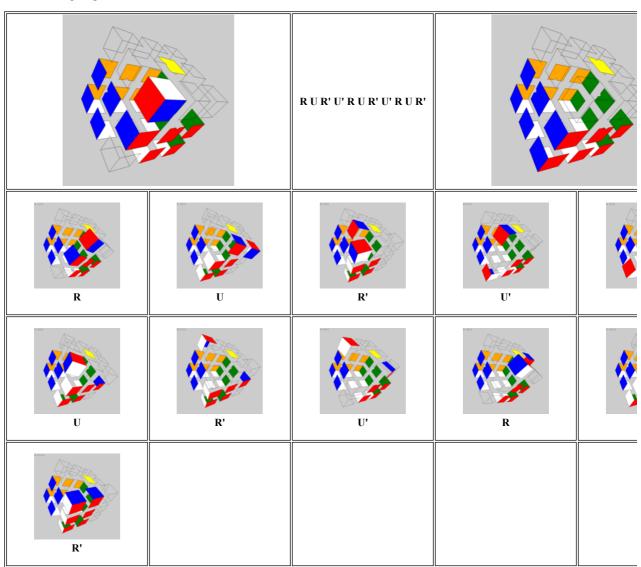
corner white right, edge set



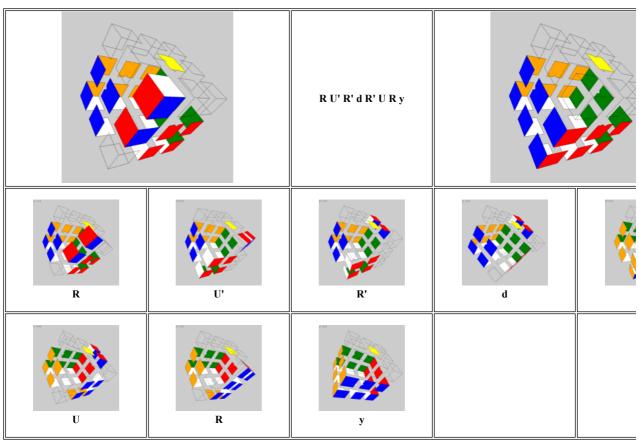
corner white right, edge unset



corner white up, edge set



corner white up, edge unset



Corner up, Edge up

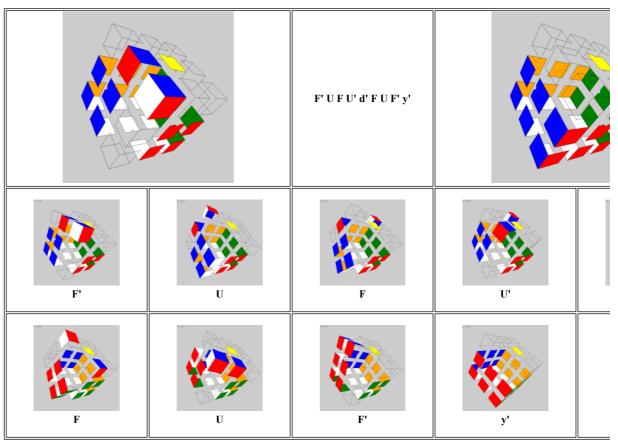
There are 24 different algorithms: 3 positions for the corner and for each corner 4 locations for the edge (front, right, back and left) and again 2 positions (set/unset). $3 \times 4 \times 2$. To ease the access, we consider 3 groups of algorithms federated by the spin position of the corner.

Naming

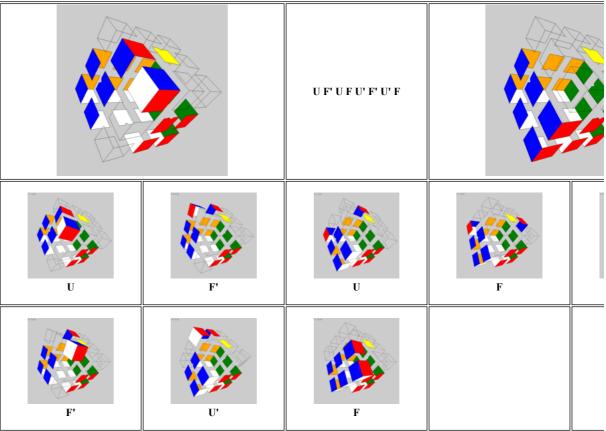
- corners: its location is above the final destination, we define its spin with respect to the white face that can be (u)p, (r)ight or (f)ront
- edge: on the upper layer, its location can be (F)ront, (L)eft, (B)ack or (R)ight, and its orientation is defined with respect to its top sticker's color: like (f)ront of (r)ight center's color

Corner up with white on front, Edge up

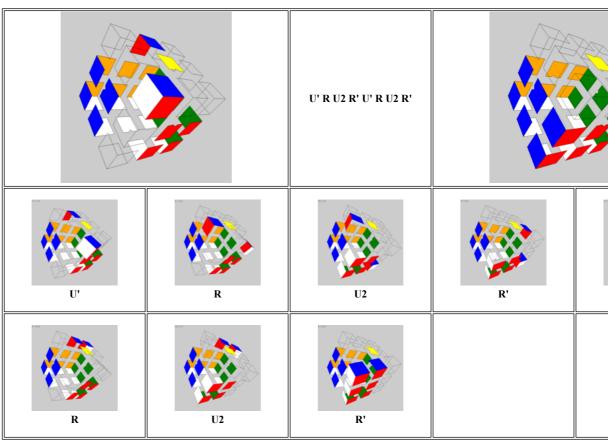
edge front, top sticker like front center



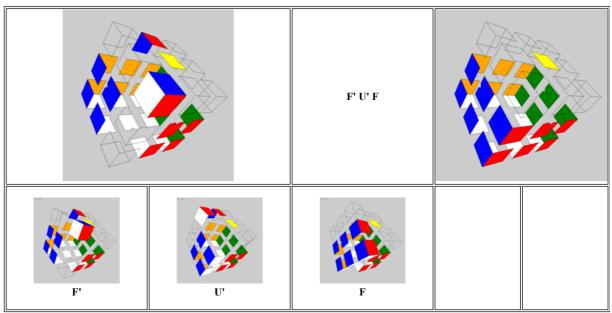
edge front, top sticker like right center



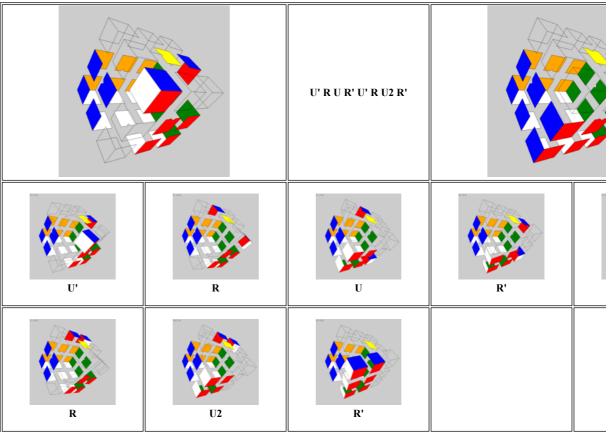
edge left, top sticker like front center



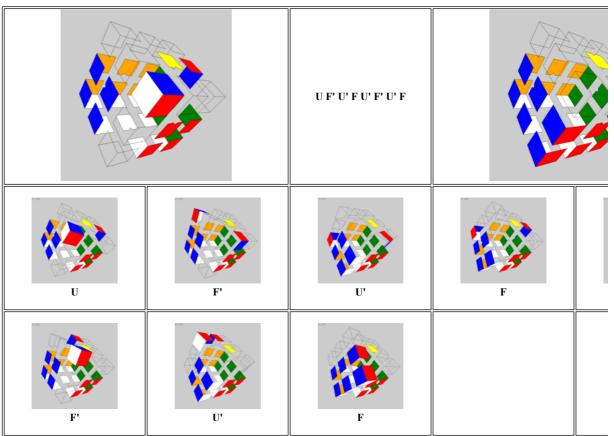
edge left, top sticker like right center



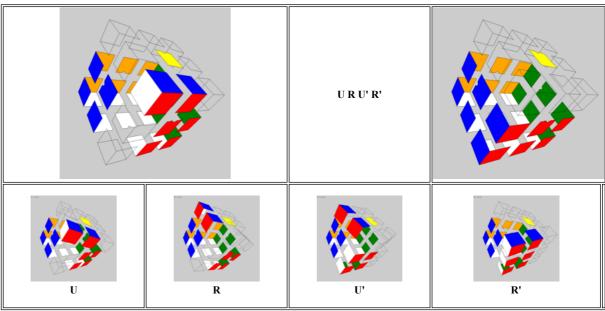
edge back, top sticker like front center



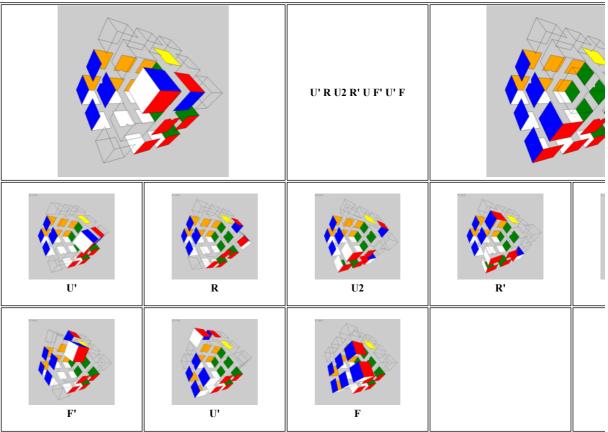
edge back, top sticker like right center



edge right, top sticker like front center

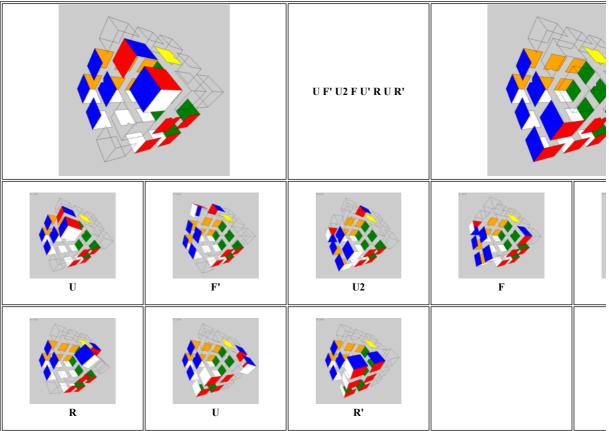


edge right, top sticker like right center

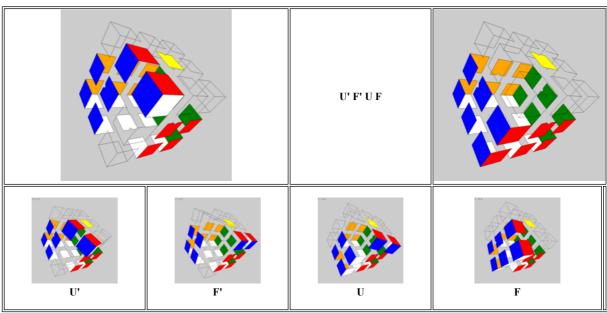


Corner up with white on right, Edge up

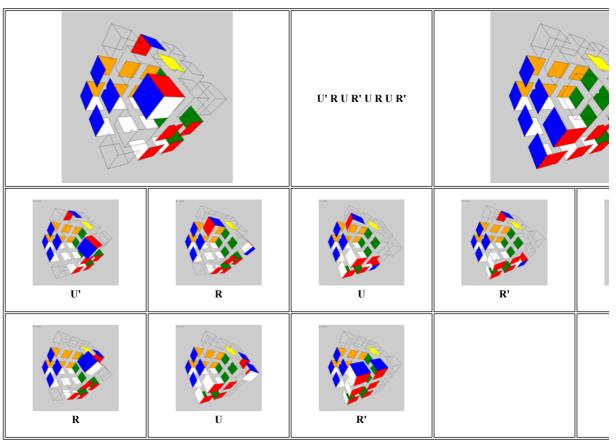
edge front, top sticker like front center



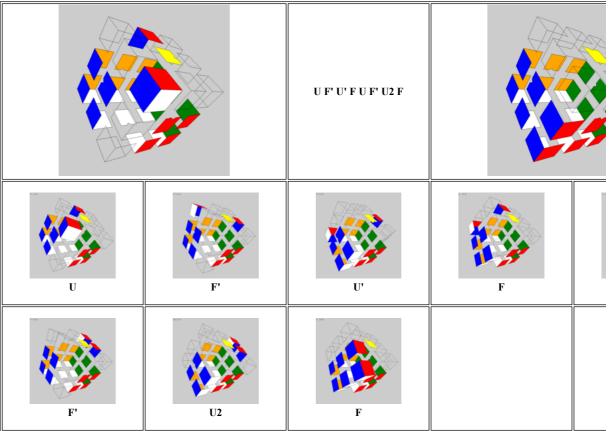
edge front, top sticker like right center



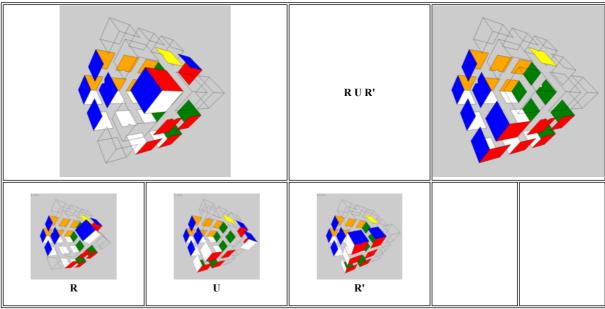
edge left, top sticker like front center



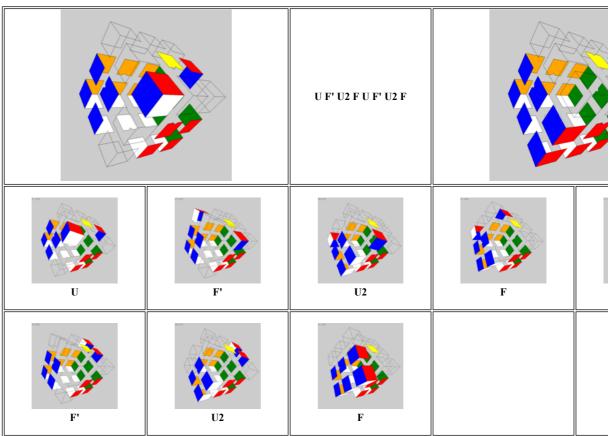
edge left, top sticker like right center



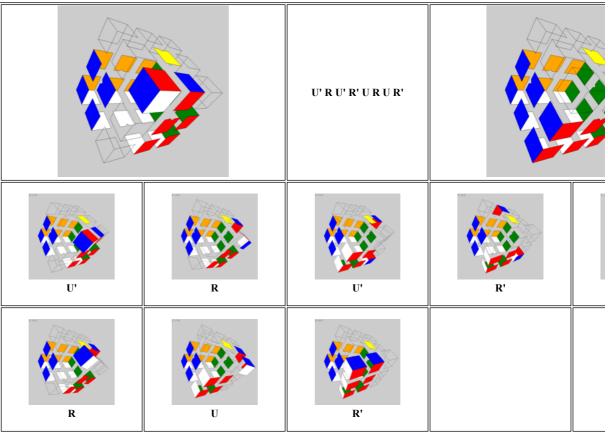
edge back, top sticker like front center



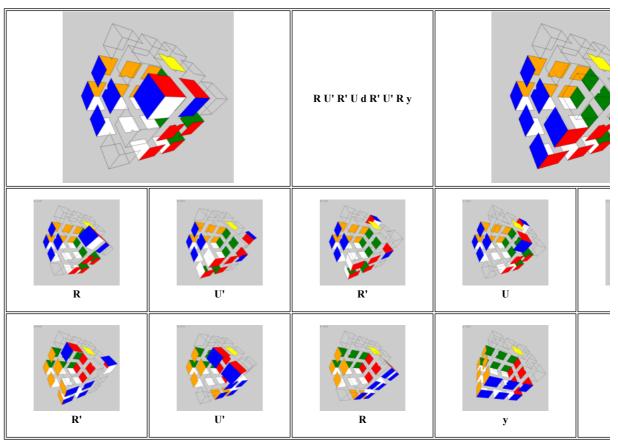
edge back, top sticker like right center



edge right, top sticker like front center

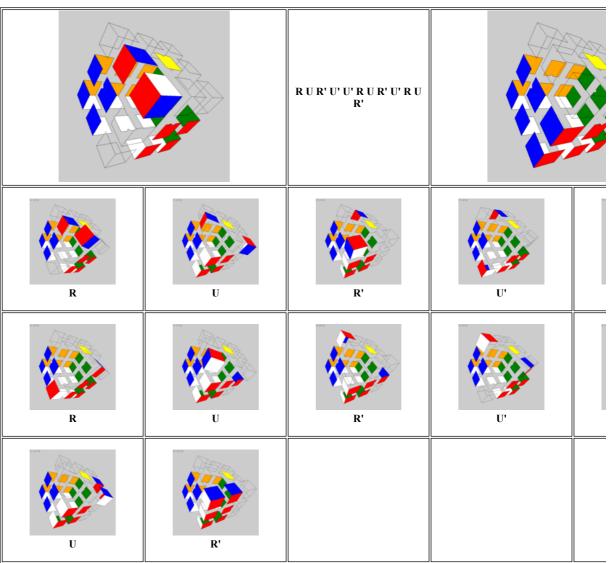


edge right, top sticker like right center

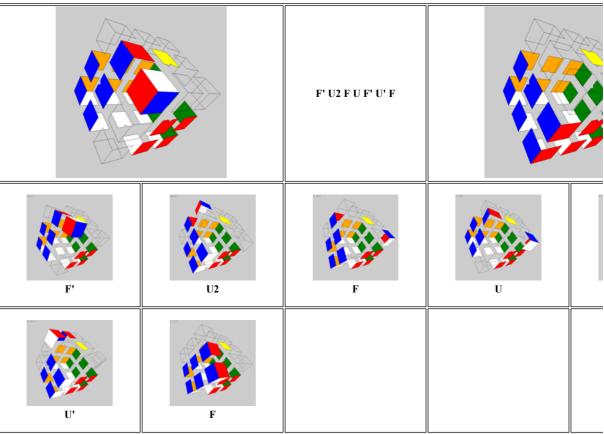


Corner up with white on up, Edge up

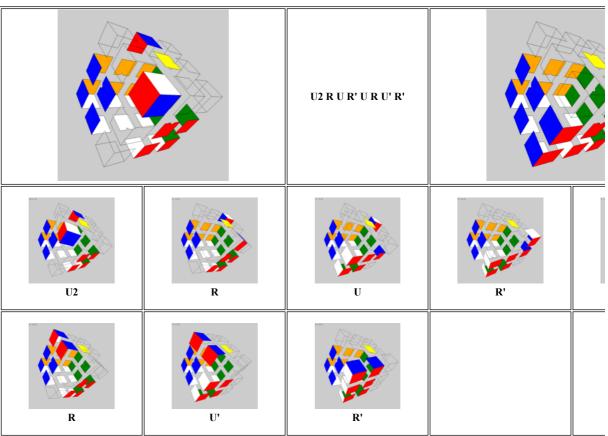
edge front, top sticker like front center



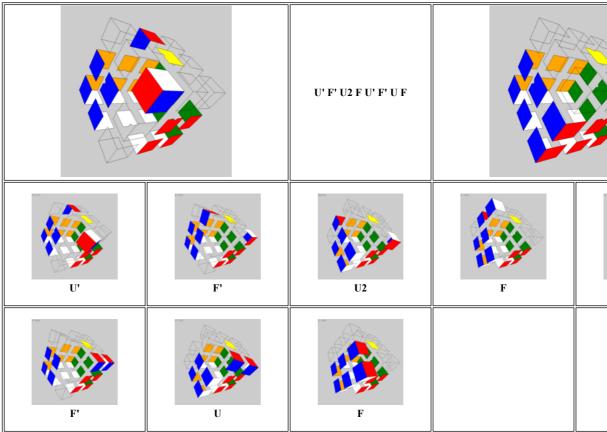
edge front, top sticker like right center



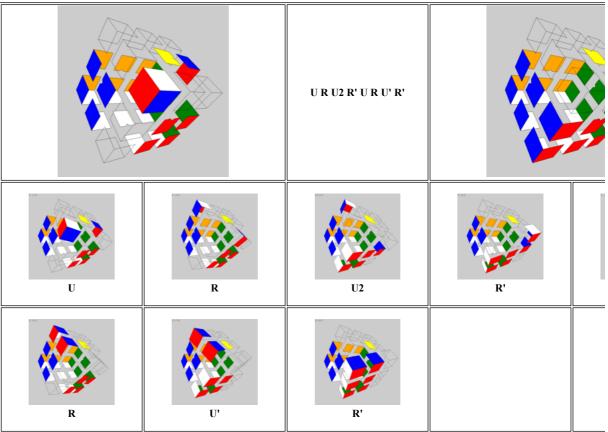
edge left, top sticker like front center



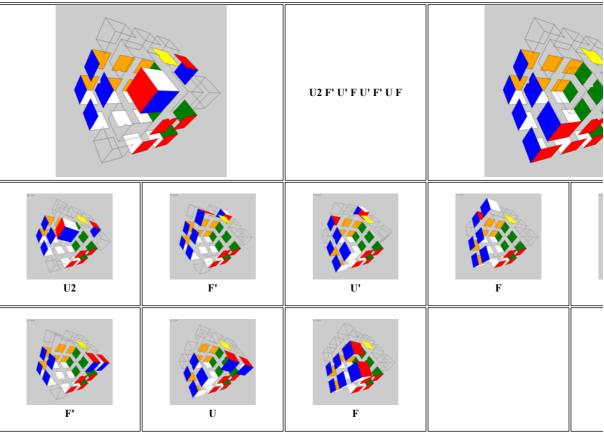
edge left, top sticker like right center



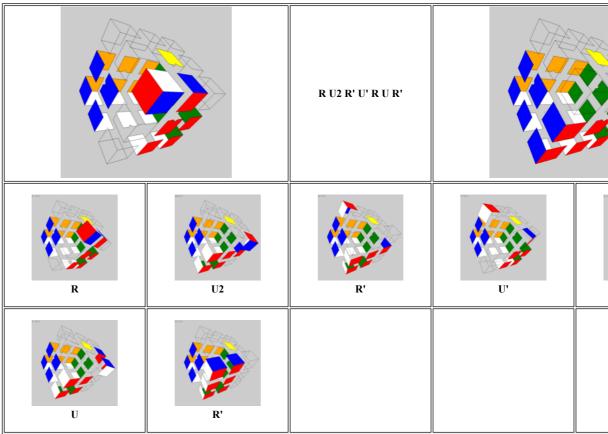
edge back, top sticker like front center



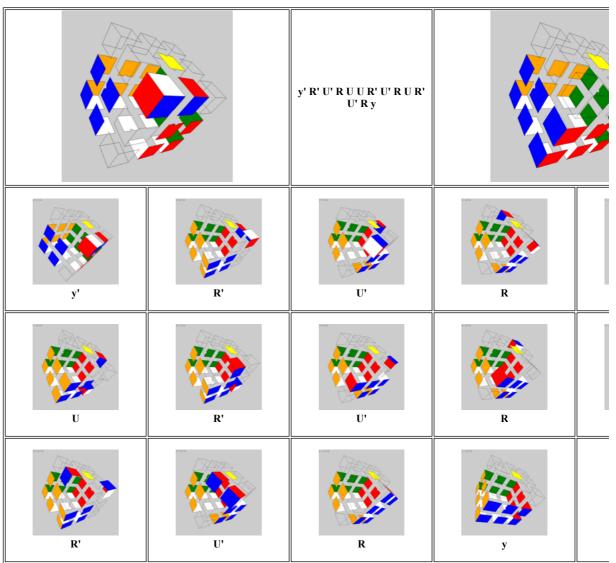
edge back, top sticker like right center



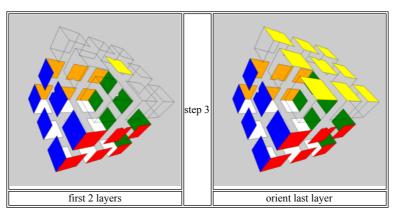
edge right, top sticker like front center



edge right, top sticker like right center



Step 3: Orient Last Layer



This step consists in orienting the Last Layer (up) so the face is entirely yellow, even though usually cubes are not on the right location.

Algorithms

They are named after the patterns that yellow or non yellow stickers define (with a bit of imagination) on a 3x3 matrix. Besides the pattern on the yellow face, the position of the yellow sticker for unset (misoriented) cube define sub-cases. I explain each pattern's name as a cathartic confession;)

Corners:

The four corners are numbered C1 (left front), C2 (left back), C3 (right front) and C4 (right back). Unset corners orientation (spin) may be clockwise (+) or counterclockwise (-) with respect to set position. If 2 or 4 corners are unset, they are paired and one element of the pair will be spun clockwise and its counterpart counterclockwise. If 3 corners are unset, the 3 are collectively either spun clockwise or counterclockwise. It is impossible to have only one corner unset.

Edges

The case of edges is simpler, because they are either set (yellow sticker already up) or unset. So we won't have to number them because the algorithm just set them with different initial setup.

There are 57 different algorithms for this step.

4 unset corners

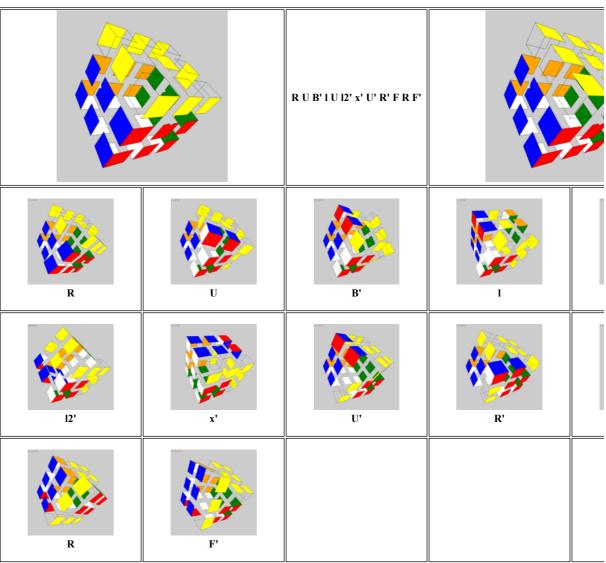
Patterns are oriented such as the 2 pairs of corners are C12 and C34. For identification, C12 spin will be identified from C1's, and C34 from C3's. For instance, C1-C2+ and C3+C4- will be C12 counterclockwise and C34 clockwise.

C12 counterclockwise will have both yellow stickers oriented to the observer, whereas C34 clockwise will have both yellow stikers oriented opposite to the observer.

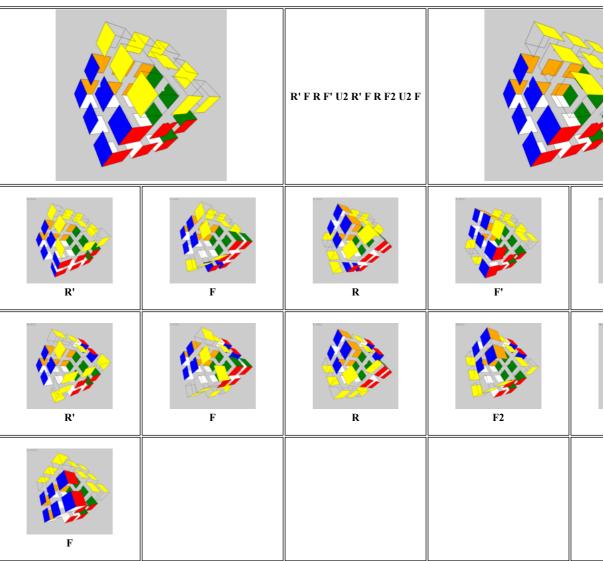
Hub: no edges (bi-symmetric)



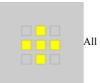
C12 counterclockwise, C34 clockwise



C12 clockwise, C34 clockwise

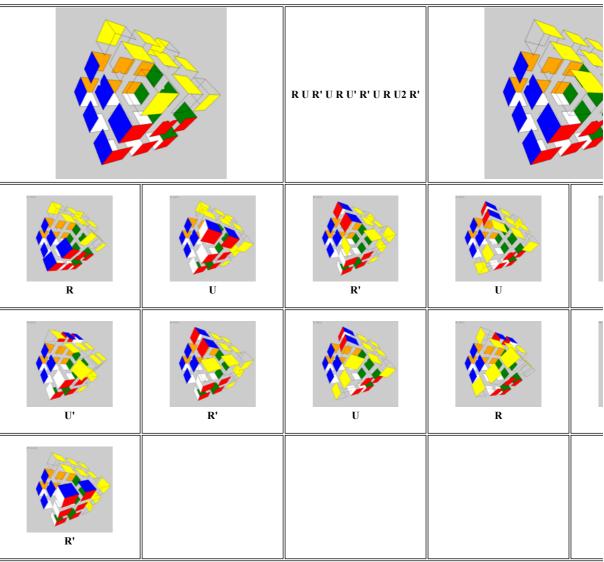


Cross Edge (bi-symmetric)

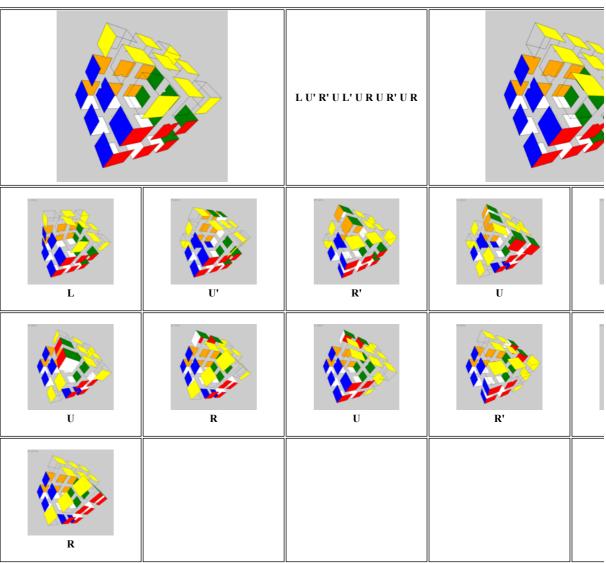


All edges are set, forming a cross.

C12 counterclockwise, C34 clockwise



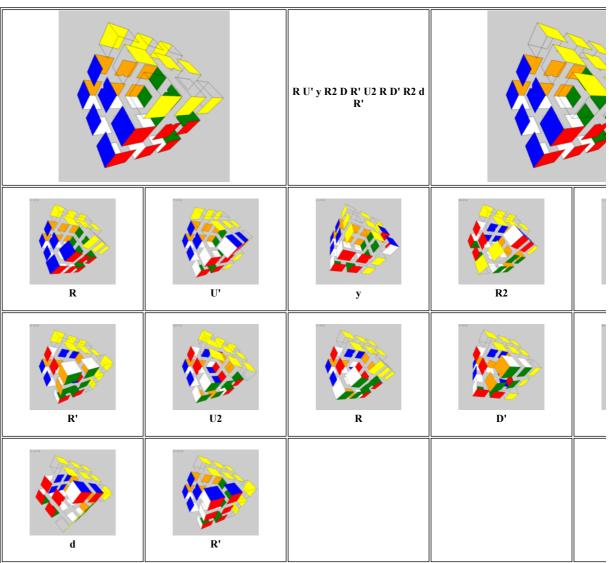
C12 clockwise, C34 clockwise



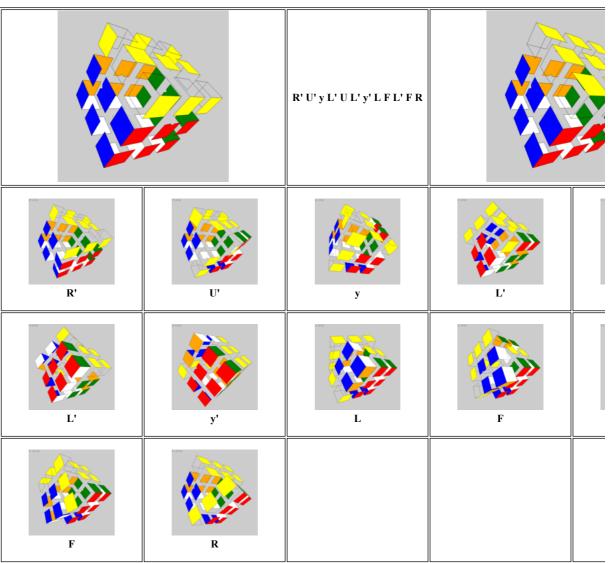
Vertical line: 2 edges (symmetric)



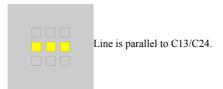
C12 counterclockwise, C34 clockwise



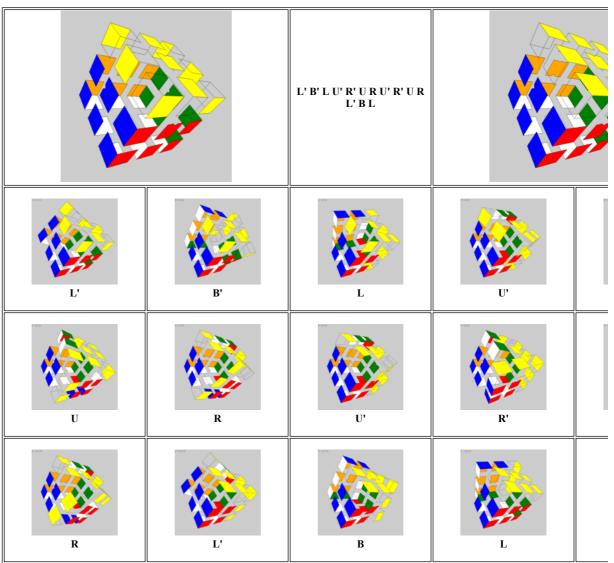
C12 clockwise, C34 clockwise



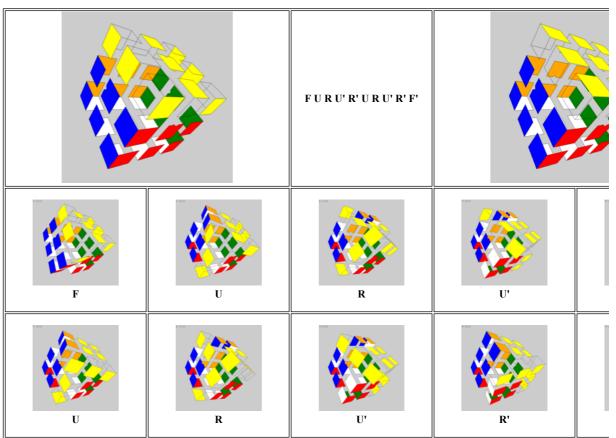
Horizontal line: 2 edges (symmetric)



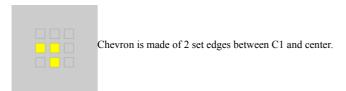
C12 counterclockwise, C34 clockwise



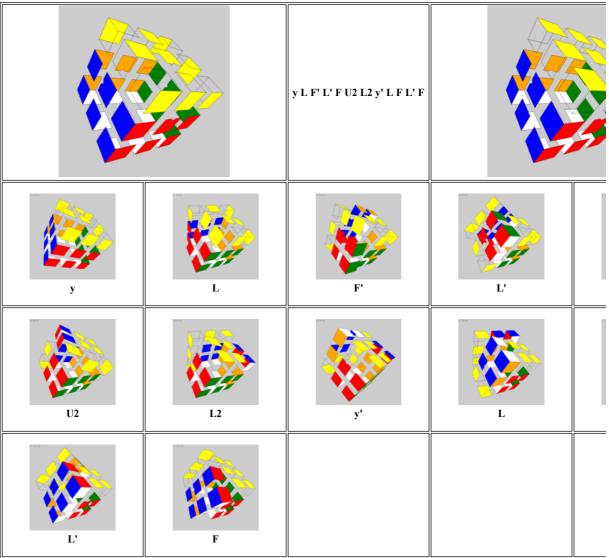
C12 clockwise, C34 clockwise



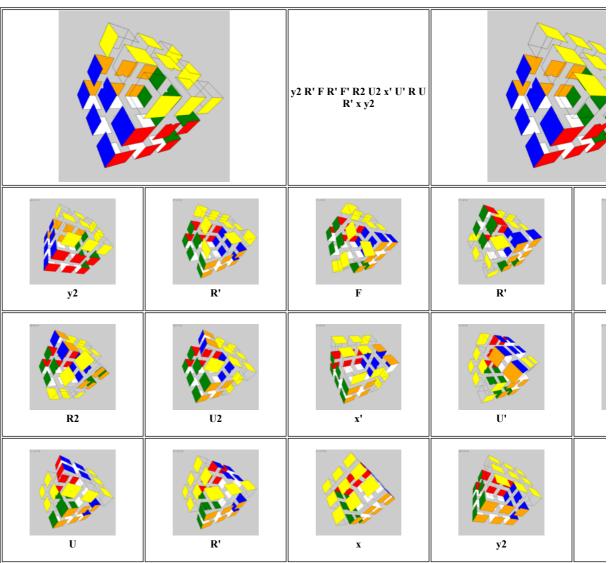
Chevron centered on C1: 2 edges (non symmetric)



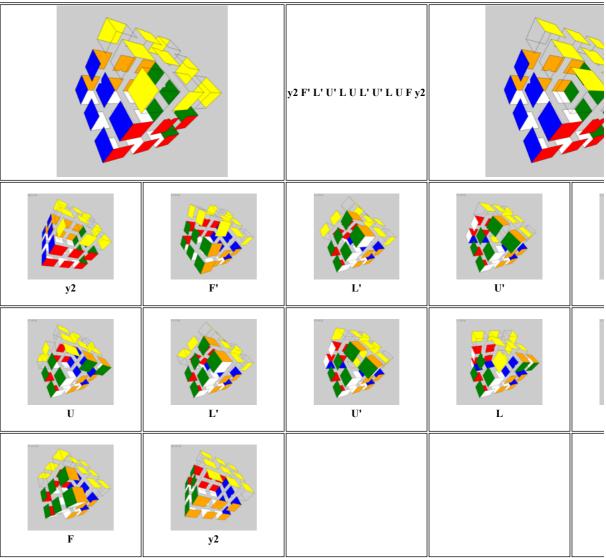
C12 counterclockwise, C34 clockwise



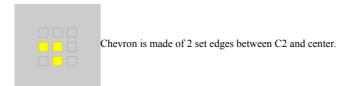
C12 clockwise, C34 clockwise



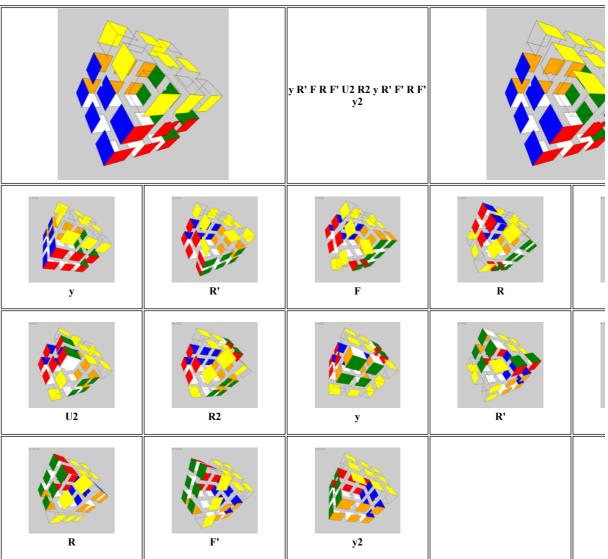
C12 counterclockwise, C34 counterclockwise



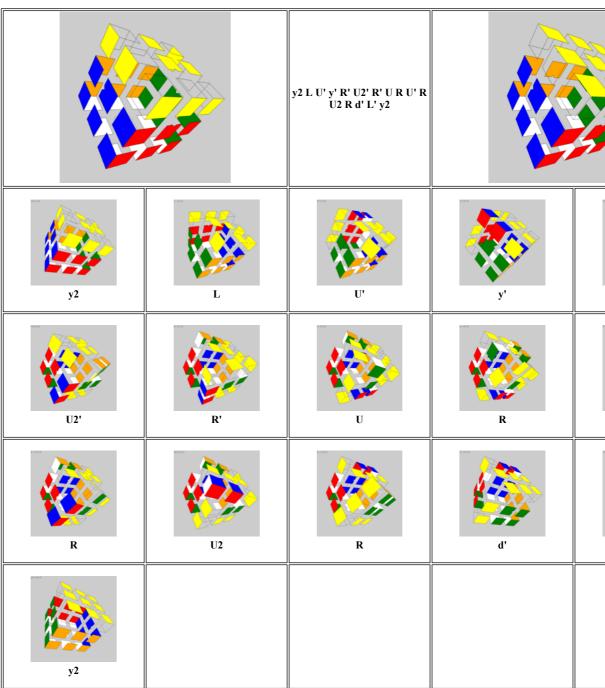
Chevron centered on C2: 2 edges (non symmetric)



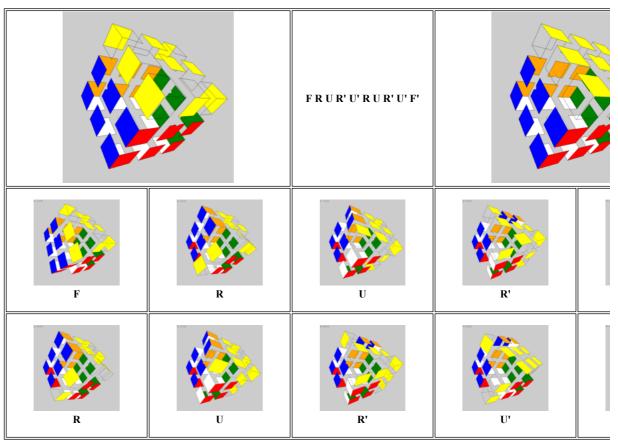
C12 counterclockwise, C34 clockwise



C12 clockwise, C34 clockwise



C12 counterclockwise, C34 counterclockwise



3 unset corners

The 3 unset corners must spin in the same direction, they are collectively spun clockwise + or counterclockwise - so no need for special numbering, and only one spin identification is enough to qualify the collective spin. C2 is always set and the other unset.

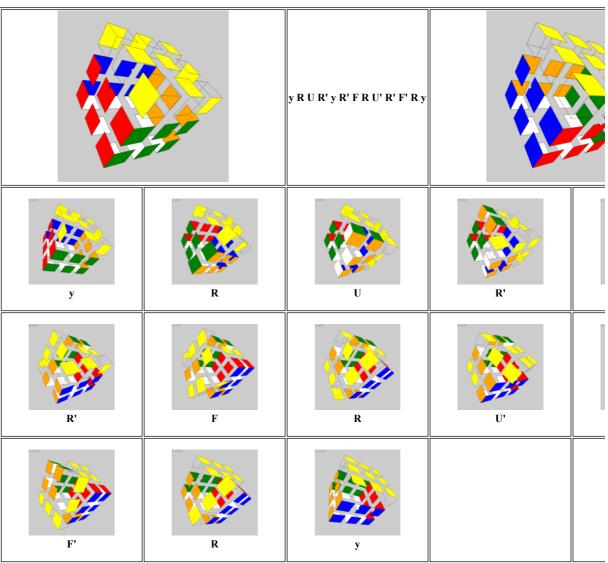
Chicken: 2 edges (symmetric)



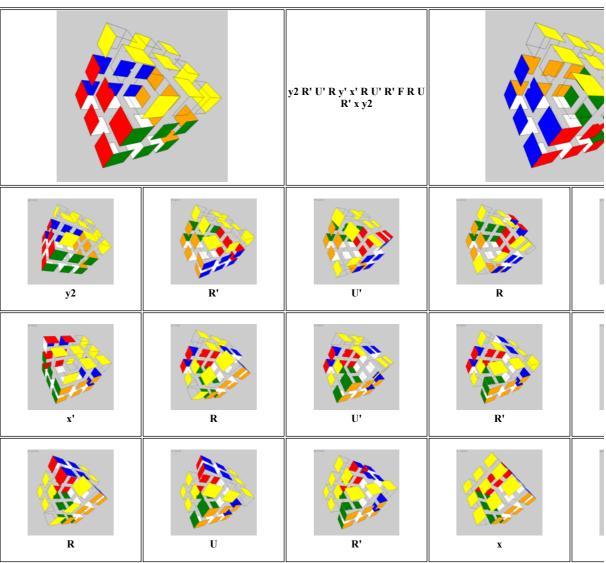
The chicken's head is C2, legs horizontal and wings vertical, so looking to the left. But we can also imagine legs are

vertical and wings horizontal, hence looking to the back face.

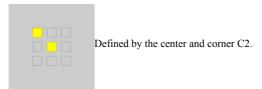
C134 counterclockwise



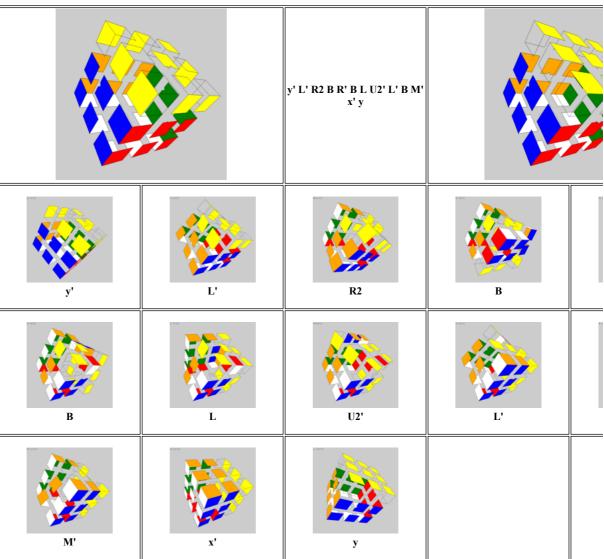
C134 clockwise



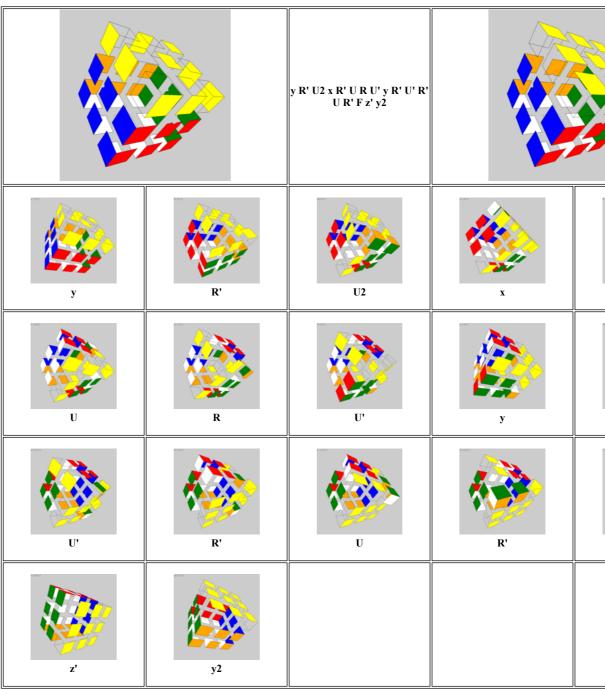
Slash: no edges (symmetric)



C134 counterclockwise



C134 clockwise

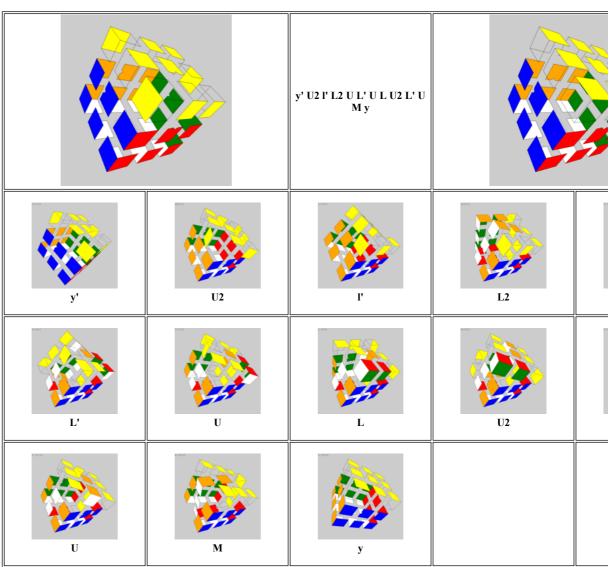


Cactus: 2 edges (not symmetric)

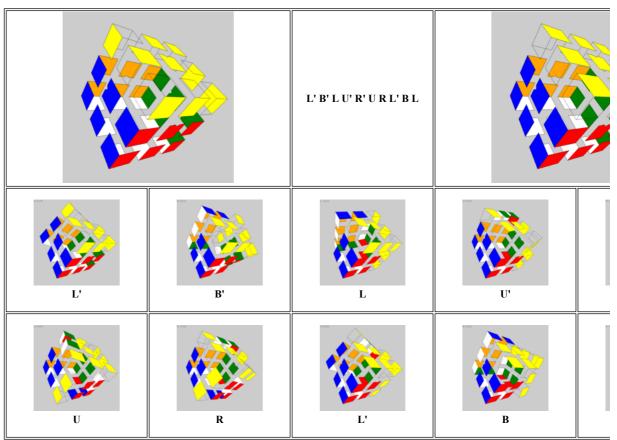


The trunk is vertical, so is the branch that stems from it.

C134 counterclockwise



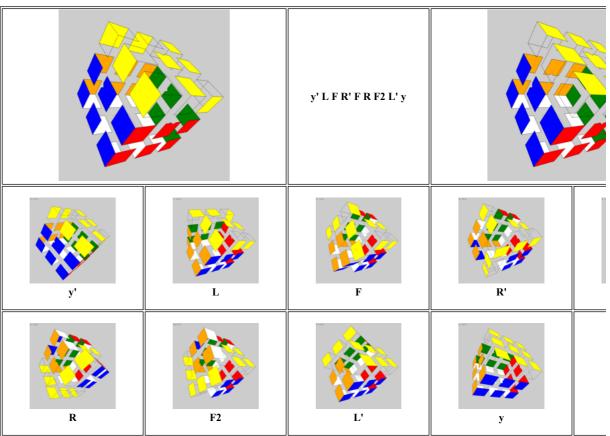
C134 clockwise



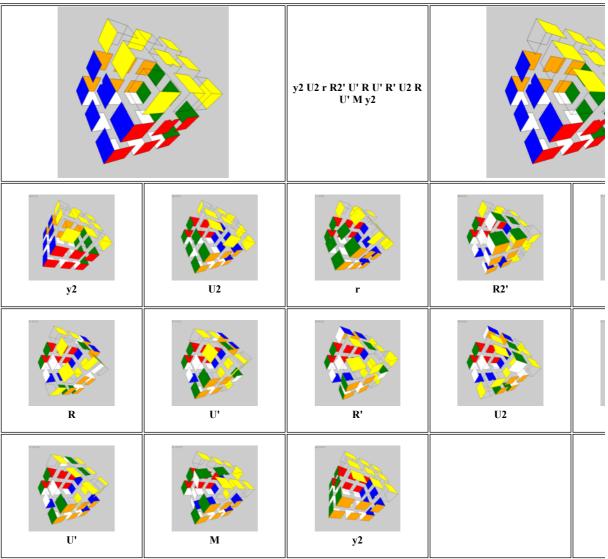
Tetris: 2 edges (not symmetric)



Actually this could be the reverse cactus, but it definitely looks like an iconic Tetris block.



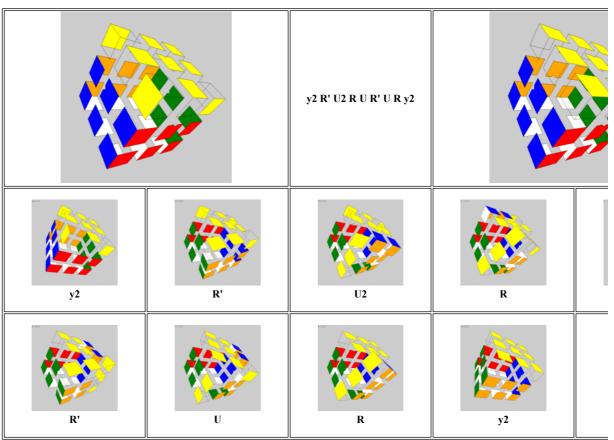
C134 clockwise



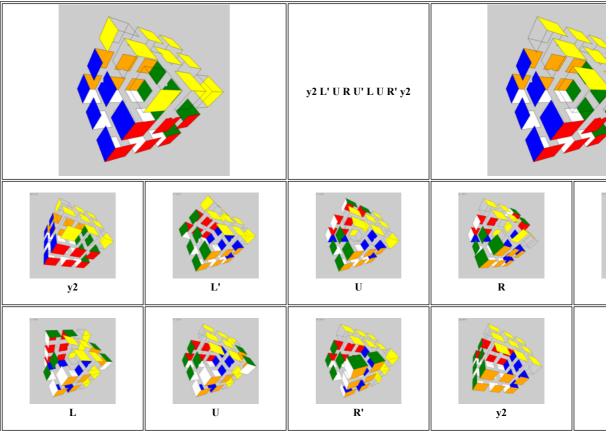
Drone: 4 edges (not symmetric)



A clumsy plane like shape with the head as a square and the tail with 2 edges. Oriented on a diagonal.



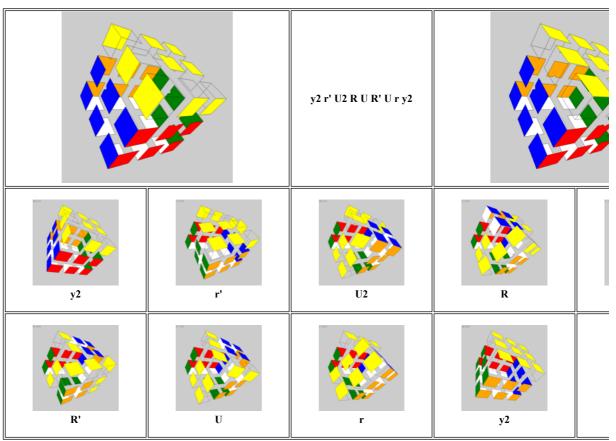
C134 clockwise



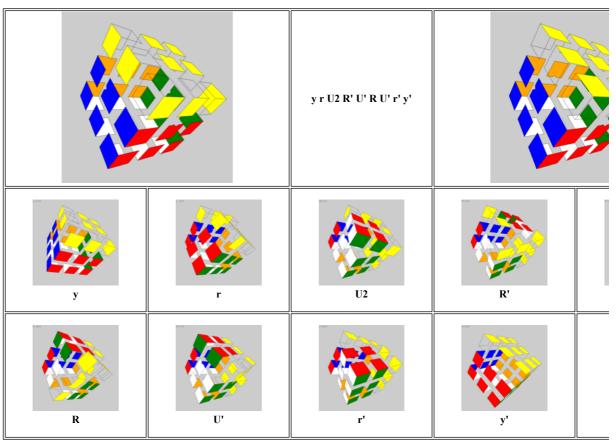
Square: 2 edges (not symmetric)



A square center on C2 and oriented on a diagonal.

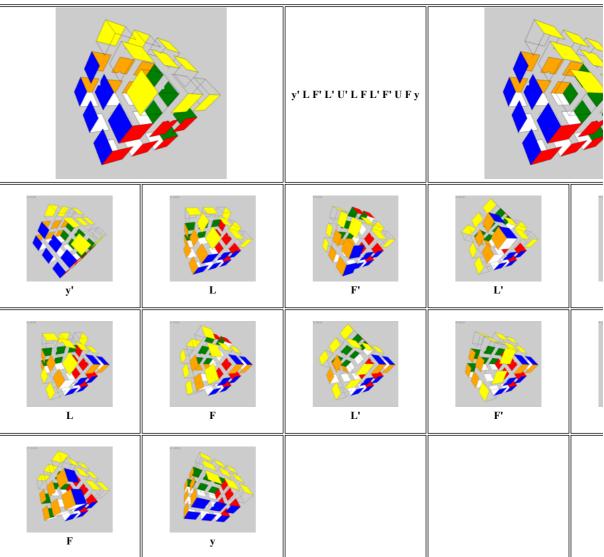


C134 clockwise

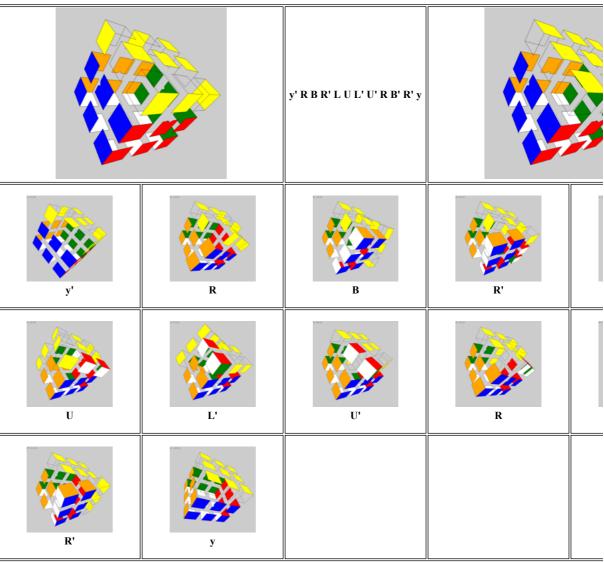


Letter L: 2 edges (not symmetric)

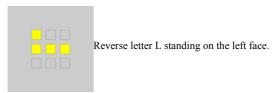


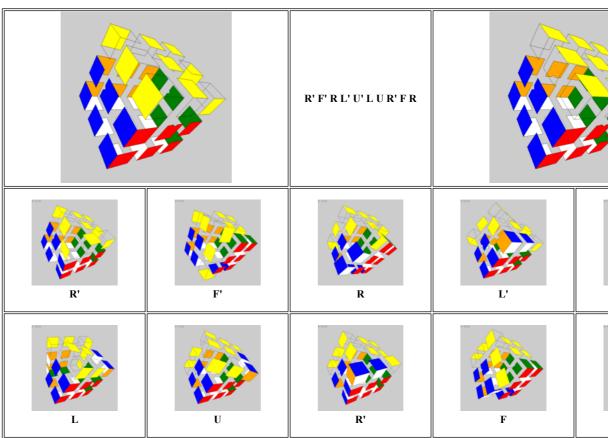


C134 clockwise

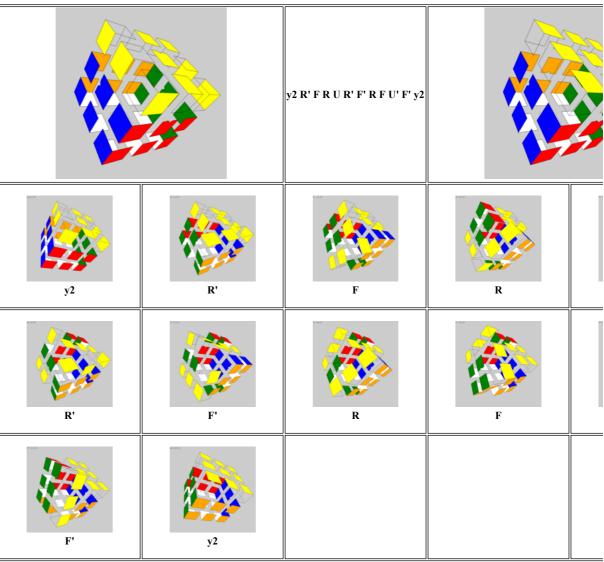


Gallow: 2 edges (not symmetric)





C134 clockwise



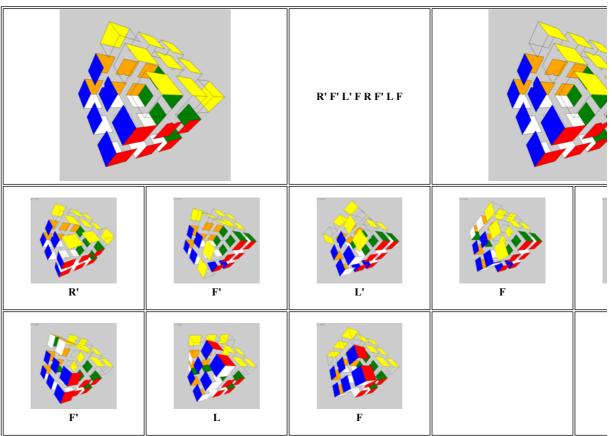
2 set corners

2 unset corners, one clockwise and one counterclockwise, which is C1 and C3 or C4 depending on the pattern (C1 is always unset, C2 is always set). When the pattern is symmetric, there is only one instance because the role clockwise can be obtained from counterclockwise with an Y2 rotation. This is C1 that defines the spin.

Butterfly: 4 edges (symmetric)



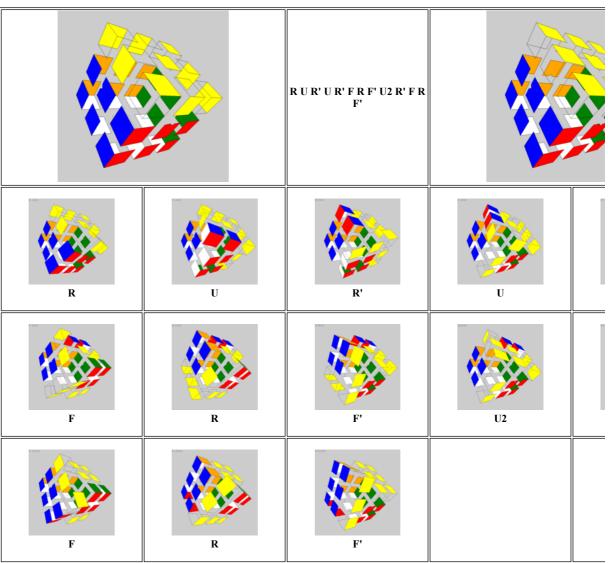
2 wings with general diagonal direction.



Dice 3 dots: 2 edges (symmetric)



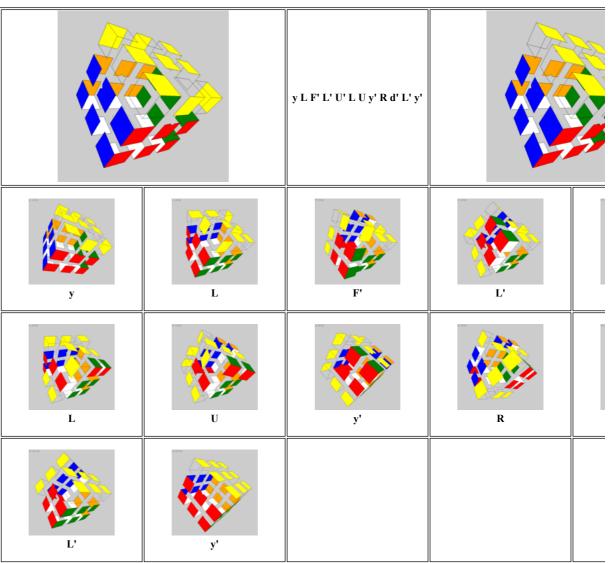
A diagonal line that looks like a dice sitting of its 3 dotted face, especially because the Rubik's cube is exploded.



Letter Z: 2 edges (symmetric)



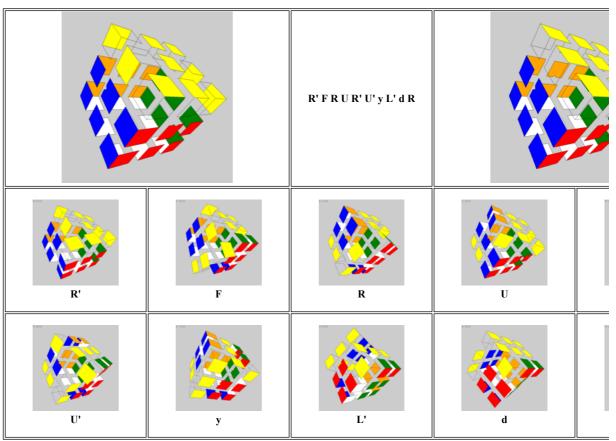
A stylized Z standing on the front face (or back since it is symmetric).



Letter S: 2 edges (symmetric)



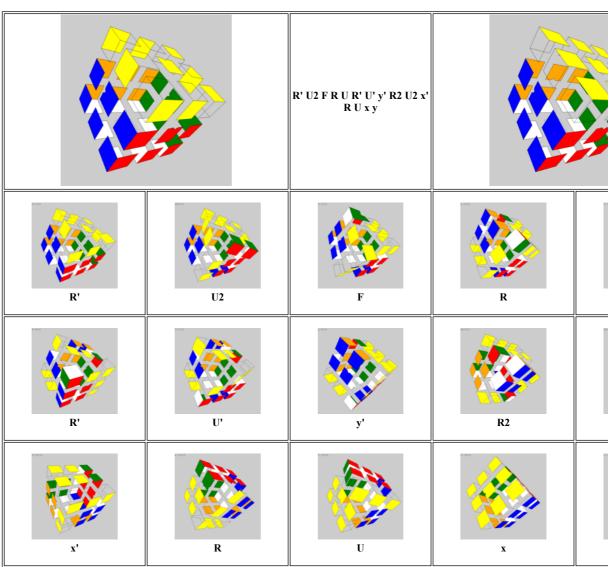
A stylized S (or reverse Z) standing on the right face (or left since it is symmetric).



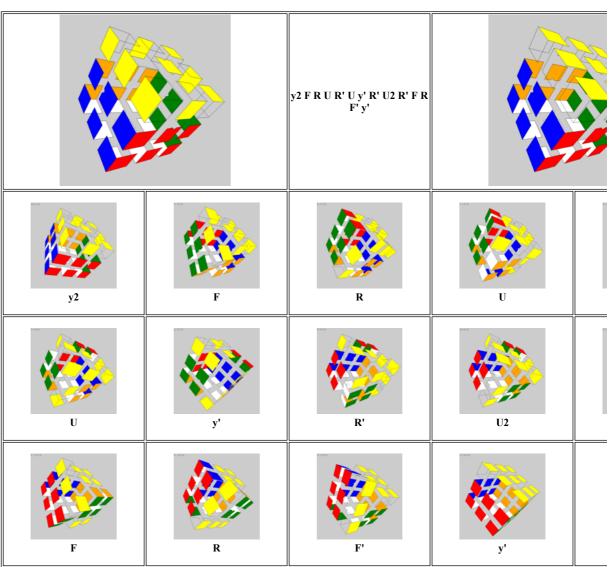
Letter V: no edges (non symmetric)



A pure letter V oriented to the back face.



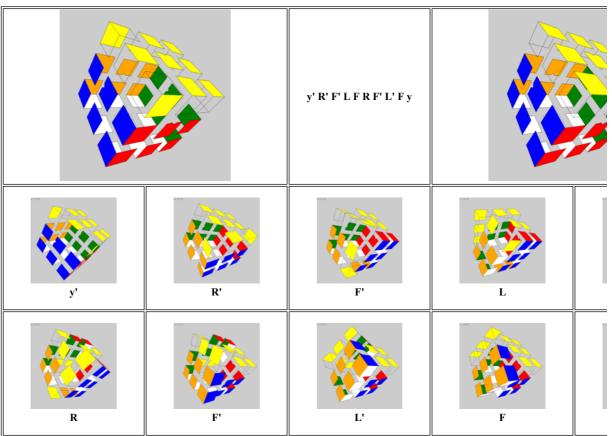
C1 clockwise



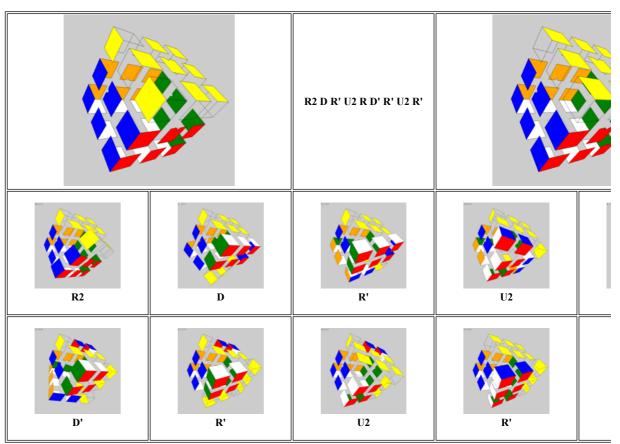
Tree: 4 edges (non symmetric)



A dense tree with thick foliage, oriented to the back face, hence roots on the front one.



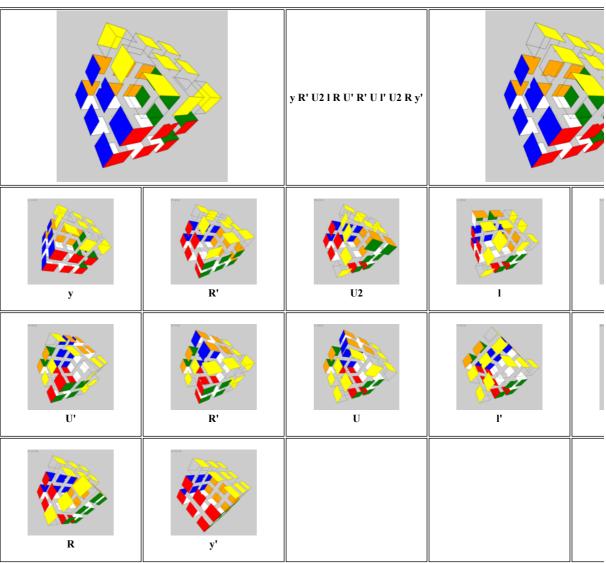
C1 clockwise



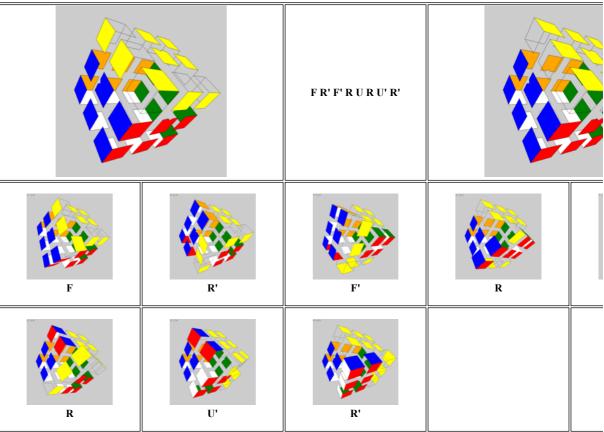
Paddle: 2 edges (non symmetric)



A short handled racket, oriented on the diagonal.



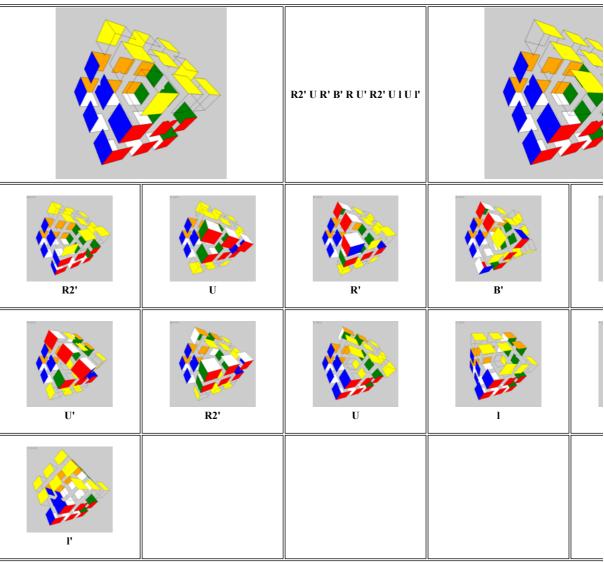
C1 clockwise



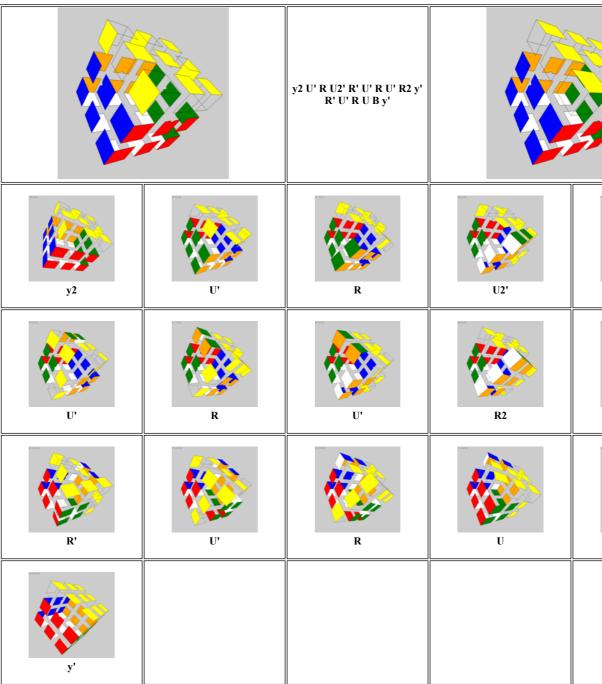
Squirrel: 2 edges (non symmetric)



The squirrel's head is C2, legs and tail horizontal, looking to the left.



C1 clockwise

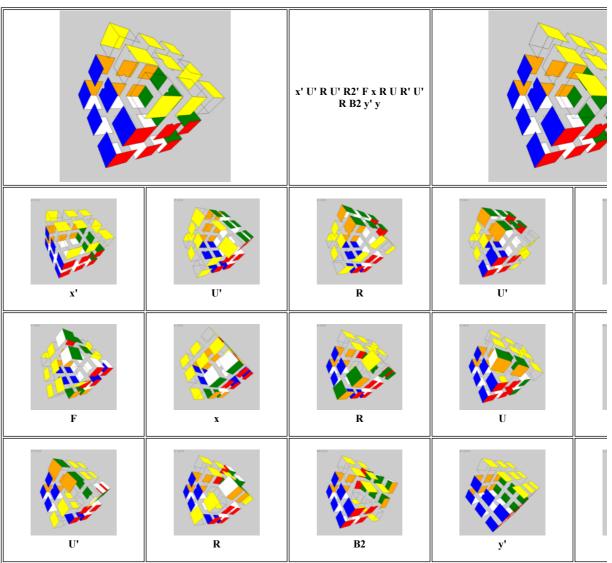


Squirrel reverse: 2 edges (non symmetric)

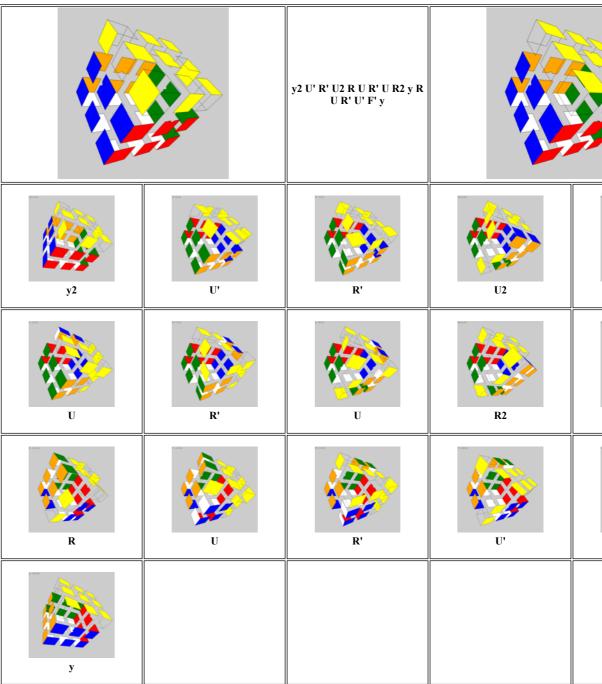


The squirrel's head is C4, legs and tail horizontal, looking to the right.

C1 counterclockwise



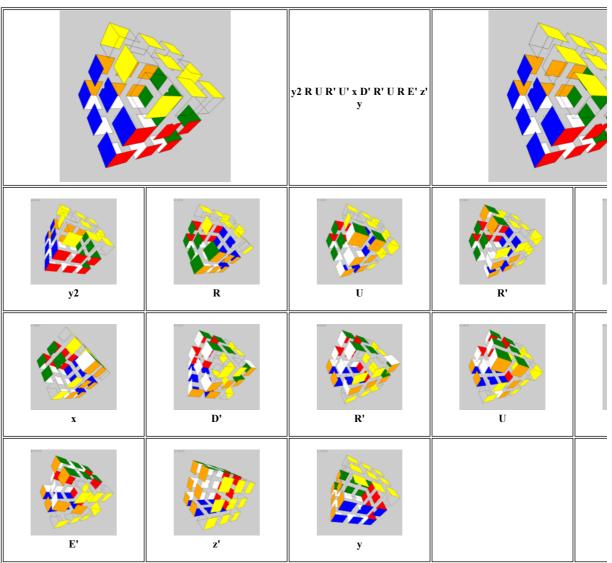
C1 clockwise



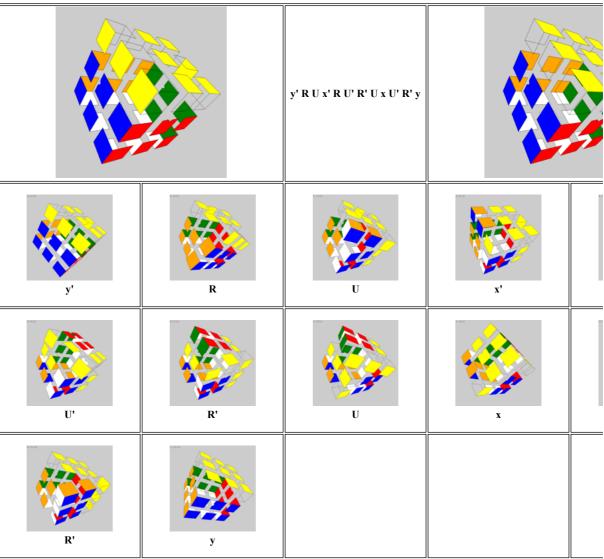
Letter C: 2 edges (non symmetric)



C1 counterclockwise



C1 clockwise

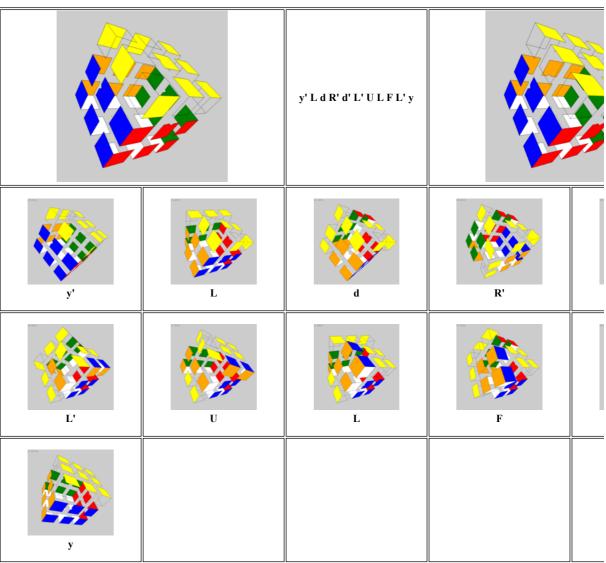


Letter P: 2 edges (non symmetric)

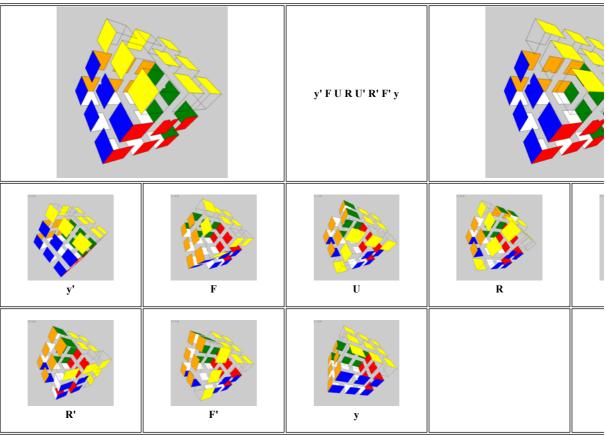


P letter upside down with the foot on the left face.

C1 counterclockwise



C1 clockwise

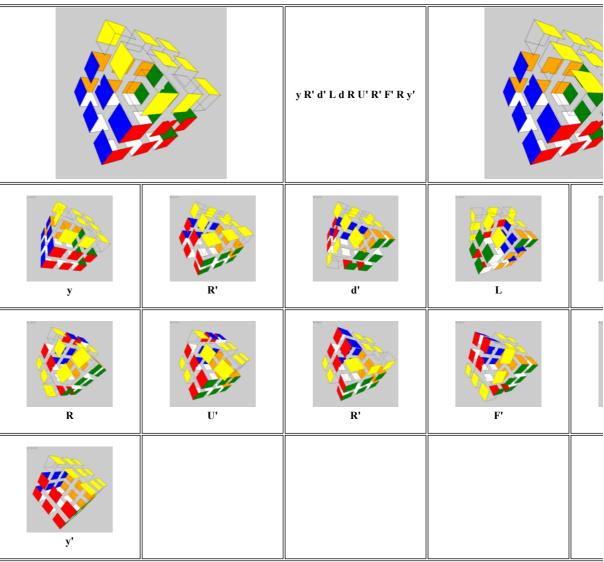


Letter P reverse: 2 edges (non symmetric)

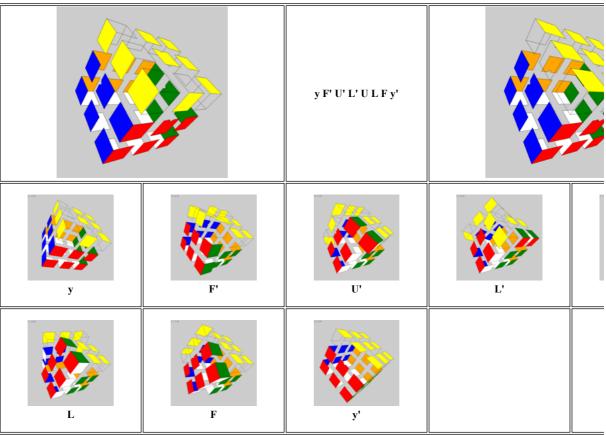


Reversed P letter with the foot on the right face.

C1 counterclockwise



C1 clockwise

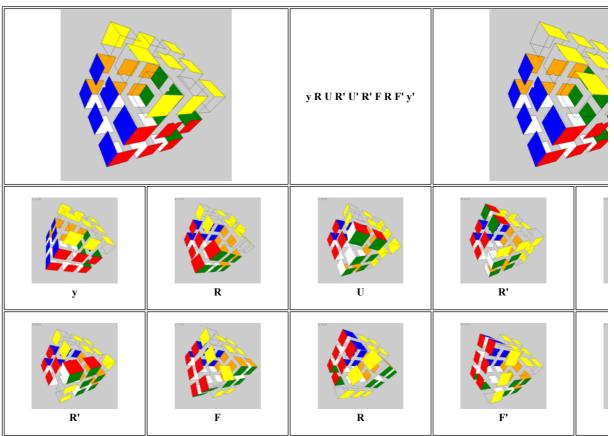


Letter T: 2 edges (non symmetric)

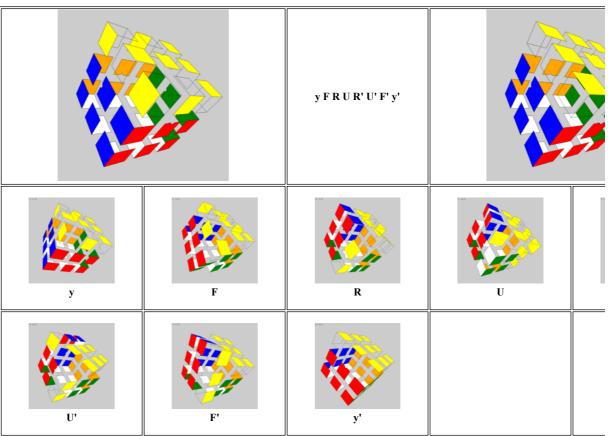


Pure letter T with the foot on the front face.

C1 counterclockwise



C1 clockwise

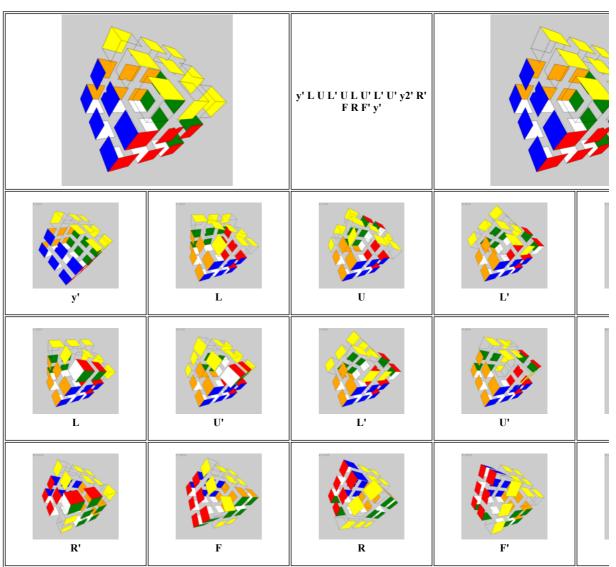


Letter W (non symmetric)

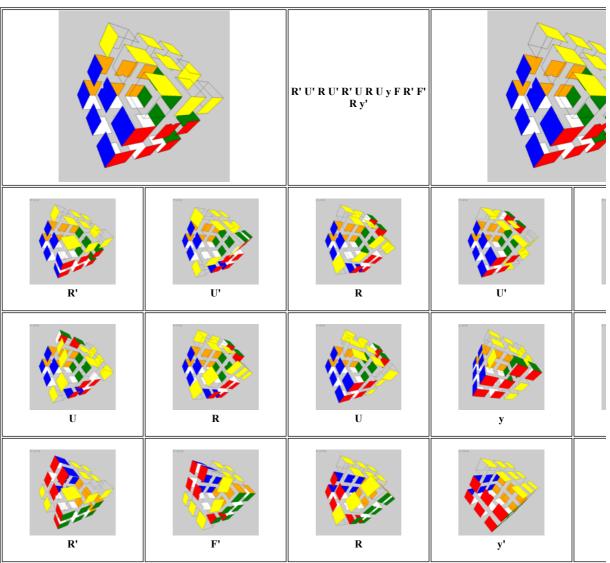


Stylized W oriented on the diagonal, and sitting on left face.

C1 counterclockwise



C1 clockwise



all corners set

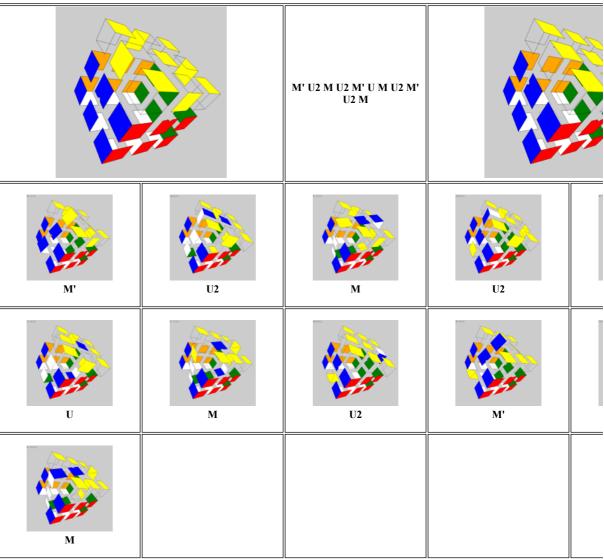
Since all the corners are set, only the edges are unset by pair/s (2 or 4 unset edges).

Dice 5 dots: 2 pairs of unset edges (symmetric)

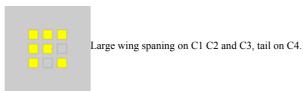


A diagonal cross that looks like a dice sitting of its 5 dotted face, especially because the Rubik'cube is exploded.

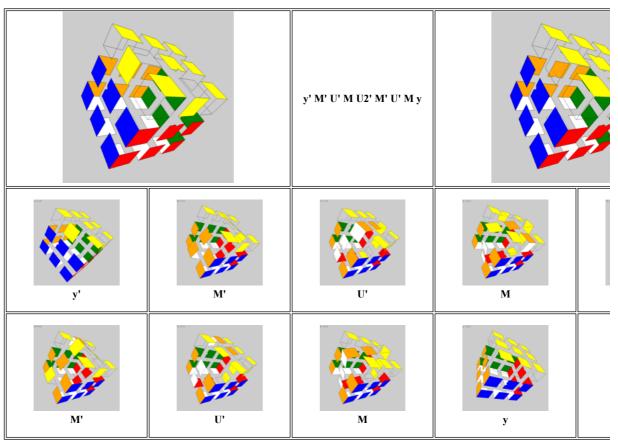
4 unset edges



Kite: 1 pair of opposite unset edges (not symmetric)



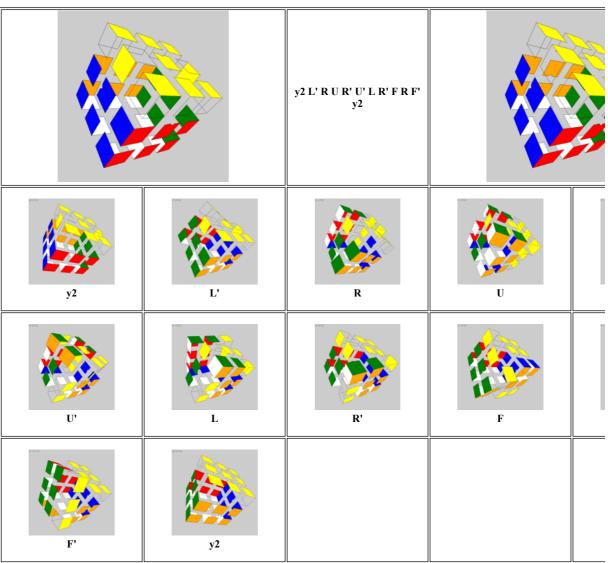
2 opposite unset edges



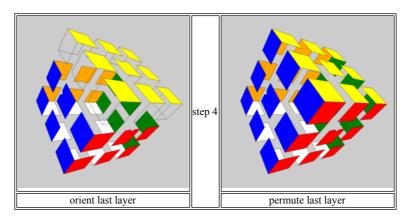
Letter H: 1 pair of adjacent unset edges (symmetric)



2 adjacent unset edges



Step 4: Permute Last Layer



This step is the final permutation of yellow face to complete the cube. It requires one algorithm out of 21.

All corner ternary permutations are centered on the upper left corner (involving upper right and lower left as well) All edge ternary permutations are either up (horizontal involving up left and right), left (vertical involving up left and down) Edges are numbered E1 for left, E2 for back, E3 for front and E4 for right.

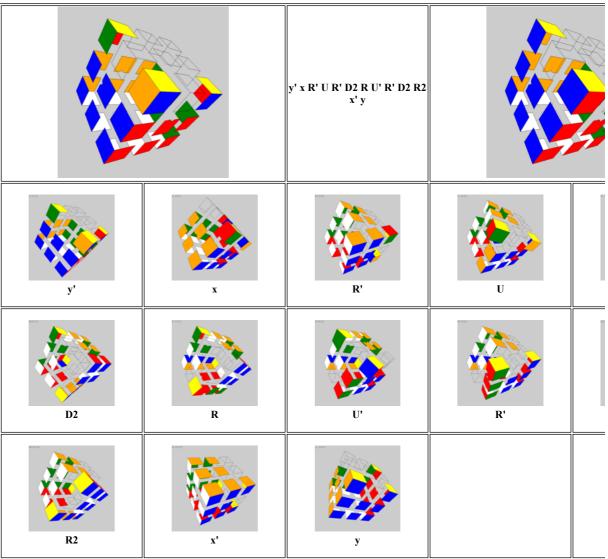
These algorithms describe either circular permutations when they involve 3 cubes (circ), or binary permutations when they involve 2 cubes. Only 2 swaps or 2 circs can be combined in the same algorithm.

Circs are defined by the center/edges that does not move (fix), whereas swaps are qualified by (opp)osite or (adj)acent and defined by the name of corner/edges involved.

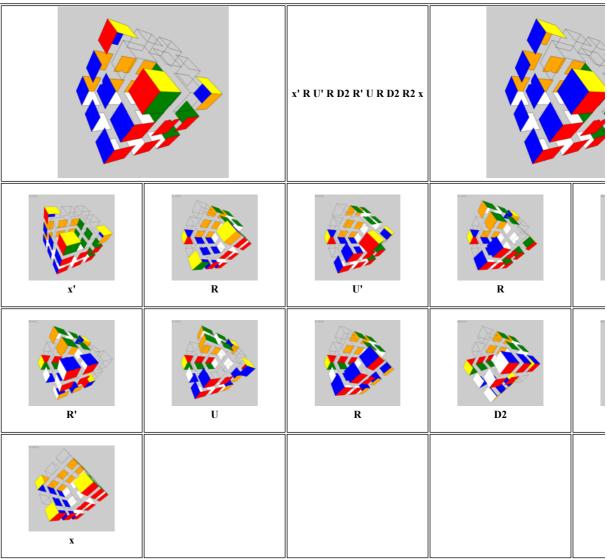
3 corner circular permutations

C2 is the only fix corner.

circular corners clockwise C2 fixed



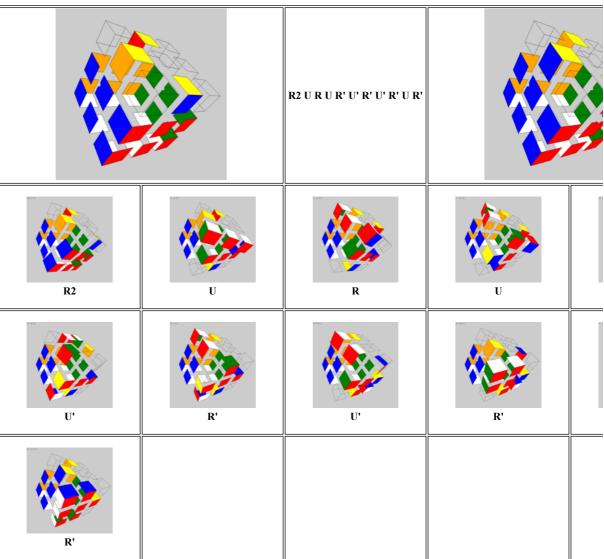
circular corners counterclockwise C2 fixed



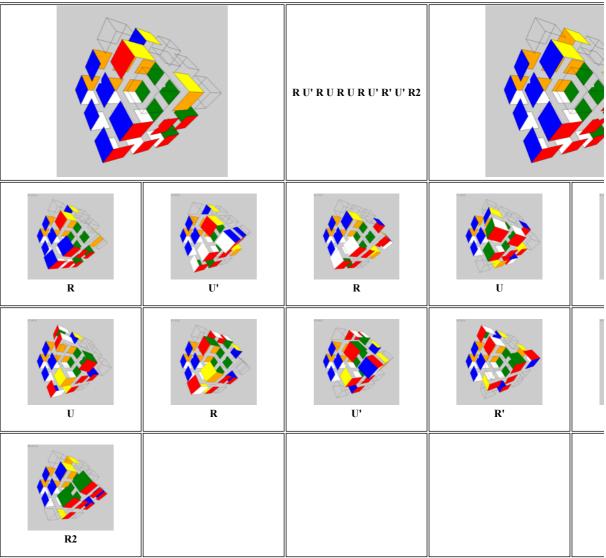
edge circular permutations

E2 is the only fix edge.

circular edges clockwise E2 fixed



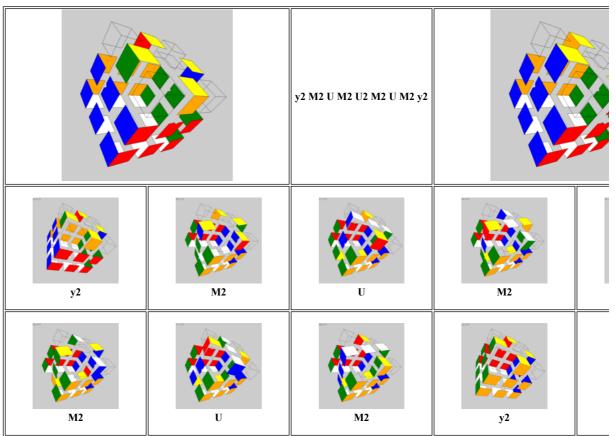
circular edges counterclockwise E2 fixed



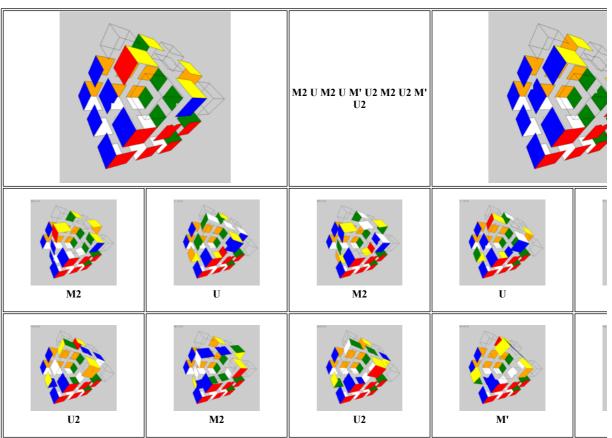
2 binary swaps of 2 pairs of edges

2 pairs or edges are swapped, opposite (forming a cross) or adjacent (forming 2 parallel slash).

swap opposite horizontal E1-E4, opposite vertical E2-E3



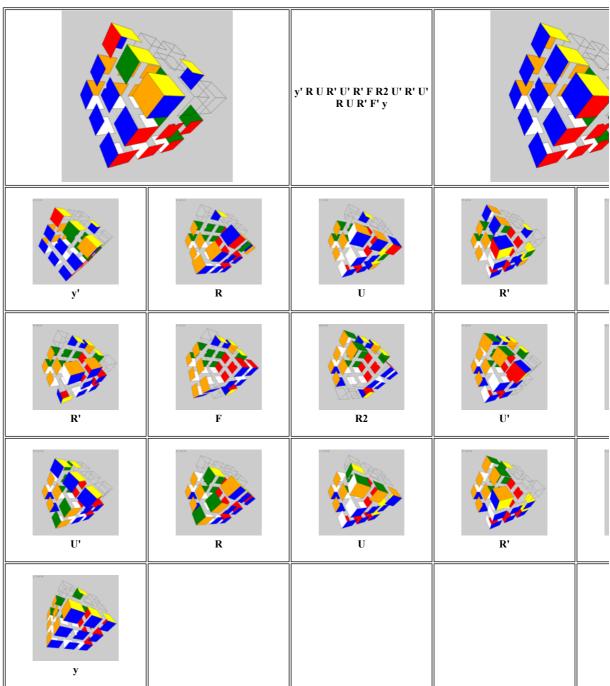
swap adjacent C2 centered E1-E2, adjacent C3 centered E3-E4



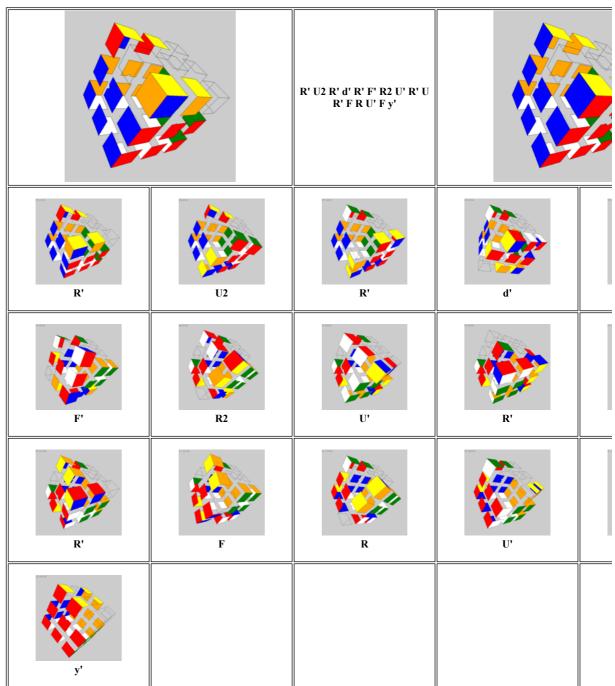
2 binary swaps: a pair of adjacent corners and a pair of edges

Corners are adjacent and always C13, edges are either opposite (2 cases) or adjacent (4 cases centered on 4 corners).

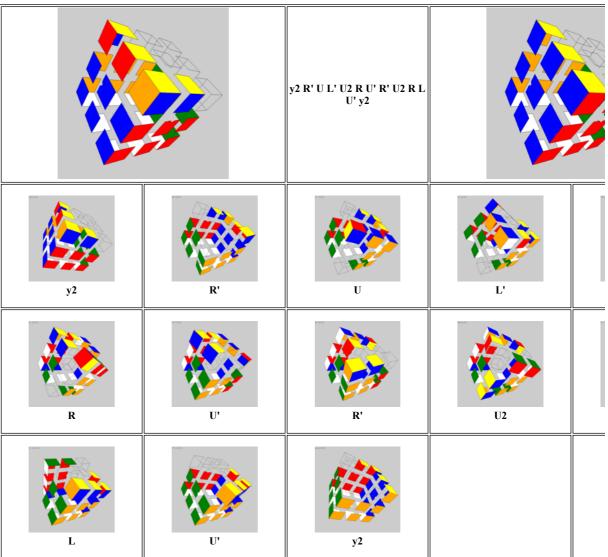
$swap\ adjacent\ horizontal\ C1\text{-}C3,\ opposite\ vertical\ E2\text{-}E3$



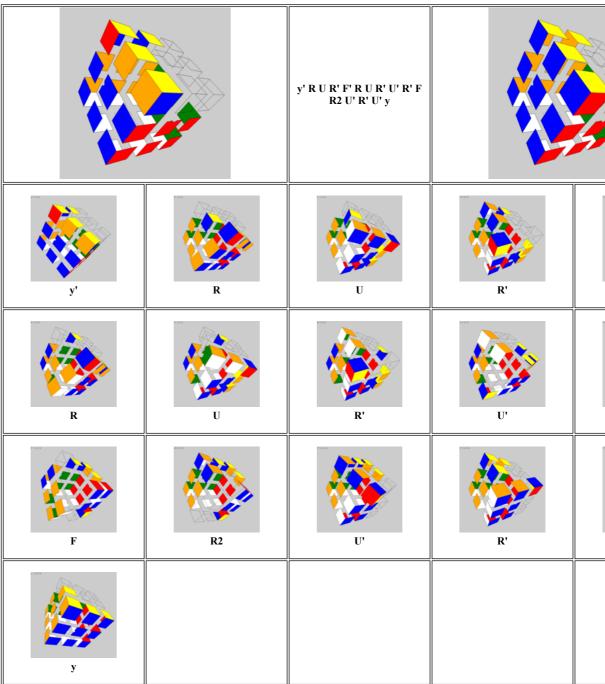
swap adjacent horizontal C1-C3, opposite horizontal E1-E4



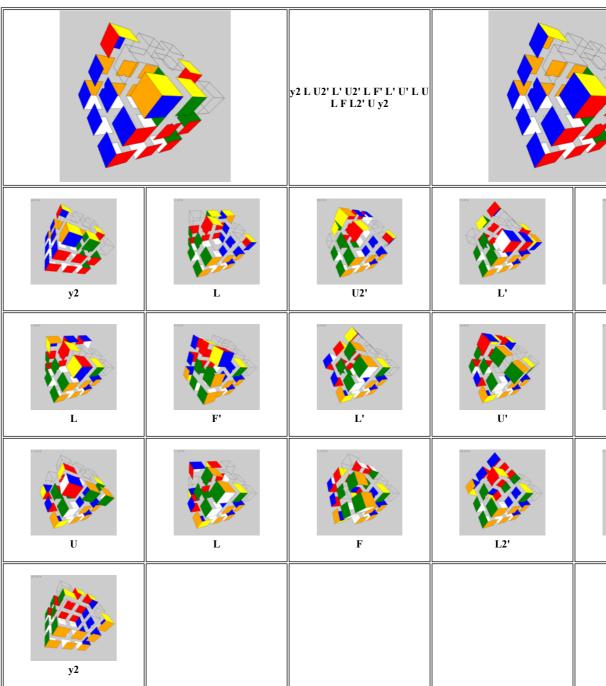
swap adjacent horizontal C1-C3, adjacent C1 centered E1-E3



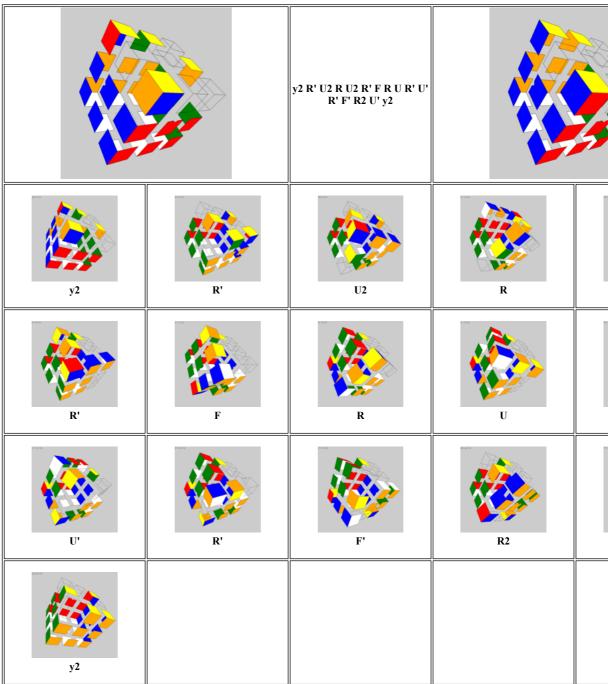
swap adjacent horizontal C1-C3, adjacent C2 centered E1-E2



swap adjacent horizontal C1-C3, adjacent C3 centered E3-E4



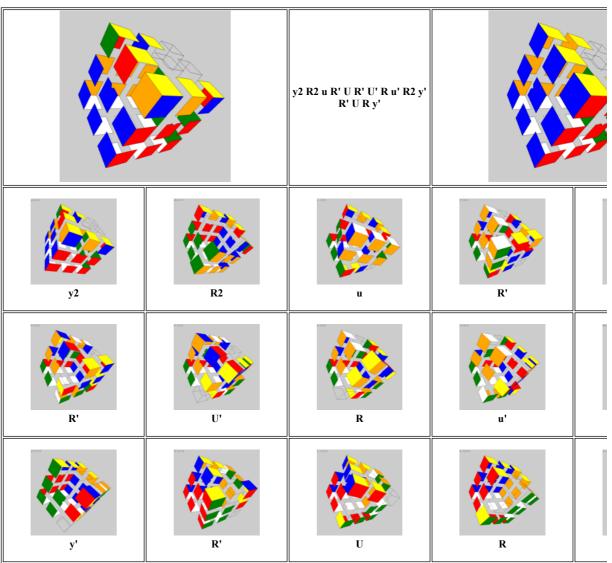
swap adjacent horizontal C1-C3, adjacent C4 centered E2-E4



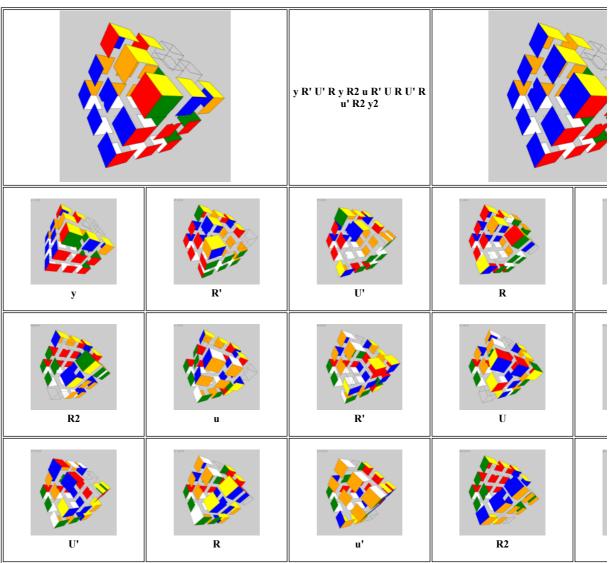
2 ternary permutations: Corner and edge

Corners and edges rotate in opposite directions: if corners rotate clockwise, edges rotate counterclockwise. C2 is always the fix corner, whereas either E1 or E2 are fix.

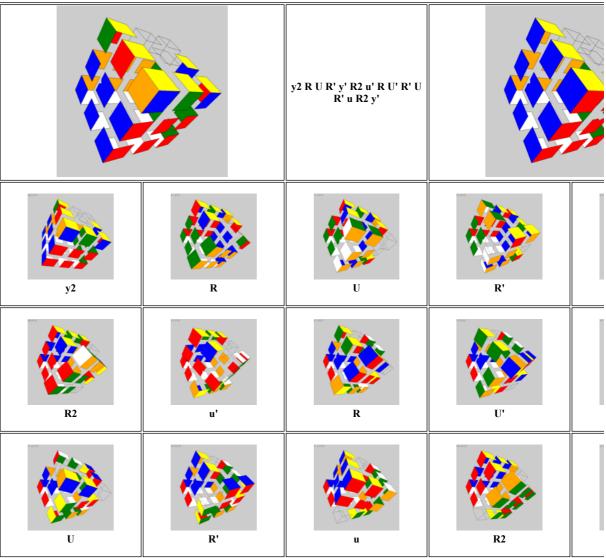
circular clockwise corners C2 fix, counterclockwise edges E1 fixed



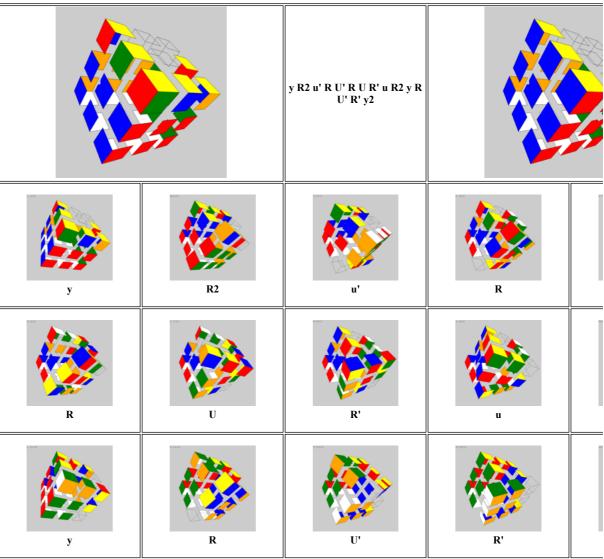
circular counterclockwise corners C2 fix, clockwise edges E1 fixed



circular clockwise corners C2 fix, counterclockwise edges E2 fixed



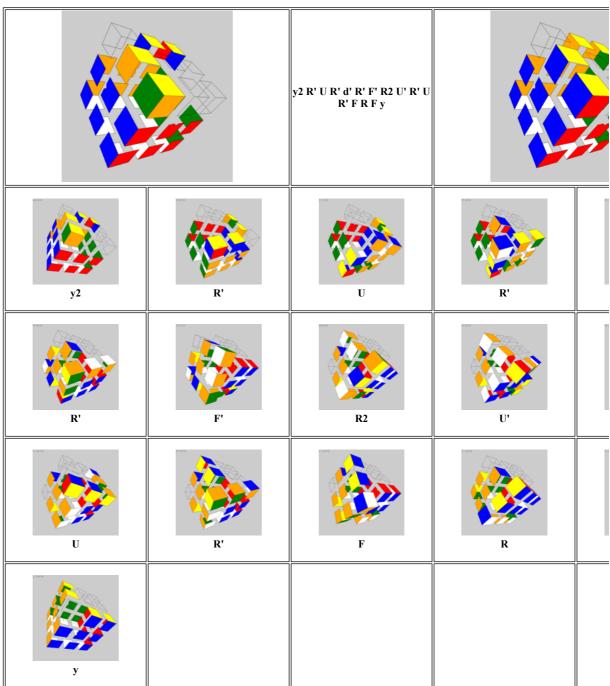
circular counterclockwise corners C2 fix, clockwise edges E2 fixed



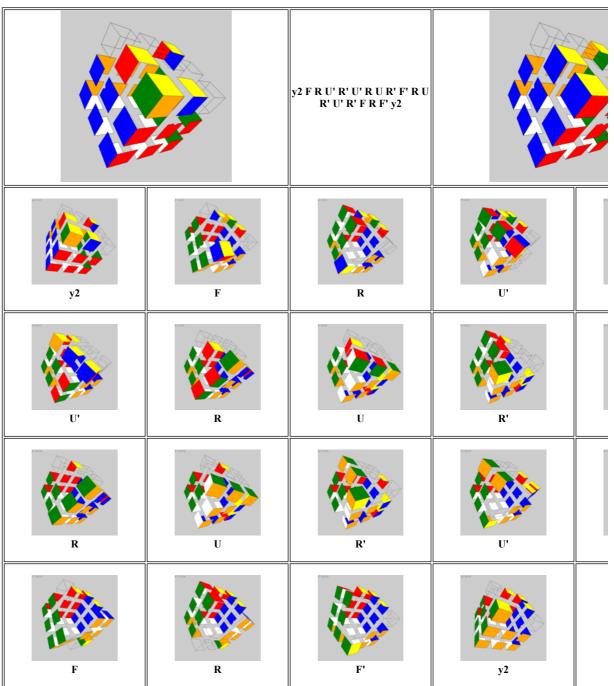
2 binary swaps: a pair of opposite corners and a pair of edges

Corners are opposite and always C23, edges are adjacent (2 cases) or opposite (2 cases). The 2 other adjacent edge cases can be obtained by an Y2 rotation.

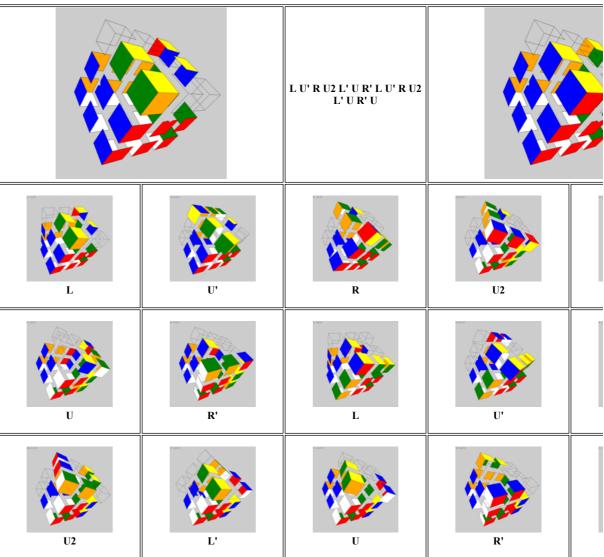
swap opposite C3-C2, adjacent C1 centered E1-E3



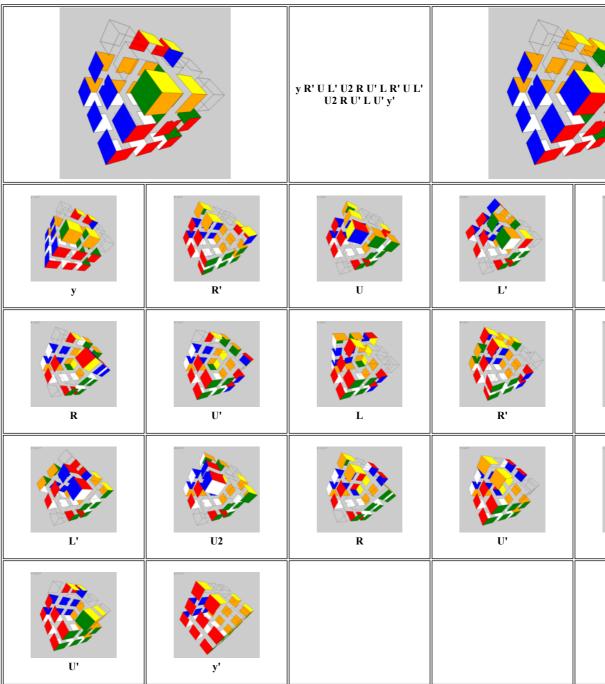
swap opposite C3-C2, adjacent C3 centered E3-E4



swap opposite C3-C2, opposite vertical E2-E3



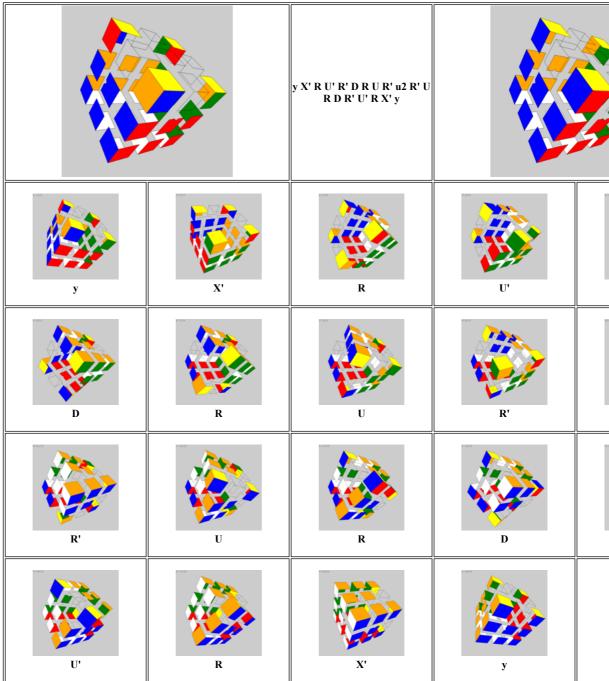
swap opposite C3-C2, opposite horizontal E1-E4



2 binary swaps: 2 pairs of corners

Corners are adjacent and the direction is vertical (parallel to front and back face). Apparently it is not possible to have 2 opposite corners to swap.

swap adjacent horizontal C1-C3, adjacent horizontal C2-C4



Annexe 1: Permutation Notation

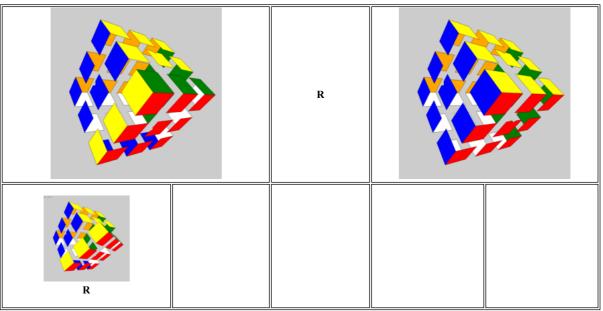
By default, rotations are clockwise by 90°, but they can be anticlockwise when suffixed by a quote """, or a U-turn when suffixed by the digit "2".

Faces

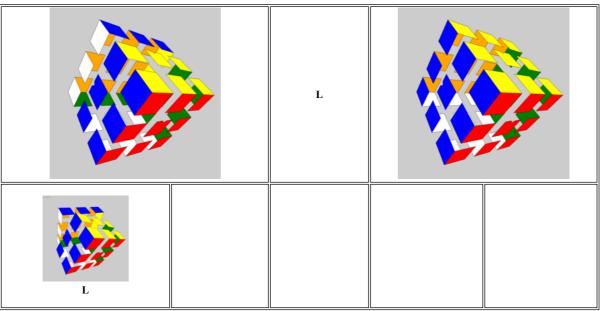
Simple face rotation: Right, Left, Up, Down, Front and Back

Directions are considered with respect to an observer in front of the face.

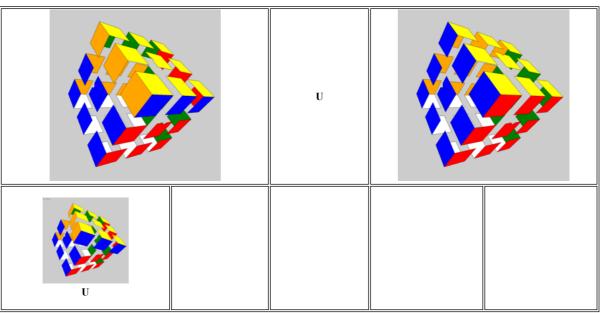
right face clockwise



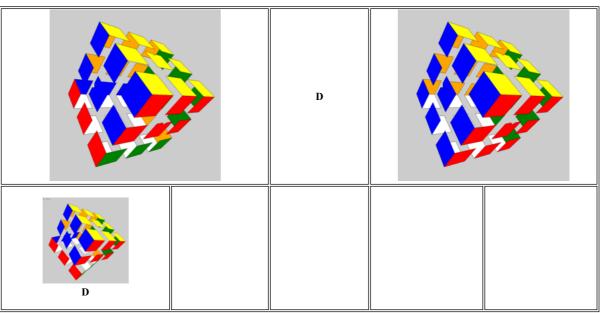
left face clockwise



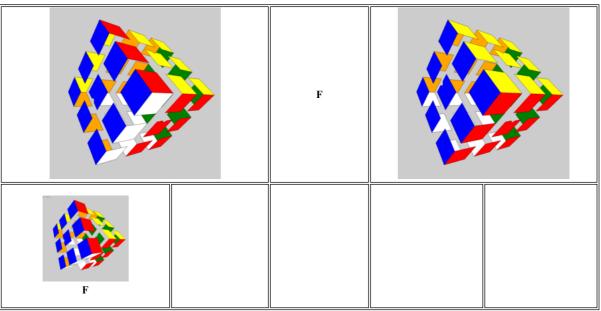
up face clockwise



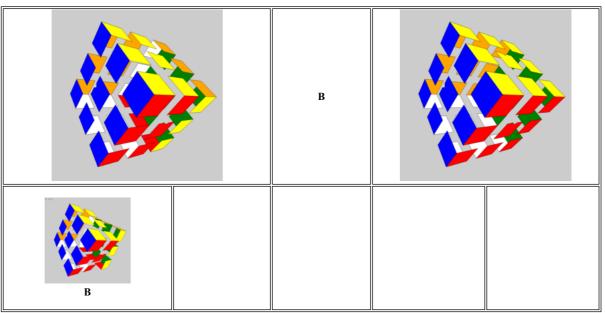
down face clockwise



fron face clockwise



back face clockwise



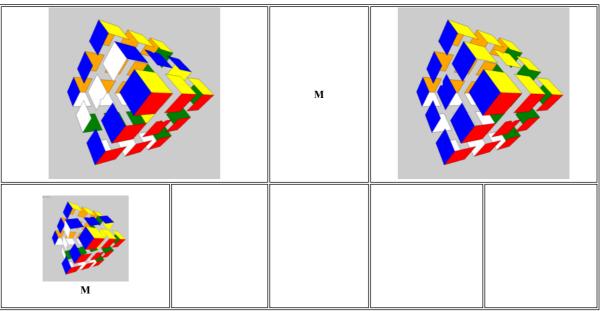
back to menu

Slices

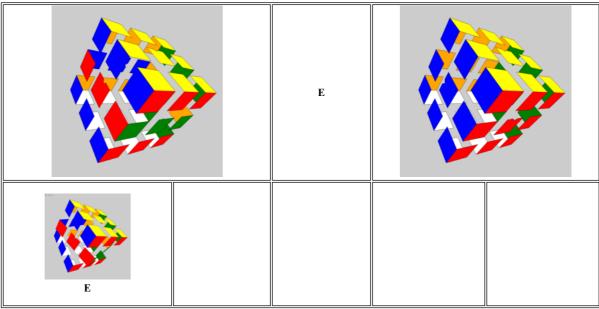
Slices between 2 opposite faces: Middle (between Left and Right), Equator (between Up and Down) and Standing (between Front and Back)

Direction follows Left for Middle, Down for Equator and Front for Standing.

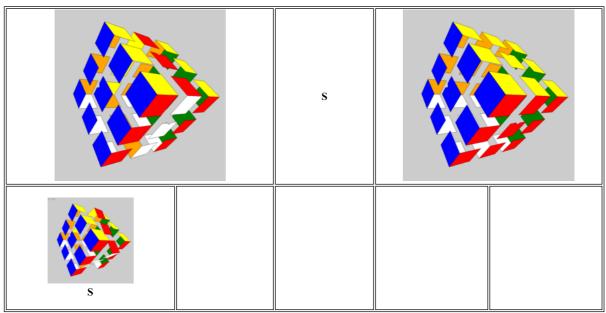
slice between right and left, following left face



slice between up and down, following down face



slice between front and back, following front face



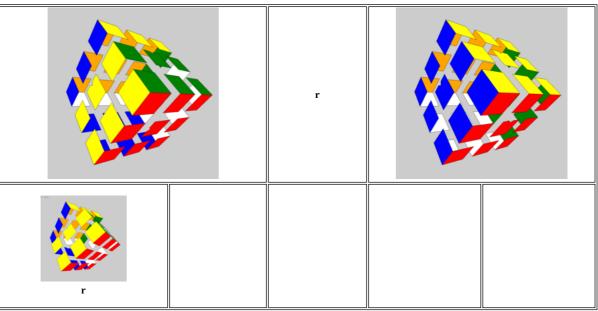
back to menu

Stacked faces and slices

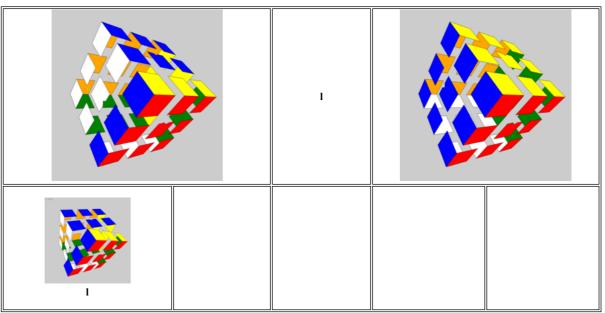
Stack of one face and its adjacent slice: right, left, up, down, front and back

Naturally, direction follow face's.

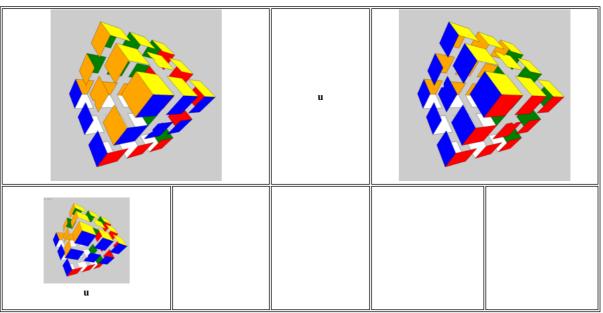
stack of middle slice and right face



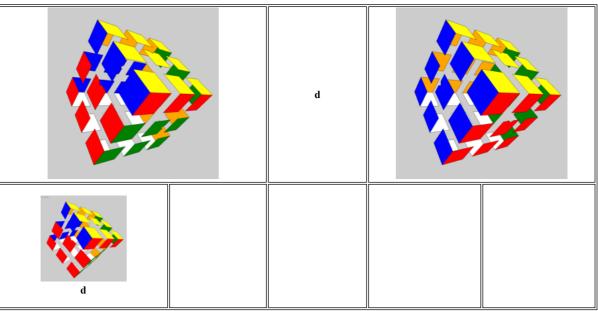
stack of middle slice and left face



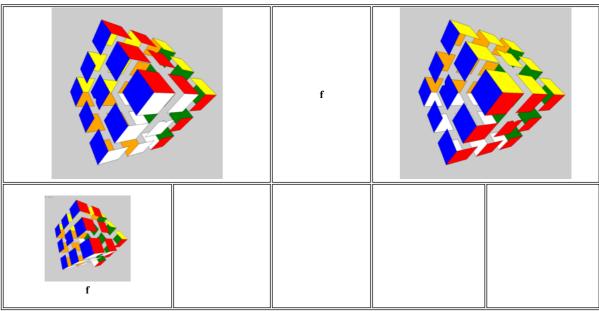
stack of equator slice and up face



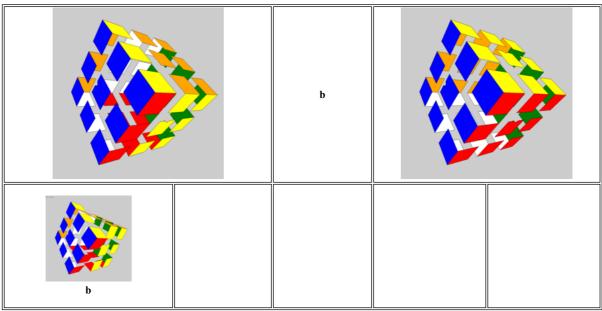
stack of equator slice and down face



stack of standing slice and front face



stack of standing slice and back face



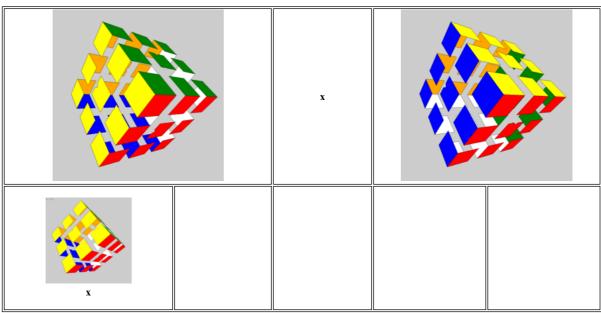
back to menu

Reorientation of the entire cube

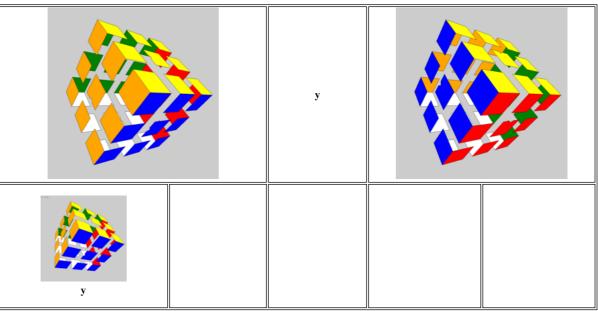
 $Change\ complete\ orientation\ with\ respect\ to\ X\ (Left/Right),\ Y\ (Up/Down)\ and\ Z\ axis\ (Front/Back).$

Direction follows Right for X, Up for Y and Front for Z.

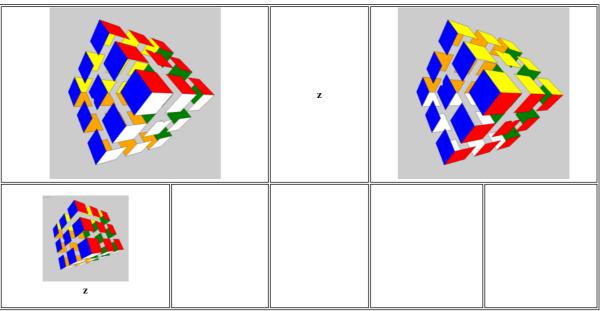
full cube following right face



full cube following up face



full cube following front face



Annexe 2: Summary of Algorithms

num	mnemonic		algorithm First 2 Layers	
1	f2l_cD0_eE-	R U'	R' d R' U2 R U R' U2 R y	
2	f2l_cD+_eE0	R U'	R' U' R U R' U' R U2 R'	
3	f2l_cD+_eE-	R U'	R' d R' U' R U' R' U' R y	
4	f2l_cDeE0		R' U R U2 R' U R U' R'	
5	f2l_cDeE-	RUI	R' U' R U' R' U d R' U' R y	
6	f2l cD0 eUF		J' R' U' F' U F	
7	f21_cD0_eUR		U F U R U' R'	
8	f2l_cD+_eUF		FUF'U'F	
9	f2l cD+ eUR	R U'	R' U R U' R'	
10	f2l cD- eUF		FU'F'UF	
11	f2l cD- eUR	===	R' U' R U R'	
12	f2l cUf eE0		U' R' U' R U2 R'	
13	f2l_cUf_eE-	===	U R' d R' U' R y	
14	f2l_cUr_eE0		U F U F' U2 F	
15	f2l_cUr_eE-		U' F d' F U F' y'	
16	f2l_cUu_eE0		R' U' R U R' U' R U R'	
17	f2l_cUu_eE-		R' d R' U R y	
18	f2l_cUf_eUBf		U R' U' R U2 R'	
19	f2l_cUf_eUBr		U' F U' F' U' F	
20	f21_cUf_eUFf		F U' d' F U F' y'	
21	f2l_cUf_eUFr		U F U' F' U' F	
22	f2l_cUf_eULf		U2 R' U' R U2 R'	
23	f2l_cUf_eULr	F' U'		
24	f2l_cUf_eURf	URU	J' R'	
25	f2l_cUf_eURr		U2 R' U F' U' F	
26	f2l_cUr_eUBf	RUI		
27	f2l_cUr_eUBr	U F'	U2 F U F' U2 F	
28	f2l_cUr_eUFf	U F'	U2 F U' R U R'	
29	f2l_cUr_eUFr	U' F'		
30	f2l_cUr_eULf	U' R	U R' U R U R'	
31	f2l_cUr_eULr	U F'	U' F U F' U2 F	
32	f21_cUr_eURf	U' R	U'R'URUR'	
33	f2l_cUr_eURr	R U'	R' U d R' U' R y	
34	f2l_cUu_eUBf	URU	J2 R' U R U' R'	
35	f2l_cUu_eUBr	U2 F	U' F U' F' U F	
36	f2l_cUu_eUFf	RUI	R' U' U' R U R' U' R U R'	
37	f2l_cUu_eUFr	F' U2	FUF'U'F	
37	f2l_cUu_eULf	U2 R	U R' U R U' R'	
39	f2l_cUu_eULr	U' F'	U2 F U' F' U F	
40	f2l_cUu_eURf	R U2	R' U' R U R'	
41	f2l_cUu_eURr y' R'		U' R U U R' U' R U R' U' R y	
num	mnemonic		algorithm Orient Last Layer	
1	oll_Hub_C-+C+		R U B' l U l2' x' U' R' F R F'	
2	oll_Hub_C+-C+		R' F R F' U2 R' F R F2 U2 F	
3	oll_LineV_C-+C+		R U' y R2 D R' U2 R D' R2 d R'	
4	oll_LineV_C+-C+		R' U' y L' U L' y' L F L' F R	
5	oll_LineH_C-+C+		L' B' L U' R' U R U' R' U R L' B L	
6	oll_LineH_C+-C+		F U R U' R' U R U' R' F'	
7	oll_CrossEdge_C+-C+		L U' R' U L' U R U R' U R	
8	oll_CrossEdge_C-+C+		R U R' U R U' R' U R U2 R'	
9	oll_ChevronC1_C-+C+		y L F' L' F U2 L2 y' L F L' F	
10	oll_ChevronC1_C+-C+		y2 R' F R' F' R2 U2 x' U' R U R' x y2	
11	oll_ChevronC1_C-+C-+_		y2 F' L' U' L U L' U' L U F y2	
12	oll_ChevronC2_C-+C+		y R' F R F' U2 R2 y R' F' R F' y2	
13	oll_ChevronC2_C+-C+		y2 L U' y' R' U2' R' U R U' R U2 R d' L' y2	
14	oll_ChevronC2_C-+C-+_		FRUR'U'RUR'U'F'	
15	oll_Chicken_C3		y R U R' y R' F R U' R' F' R y	
16	oll_Chicken_C3+_		y2 R' U' R y' x' R U' R' F R U R' x y2	
17	oll_Slash_C3		y' L' R2 B R' B L U2' L' B M' x' y	
18	oll_Slash_C3+_		y R' U2 x R' U R U' y R' U' R' U R' F z' y2	
19	oll_Cactus_C3		y' U2 I' L2 U L' U L U2 L' U M y	
II	II .			

lan			ng: advanced Fridrich method
20	oll_Cactus_C3+_ oll Tetris C3-		B' L U' R' U R L' B L L F R' F R F2 L' y
22	oll Tetris C3+		U2 r R2' U' R U' R' U2 R U' M y2
23	oll Drone C3-		R' U2 R U R' U R y2
24	oll Drone C3+		L' U R U' L U R' y2
25	oll_Square_C3		r' U2 R U R' U r y2
26	oll_Square_C3+_		U2 R' U' R U' r' y'
27	oll LetterL C3-		LF'L'U'LFL'F'UFy
28	oll LetterL C3+		R B R'L U L'U'R B'R'y
29	oll Gallow C3-		F'RL'U'LUR'FR
30	oll Gallow C3+		R' F R U R' F' R F U' F' y2
31	oll Butterfly C-		F'L'FRF'LF
32	oll Dice3 C-		J R' U R' F R F' U2 R' F R F'
33	oll LetterV C-		U2 F R U R' U' y' R2 U2 x' R U x y
34	oll LetterV C+		F R U R' U y' R' U2 R' F R F' y'
35	oll Tree C-		X' F' L F R F' L' F y
36	oll Tree C+		D R' U2 R D' R' U2 R'
37	oll Paddle C-		L' U2 I R U' R' U I' U2 R y'
38	oll Paddle C+		L' F' R U R U' R'
39	oll_Squirrel_C		' U R' B' R U' R2' U I U I'
40	oll Squirrel C+		U' R U2' R' U' R U' R2 y' R' U' R U B y'
41	oll_SquirrelRev_C		J' R U' R2' F x R U R' U' R B2 y' y
42	oll_SquirrelRev_C+_		U' R' U2 R U R' U R2 y R U R' U' F' y
43	oll_LetterC_C		R U R' U' x D' R' U R E' z' y
44	oll_LetterC_C+_		R U x' R U' R' U x U' R' y
45	oll_LetterP_C		d R' d' L' U L F L' y
46	oll_LetterP_C+_	y' I	FURU'R'F'y
47	oll_LetterPRev_C	y R	' d' L d R U' R' F' R y'
48	oll_LetterPRev_C+_	y F	'U'L'ULFy'
49	oll_LetterT_C	y R	. U R' U' R' F R F' y'
50	oll_LetterT_C+_	y F	R U R' U' F' y'
51	oll_LetterW_C	y' I	L U L' U L U' L' U' y2' R' F R F' y'
52	oll_LetterW_C+_		U' R U' R' U R U y F R' F' R y'
53	oll_LetterZ_C		F' L' U' L U y' R d' L' y'
54	oll_LetterS_C		F R U R' U' y L' d R
55	oll_Dice5_		U2 M U2 M' U M U2 M' U2 M
56	oll_Kyte_		M' U' M U2' M' U' M y
57	oll_LetterH_	y2	L' R U R' U' L R' F R F' y2
num	mnemonic		algorithm Permute Last Layer
1	pll_1cr_c+Fx2		y' x R' U R' D2 R U' R' D2 R2 x' y
2	pll_1cr_c-Fx2		x' R U' R D2 R' U R D2 R2 x
3	pll_1cr_e+Fx2		R2 U R U R' U' R' U' R' U R'
4	pll_1cr_e-Fx2		R U' R U R U R U' R' U' R2
5	pll_2sw_eOpp14Hrz_eOpp23Vrt		y2 M2 U M2 U2 M2 U M2 y2
6	pll_2sw_eAdj12Crt2_eAdj34Crt3		M2 U M2 U M' U2 M2 U2 M' U2
7	pll_2sw_cAdj13Hrz_eOpp23Vrt		y' R U R' U' R' F R2 U' R' U' R U R' F' y
8	pll_2sw_cAdj13Hrz_eOpp14Hrz		R' U2 R' d' R' F' R2 U' R' U R' F R U' F y'
9	pll_2sw_cAdj13Hrz_eAdj13Crt1		y2 R' U L' U2 R U' R' U2 R L U' y2
10	pll_2sw_cAdj13Hrz_eAdj12Crt2		y' R U R' F' R U R' U' R' F R2 U' R' U' y
11	pll_2sw_cAdj13Hrz_eAdj34Crt3		y2 L U2' L' U2' L F' L' U' L U L F L2' U y2
12	pll_2sw_2Adj13Hrz_eAdj24Crt4		y2 R' U2 R U2 R' F R U R' U' R' F' R2 U' y2
13	pll_2cr_c+Fx2_e-Fx1		y2 R2 u R' U R' U' R u' R2 y' R' U R y'
14	pll_2cr_c-Fx2-e+Fx1		y R' U' R y R2 u R' U R U' R u' R2 y2 y 2 D
	Inll 2or c+Ev2+c Ev2		y2 R U R' y' R2 u' R U' R' U R' u R2 y'
15	pll_2cr_c+Fx2+e-Fx2		
15 16	pll_2cr_c-Fx2-e+Fx2		y R2 u' R U' R U R' u R2 y R U' R' y2 v2 R' R' R' F' R2 R' R' F R F v
15 16 17	pll_2cr_c-Fx2-e+Fx2 pll_2sw_cOpp32_eAdj13Ctr1		y2 R' U R' d' R' F' R2 U' R' U R' F R F y
15 16 17 18	pll_2cr_c-Fx2-e+Fx2 pll_2sw_cOpp32_eAdj13Ctr1 pll_2sw_cOpp32_eAdj34Ctr3		y2 R' U R' d' R' F' R2 U' R' U R' F R F y y2 F R U' R' U' R U R' F' R U R' U' R' F R F' y2
15 16 17 18	pll_2cr_c-Fx2-e+Fx2 pll_2sw_cOpp32_eAdj13Ctr1 pll_2sw_cOpp32_eAdj34Ctr3 pll_2sw_cOpp32_eOpp23Vrt		y2 R' U R' d' R' F' R2 U' R' U R' F R F y y2 F R U' R' U' R U R' F' R U R' U' R' F R F' y2 L U' R U2 L' U R' L U' R U2 L' U R' U
15	pll_2cr_c-Fx2-e+Fx2 pll_2sw_cOpp32_eAdj13Ctr1 pll_2sw_cOpp32_eAdj34Ctr3		y2 R' U R' d' R' F' R2 U' R' U R' F R F y y2 F R U' R' U' R U R' F' R U R' U' R' F R F' y2