Efficient Simulation of Knitted Cloth Using Persistent Contacts

Supplementary Document: Stitch Wrapping Force and its Jacobians

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All notation refers to Fig. 3 in the main document.

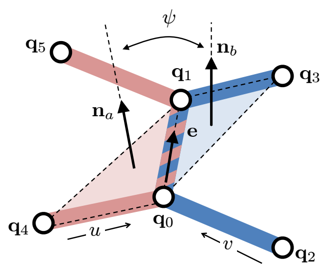

Figure 1:

The stitch direction is defined by a vector

$$\mathbf{e} = \frac{\mathbf{x}_1 - \mathbf{x}_0}{\|\mathbf{x}_1 - \mathbf{x}_0\|}, \tag{1}$$

with derivatives

$$\frac{\partial \mathbf{e}}{\partial \mathbf{x}_0} = -\frac{1}{\|\mathbf{x}_1 - \mathbf{x}_0\|}\left(\mathbf{I} - \mathbf{e}\,\mathbf{e}^T\right) \quad \text{and} \quad \frac{\partial \mathbf{e}}{\partial \mathbf{x}_1} = \frac{1}{\|\mathbf{x}_1 - \mathbf{x}_0\|}\left(\mathbf{I} - \mathbf{e}\,\mathbf{e}^T\right). \tag{2}$$

Two triangles $(\mathbf{q}_0, \mathbf{q}_1, \mathbf{q}_4)$ and $(\mathbf{q}_0, \mathbf{q}_3, \mathbf{q}_1)$ have normal vectors

$$\mathbf{n}_a = \frac{\mathbf{v}_a}{\|\mathbf{v}_a\|}, \quad \text{with } \mathbf{v}_a = (\mathbf{x}_4 - \mathbf{x}_1) \times (\mathbf{x}_0 - \mathbf{x}_1). \tag{3}$$

$$\mathbf{n}_b = \frac{\mathbf{v}_b}{\|\mathbf{v}_b\|}, \quad \text{with } \mathbf{v}_b = (\mathbf{x}_0 - \mathbf{x}_1) \times (\mathbf{x}_3 - \mathbf{x}_1). \tag{4}$$

It is convenient to define the auxiliary vectors

$$\mathbf{x}_{a0} = \mathbf{x}_4 - \mathbf{x}_1, \quad \mathbf{x}_{a1} = \mathbf{x}_0 - \mathbf{x}_4, \quad \text{and } \mathbf{x}_{a4} = \mathbf{x}_1 - \mathbf{x}_0. \tag{5}$$

$$\mathbf{x}_{b0} = \mathbf{x}_1 - \mathbf{x}_3, \quad \mathbf{x}_{b1} = \mathbf{x}_3 - \mathbf{x}_0, \quad \text{and } \mathbf{x}_{b3} = \mathbf{x}_0 - \mathbf{x}_1. \tag{6}$$

The derivatives of these terms, $\frac{\partial \mathbf{x}_{ai}}{\partial \mathbf{x}_j}$ and $\frac{\partial \mathbf{x}_{bi}}{\partial \mathbf{x}_j}$, can take the values $\{\mathbf{I}, -\mathbf{I}, \mathbf{0}\}$.

The wrapping angle between the triangles is

$$\psi = \arccos(\mathbf{n}_a^T \mathbf{n}_b), \tag{7}$$

and its derivatives take the form

$$\frac{\partial \psi}{\partial \mathbf{x}_i} = \frac{1}{\|\mathbf{v}_b\|}\,\mathbf{n}_b^T\,\mathbf{e}^T\,\mathbf{x}_{bi} - \frac{1}{\|\mathbf{v}_a\|}\,\mathbf{n}_a^T\,\mathbf{e}^T\,\mathbf{x}_{ai}. \tag{8}$$

Stitch wrapping energy:

$$V = \frac{1}{2}\,k_w\,L\,\psi^2. \tag{9}$$

Forces on stitch end points ($i \in \{0, 1, 3, 4\}$):

$$\mathbf{F}_{\mathbf{x}_i} = -k_w\,L\,\psi\left(\frac{1}{\|\mathbf{v}_b\|}\,\mathbf{x}_{bi}^T\,\mathbf{e}\,\mathbf{n}_b - \frac{1}{\|\mathbf{v}_a\|}\,\mathbf{x}_{ai}^T\,\mathbf{e}\,\mathbf{n}_a\right). \tag{10}$$

Force Jacobians:

$$\begin{aligned}\frac{\partial \mathbf{F}_{\mathbf{x}_i}}{\partial \mathbf{x}_j} = &-k_w\, L\, \left(\frac{1}{\|\mathbf{v}_b\|}\,\mathbf{x}_{bi}^T\,\mathbf{e}\,\mathbf{n}_b - \frac{1}{\|\mathbf{v}_a\|}\,\mathbf{x}_{ai}^T\,\mathbf{e}\,\mathbf{n}_a\right)\,\left(\frac{1}{\|\mathbf{v}_b\|}\,\mathbf{x}_{bj}^T\,\mathbf{e}\,\mathbf{n}_b - \frac{1}{\|\mathbf{v}_a\|}\,\mathbf{x}_{aj}^T\,\mathbf{e}\,\mathbf{n}_a\right)^T \\ &- k_w\, L\,\psi\,\left(\frac{1}{\|\mathbf{v}_b\|^2}\,\mathbf{x}_{bi}^T\,\mathbf{e}\,\left(\mathbf{I} - 2\,\mathbf{n}_b\,\mathbf{n}_b^T\right)\,\mathbf{x}_{bj}^* + \frac{1}{\|\mathbf{v}_b\|}\,\mathbf{n}_b\,\mathbf{x}_{bi}^T\,\frac{\partial \mathbf{e}}{\partial \mathbf{x}_j} + \frac{1}{\|\mathbf{v}_b\|}\,\mathbf{n}_b\,\mathbf{e}^T\,\frac{\partial \mathbf{x}_{bi}}{\partial \mathbf{x}_j}\right) \\ &- k_w\, L\,\psi\,\left(-\frac{1}{\|\mathbf{v}_a\|^2}\,\mathbf{x}_{ai}^T\,\mathbf{e}\,\left(\mathbf{I} - 2\,\mathbf{n}_a\,\mathbf{n}_a^T\right)\,\mathbf{x}_{aj}^* - \frac{1}{\|\mathbf{v}_a\|}\,\mathbf{n}_a\,\mathbf{x}_{ai}^T\,\frac{\partial \mathbf{e}}{\partial \mathbf{x}_j} - \frac{1}{\|\mathbf{v}_a\|}\,\mathbf{n}_a\,\mathbf{e}^T\,\frac{\partial \mathbf{x}_{ai}}{\partial \mathbf{x}_j}\right),\end{aligned} \tag{11}$$

where $\mathbf{u}^*$ denotes the cross product matrix for vector $\mathbf{u}$.